Masterthesis in the study program Informatik – Software and Information Engineering

Influence of network-topologies on equilibrium in continuous double-auctions

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Statuatory Declaration

I declare that I have developed and written the enclosed work completely by myself, and have not used sources or means without declaration in the text. Any thoughts from others or literal quotations are clearly marked. This Master Thesis was not used in the same or in a similar version to achieve an academic degree nor has it been published elsewhere.

Widmung

Ich widme diese Arbeit meinen beiden liebevollen Eltern, die den verlorenen Sohn nach 11 Jahren in Wien wie selbstverständlich wieder mit offenen Armen zu Hause in Vorarlberg aufgenommen haben und ihm so das Masterstudium ermöglichten und ihm dadurch halfen ein völlig neues Kapitel in seinem Leben aufzuschlagen.

Contents

Abstract

In the paper of Breuer et al. (2015) a model for endogenous leverage in a continuous double-auction is introduced and it is shown under which circumstances holdings and trading prices approach an equilibrium. A main criteria is the trading network the agents use where the authors examine only two topologies and report that the prices come to an equilibrium only in the case of a fully connected network. They leave the question open on how the model behaves with different kind of networks and which network topology exactly allows an equilibrium to be reached for further research. This thesis builds upon this model and gives a hypothesis for the necessary property a network must satisfy to allow the model to approach theoretical equilibrium as reported in Breuer et al. (2015). Then a few network-topologies are examined in regard of their ability to allow equilibria to be reached or not through computer-driven simulation. As will be shown in this thesis through validation by computer-driven simulation the hypothesis turns out to be correct only after extending the model by an additional market.

Chapter 1

Introduction

In 2008 the so called *Subprime Mortgage Crisis* struck the world. It was caused by declining house prices which rose during the US Housing Market Bubble in 2006 to an all-time high. Borrowers used their asset as collateral for the mortgage which constantly increased in value which guaranteed them a low payment-rate because the rate was coupled to the value of the asset. Banks granted "subprime" mortgages to more and more highly risky borrowers. In 2007 borrowers started to default which led to falling prices as the banks reclaimed the collateral and wanted to sell it again on the market to compensate for the loss. This led to a flood of assets on the market which led to a decline of housing prices overall. As the prices fell dramatically the payment-rates rose dramatically to compensate for the cheaper asset. This in turn resulted in even more borrowers going default because of margin requirements which resulted in a dramatic downward spiral. Even worse the banks were selling these collateralized products between each other and even insured themselves against defaults of borrowers which led to an even more dramatic kick-back. The mechanism of borrowing money to buy goods which in turn act as a security for the borrowed money is called leverage which was determined as the primary driving force behind systemic risk in the aftermath of the *Subprime Mortgage Crisis*.

1.1 Motivation

In the classic economics literature leverage was always an exogenous parameter in the works on collateralized credit but recently Geanakoplos (2009) and Geanakoplos and Zame (2014) proposed an equilibrium framework which endogenized leverage. Breuer et al. (2015) built upon this findings and developed a simulation on top of the equilibrium frameworks in which zero-intelligence agents trade assets and bonds in a continuous double-auction. Breuer et al. asked

> "whether the competitive theory of trade in leveraged assets has descriptive and predictive power in a double auction environment."

and wanted to better understand the dynamic of such an equilibrium process, how prices develop and whether they approach the equilibrium predicted in the framework or not. They made three contributions:

1. Continuous double-auction for leveraged assets is new.
2. Institutional specification matter a lot.
3. Robustness tests were conducted to show under which circumstances equilibrium cannot be reached.

The authors could show that in their simulation trading prices and wealth-distribution approach the theoretical equilibrium of Geanakoplos (2009). They investigated a fully connected network and a hub-network of agents where the equilibrium was only reached in the case of the fully connected network.

1.2 Objectives

This thesis investigates additional topologies of networks and their convergence towards theoretical equilibrium. Furthermore it presents a hypothesis about which properties a topology of a network must satisfy to reach the same equilibrium as in Breuer et al. (2015). For experimental investigation a software was built for this thesis which implemented the exact simulation model of Breuer et al. (2015) but extended it further to be applicable to arbitrary topologies.

1.3 Struture

The thesis starts with chapter 2 where the theoretical background involved with this thesis is handled. In chapter 3 a short overview of the model which is used in the simulation of this thesis is presented. In chapter 4 the hypothesis which is the motivation for this thesis is derived and proven. Chapter 5 then gives an in-depth explanation of the implementation of the computer-driven simulation built for this thesis. Chapter 6 validates the previously described thesis-software against the results found in Breuer et al. (2015) and reports the results for the additional network-topologies. Chapter 7 connects the content of the previous chapters to verify whether the hypothesis of chapter 4 is valid or not. The hypothesis is not valid and needs an adjustment in the form of a new market which is introduced and discussed with additional results in chapter 8. Chapter 9 gives conclusions on the findings of the thesis and outlines topics for further research.

Chapter 2

Theory

2.1 Equilibrium

Equilibrium is a fundamental property in all kind of complex systems and indicates a state where the change-rates of all time-dependent variables are 0 and stay at 0 from some point t on - the system does not change any more over time, it has come to a halt. This thesis investigates whether the simulation-model presented in chapter 3 reaches equilibrium for a given configuration or not. Because the model upon which the simulation is based is rooted in economics a short definition of equilibrium in economics is necessary.

2.1.1 Economics

Equilibrium theory in economics is the theory of finding prices which will clear markets. Clearing is the process of finding a price in which all demands are matched to the given supplies thus clearing the market by leaving no unmatched demands or supplies. In other words it tries to find a price which satisfies all offers. Thus equilibrium in economics is reached when supplies equals demands.

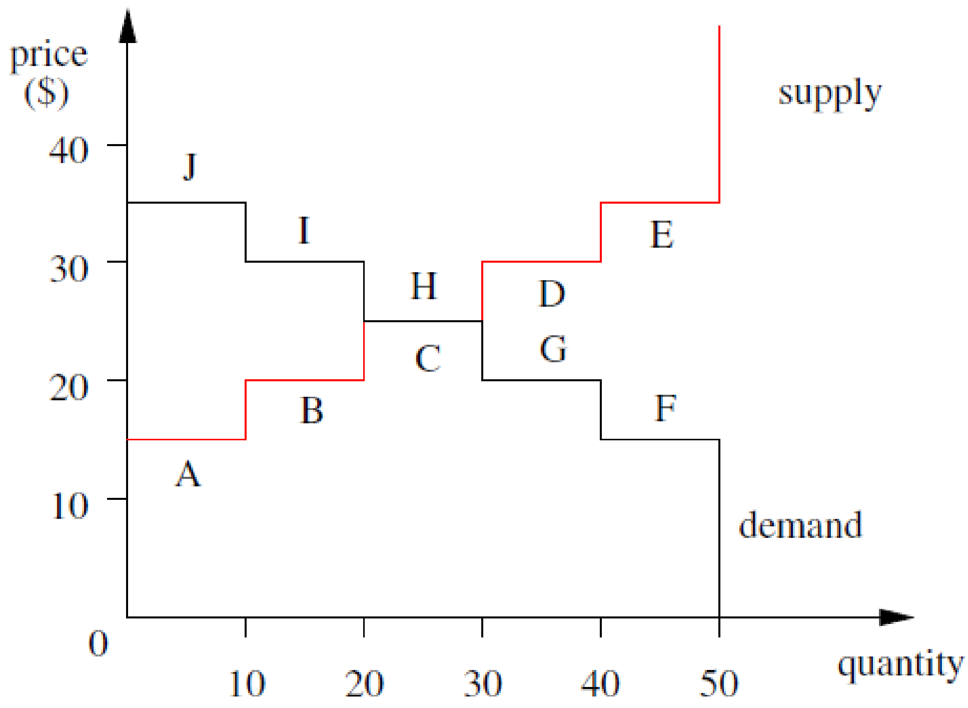

Figure 1: Illustrative supply and demand curves. Parsons et al. (2006)

In diagram 1 the equilibrium price which clears the market is at \$25 because this price satisfies the constraint buyer-price $\geq$ seller-price for all traders, thus all traders match and the market is cleared. The constraint buyer-price $\geq$ seller-price is a fundamental law of economics and imposes an ordering on the prices of the market and is rooted in the fact that a buyer values the good it wants to buy more than the seller and has to pay at least the amount of money the seller wants for it, or more.

2.1.2 Equilibrium theory vs. dynamic process

It is important to note that equilibrium theory in economics is inherently non-dynamic and models no dynamic process of any kind over time. It is just a framework by providing formulas for calculating equilibrium prices but does not describe the dynamic process of how these equilibrium prices are settled. However the simulation in this thesis and the model by Breuer et al. (2015) upon which it is built as defined in chapter 3 is a dynamic process which tries to approach the equilibrium predicted by the economics theory through a trading-process over time as defined in the following section and in chapter 3. Thus both parts are necessary: the equilibrium theory to predict the theoretical equilibrium prices and the dynamic simulation-process in investigating whether the equilibrium prices can actually be reached or not and how these prices are settled over time.

2.2 Continuous Double Auction

The continuous double-auction (CDA) is a type of auction upon which the model of Breuer et al. (2015) presented in chapter 3 and thus the thesis-software is based. The reasons why they chose the continuous double-auction as the auction-mechanism is:

> "Experimental economists believe that the continuous double auction is a trading institution that comes close to an environment which abstract equilibrium theories of competitive trading try to describe. It is an institution that allows for competitive bidding and trade on both sides of the market over time. One of the discoveries of experimental economists is that in many experiments double auctions converge to states where trading activity comes to a halt. In these final states prices and allocations often are similar to what equilibrium theory predicts." Breuer et al. (2015)

2.2.1 Definition

To explain the details of a *continuous* double-auction one has to start with the double-auction (DA) alone. Generally speaking the DA is a market institution which defines rules how traders can exchange good for some numeraire between each other. It is an auction process which coordinates messages between traders which include some price information. Thus a DA is a multilateral process in which during multiple round TODO: plural traders can enter offer TODO: plural into an offer-book and accept offers made by others. Traders can be distinguished between seller and buyer and send their messages/place their offers in a given price-range according to their limit-price. Depending on the type of the DA at some point a clearing of the market happens thus leading to the actual exchange between the traders and thus a change in the allocation of their goods and cash. Parsons et al. (2006)

2.2.2 Characterization

It is important to note that there does not exist such a single thing as the "double auction" as there are many variants of it where they can all be differentiated in the way when and how traders place their offers and when and how the auction clears the market. According to Parsons et al. (2006) the following questions must be answered when characterizing a double-auction instance:

- When does the clearing happen? Is it periodic or continuous?
- When do offers of traders arrive? Do offers arrive over time?
- What information is available to each trader about current offers and other traders?
- How are unmatched offers treated? What happens to unmatched bids and asks when a match occurs?
- How are the trades priced? Are trades priced using discriminatory or uniform pricing and how are the uniform or discriminatory prices determined?
- Are there one or multiple trading-periods? Is the market one-shot or repeated?

When does the clearing happen? Is it periodic or continuous? Clearing happens at the end of a round where the first match of two random traders on a random market is searched. It is continuous because traders agree on each others offers and exchange the traded goods immediately. In a periodic DA clearing happens at discrete time slots during the trading-process after multiple rounds.

When do offers of traders arrive? Do offers arrive over time? Traders place their offers simultaneously at the beginning of each round. Offers do not arrive over time as time advances only from round to round and is not modelled explicitly as a time-flow.

What information is available to each trader about current offers and other traders? None. The traders are not able to look into the offer-book or to communicate with other traders. They act only as zero-intelligence agents as introduced in section 2.2.4.

How are unmatched offers treated? What happens to unmatched bids and asks when a match occurs? They are deleted. Prices are placed randomly as will be seen in 3 and if they haven't matched in the current round the won't match in the future ones thus they are all deleted from the offer-book.

How are the trades priced? Are trades priced using discriminatory or uniform pricing and how are the uniform or discriminatory prices determined? Discriminatory pricing is used. In uniform pricing one price is chosen and applied to all trades which clears the market where in discriminatory pricing the prices are determined individually for each trade. The transaction-price where the buyer and seller meet is the half-way price between the offers of both. Another possibility for the transaction-price as reported in Gode and Sunder (1993) is to select the price of the offer which was placed first.

Are there one or multiple trading-periods? Is the market one-shot or repeated? A repeated DA comprises of multiple trading-periods where traders are endowed with new allocations and may or may not keep their final allocations after each period. This is not the case in this thesis where only one trading-period is simulated thus the market is one-shot. Note that this is not to be confused with Round as there are many rounds within one trading-period.

2.2.3 The continuous double-auction process

The following points summarise the workings of the instance of CDA used in this thesis:

- Endow all traders with initial goods and numeraire.
- Open all markets.
- Execute rounds as long as traders are able to trade.
- In each round every trader is allowed to place one buy and one sell offer on all opened markets.
- After all offers have been placed the auction clears the markets.
- During clearing the first match between random traders on a random market is searched where buyer-price $\geq$ seller-price.

- On a match the offered amount is transferred and both traders meet at the half-way price.
- Upon a match all the other offers on all markets are deleted and a new round starts.

2.2.4 Zero-intelligence agents

The traders in this thesis are modelled as zero-intelligence agents as introduced by Gode and Sunder (1993). These are traders which place offers strictly in a range which increases their utility and do neither learn nor can adopt to the behaviour of other agents or changing conditions on the market. They are completely deterministic in a way that they never change their behaviour.

When using zero-intelligence agents the question about the allocative efficiency of the market must be raised: "Are zero-intelligence agents able to achieve or come close to 100% allocative efficiency?". According to Gode and Sunder (1993) this is the case if:

> "Imposing a budget constraint [...] is sufficient to raise the allocative efficiency of these auctions close to 100 percent. Allocative efficiency of a double auction derives largely from its structure, independent of traders' motivation, intelligence, or learning."

Because the model used in this thesis which is presented in chapter 3 is "imposing a budget constraint" the potential of coming close to 100% allocative efficiency is given. Results given in chapter 6 show that this is really the case.

2.3 Leverage

Leverage in economics is a major part in the model of this thesis as defined in chapter 3 and thus a short introduction and overview of its meaning and implications is given.

2.3.1 Definition

Leverage is "any technique to multiply gains and losses" as defined by Brigham (2012). There are different types of leverage where in this context only the so called *financial leverage* is of interest. In leverage money is borrowed to buy some goods where the leverage is the ratio of the total debt to the traders equity. The greater the debt or the smaller the equity of the trader the higher the leverage.

$$leverage = \frac{total\ debt}{trader\ equity} \tag{2.1}$$

As an example an agent wants to buy a house for 500€ but has only 50€ in cash. Thus the agent borrows 450€ to finance the house thus creating a leverage of $\frac{500}{50} = 10$.

2.3.2 Dangers

As stated above leverage not only multiplies gains but also losses. If house prices rise by 25% this will result in a gain of 125€ or 250% return on the agents investment when selling the house. However if house prices decline by 15% this will result in a loss of 75€ or 150% loss on the agents investment - in this case the agent has lost more money than it initially invested. The leveraged loss can become a serious issue in financed housing because of margin requirements towards the lender. The margin requirement is the minimum margin the house-buyer has to maintain independent of the gains or losses of the house. Thus if the house-price drops to 475€ then the net value of the margin is 25€ and the agent needs to bring in back 25€ to meet the margin requirements. If the losses are higher or the agent is not able to compensate them the house has to be sold. TODO: quelle

2.4 Complex Networks

This thesis lays its main focus on the influence of network-topologies on the equilibrium found in continuous double-auction. The networks define the neighbourhood between agents and determine which pair of agents can trade with each other. All graph-related definitions in the following sections are provided through Drmota et al. (2007).

A network is a graph *G = (V,E)* which has a finite set of vertices *V = V(G)* and a finite set of edges *E = E(G)*. The vertices resemble the agents and the edges connecting them resemble the neighbourhood between agents or the knowledge of each other. Two agents know each other and can trade with each other if there exists an edge between them. In this context only undirected graphs consisting of undirected edges $e \in E(G)$, $e = v_1, v_2$ between two vertices v_1, $v_2 \in V(G)$ without multi- and self-edges are of interest.

- Undirected: if one agent v_1 knows another agent v_2 then v_2 knows v_1 too - trading is always possible in both directions.
- No multi-edges: one neighbourhood connection is enough as edges have no additional properties or weights.
- No self-edges: agents are not allowed to trade with themselves.

This thesis also investigates the equilibrium in so called *complex networks* which are a special kind of random networks which could exhibit small-world properties and could follow a power-law distribution which are discussed below. In the following sections a short overview of the development of network-topologies and recent findings in this research-field is given and the complex networks used in this thesis are discussed. See appendix A for a complete catalogue of network-topologies investigated in this thesis - the complex ones are:

- Erdos-Renyi
- Barbasi-Albert
- Watts-Strogatz

The main sources for the following sections is the paper Newman (2003) and the books Jackson (2008) and Easley and Kleinberg (2010). Note that in this thesis only static networks are of interest thus no models of network-growth or processes in and on networks are discussed.

2.4.1 Overview

Since the first proof in network-theory by Euler in 1735 the analysis of networks has had a long tradition. Up until a few years ago the analysis of small graphs and properties of individual vertices or edges dominated the field of network-research but over the last years the focus shifted towards large-scale statistical properties. This transition of focus was made possible by the availability of an ever increasing amount of processing power through computers which allow the investigation of networks with millions of vertices.

2.4.2 Random graphs

One of the simplest network models studied was the random graph which was investigated first by Erdős and Renyi (1959). The motivation behind random graphs is to assume a random-process of network formation and to study the resulting networks. Such networks are constructed by having N unconnected vertices and then adding at random each possible edge with a given probability p where the distribution of p follows a specific distributions e.g. uniform-, poisson-, gaussian-distribution. Thus random graphs can be reduced to a "binomial model of link formation" Jackson (2008) where out of all the possible random graphs with N nodes one graph is selected with a probability of

$$p^m(1-p)\frac{n(n-1)}{2} - m \tag{2.2}$$

as reported in Jackson (2008).

2.4.3 Small-World effects

In 1960s Stanley Milgram conducted an experiment in which he demonstrated that a letter can reach a destination person by an average of just 6 intermediate steps in between. In his papers on this experiment Travers and Milgram (1969) and Milgram (1967) he termed this phenomenon the *small-world effect*. Although the results of his work have been questioned - more specifically that the world is really a small one e.g. by Kleinfeld (2002) - it had big influences within the network-research community and led to the development of the small-world property. It is of very importance e.g. in social networks because this property implies that information spreads very quickly in the network as it needs very few steps to reach all nodes. This can also be applied to trading networks where it enables traders to trade goods within very few intermediary steps to traders which value the good the most.

To calculate whether a network has the small-world property one starts with the formula given by Newman (2003) for calculating the average path length in a network

$$\ell = \frac{1}{\frac{1}{2}n(n+1)} \sum_{i \geq j} d_{ij} \tag{2.3}$$

where d_{ij} is the shortest distance from vertex i to vertex j. Networks then exhibit the small-world property if ℓ scales at max logarithmically with network size of mean degree.

2.4.4 Scale-free networks

Albert and lászló Barabási (2002) found that real-world networks in contrast to random graphs are non-random which led to the discovery of the phenomena of scale-free networks which are networks whose vertex-degree distribution follows a power-law.

Degree distribution

In an undirected Graph G the edges adjacent to $v \in V(G)$

$$\Gamma(v) = w \in V(G) | vw \in E(G) \tag{2.4}$$

are the neighbours. The quantity of the neighbours of $v \in V(G)$

$$d(v) = |\Gamma(v)| = |w \in V(G) | vw \in E(G)| \tag{2.5}$$

is the degree of a vertex $v \in V(G)$. When regarding the adjacent-matrix of a graph the degree of a vertex $v_i \in V(G)$ is given by

$$d(v_i) = \sum_{j=1}^{n} a_{ij} \tag{2.6}$$

Having defined the degree of a vertex one can calculate the degree distribution of of a given network simply by counting the number of nodes which have a given degree k:

$$P_{deg}(k) = \textit{fraction of nodes in the graph with degree } k \tag{2.7}$$

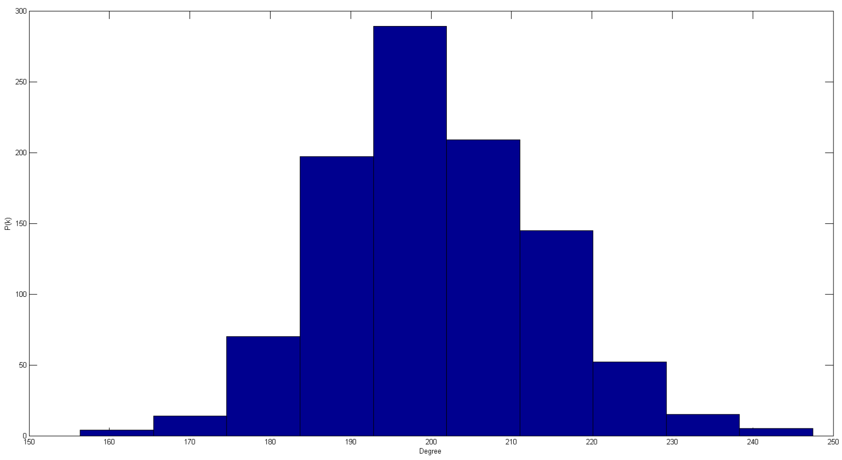

Figure 2: Histogram of the degree-distribution of a random Erdos-Renyi network with 1000 agents generated by the thesis-software. The average degree is 200.44.

Power-law distribution

In figure 3 the degree-distribution of a scale-free network is shown.

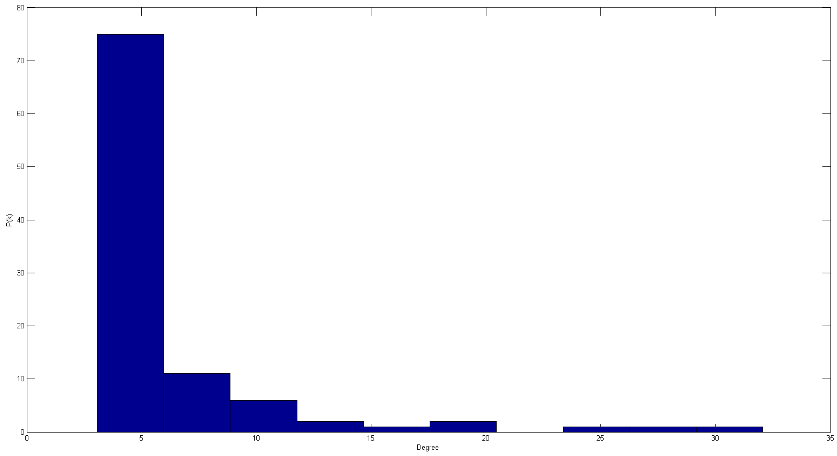

Figure 3: Histogram of the degree-distribution of a Barbasi-Albert network with 100 agents generated by the thesis-software. The average degree is 4.1. Note that the Barbasi-Albert model creates scale-free networks.

What is striking about this figure is the long right tail. This shows that there exist few vertices in this network which have a very high degree far beyond the average degree. This means that many other vertices are connected to them - they act as a kind of hub. Most of the vertices though have a low degree around 5 which in combination with the few high-degree vertices results in the long tail of this distribution which follows a power-law

$$P_{deg}(k) \sim k^{-\gamma} \tag{2.8}$$

Note that due to the power-law the distribution of such networks remains unchanged when scaling k with a given factor a. Thus those networks are called *scale-free*.

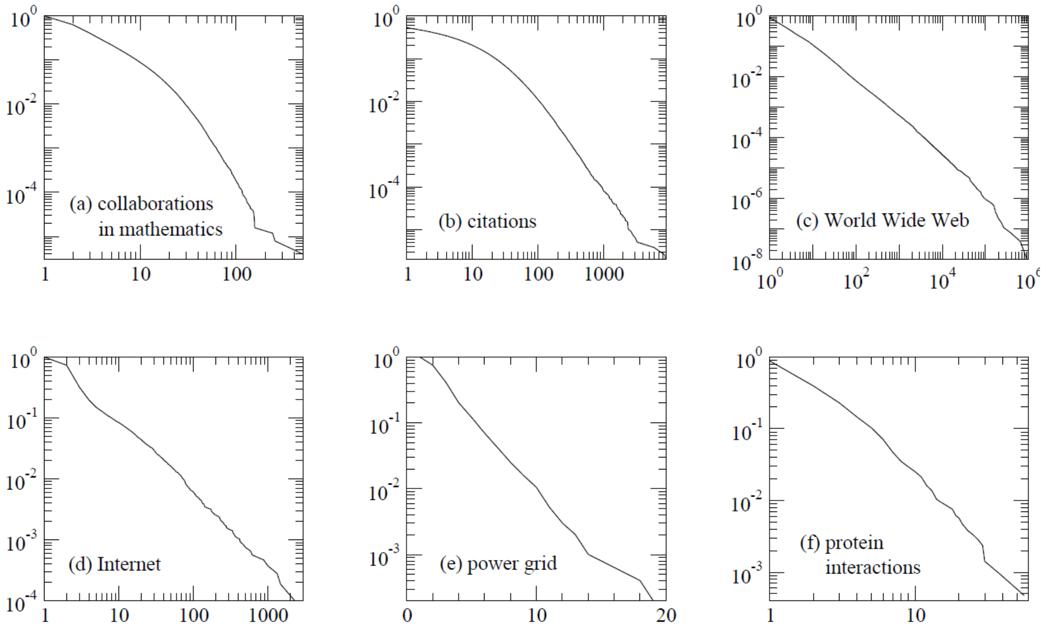

Figure 4: Examples for degree-distributions of real-world networks. Note that a straight line in a log-log plot is evidence of a power-law distribution thus only (c), (d) and (f) appear to have power-law distributions. Newman (2003)

The strengths of scale-free networks is their resilience against *random* removal of vertices.

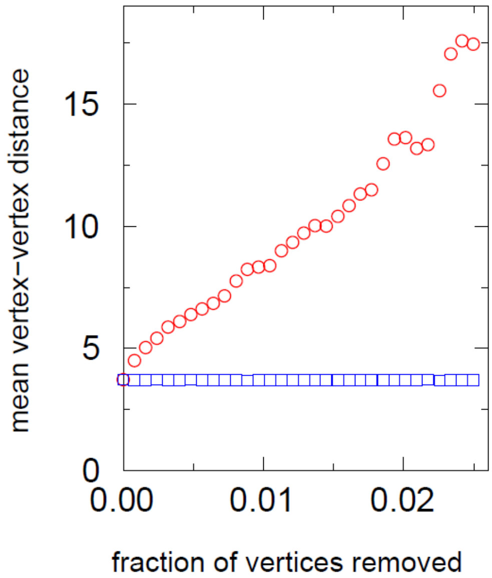

Figure 5: Random removal of vertices increases the average path length very slightly. Removing vertices with highest degree first increases the average path length rapidly. Newman (2003)

This resilience can be of benefit in a scale-free trading network where the inability of a random trader to trade because it has no more goods or cash won't impair the overall trading ability. On the other hand if an important trader which acts as a hub becomes unable to trade then the overall trading process may be affected.

2.4.5 Complex Network examples

To summarize complex networks are random networks that could have small-world properties and could be scale-free depending on the algorithms used to create them. There exist a few models to create complex networks where three well known models are implemented in this thesis. See appendix A for examples of the following networks created by the thesis-software with varying parametrization.

Erdos-Renyi

The Erdos-Renyi model generates a pure random-graph which exhibits *no* small-world properties. It starts with N vertices and selects out of all the possible graphs with n vertices one by random by iterating over all possible edges and including each with a given probability p. Erdős and Renyi (1959) and Erdős and Renyi (1960)

Watts-Strogatz

The Watts-Strogath model generates a random-graph which exhibits small-world properties. One starts with N unconnected vertices and creates a regular lattice where each vertex is connected to K neighbours with $\frac{K}{2}$ on each side. Then iterate through all vertices and and rewire each edge which is connected to a vertex already visited with a given probability b to another vertex from all the possible vertices - self-loops and link-duplication is not allowed. Watts and Strogatz (1998)

Barbasi-Albert

The Barbasi-Albert model generates a random-graph which exhibits small-world and scale-free properties which is achieved through preferential attachment. To create a network with N vertices one starts with m_0 vertices and adds $N - m_0$ vertices. Each new vertex is connected to m existing vertex where the probability to be connected to a given vertex is proportional to its degree. When choosing m = 1 then one creates a hub-structure. Albert and lászló Barabási (2002)

Chapter 3

The Leverage Cycle

In this chapter the model for the simulation is given. All the following chapters build upon this model where the thesis-software is an implementation of it.

3.1 Geanakoplos

The model of Breuer et al. (2015) which is discussed in the next section is based upon the works of John Geanakoplos article Geanakoplos (2009) "The Leverage Cycle" thus for a better understanding a short overview of the innovations and influences found therein is given.

The work of Geanakoplos focuses on asset-pricing, the influence of leverage on asset-prices and how leverage affects crises. He claims that because of leverage during boom times the asset-prices are too high because of massive leverage and during bad times the asset-prices are too low because of a massive drop in leverage. That is what he terms "the leverage cycle". Further he predicts that leverage cycles will occur although people remember past ones unless the central bank tries to stop those cycles by regulating leverage. Geanakoplos proposes a theory of equilibrium leverage and asset pricing which gives a central bank a tool for regulating leverage during boom times to prevent asset-prices skyrocketing and reinforce leverage in down times to lift the asset-prices which are too low to a reasonable level.

3.1.1 The natural buyer

For Geanakoplos all crises start with scary bad news which are then the reason why asset-prices drop below a price which is lower than everyone expected. He introduces the so-called "natural buyer" which is an agent who values the asset more than the public. This can be because the agent is less risk averse, get more utility out of it, use the asset more efficient. In the end the details matter not, the natural buyer is just more optimistic than the public. To prevent a too specific distinction between the natural buyer and the public Geanakoplos introduces a range of optimism $h \in H = [0..1]$ in which all agents are ordered by their optimism h where the extreme pessimists reside at the lower end of 0 and the extreme optimists ate the upper end of 1. Each agent assigns the probability that good news will occur according to its optimism h where the extreme optimist thinks that good news will happen for sure and the extreme optimist think that it will never occur thus the more optimistic an agent, the more a natural buyer it is. If the natural buyers drops out of the business then the asset-prices drop as the natural buyers are the only ones willing to drive asset-prices up through leverage as they value the asset-prices the most. Thus the natural buyers buy as many assets they can both by cash and through borrowing and using the assets as security thus creating leverage. Because of this mechanics Geanakoplos emphasises that it is of very importance who lost money in a crisis - the public or the natural buyers whereas a loss for the natural buyers is the real catastrophe as no one is willing to drive up the asset-prices any more.

3.1.2 Two-period economy

Geanakoplos then introduces a two-period economy. In the first period each agent of the previously mentioned continuum H is endowed with one consumption good C and one asset Y and can then trade with each other. The second period can be one of two states: U(p) and D(own) whereas in the up-state the asset Y is worth 1.0 and in the down-state only 0.2. The agents differ only in their optimism h by which they assign the probability that the up-state will happen tomorrow in the second period. According to this they trade on the market according to their utility-function which depends on their optimism h. The following formula gives the utility-function of an agent according to Geanakoplos. It gives the price of an asset, that is how much an agent values it - obviously the more optimistic the higher the price.

$$price = h + (1 - h)0.2 \tag{3.1}$$

Now if *price* is larger than some offered price p then the agent is going to buy the asset for the offered price p as the agent values it more thus when buying the asset the agent will make an expected profit. If the *price* is less than the offered price p the agent is going to sell the asset as the value the agent assigns to it is lower than the offered price p thus the agent can make an expected profit in selling it.

3.1.3 Loan market

Geanakoplos introduces a loan market where agents can lend and borrow money through loans to be able to further buy assets after they have ran out of cash. A loan can be sold and bought for $j = 0.2$ and needs to be paid back at the beginning of the second state. Because lenders worry about default each loan needs to be backed up by an asset as security.

In the up-state the borrower will pay back $j = 0.2$ and in the down-state the borrower will pay back either $j = 0.2$ or the asset which is worth of 0.2 in the down-state. Thus a loan which is bought for $j = 0.2$ and pays back the same amount is a risk-less loan as the lender can not loose money because independent of the occurring state always $j = 0.2$ will be given back.

Geanakoplos then predicts the so called *marginal buyer* around $h = 0.69$. All agents with $h < 0.69$ are pessimists and sell their assets. All agents above $h > 0.69$ are optimists and buy all the assets the pessimists sell either through cash or buy borrowing money from the pessimists through loans and using the borrowed money to buy further assets which then in turn act as security - the leverage is endogenous.

Geanakoplos then introduces loans with $j > 0.2$ where in the up-state they promise their initial value j and in the down-state they deliver only 0.2. Thus loans with $j > 0.2$ are risky loans because a lender can loose money depending on the occurring state. If a lender granted a bond of type $j = 0.5$ and the down-state will occur the borrower will either return 0.2 cash or the security-asset which is now only worth 0.2 - the lender has lost 0.3 cash.

In the classic equilibrium theory as outlined in chapter 2 the only equilibrating variables are prices. Geanakoplos argues that the problem with the classic model is that for determining the equilibrium of loans one needs two variables: the promise j and the collateral requirement which is impossible to solve with just one equation. The solution of Ganakoplos to modelling collateral is to

> "... think of many loans, not one loan. Conceptually we must replace the notion of contracts as promises with the notion of contracts as ordered pairs of promises and collateral. Each ordered pair-contract will trade in a separate market with its own price"

$$Contract_j = (Promise_j, Collateral_j) = (A_j, C_j) \tag{3.2}$$

He then shows that if there exist markets for all type of bonds which include the risk-less bond $j = 0.2$ then only the risk-less bond will be traded. The case with only risky bonds available are excluded by assumption.

Note that this is only a small part of the quite involved economic theory. Geanakoplos does not stop at this point but this overview is already enough to get the basic influences found in the work of Breuer et al. (2015).

3.2 Breuer et al.

As already outlined the model of Breuer is heavily influenced by the work of Geanakoplos with the major difference that it is not a pure static equilibrium theory but is a simulation-process which approaches the equilibrium iteratively over time. Also a major achievement is that not only assets and bonds are traded against cash but the model has been extended by an additional market which allows collateralized assets to be traded. According to Breuer et al. (2015) this is the first time that the trading of leveraged assets was investigated in a continuous double-auction environment. It is also of very importance to note that although the up- and down-states are part of this model too they are actually never realized, act only as a model and thus only the first period is simulated TODO: dieser satz macht wenig sinn, anders formulieren. Furthermore agents are not an infinite continuum but finite entities because the equilibrium-solving is done as an iterative simulation-process in software and thus finite agents are required.

The major differences to the approach of Geanakoplos are:

1. Collateralized assets are traded in addition to the other markets.
2. Up- and down-states are never realized but only the first period is simulated.
3. Equilibrium in the case of only a risky-bond available is treated.
4. There is a finite number of agents as opposed to a continuum.
5. It is an auction-process over time which iteratively approaches theoretical equilibrium where Geanakoplos is a static equilibrium theory. The mechanism used in this auction is a continuous double-auction as introduced in chapter 2

In the following sections some details which are different to the model of Geanakoplos or need more explanation are discussed.

3.2.1 States

Both the up- and the down-state are the same as in Geanakoplos where the up-state is denoted with pU and the down-state is denoted as pD and assets are worth 1 in pU and 0.2 in pD. Agents are endowed with 1 unit of cash and 1 unit of assets today and are then able to trade between each other. The tomorrow-state will not be drawn, agents trade only today.

3.2.2 Markets and limit-functions

Asset/Cash market The asset market is the same as in Geanakoplos (2009) where assets are just bought and sold against cash.

$$\ell_{asset} = h + (1 - h)pD \tag{3.3}$$

Bond/Cash market The bond market acts the same way as described in Geanakoplos (2009). Collateral acts as enforcement of financial promises and thus for a given amount of loans the same amount of assets must be held as securities. A loan can be bought for a given price which is the face-value V. This face-value has to be paid by the borrower in up-state whereas only pD has to be paid in down-state. Again note that up- and down-states are never realized but influence the utility-functions. Risk-less as well as risky bonds are available in the model of Breuer et al. (2015) where the risky bonds are those with face-value $V > 0.2$. Although Breuer et al. allowed more than one bond-types simultaneously in their model this is not implemented in the thesis-software as it is not the primary focus of this work and would have required substantial changes in the software.

$$\ell_{bond} = hV + (1 - h)pD \tag{3.4}$$

Asset/Bond market The Asset/Bond market trades assets against bonds thus the utility-function is just the ratio of the Asset/Cash utility to the Bond/Cash utility which gives the amount of bonds one asset is worth for a given optimism h.

$$\ell_{asset/bond} = \frac{\ell_{asset}}{\ell_{bond}} \tag{3.5}$$

3.2.3 Agent Utilitiy

TODO: move to next section of utility The utility of an agent is a generic measure to define . It is defined as an equation into which the holdings of the agent are multiplied by the expected value the agent assigns to each.

In this section the utility-functions of an agent for each market are given. See chapter 5 for more details on the markets. TODO: the functions below are NOT the utility-function but the limit-functions of an agent on each market. The utility-function is just a SINGLE one (no plural) and give a single utility-value. it includes the current holdings of cash, loan and assets and multiplies it by the limit-value of each market. The limit-functions are defined first, then the utility-function

total utility

$$u_{agent} = u_{cash} + u_{asset} + u_{bond} \tag{3.6}$$

Cash utility

utility cash = (assets selling - assets buying) * asset-price + (bonds giving - bonds taking) * bond-price + cash endowment

$s_{asset} \ldots selling\ amount\ asset$
$g_{bond} \ldots giving\ amount\ bond$
$b_{asset} \ldots buying\ amount\ asset$
$t_{bond} \ldots taking\ amount\ bond$
$p_{asset} \ldots price\ asset$
$p_{bond} \ldots price\ bond$
$e_{cash} \ldots endowment\ cash$

$$u_{cash} = (s_{asset} - b_{asset})p_{asset} + (g_{bond} - t_{bond})p_{bond} + e_{cash} \tag{3.7}$$

Asset utility

utility asset = asset holdings * asset limit

$h_{asset} \ldots holdings\ asset$

$$u_{asset} = h_{asset} * \ell_{asset} \tag{3.8}$$

Bond utility

utility bond = bond holdings * bond limit

$h_{bond} \ldots holdings\ bond$

$$u_{bond} = h_{bond} * \ell_{bond} \tag{3.9}$$

3.2.4 Collateralized asset market

One of the major inventions of the work of Breuer et al. is the introduction of a market for collateralized assets which has never been studied in continuous double-auction simulations so far. This market enables an agent which is out of cash but high on assets to buy additional assets by selling bonds and thus borrowing money which the agent uses to buy the wanted assets and in return using them as security for collateral constraints. When implementing this mechanism Breuer et al. had to overcome two major difficulties.

1. Coordination of asset and bond markets - the buying of an asset and the selling of a bond needs to be coordinated across both markets and must happen at the same time.

2. Reversibility of suboptimal trades - earlier trades could have been suboptimal for an agent because it couldn't fully anticipate the behaviour of other agents and thus needs to get out of old trades. Technically speaking this would require to free collateralized assets by unlocking them and transferring them in to the state of a real asset - no more collateralized, thus being completely owned by the agent.

Breuer et al. proposed solutions to this two difficulties:

ABM mechanism The straight-forward solution to the coordination of the asset and bond markets would be to condition a buy offer of an asset to a sell offer of a bond. Breuer et al. reported that "Separate utility improvement in each of the coupled trades is more restrictive than a net sum utility improvement of all coupled trades." which prevents theoretical equilibrium to be reached. Thus they define the market to traded assets directly against bonds thus reducing the involved agents from three to two and removing the coordination-problem because only one product with one price is traded. This resolves the problem with the restrictiveness of utility in the case of two products.

Bond pledgeability The problem with the reversibility of suboptimal trades was solved in allowing to uncollateralize an asset by buying a bond. Breuer et al. called this mechanism "bond pledgeability" (BP) and showed that without this mechanism the simulation never converges towards the theoretical equilibrium. See chapter 5 ”Implementation” for details on the implementation of this mechanism.

3.2.5 Auction Mechanism

The auction mechanism used is a continuous double-auction on all markets open at the same time with a finite number of agents.

Bidding To prevent a bias one agent is picked at random and then submits offerings on all markets while respecting the following constraints.

- If the agent has no more assets it can't sell them either through cash or bonds.
- The agent cannot buy more assets or bonds than it owns cash.
- When placing sell-offers of bonds or assets there must not remain bonds which have no collateral as security.

Matching Again to prevent a bias pick one market at random and pick at random the buy or sell offers on this market and compare them with the offers of the previously selected random agent. A match occurs only if:

$$\textit{buy-price} \geq \textit{sell-price} \tag{3.10}$$

In this case the offers of all other agents which have not matched are deleted from the offering-book and the matching-price is calculated at the half-way price:

$$\textit{matching-price} = \frac{\textit{buyer-price} + \textit{seller-price}}{2} \tag{3.11}$$

If no match occurs with the current random agent pick another agent at random and continue as above with submitting its offers on all markets.

3.2.6 Equilibrium

Breuer et al. reported equilibria for prices and allocations both of bonds and assets where the equilibria are fundamentally different whether a risk-free bond is available or not.

Risk-free bond If a risk-free bond with a face-value of $V \leq 0.2$ is available then the agents are divided into two subgroups by i^*:

1. Agents with $0 < i \leq i^*$ are pessimists and hold only cash or the risk-free bond with highest face-value.
2. Agents with $i^* < i \leq 1$ are optimists and are maximally short in risk-free bonds with highest face-value and hold only assets.

Below the formulas reported in Breuer et al. (2015) are given for calculating i^*, the asset-price p and the bond-price q in equilibrium.

$$i^* = \frac{p - 0.2}{0.8} \tag{3.12}$$

$$p = \frac{1 + q - i^*}{i^*} \tag{3.13}$$

$$q = 0.2 \tag{3.14}$$

Risky bond When only a risky bond with face-value $V > 0.2$ is available then the agents divide into three instead of two subgroups separated by i_1 and i_2:

1. Agents with $0 < i \leq i_1$ are pessimists and hold only cash.
2. Agents with $i_1 < i \leq i_2$ are median agents and hold only bonds with the lowest face-value.
3. Agents with $i_2 < i \leq 1$ are optimists and hold only assets and are maximally short in risky bonds with the lowest face-value.

Below the formulas reported in Breuer et al. (2015) are given for calculating i_1, i_2, the asset-price p and the bond-price q in equilibrium. Note that in this thesis equilibria are always calculated for a risky bond with a face-value of $V = 0.5$.

$$i_1 = \frac{q - 0.2}{V - 0.2} \tag{3.15}$$

$$i_2 = \frac{0.2(p - q)}{0.8q - (V - 0.2)p} \tag{3.16}$$

$$p = \frac{1}{i_1} - 1 \tag{3.17}$$

$$q = p\frac{i_2 - i1}{1 - i_1} \tag{3.18}$$

Note that this case is not discussed in Geanakoplos (2009) where it is excluded by assumption.

Calculating theoretical Equilibrium

Theoretical equilibrium can be calculated through the previously given equations for an infinite number of agents. In the simulation a finite set of agents is used for which the theoretical equilibrium must be found too to compare the results of the simulation to the theoretical equilibrium. For this purpose Breuer et al. (2015) developed an algorithm in MATLAB which searches the finite solution-space for the given equilibrium. Mr. Martin Jandacka wrote a short, unpublished documentation on the approach for risky bonds which is summarized here.

For given asset prices q and bond-prices q each agent optimises its expected utility. As can be seen in the section 3.2.2 the utility-functions are linear which makes this optimization problem a linear one which can be solved by Linear Programming (LP). Thus the two agents *i1* and *i2* are searched where *i1* marks the end of the pessimists and *i2* the beginning of the optimists. This is done by iterating through all possible combinations of *i1* and *i2* and checking if they generate equilibrium on the market or not. Thus time dependence is $O(N^2)$ where N is the amount of agents.

3.2.7 Endogenous leverage

Endogenous leverage is the central topic of the models both of Geanakoplos and of Breuer et al. Because it may seem not immediately clear where and how leverage is endogenous in the model of Breuer et al. this section outlines where this is the case and how it is implemented.

In the work of Breuer et al. (2015) it is noted that

> "In this theory the amount that can be borrowed against a particular asset to purchase it is determined in the market."

and furthermore

> "Leverage, the percentage of the value of the real asset that can be borrowed to purchase it, is determined by contract selection through the market. Leverage is endogenous."

Contract selection amounts to the selection of the bond-types used by the agents to finance their trades. Geanakoplos and Breuer report that if multiple bonds are available which include risk-free bonds the agents will select the risk-free bond with the highest face-value which is 0.2. The reason is that buyers which are more optimistic towards the up-state expect that they would have to pay the face-value so they try to select a bond with the lowest possible face-value the sellers would accept. The sellers which are more pessimistic towards the up-state expect that they will more likely get back the down-state value of 0.2 and don't want to trade above this value as they expect to loose money in this case. So buyers and sellers select the risk-free bond with face-value of 0.2 endogenously through the mechanics of the model and not by parameters which are set exogenous by an experimenter. Thus Leverage is regarded as endogenous, coming from within the simulation-model itself.

Note that the prices of assets and debt are distorted through leverage because optimists value the goods more and are willing to drive the prices up through the use of leverage. This was a major finding by both Geanakoplos and Breuer.

3.2.8 Equilibrium of trading-process

As already noted the model given above is a dynamic process which approaches an equilibrium over time. The equilibrium is established if the system does not change any more over time: the process has come to a halt, all time dependent variables stay constant. Due to its design the process of this model will always come to a halt at some point - and will thus have some equilibrium - which is the case when all traders have become unable to trade:

- Due to collateral constraints.
- They cannot place utility-increasing offers any more - utility-reduction is not allowed in this model.

It is of very importance to note that if the process has come to a halt *some* equilibrium has been established but that the established equilibrium *must not* be necessarily be the theoretical one as given by the equations in section 3.2.6.

Chapter 4

Hypothesis

In this chapter the question of the importance of fully-connectedness for reaching the equilibrium is raised where the question is whether it is really necessary to have a fully-connected network to reach equilibrium or not. We challenge this and claim that a much lower connectivity but a very special one is required and present a definition for it in a hypothesis which states that if a network satisfies the postulated properties of the hypothesis then using this network in the simulation will result in approaching the same equilibrium as found in fully-connected networks.

4.1 Definition

$$\begin{aligned} &\textit{fully-connectedness equilibrium} \iff \\ \forall\ &\textit{agent-pairs}\,(a_1, a_n)\ \exists\ \textit{path}\, P\ \{a_1, a_2, ..., a_{n-1}, a_n\} \mid h(a_i) < h(a_{i+1}) \end{aligned} \tag{4.1}$$

Conjecture 1. *If and only if for all agents exists a path between two agents in which each visited agent has a larger optimism factor than the previous one then the same equilibrium as in fully-connectedness will be reached.*

The most minimal configuration of agents which satisfies this hypothesis is a graph of agents where each agent is connected to the agent with the next higher optimism-factor. This is the same as the Ascending-Connected topology - see appendix A - which is the major network of interest in this thesis (besides the fully-connected one) as it is the most minimal topology which satisfies the hypothesis.

4.2 Motivation

The motivation behind the hypothesis is the fact that according to the double-auction definition - see chapter 2 - for a match to happen the buyer-price must be larger or equal to the seller-price. This can only be the case when the buyer has a higher optimism-factor than the seller due the way agents place their prices - see chapter 3.

4.2.1 Proof

The proofs are given for the Asset/Cash market only because the equations of the Loan/Cash market work exactly the same way where only the absolute numbers are different. The Asset/Bond market is the same too as the limit-price is just the ratio of the limit-prices of asset and bond thus resulting in the same linear ordering of the limit-prices. TODO: stimmt nicht

Illustrating the matching ranges In figure 6 the price-ranges of both a seller and buyer are given where h(s) and h(b) denote the optimism-factors of the seller and buyer respectively. The seller places its offerings in the price-range of [h(s)..pU] as it wants to sell the good above the expected price to make a profit. The buyer places its offerings in the price-range of [pD..h(b)] as it wants to buy the goods below the expected price to make a profit. The resulting matching-range on which the prices can meet - again buyer-price $\geq$ seller-price - is marked by the red rectangle. It is easy to see that a match with these mechanics can occur only if the optimism-factor of the buyer h(b) is strictly higher than the one of the seller h(s).

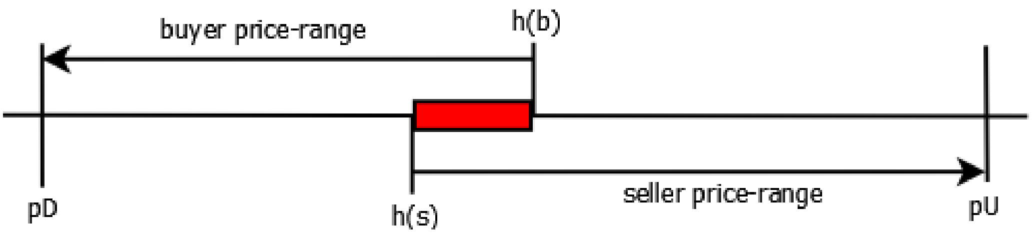

Figure 6: Matching of buyers and sellers price-ranges. The red rectangle marks the matching-range.

Formal proof Proofing optimism of buyer is greater or equal than the optimism of the seller $h_B \geq h_S$ by showing the limit-price of the buyer is greater of equal the limit-price of the seller $limit_B \geq limit_S$.

either through proof by contradiction: assume a seller with a larger optimism than the buyer and show that the assumption that the utility-function must increase fails.

or through difference between the two utility functions of a buyer and a seller: difference is always larger or smaller (dependeing on the direction of subtraction) than zero.

TODO: hypothese beweisen 1. zeigen dass hypothese auf minimale vernetzung reduziert werden kann bzw. dass diese minimale vernetzung ausreicht. (über den vorhergehenden formal proof und dass limit-prices monoton steigen) 2. zeigen, dass lücken bei minimalster vernetzung an beliebiger stelkle zu widerspruch führt: gleichgewicht wird nicht erreicht.

Asset/Cash

$limit_B = h_B + \frac{1}{5} - \frac{h_B}{5}$
$limit_S = h_S + \frac{1}{5} - \frac{h_S}{5}$

Proof.

$$
\begin{aligned}
limit_S - limit_B &\leq 0 \\
(h_S + \frac{1}{5} - \frac{h_S}{5}) - (h_B + \frac{1}{5} - \frac{h_B}{5}) &\leq 0 \\
h_S + \frac{1}{5} - \frac{h_S}{5} - h_B - \frac{1}{5} + \frac{h_B}{5} &\leq 0 \\
5h_S + 1 - h_S - 5h_B - 1 + h_B &\leq 0 \\
4h_S - 4h_B &\leq 0 \\
h_S - h_B &\leq 0 \qquad \text{can only hold if } h_B \geq h_S
\end{aligned}
$$

$\square$

Bond/Cash

$limit_B = h_B V + \frac{1}{5} - \frac{h_B}{5}$
$limit_S = h_S V + \frac{1}{5} - \frac{h_S}{5}$

Proof.

$$\begin{aligned}
limit_S - limit_B &\leq 0 \\
(h_S V + \frac{1}{5} - \frac{h_S}{5}) - (h_B V + \frac{1}{5} - \frac{h_B}{5}) &\leq 0 \\
h_S V + \frac{1}{5} - \frac{h_S}{5} - h_B V - \frac{1}{5} + \frac{h_B}{5} &\leq 0 \\
5h_S V + 1 - h_S - 5h_B V - 1 + h_B &\leq 0 \\
5h_S V - 5h_B V - h_S + h_B &\leq 0 \\
5V(h_S - h_B) - (h_S - h_B) &\leq 0 && \text{substituting } u = (h_S - h_B) \\
5Vu - u &\leq 0 && \text{assuming } h_S > h_B \\
\Rightarrow u > 0 \Rightarrow 5Vu - u &> 0 && \text{violates the original assumption} \\
\textit{proof by contradiction} \Rightarrow h_B &\geq h_S
\end{aligned}$$

□

Asset/Bond

$limit_B = \frac{h_B + \frac{1}{5} - \frac{h_B}{5}}{h_B V + \frac{1}{5} - \frac{h_B}{5}}$
$limit_S = \frac{h_S + \frac{1}{5} - \frac{h_S}{5}}{h_S V + \frac{1}{5} - \frac{h_S}{5}}$

Proof.

$$limit_S - limit_B \leq 0$$
$$\frac{h_S + \frac{1}{5} - \frac{h_S}{5}}{h_S V + \frac{1}{5} - \frac{h_S}{5}} - \frac{h_B + \frac{1}{5} - \frac{h_B}{5}}{h_B V + \frac{1}{5} - \frac{h_B}{5}} \leq 0$$
$$(h_S + \frac{1}{5} - \frac{h_S}{5})(h_B V + \frac{1}{5} - \frac{h_B}{5}) - (h_B + \frac{1}{5} - \frac{h_B}{5})(h_S V + \frac{1}{5} - \frac{h_S}{5}) \leq 0$$
$$substituting\ S = \frac{1}{5} - \frac{h_S}{5}, B = \frac{1}{5} - \frac{h_B}{5}$$
$$(h_S + S)(h_B V + B) - (h_B + B)(h_S V + S) \leq 0$$
$$h_S h_B V + h_S B + S h_B V + BS - (h_B h_S V + h_B S + B h_S V + BS) \leq 0$$
$$h_S h_B V + h_S B + S h_B V + BS - h_B h_S V - h_B S - B h_S V - BS \leq 0$$
$$h_S B + S h_B V - h_B S - B h_S V \leq 0$$
$$h_S B - h_B S + V(h_B S - h_S B) \leq 0$$
$$h_S(\frac{1}{5} - \frac{h_B}{5}) - h_B(\frac{1}{5} - \frac{h_S}{5}) + V(h_B(\frac{1}{5} - \frac{h_S}{5}) - h_S(\frac{1}{5} - \frac{h_B}{5})) \leq 0$$
$$\frac{h_S}{5} - \frac{h_S h_B}{5} - \frac{h_B}{5} + \frac{h_B h_S}{5} + \frac{h_B V}{5} - \frac{h_B h_S V}{5} - \frac{h_S V}{5} + \frac{h_S h_B V}{5} \leq 0$$
$$\frac{h_S}{5} - \frac{h_B}{5} + \frac{h_B V}{5} - \frac{h_S V}{5} \leq 0$$
$$h_S - h_B + h_B V - h_S V \leq 0$$
$$h_S(1 - V) + h_B(-1 + V) \leq 0$$
$$substituting\ x = h_S(1 - V), y = h_B(-1 + V)$$
$$x + y \leq 0$$
$$assume\ V\ linear\ in\ range\ of\ [0..1]$$
$$x = [h_S..0] \Rightarrow x \geq 0$$
$$y = [-h_B..0] \Rightarrow y \leq 0$$
$$\Rightarrow x + y \leq 0 \iff h_B \geq h_S$$

□

4.2.2 Proof of monotony of limit-functions

Asset/Cash market

Proof.

$$\begin{aligned} limit_{asset} &= h\,pU + (1-h)pD && \text{pU = 1, pD = 0.2} \\ &= h + (1-h)0.2 \\ &= h + \frac{1}{5} - \frac{h}{5} && \frac{\mathrm{d}}{\mathrm{d}h} \\ &= 1 - \frac{1}{5} = \frac{4}{5} \end{aligned}$$

constant, positive slope implies a monotony increasing limit-function over the range of the real numbers. QED □

Bond/Cash market

Proof.

$$\begin{aligned} limit_{bond} &= h\,V + (1-h)pD && \text{pD = 0.2} \\ &= h\,V + (1-h)0.2 \\ &= h\,V + \frac{1}{5} - \frac{h}{5} && \frac{\mathrm{d}}{\mathrm{d}h} \\ &= V - \frac{1}{5} \end{aligned}$$

V is constant =¿ constant, positive slope implies a monotony increasing limit-function were $V >= \frac{1}{5}$. QED □

Asset/Bond market

Proof.

$$\begin{aligned} limit_{asset/bond} &= \frac{h\,pU + (1-h)pD}{h\,V + (1-h)pD} && \text{pU = 1, pD = 0.2} \\ &= \frac{h + \frac{1}{5} - \frac{h}{5}}{h\,V + \frac{1}{5} - \frac{h}{5}} && \frac{\mathrm{d}}{\mathrm{d}h} \\ &= -\frac{5(V-1}{(h(5V-1)+1)^2} && \text{assume h and V in range [0..1]} \\ &\Rightarrow 5(V-1) <= 0 \\ &\Rightarrow (h(5V-1)+1)^2 \geq 0 \\ \Rightarrow -\frac{5(V-1}{(h(5V-1)+1)^2} &>= 0 && \text{for h and V in range [0..1]} \end{aligned}$$

positive slope implies a monotony increasing limit-function

□

4.3 Predictions

The following topologies found in appendix A satisfy the definition of the hypothesis:

- Fully-Connected
- Half-Fully connected
- Ascending-Connected
- Ascending-Connected with all kind of short-cuts
- Erdos-Renyi and Watts-Strogatz with the correct parametrization by pure chance.

It is expected that according to the hypothesis all of these topologies will reach the equilibrium found in fully-connectedness. All other topologies do not satisfy the hypothesis and are expected to clearly fail reaching the equilibrium of the Fully-Connected topology.

See chapter 6 and 7 whether the results reflect the hypothesis or not.

Chapter 5

Implementation

For this thesis a software was written to be able to investigate the behaviour and results of the different types of networks and get visual and numerical results to be embedded in this written text. In this chapter the implementation-details of the software are discussed.

5.1 Requirements

The author of this thesis had access to the software of Breuer et al. (2015) which was written in C++ therefore the question is why a new software had to be written and why not the original could be used. The reason for it was that the original software supported only a very narrow feature-set focusing only on the numerical results of a fully-connected network. Thus a complete redevelopment in Java was the option used. The following requirements were identified:

- Java based.
- Comprehensive GUI functionality.
- Emulate the functionality of Breuer et al. (2015) and its results.
- Represent arbitrary networks in the simulation.
- Step-through forward and backwards in a simulation-run.
- Run replications of simulations.
- Store results of replications to be opened again for later usage.
- Command-line mode to run previously defined number of replications.

Java based The original software was written in C++ which offers very much power and highest speed if used correctly but comes with a very high responsibility regarding memory-management. Java offers a much more relaxed programming model regarding memory-management as it is garbage collected. This does not mean that the programmer can waste memory without giving thought to it but that not as many aufwand into it. debugging is easier. The most compelling argument for Java are the vast libraries which are included in the JDK which are missing in standard C++. As complete GUI-functionality for the whole software is a requirement Java is the way to go. Although multi-purpose libraries like Boost and GUI-frameworks like Qt are available for C++ too, it takes quite some time to set them up correctly for the target platform one develops for which implies that in C++ the development would always have been for just one platform whereas Java runs on every platform - if not platform-dependent stuff was used - without recompilation. As one will see later in the "Command-line mode"-Feature this is a major requirement to make this feature practical. Thus the reasons for using Java were:

- Platform-independence which applies to 3rd party libraries too.
- GUI-framework provided by JDK.
- Relaxed memory-management which emphasises fast iterations and easy debugging.
- Support for smooth XML-Serialization.
- 3rd party libraries for network-modelling and -visualization.

GUI functionality All functionality should be accessible through a GUI where some features e.g. "Inspection" are only possible to use through a GUI.

Emulate Breuer et al. (2015) functionality Of course the whole software should be a super-set of the functionality of the one found in Breuer et al. (2015) so this was the point to start from.

Arbitrary networks It should be possible to restrict trading between the agents to arbitrary networks. As network-modelling and -visualization library JUNG is used.

Step-through simulation The software should support to go through a simulation-run step-by-step and storing all steps of the simulation to jump back and forth between steps.

Real-time visualisation and information The original C++ software didn't provide real-time information about the current wealth-distribution and market-dynamics and provided the user just with the numerical results in the end through the means of a command-line output. For better understanding of dynamics of both wealth and markets a real-time visualisation of both are necessary together with extensive information on the current state like the offering-book, agent-information, network-activity, history of matches and the current equilibrium. Also the real-time visualisation is necessary to provide this written part of the thesis with diagrams of various results and processes.

Replications Because the whole trading-process includes randomness the results are subject to noise thus replications are an absolute must-have feature to be able to give reliable results. Each replications is independent from all others thus it is a candidate for parallel programming to speed up the already very time-consuming process of running replications. According to the number of CPU-cores the software should spawn threads up to the number of cores and run replications in parallel thus speeding up by a considerable amount of time.

Store results When running a bunch of replications for a given set-up the results of it should be automatically stored as XML to be accessible for later inspection thus conserving state and eliminating the necessity to re-run time-consuming simulation-runs with a high number of replications.

Command-line mode As already described replications are required to be implemented for parallel processing where up to the number of CPU-cores replications can run in parallel. When running a vast number of replications one does not need to GUI-functionality and most probably the machine is so occupied by the heavy work-load that a fließender usage of GUI would be not possible any-more. Thus replications should be runnable through a separate command-line mode of the thesis-software which reads information from a XML-File in which one can specify multiple simulation set-ups for which replications should be run. The command-line mode iterates through all configurations and runs the required replications and writes the result out for a later inspection. Obviously the more CPU-cores the faster a simulation-run with e.g. 50 replications finishes. For this reason most of the final replication-runs were done on a 40-core machine of the FH Vorarlberg which runs on Linux on which the thesis-software could be run easily because of Java's platform-independence.

5.2 Functionality

In this section the functionality of the thesis-software is explained to get an understanding of the implemented features. All features are available through the GUI unless stated otherwise. For inspection, replications and experimenter an individual tab is available inside the main window so they can be used in parallel. The features were implemented in combination with emulating the functionality of the model introduced in chapter 3 thus this thesis-software is a super-set of the software used in Breuer et al. (2015).

5.2.1 Inspection

Inspection allows for a given simulation-configuration to step through the whole process match-by-match, keep track of the successful matches and the wealth-distribution of the agents at this time and to jump back to each successful match. Through an offer-book the state and current offerings of all agents at a given time of the simulation (including past successful matches) can be inspected and compared by opening multiple offer-book windows. Furthermore this feature provides the user with statistics of successful, failed and total matches and the current equilibrium. Very much attention was paid to the implementation of the real-time visualisation of the agents wealth-distribution and the market-activities as visualisation is of very importance for a successful inspection and interpretation of a dynamic process. To be able to compare the visual results of 2 different simulation-configurations there exist 2 "Inspection" tabs in the main window.

5.2.2 Networks

This functionality allows to model different kind of network-topologies as described in appendix A to restrict trading only to this kind of neighbourhood. Obviously network-topologies are graphs and thus it would have been required to implement a graph-library but that was not implemented self but the graph-library JUNG was used as it supported all the required features like visualisation, neighbourhood- and path-queries.

5.2.3 Replications

To get robust results replications are used to reduce the influence of random noise. This feature allows to let run a specific simulation configuration for a given number of replications in parallel. The specific simulation-configuration serves as a template - especially the network-topology - and each replication does a clone of the network together with the agents to be able to run in parallel without the need of synchronization. It was tried to implement the network as a shared network between all replications but because the agents serve as nodes this was not feasible and would have required a very fundamental re-factoring of already existing functionality. Thus the higher requirements in memory and time for cloning was accepted for a more elegant solution. When replications are processed running ones can be inspected and information can be queried using the GUI. It is possible to cancel one replication, cancel a whole task which reduces thread-load, inspect wealth-distribution, see the failed and successful matches and statistics of already finished replications. Whenever a replication is finished it is added to the pool of results and the median result is re-calculated and visualized and informations updated. When all replications have finished the results are written to a new XML-file in the results-folder of the software. This result-file can be opened using the experiments-feature which is described next. A simulation-configuration can be saved as a separate new experiment-file or added to an existing experiment-file - see next section for experiments.

5.2.4 Experiments

The experiments-feature allows to open experiment-files and result-files of replication- and experiment-runs. An experiment-file is just a collection of simulation-configurations to be run for a given number of replications. When an experiment-file is opened all simulation-configurations are displayed in a list an each of them can be opened as a new replication-tab which in turn can be run as explained in section "Replications". To be able to coneveniently run all simulation-configurations of an experiment-file without needing user-interaction a command-line feature was implemented. For this purpose the jar-file LeverageCycleCMD.jar was built. Start an experiment-run by using the following command:

java -jar LeverageCycleCMD.jar PATH-TO-EXPERIMENT-FILE MAX-THREADS-OPTIONAL

Note that the number of threads to use is optional and when omitted the number of CPU-cores found on the system is used.

Each replication-run of a given simulation-configuration in the experiment-file produces a result-file with the name provided in the configuration. These result-files are dubbed result-files of experiment-runs as previously stated but are the same as the result-file of a replication-run as discussed in the section "Replications" When opening a result-file a new tab is added to the main-window and the statistics of the run together with equilibrium-statistics are displayed. Furthermore the wealth-distirbution and market-acitivities are visualized and it is possible to view the statistics of each individual replication. Furthermore the user can display the network-topology used for this simulation-configuration which is important as random networks are different when created new.

5.3 Architecture

In this section the basic architecture of the thesis-software is discussed. It is important to note that this thesis is a fat-client and has its major emphasis not on the software-development aspect but on the visual- and numerical results where the accompanying software is just a tool for the means to calculate theses required results. Thus the architecture is guided by a simple division of layers into front-end, controller and back-end. The front-end is responsible for input and output of the user through GUI or command-line. The controller-layer provides encapsulated chunks of functionality of the back-end to the front-end. It is necessary to abstract, encapsulate and combine the stateful nature of communication with the back-end into a separate layer instead of polluting the front-end with it and creating unnecessary dependencies. The back-end layer provides the functionality where the real work is done e.g. simulation is executed. Theoretically the dependencies are top-down where the front-end includes only controller-functionality, the controller includes only back-end functionality and the back-end has no dependencies to the preceding layers. In this thesis-software a more pragmatic approach was chosen so this dogma was not followed where over-engineering and over-complication would have resulted when sticking strictly to the separation of dependencies. Thus in very few cases the controller- and back-end layer include front-end layer functionality to create different types of network topologies in a more convenient way. Also the front-end accesses instances of pure back-end classes for graphical visualisation and information purposes. Despite the seemingly flawed architecture the development-process has proven to be very smooth and expansions and refactorings went quite smoothly and always resulted in a better and clearer structure with reduced code-smell which is always a sign for a good architecture.

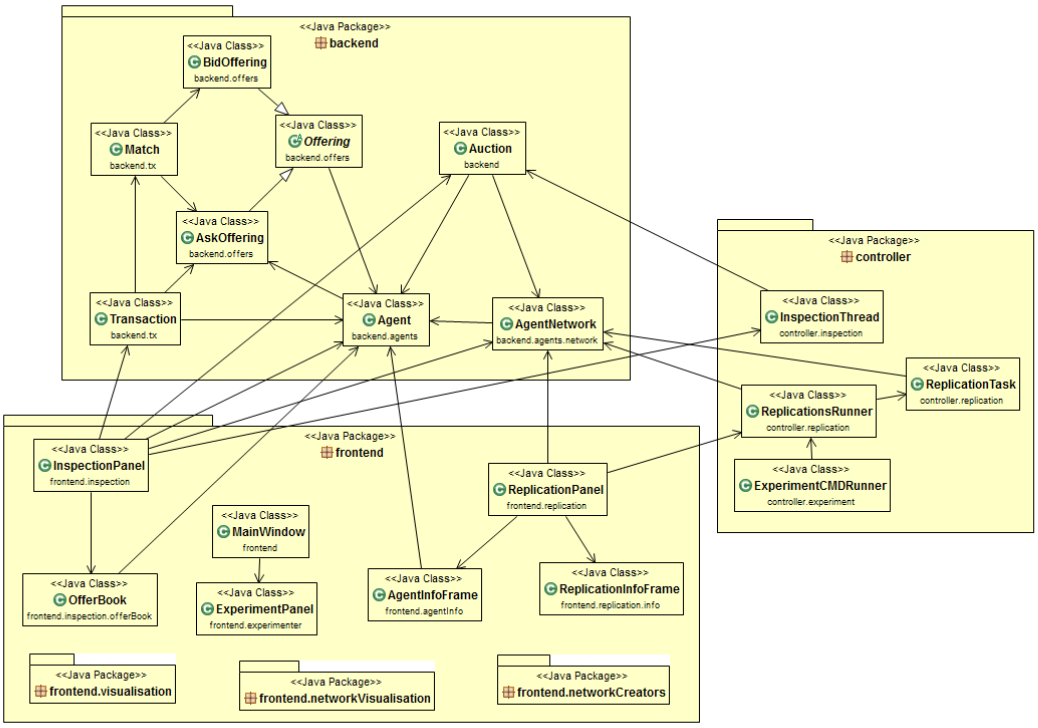

Figure 7: Package-diagram with most important classes and associations illustrating the architecture. (Note that the software contains many more classes and associations which are omitted for clarity reasons as this diagram should provide only a basic overview and would contain by far too much information if everything would have been included.)

5.3.1 Frontend

The front-end contains the GUI functionality through which the user interacts with the program and the real-time visualisation of wealth-distribution and market-activity. Because data like the offer-book, agent/replication/equilibrium-information and network-visualization is required in different contexts much emphasis was put on massive re-use of GUI components through the use of sub-classing and aggregation already provided by SWING. Thus many small components were implemented and aggregated together to create bigger chunks of logical functionality. In the package-diagram 7 only the most important panels are shown. Those provide the main entry-point for the user to the given functionality as indicated by their names.

- InspectionPanel - Is the entry-point for the inspection-functionality. There exist 2 instances during run-time to be able to compare two inspection-runs.

- ReplicationPanel - Is the entry-point for the replication-functionality.
- ExperimentsPanel - Is the entry-point for the experiments-functionality. Results of replications and experiment-files can be loaded there.
- AgentInfoFrame - Displays the information of agents: current wealth, optimism and its price-ranges.
- OfferBook - Is only available in the inspection-functionality and allows to inspect the current offerings of each agent on all markets and the agent-information of each agent the same way as in *AgentInfoFrame*. Multiple instances can be created simultaneously by the user to compare the data of multiple agents.

5.3.2 Controller

The controller package is rather slim and contains the steering functionality to drive inspections, replications and experiments.

- InspectionThread - Inspections are driven by this class which allows to advance the simulation step-by-step using the consumer-producer paradigm through lock and wait functionality provided by Java through the use of its monitors.
- ReplicationsRunner and ReplicationTask - These classes encapsulate functionality to run a number of replications for a given simulation-configuration in parallel, calculating the current median for all important values on-line and writing results to xml-files.
- ExperimentCMDRunner - Experiments are executed from the command-line through the use of this class where it cycles through all simulation-configurations of a experiment-configuration and uses *ReplicationsRunner* to execute it in parallel.

5.3.3 Backend

The back-end contains the domain-specific functionality for the simulation. The important classes can be seen in the package-diagram 7 but for better clarity each one is introduced briefly.

- Auction - holds the state and operations for a single simulation-run. This class does the sweeping and clearing discussed in section "Simulation", determines whether trading is still possible or not and calculates current equilibrium. allows to run a matching-round

- Agent - encapsulates the functionality and state necessary for the agents in the simulation. See next section "Agents" for a more in-depth discussion.
- AgentNetwork - holds the connections between the agents and provides functionality to query neighbourhood, paths and connections between agents and random and sequential in-order iterator over all agents.
- Offering - encapsulates the data necessary for offerings where there are two subclasses *BidOffering* and *AskOffering* to differentiate between the two types of offerings. See the section "Markets" on details of offerings.
- Match - provides the functionality to create a match out of given offers and returning an instance of *Match* through a factory-method. The *Match* instance holds the price, amount and the offers which have matched. See section "Sweeping and Matching" on more details.
- Transaction - encapsulates functionality to search for a match within a given neighbourhood, is used by *Auction* class to sweep through the agents and is returned after a matching-round and provides necessary information about whether it was successful or not.

5.4 Agents

Although agents are used in this software it is not an agent-based simulation in the classical way as these agents are zero-intelligence ones and have no states of behaviour. That is each agent makes bid- and ask-offers on all markets if the constraints allow it where the prices are selected from random ranges which improve the utility of the agent - that means it makes always offers which would result in a profit.

Each agent is characterized by its state where the main variables are:

- Id - the unique id of the agent in the range of the natural numbers in the range of [1..number of agents].
- Optimism-factor h - defines how optimistic the agent is in the range of [0..1] where 0 is most pessimistic and 1 is most optimistic. The distribution of the optimism-factor among the agents is linearly ascending with the id and is defined through the following equation:

$$\textit{optimism-factor} = \frac{id}{number\ of\ agents + 1} \tag{5.1}$$

- Cash holdings - the current cash holdings of the agent.
- Assets holdings - the current asset holdings of the agent not including the assets granted as securities for giving loans.
- Loan given - the amount of loans bought from / granted to other agents. For a given amount of loans the equal amount of assets are granted as security to the buyer of the loan. Thus this variable increases the amount of assets the agent can trade with.
- Loan taken - the amount of loans sold to other agents. For a given amount of loans the equal amount of assets the equal amount of assets are granted as security to the buyer of the loan. Thus this variable decreases the amount of assets the agent can trade with as this amount of loans needs to be kept as securities.

There are three important derived variables which are calculated from the previous ones:

- Collateralized assets - is the amount of assets which are bound through collateral obligations because of taken bonds.

$$\textit{collateralized assets} = max(0, \textit{loans taken} - \textit{loans given}) \tag{5.2}$$

- Free assets - is the amount of assets which are unbound and act not as security and are owned completely by the agent.

$$\textit{free assets} = \textit{assets holding} - \textit{collateralized assets} \tag{5.3}$$

- Loans - is the net number of loans and calculated through

$$\textit{loans} = \textit{loans given} - \textit{loans taken} \tag{5.4}$$

  Thus this value is positive if the agent has granted more loans to other agents than received and it is negative if the agent has received more loans from other agents than granted.

Note that all variables cannot go negative except loans.

5.5 Markets

In this section all markets and their implementations are described. Each market is completely characterised by the following points:

- Products - the products traded on the corresponding market.
- Price-ranges - the price-ranges in which an agent places profit-making offers on the market. A range is defined by a minimum and maximum where the limit-prices vary across this range. This implies that the most pessimistic agent is at the minimum end of the range and the most optimistic agent is at the maximum end of the range.
- Bid- and Ask-Offerings - define the pre-conditions for an agent to make offerings on the market and the amount and the prices generated during an offer. Note that in case of bid-offerings the price of the profit-making offer is drawn randomly between the minimum price and the limit-price because when buying one wants to buy below the expected value to make a profit where in case of ask-offerings the price is drawn randomly between the limit-price and maximum price because when selling one wants to sell above the expected value to make a profit.
- Match-Table - In case of a match between two agents the wealth-exchange is declared where the wealth is increased/decreased as indicated by the +/- signs.

5.5.1 Asset/Cash

Products

Free assets are traded against cash. The buyer gets a specific amount of free assets for a given amount of cash where the seller gives away the specific amount of free assets and gets the given amount of cash.

Price-Range

minimum The minimum value of one asset in cash is the down-value pD tomorrow as defined in chapter 3. This value is obviously a constant for all agents.

$$\textit{min asset-price} = pD \tag{5.5}$$

maximum The maximum value of one asset in cash is the up-value pU tomorrow as defined in chapter 3. This value is obviously a constant for all agents.

$$\textit{max asset-price} = pU \tag{5.6}$$

limit The limit-price of an asset in cash depends on the optimism-factor h of the agent where the most optimistic agent assigns pU and the most pessimistic pD as the price. Thus applying linear interpolation one receives the following equation.

$$\textit{limit-price asset} = h * pU + (1.0 - h) * pD \tag{5.7}$$

Bid-Offering

As amount one TRADING-UNIT of assets is selected but if there is not enough cash left to buy one TRADING-UNIT of assets then no bid-offer is made.

Table 1: Bid-Offering parameters of Asset/Cash market

Pre-Condition	*cash holdings* > *price of TRADING-UNIT assets*
Asset-Price	random(*min asset-price*, *limit-price asset*)
Asset-Amount	*TRADING-UNIT*

Ask-Offering

Ask offers are generated only when the agent has at least one TRADING-UNIT of free assets left. As amount one TRADING-UNIT of assets is selected - in the thesis-implementation 0.1 - but if there are fewer free assets left then no offer is made.

Table 2: Ask-Offering parameters of Asset/Cash market

Pre-Condition	*free assets* > *TRADING-UNIT*
Asset-Price	random(*limit-price of asset*, *max asset-price*)
Asset-Amount	*TRADING-UNIT*

Match

Table 3: Wealth-Exchange during a match on Asset/Cash market

	Seller	Buyer
Assets holdings	- matching-amount	+ matching-amount
Cash holdings	+ matching-price	- matching-price

5.5.2 Bond/Cash

Products

Bonds are traded against cash. The buyer grants the seller a loan in buying a bond from the seller thus the buyer gets a given bond-amount from the seller and gives a given cash-amount to the seller. For a given amount of sold bonds the equal amount of assets need to be hold as securities.

Price-Range

minimum The minimum value of one bond in cash is the down-value pD tomorrow as defined in chapter 3. This value is obviously a constant for all agents.

$$\textit{min bond-price} = pD \tag{5.8}$$

maximum The maximum value of one bond in cash is the face-value V tomorrow as defined in chapter 3. This value is obviously a constant for all agents.

$$\textit{max bond-price} = \textit{facevalue}\ V \tag{5.9}$$

limit The limit-price of a bond in cash depends on the optimism-factor h of the agent where the most optimistic agent assigns *facevalue* V and the most pessimistic pD as the price. Thus applying linear interpolation one receives the following equation.

$$\textit{limit-price bond} = h * V + (1.0 - h) * pD \tag{5.10}$$

Bid-Offering

Bid offers are generated only when the agent has any cash holdings. As amount one TRADING-UNIT of bonds is selected but if there is not enough cash left to buy one TRADING-UNIT of bonds then the amount of bonds is selected which can be bought with the remaining cash holdings.

Table 4: Bid-Offering parameters of Bond/Cash market

Pre-Condition	$\textit{cash holdings} > 0$
Bond-Price	random(*min bond-price*, *limit-price of bonds*)
Bond-Amount	$\min(\frac{\textit{cash holdings}}{\textit{Bond-Price}}, \textit{TRADING-UNIT})$

Ask-Offering

Ask offers are generated only when the agent has any free assets because when selling the agent needs to reserve as many free assets as security as bonds sold. As amount one TRADING-UNIT of bonds is selected but if there are fewer free assets left then the remaining amount of free assets is selected.

Table 5: Ask-Offering parameters of Bond/Cash market

Pre-Condition	$\textit{free assets} > 0$
Bond-Price	random(*limit-price of bonds*, *max bond-price*)
Bond-Amount	min(*free assets*, *TRADING-UNIT*)

Match

Table 6: Wealth-Exchange during a match on Bond/Cash market

	Seller	Buyer
Loan Given	N/A	+ matching-amount
Loans Taken	+ matching-amount	N/A
Cash holdings	+ matching-price	- matching-price

5.5.3 Asset/Bond

Products

Assets are traded against bonds. The buyer gets a specific amount of free assets for a given amount of bonds where the seller gives away the specific amount of free assets and gets the given amount of bonds.

Price-Range

minimum The minimum value of one asset in bonds is defined as the ratio of the minimum asset-price in cash to the minimum bond-price in cash. This value is obviously a constant for all agents.

$$\textit{min asset/bond price} = \frac{\textit{minimum asset-price}}{\textit{minimum bond-price}} = \frac{pD}{pD} = 1 \tag{5.11}$$

maximum The maximum value of one asset in bonds is defined in the ratio of the maximum asset-price in cash to the maximum bond-price in cash.

$$\textit{max asset/bond price} = \frac{\textit{maximum asset-price}}{\textit{maximum bond-price}} = \frac{pU}{V} \tag{5.12}$$

limit The limit-price in bonds of an asset in bonds depends on the optimism-factor h and is just the ratio of the limit-price of the asset to the limit-price of the loans.

$$\textit{limit-price asset/bond} = \frac{\textit{limit-price asset}}{\textit{limit-price bonds}} \tag{5.13}$$

Bid-Offering

Bid offers are generated only when the agent is not negative on free assets after the trade. As amount one TRADING-UNIT of a assets is selected. To calculate the free assets after a match the following steps are performed:

1. Start with current free assets holdings.
2. Subtract current collateral obligations.
3. Subtract taken loans after trade.
4. Add assets bought through trade.

Table 7: Bid-Offering parameters of Asset/Bond market

Pre-Condition	*free assets >= 0 after trade*
Asset/Bond-Price	random(*min asset/bond price*, *limit-price of asset/bond*)
Asset/Bond-Amount	*TRADING-UNIT*

Ask-Offering

Bid offers are generated only when the agent is not negative on free assets after the trade. As amount one TRADING-UNIT of bonds is selected. To calculate the free assets after a match the following steps are performed:

1. Start with current free assets holdings.
2. Subtract current collateral obligations.
3. Add given loans after trade.
4. Subtract assets sold through trade.

Table 8: Bid-Offering parameters of Asset/Bond market

Pre-Condition	*free assets >= 0 after trade*
Asset/Bond-Price	random(*limit-price of asset/bond*, *max asset/bond price*)
Asset/Bond-Amount	*TRADING-UNIT*

Match

Table 9: Wealth-Exchange during a match on Asset/Loan market

	Seller	Buyer
Loan Given	+ matching-amount	N/A
Loans Taken	N/A	+ matching-amount
Asset holdings	- matching-price	+ matching-price

5.6 Bond pledgeability (BP) mechanism

As already defined in chapter 3 the BP-mechanism allows to free a collateralized asset by buying a bond. Breuer et al. (2015) define it technically:

> This amounts to a collateral constraint requiring the negative *net* number of bonds (short minus long) to be less or equal the number of assets.

Thus to implement the BP-mechanism one must be able to differentiate between the amount of loans the agent has taken (short) and the amount of loans the agent has given (long). The collateralized assets are then calculated by subtracting the long bonds from the short bonds where the result must be ≥ 0 to satisfy the collateral constraint. As can be seen in section 5.4 this is already implemented. An agent has two loan-variables: *loan given* and *loan taken* and thus the collateralized assets are calculated as given in equation 5.2. When ignoring the BP-mechanism this equation would skip the subtraction of the *loans given* which in this case ignores the "pledgeability"-part. In this case the loan-obligations can only increase - and thus the amount of collateralized assets - but never be reduced because *loan given* is not included any more in this equation. Thus the only difference when ignoring the BP-mechanism is a change in the equation of the collateral constraint of an agent. The equation without BP is:

$$collateralized\ assets = max(0, loans\ taken) \tag{5.14}$$

5.7 Simulation

In this section a few details of the simulation-implementation are discussed as they are not so obvious but interesting and important enough to be mentioned.

As noted above in the section "Markets" bonds and assets are always traded in chunks of TRADING-UNITS which is 0.1 in case of assets and 0.2 in case of bonds. Note that it is nothing unusual to scale the initial wealth-endowments to 1.0 and trade fractions of them as in the end the absolute endowment does not matter but the distribution over the agents so a transformation to natural numbers would not gain anything. What is much more important is that the system conserves wealth: no wealth is lost and no wealth is created *ex nihilo*. This is guaranteed by the equations and pre-conditions found in section "Markets".

5.7.1 Sweeping and Matching

Sweeping Sweeping is the process of generating a successful match by running through the agents and apply matching between their offers. Because offering-prices are created randomly in specific ranges it could happen that no offers match. To elevate this problem not only 1 but up to 500 sweeps are done when calculating the next successful match.

Matching Matching is the process of finding a match of a given agent in the sweeping-process with the offerings of its neighbourhood. Note that a match

Algorithm 1 Sweeping Pseudocode

```
clear best offerings of all agents
while sweeps < 500 do
    clear global offerings of previous sweep
    shuffle agents
    generate offerings for all agents
    for all shuffled agents do
        find match in neighbourhood
        if match found then
            execute match
            exit sweeping
    if no trading possible then
        exit sweeping
```

can only occur on one specific market between two agents thus satisfying only one buy- and one sell-offer.

Algorithm 2 Matching Pseudocode

```
get best offerings of neighbourhood
for all neighbourhood offerings do
    randomly check sell or buy offerings first
    if buy-price ≥ sell-price then
        m ← new Match
        m.price = (buy-price+sell-price) / 2
        m.amount = min(buy-amount, sell-amount)
        m.normPrice = m.price * m.amount
        exit matching
```

Note that as defined in the double-auction theory - see chapter 2 - the agents meet at the half-way price. This guarantees that not more than the previously defined amount is traded which again conserves the wealth. Care must be taken when calculating the real price of the match as all offering prices are for 1.0 Unit of the traded good. Thus to obtain the real matching-price the previously calculated half-way price needs to be scaled by the matching-amount to obtain the price for the given matching-amount. Also note that if one match is found on any market then the sweeping and matching is terminated and a successful match is returned thus the agents can place offers on all markets although the sum of the offers would be infeasible because only one offer will be satisfied and the other ones are thrown

Algorithm 3 Get Best Offerings of Neighbourhood Pseudocode

```
bestOfferings ← null
a ← current agent
for all neighbours of a do
    n ← next neighbour of a
    neighbourOfferings ← n.currentOfferings
    for all neighbourOfferings do
        nO ← next offering in neighbourOfferings
        for all bestOfferings do
            bO ← next offering in bestOfferings
            if nO.price dominates bO.price then
                bO ← nO
```

away.

5.8 Performance improvement

5.8.1 Local and Global Offer-book

It is easy to see that the calculation of the best neighbourhood can be quite costly the more neighbours an agent has. For of the ascending-connected topology this is not a big deal but in the case of the fully-connected topology the impact is serious. Because fully-connected is an important topology as it serves as a benchmark for the others an optimisation was implemented specially suited for this kind of topology: the introduction of a global offer-book. Instead of calculating the best offerings within the neighbourhood all offers are checked against each other in a global offering-list during the time of offering-creation. After all offers have been generated this list contains only the best sell and ask offerings for each market. The step of calculating the best neighbourhood can then be omitted in the case of fully-connectedness because it would be the same as the global best-offerings list thus saving a substantial amount of processing time.

5.8.2 Matching probabilities

General matching For a match to happen the buyer-price must be larger or equal the seller-price. Thus as shown in chapter 4 a price higher than the seller-price can only be placed by a buyer which has a strictly larger optimism-factor than the seller because only then their offering-ranges have a chance to cover each other partly. Thus matching in general works between a seller with lower optimism-factor and a buyer with higher optimism-factor. As noted in chapter 3 agents place offering-prices randomly in their ranges which are specified in this chapter in section "Markets" so matching is also subject to randomness. To investigate the matching-probabilities one must look at the coverage probabilities of the matching-ranges. The limit-price of bidders increase with rising optimism-factor and thus the width of the bidding-range decreases because bidders make their offers in the range of [limit-price..pU]. The situation is the inverse for askers as their limit-price increases too but the width of their asking-range increases because they place their offers in the range of [pD..limit-price]. Thus obviously the larger the difference in the optimism-factor between a seller and a buyer - again note that buyers optimism-factors are strictly larger than those of their corresponding sellers - the more likely a match will happen as both ranges increase and cover much more total distance thus increasing the probability that prices will be drawn which match. The following formulas derive the probabilities that an asker and a bider with a given limit-price will match on the Asset/Cash market. First the coverage-range is calculated by subtracting the limit-price of the asker from the one of the bidder as the one of the bidder must be larger. Then the probabilities of the asker and of the bider to fall into the range of the coverage-range is calculated. Finally the probability that both the asker and bider range will match is calculated by multiplying the previously calculated probabilities. Note that a division by 2 needs to be done because of symmetrical reasons because it does not matter where in the range the prices will meet but because the buy-price has to be larger or equal than the seller-price only the half-range is available.

$$
\begin{aligned}
\textit{coverage-range} &= \textit{limit-price bidder} - \textit{limit-price asker} \\
p(asker) &= \frac{\textit{coverage-range}}{pU - \textit{limit-price asker}} \\
p(bider) &= \frac{\textit{coverage-range}}{\textit{limit-price bider} - pD} \\
p(asker|bider) &= \frac{p(asker) * p(bider)}{2}
\end{aligned}
\tag{5.15}
$$

Matching in ascending-connectedness When considering the Ascending-Connected topology matching-probabilities become quite an issue as each agent has only 2 neighbours where the one with lower optimism-factor is the seller and the one with higher optimism-factor is the buyer. Thus compared to fully-connectedness where agents - depending on their optimism-factor - have multiple buyers and sellers the matching-probabilities are likely to be very low. To get a better understanding of the matching-probabilities in ascending-connectedness three diagrams visualizing them are shown below. Note that for clarity only 30 agents are considered on the Asset/Cash market. The following figure 8 visualizes the probabilities of pairwise neighbours to fall into their coverage-range as calculated by the formulas given above.

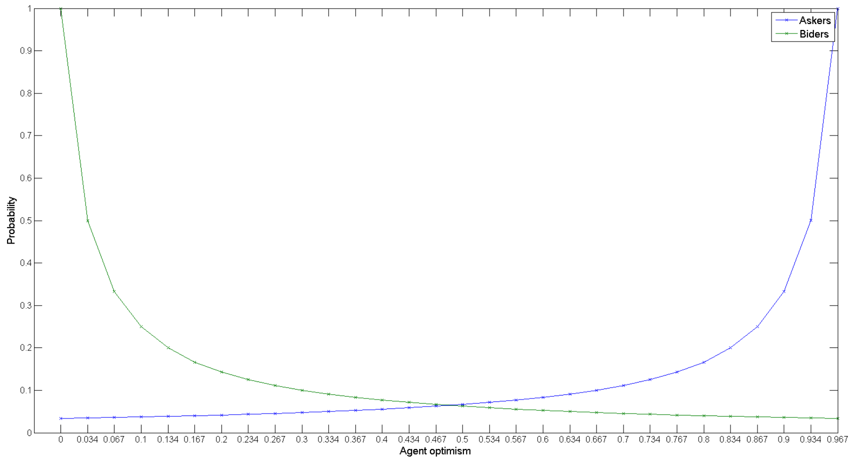

Figure 8: Pairwise probabilities of falling into coverage-range of 30 agents in Ascending-Connected topology on Asset/Cash market

The first bider with optimism-factor of 0.034 has a probability of 1.0 that the generated buy-prices fall into the range of its asker-neighbour with optimism-factor of 0. In turn the first asker with optimism-factor of 0 has a probability of 0.033 that its generated sell-prices fall into the range of its bid-neighbour with optimism-factor of 0.034. Thus the probabilities decrease for the bidders with increasing optimism-factor as their offering-range gets smaller and smaller where the probabilities increase for the askers with increasing optimism-factors as their offering-range gets wider and wider. This leads to the highest matching-probabilities at the edges and the lowest around the center.

The following figure 9 visualizes the combined probabilities that asker and bider match their ranges.

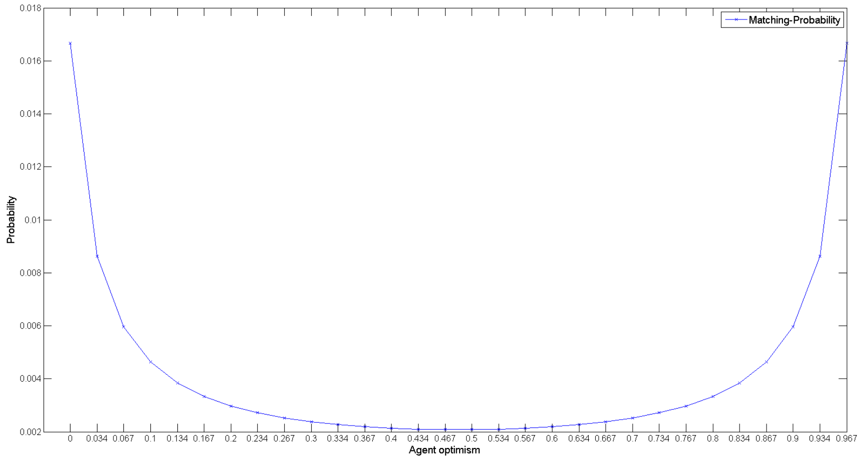

Figure 9: Pairwise matching-probabilities of 30 Agents in Ascending-Connected topology on Asset/Cash market

To see the influence of incresing number of agents the following figure 10 visualizes the combined probabilities of 50 agents.

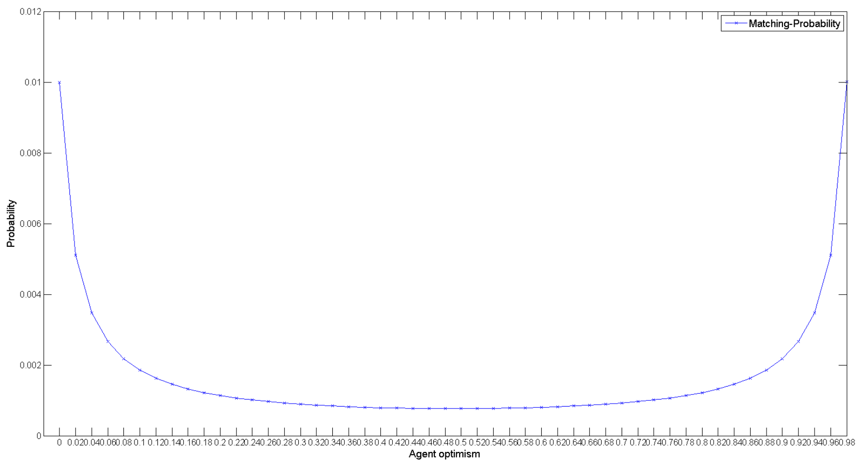

Figure 10: Pairwise matching-probabilities of 50 Agents in Ascending-Connected topology on Asset/Cash market

Looking at these diagrams one can derive the following facts about the matching-probabilities in Ascending-Connected topology:

1. Matching-probabilities are continuous and symmetric, are reduced towards the center of the optimism-scale and have their maximum at the very edges.

2. The more agents in the network the lower the matching-probabilities.

The lower matching-probabilities result in longer simulation-runs and more inefficiencies towards the end where free assets are kept by optimists instead of traded as the matching-probabilities decrease so much that no more trading will occur. To elevate this problem one can adjust the matching-probabilities by transforming the price-ranges. It is of very importance that the shape of the matching-probabilities distribution must be exactly the same. Only the absolute values of the probabilities are increased where the edges are at the maximum of 1.0. This is mathematically quite involved and was not developed by myself but by Supervisor Mr. Vollbrecht. The formulas and explanations have been moved to appendix C "Increasing the matching-probabilities in Ascending-Connected topology" This variant is dubbed "Ascending-Connected topology with Importance Sampling". See chapter 6 for the performance and results of this variant.

Chapter 6

Results

In this chapter the results of the experiments are given. Each topology-type introduced in appendix A was simulated where in this chapter only Fully-Connected and Ascending-Connected topologies are handled as the Ascending-Connected topology - both with and without importance sampling - is the most minimal network which satisfies the requirements for the hypothesis. The results for the other topologies can be found in appendix B for Hub-Based, Scale-Free and Small-World Topologies".

Note: The numbers in tables resemble always a median-value with the standard-deviation given in parentheses.

6.1 Validating simulation results

As a point-of-reference and as an experimental proof for the correctness of the implementation of the thesis-software the results of a validation against both the theoretical equilibrium and the equilibrium found in Breuer et al. (2015) are given. Because equilibrium differs across the number of agents and the type of bond traded to be comparable the same amount of agents and the same bond-type has to be used in the experiments. Thus 1000 agents and a bond with face-value of 0.5 were chosen because Breuer et al. (2015) report their equilibria for this number of agents and bonds with face-value between 0.1 to 0.5.

6.1.1 References

Table 10: Theoretical equilibrium for 1,000 agents and 0.5 bond

Asset-Price p	0.715
Bond-Price q	0.374
Marginal agent i1	0.583
Marginal agent i2	0.802

Table 11: Equilibrium in Breuer et al. (2015) for 1,000 agents and 0.5 bond

Asset-Price p	0.716
Bond-Price q	0.375
Marginal agent i1	0.583
Marginal agent i2	0.801
Pessimist Wealth	1.716
Medianist Wealth	4.578
Optimist Wealth	5.032

6.1.2 Thesis results

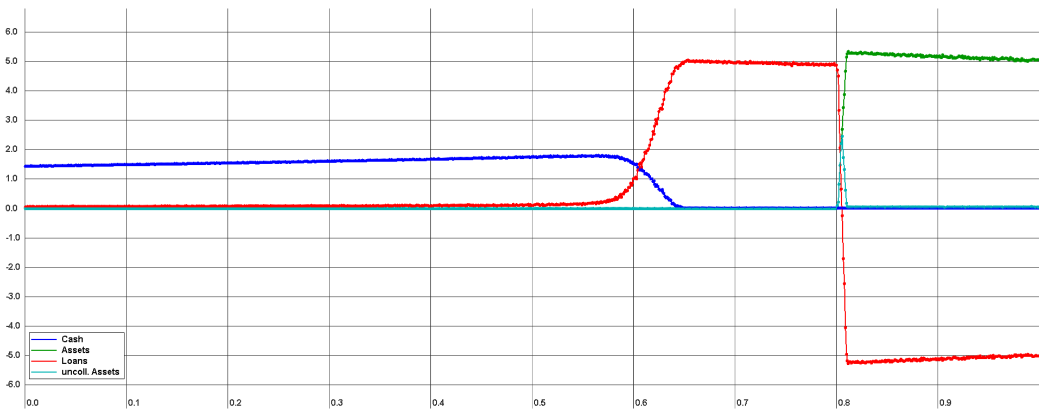

Figure 11: Wealth-Distribution of thesis-implementation of Fully-Connected topology for 1,000 agents and 0.5 bond

Table 12: Equilibrium of thesis-implementation for 1,000 agents and 0.5 bond

Asset-Price p	0.700 (0.005)
Bond-Price q	0.389 (0.002)
Marginal agent i1	0.616 (0.004)
Marginal agent i2	0.805 (0.001)
Pessimist Wealth	1.582 (0.01)
Medianist Wealth	4.578 (0.031)
Optimist Wealth	5.105 (0.025)

Table 13: Difference of Fully-Connected topology price-equilibrium as given in table 12 to theoretical equilibrium as given in table 10

	Result	Reference	difference to Reference
Asset-Price p	0.700	0.715	-2.1%
Bond-Price q	0.389	0.374	+4.0%
Marginal agent i1	0.616	0.583	+5.6%
Marginal agent i2	0.805	0.802	+0.4%

Table 14: Difference of Fully-Connected topology wealth-equilibrium as given in table 12 to wealth-equilibrium as given in Breuer et al. (2015) from table 11

	Result	Reference	difference to Reference
Pessimist Wealth	1.582	1.716	-7.8%
Medianist Wealth	4.578	4.578	0.0%
Optimist Wealth	5.105	5.032	+1.5%

Although marginal agent i1 and bond-price q are quite different than from theoretical equilibrium and the pessimists wealth is 7.8% less than given in Breuer et al. (2015) these results are nonetheless accepted as reaching the equilibrium. The differences emerge from the reasons that the thesis-simulation runs were terminated earlier than in Breuer et al. (2015) which results in the i1 and i2 edges to be not as sharp as reported in Breuer et al. (2015). It would be necessary to run the simulation an order of magnitude longer as the matching probabilities are reduced rapidly when only direct neighbours are able to trade any more within a network of 1000 agents. See section 5.8 for details on matching-probabilities.

6.1.3 Performance and termination measurements

As noted in section 5.7.1 a matching-round performs up to 500 offering-rounds where during one round all agents make an offer to find a match. If a match occurs during one offering-round the current matching-round is terminated and marked as successful. If no match occurs during all 500 offering-rounds the current matching-round is terminated too but marked as failed. Thus the following terminology is defined:

Successful matching-round a match occurred within maximal 500 offering-rounds where in each offering-round all agents make an offer.

Failed matching-round no match occurred within 500 offering-rounds where in each offering-round all agents make an offer.

Termination criteria after 1,000 successive failed matching-rounds it is expected that no more trading will occur thus the simulation is terminated.

Table 15: Performance of thesis-implementation with 1000 agents and 0.5 bond

Successful matching-rounds	19,300.04 (101.68)
Failed matching-rounds	10,306.78 (2914.11)
Total matching-rounds	29,606.82 (2938.82)
Ratio successful/total	0.65
Ratio failed/total	0.35

6.2 Experiments configuration

In the following experiments 100 agents were used, all markets (Asset/Cash, Bond/Cash, Asset/Bond) were enabled, a bond with face-value of 0.5 was selected and in each experiment 50 replications were run. A replication was terminated after 1000 failed matching-rounds in a row. Note that if trading is not possible any more before this criteria is met the simulation is terminated and thus it is possible that it halts earlier as can be seen for the Ascending-Connected Importance Sampling topology.

Breuer et al. (2015) showed that equilibrium can be reached already with 30 agents so this was the minimum number of agents to start with but for a smoother visual result 100 were chosen. Also one simulation-run takes not very much time with 100 as compared to the 1,000 agents thus it is a very good match between visual accurateness and processing-power requirements.

The 0.5 bond was selected because it is a risky one which is important as with risk-less loans which have a face-value less than or equal 0.2 the results are indifferent and not unique and won't show the characteristic distribution of equilibrium.

As already described in section 5.7.1 the whole simulation-process is a random-process with an equilibrium different for each topology as the fixed-point solution thus one needs replications to reduce noise. The number of 50 replications was chosen because it is a good match between processing-power requirements and overall reduction of noise. Thus increasing the number e.g. to 100 or 200 would not result in much better results - both visual and numerical - but would need much longer to run. All facts can already be seen and derived when using 50 replications thus for all figures 50 replications were used unless stated otherwise e.g. a single run.

Table 16: Configuration for all experiments

Agent-Count	100
Bond-Type	0.5
Replication-Count	50
Matching-Round	max. 500 offering-rounds
Terminate after	1,000 failed successive matching-rounds

Table 17: Theoretical Equilibrium for 100 agents and 0.5 bond

Asset-Price p	0.717
Bond-Price q	0.375
Marginal agent i1	0.584
Marginal agent i2	0.802

6.3 Fully-Connected

This topology serves as the major point-of-reference for the other experiments as it reaches the theoretical equilibrium for 1,000 agents as demonstrated and explained.

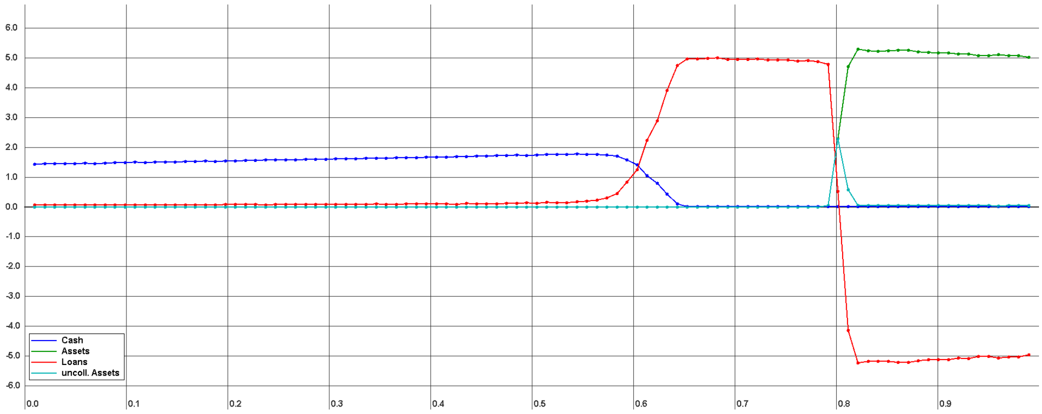

Figure 12: Wealth-distribution of Fully-Connected topology

Table 18: Equilibrium of Fully-Connected topology

Asset-Price p	0.689 (0.01)
Bond-Price q	0.384 (0.004)
Marginal agent i1	0.603 (0.007)
Marginal agent i2	0.803 (0.003)
Pessimist Wealth	1.597 (0.015)
Medianist Wealth	4.565 (0.113)
Optimist Wealth	5.021 (0.064)

Table 19: Performance of Fully-Connected topology

Successful matching-rounds	1916.14 (31.42)
Failed matching-rounds	4448.66 (1668.93)
Total matching-rounds	6364.8 (1679.21)
Ratio successful/total	0.3
Ratio failed/total	0.7

Table 20: Difference to theoretical equilibrium

	Result	Reference	difference to Reference
Asset-Price p	0.689	0.717	-3.9%
Bond-Price q	0.384	0.375	+2.4%
Marginal agent i1	0.603	0.584	+3.2%
Marginal agent i2	0.803	0.802	+0.1%

6.4 Ascending-Connected topology

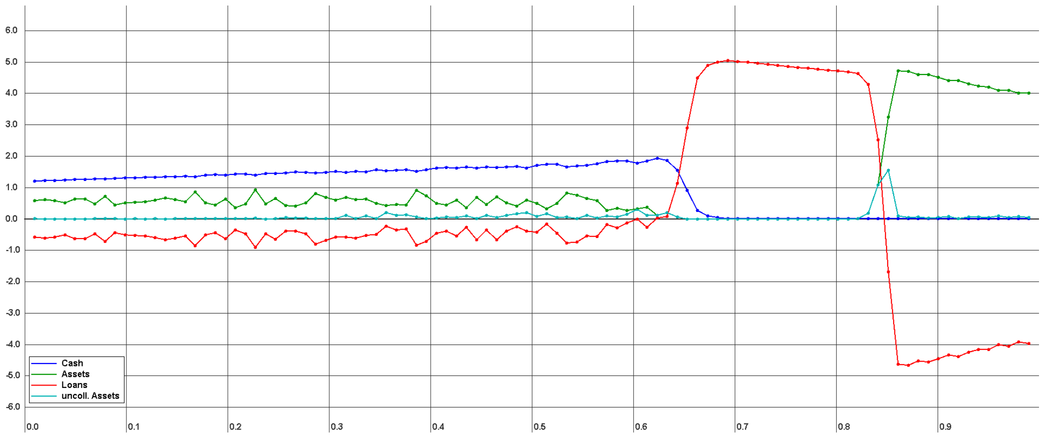

Figure 13: Wealth-distribution of Ascending-Connected topology

Table 21: Equilibrium of Ascending-Connected topology

Asset-Price p	0.711 (0.016)
Bond-Price q	0.391 (0.005)
Marginal agent i1	0.646 (0.012)
Marginal agent i2	0.850 (0.008)
Pessimist Wealth	1.166 (0.072)
Medianist Wealth	1.869 (0.243)
Optimist Wealth	4.307 (0.07)

Table 22: Performance of Ascending-Connected topology

Successful matching-rounds	36,940.96 (1948.69)
Failed matching-rounds	1176.08 (98.01)
Total matching-rounds	38,117.04 (1934.06)
Ratio successful/total	0.97
Ratio failed/total	0.03

Table 23: Difference to theoretical equilibrium

	Result	Reference	difference to Reference
Asset-Price p	0.711	0.717	-0.8%
Bond-Price q	0.391	0.375	+4.2%
Marginal agent i1	0.646	0.584	+10.6%
Marginal agent i2	0.850	0.802	+6.0%

Table 24: Difference to Fully-Connected topology equilibrium

	Result	Reference	difference to Reference
Asset-Price p	0.711 (0.016)	0.689 (0.01)	+3.2% (+60%)
Bond-Price q	0.391 (0.005)	0.384 (0.004)	+1.8% (+25%)
Marginal agent i1	0.646 (0.012)	0.603 (0.007)	+6.9% (+71%)
Marginal agent i2	0.850 (0.008)	0.803 (0.003)	+6.0% (+166%)
Pessimist Wealth	1.166 (0.072)	1.597 (0.015)	-27.0% (+380%)
Medianist Wealth	1.869 (0.243)	4.565 (0.113)	-59% (+115%)
Optimist Wealth	4.307 (0.070)	5.021 (0.064)	-14.2% (+9.3%)

6.4.1 Ascending-Connected Importance Sampling

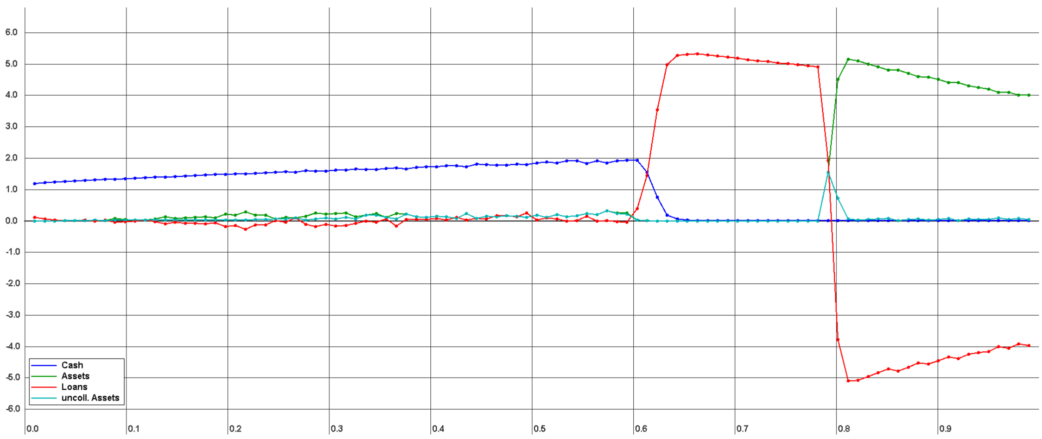

Figure 14: Wealth-distribution of Ascending-Connected Importance Sampling topology

Table 25: Equilibrium of Ascending-Connected Importance Sampling topology

Asset-Price p	0.691 (0.009)
Bond-Price q	0.383 (0.004)
Marginal agent i1	0.614 (0.009)
Marginal agent i2	0.799 (0.006)
Pessimist Wealth	1.497 (0.072)
Medianist Wealth	3.934 (0.505)
Optimist Wealth	4.519 (0.051)

Table 26: Performance of Ascending-Connected Importance Sampling topology

Successful matching-rounds	49,881.6 (1733.33)
Failed matching-rounds	1.0 (0.00)
Total matching-rounds	49,882.6 (1733.33)
Ratio successful/total	0.9999
Ratio failed/total	0.0001

Note that in this case the matching-probabilities are such that upon the first failed matching-round the equilibrium is reached as no agent can trade with each other any more which results in just on single failed matching-round.

Table 27: Difference to theoretical equilibrium

	Result	Reference	difference to Reference
Asset-Price p	0.691	0.717	-3.6%
Bond-Price q	0.383	0.375	+2.1%
Marginal agent i1	0.614	0.584	5.1%
Marginal agent i2	0.799	0.802	-0.4%

Table 28: Difference to Fully-Connected topology equilibrium

	Result	Reference	difference to Reference
Asset-Price p	0.691 (0.009)	0.689 (0.01)	+0.3% (-10%)
Bond-Price q	0.383 (0.004)	0.384 (0.004)	-0.3% (0.0%)
Marginal agent i1	0.614 (0.009)	0.603 (0.007)	+1.8% (28.6%)
Marginal agent i2	0.799 (0.006)	0.803 (0.003)	-0.5% (+100%)
Pessimist Wealth	1.497 (0.072)	1.597 (0.015)	-6.2% (+380%)
Medianist Wealth	3.934 (0.505)	4.565 (0.113)	-13.8% (+346%)
Optimist Wealth	4.519 (0.051)	5.021 (0.064)	-10% (-20.3%)

Chapter 7

Interpretation

In this chapter the interpretation of the results of chapter 6 are given and discussed where the central question is whether the Ascending-Connected topology satisfies the hypothesis or not. Thus only this topology is handled - both with and without importance sampling - because it is the most minimal network which satisfies the requirements for the hypothesis. The interpretations for the results of Hub-, Scale-Free and Small-World Topologies are handled in appendix B but only to a minimal extent as they turn out to fall far from satisfying the hypothesis and the equilibrium because almost all of them do not meet the requirements but show interesting behaviour.

7.1 Validating the Hypothesis

When comparing the results of Ascending-Connected topology with and without importance sampling from Chapter 6 of figure 14 and 13 with the results of the Fully-Connected topology of figure 12 it becomes immediately clear that the equilibrium is different from the one of the Fully-Connected network and thus theoretical equilibrium is not reached in the case of Ascending-Connected topology neither with or without importance sampling. Although the visual results come quite close to the Fully-Connected one - there is a clear distinction between pessimists, medianists and optimists and the wealth-distribution looks about the same as in fully-connected - there remain serious artefacts in the range of the pessimists. Thus the hypothesis is proven wrong by experiment.

7.2 Analysing artefacts

Obviously the artefacts in the range of the pessimists indicate a miss-allocation of wealth, which are in fact collateralized assets. Pessimists, as noted in Chapter 3, are maximally short on assets and bonds and hold only cash, thus it is clearly a miss-allocation. As will be shown it comes from the fact that the pessimists want to sell but no neighbour is able to buy any more - a scenario which is not possible in Fully-Connected topology and is thus unique to Ascending-Connected networks with and without importance sampling.

7.2.1 Dynamics of a single run

To better understand how such artefacts arise one needs to investigate the dynamics of a single run of the Ascending-Connected topology. The tools used are both the market-activity and wealth-distribution diagrams where the former one shows during which points in time - which are the successful matching-rounds - of the simulation each market is active. Being active means a successful match on a given market which implies that in a successful matching-round only one market can be active as only one match on a specific market happens during a successful matching round. Because of this a moving window of size 100 is used to create a moving-average filter over all active markets where the result is normalized and all market-activity sums to 1.0 at each point in time of the diagram. This allows for a very good visual analysis of distinct trading-stages because noise is reduced but the overall trend of a market can be still seen clearly.

3 trading-stages can be identified in the market-activity diagram of Ascending-Connected topology.

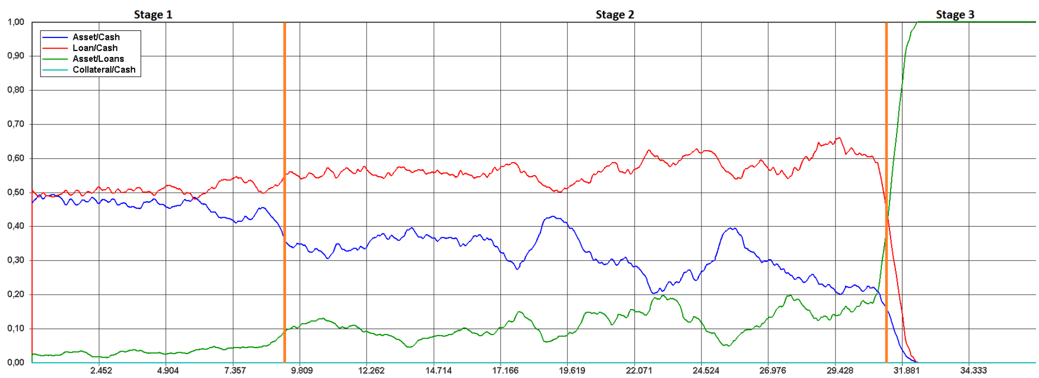

Figure 15: Market-activity stages of Ascending-Connected topology

Stage 1 The allocations are very chaotic overall but pessimists can be identified already as they sell their free assets against cash thus holding primarily cash but lots of collateralized assets are in the pessimists-range as well. Real distinction of optimists is not yet visible and medianists are far from showing up.

The Asset/Cash and Loan/Cash markets are very dominant in this stage as the pessimists try to get cash for their free assets where the Asset/Loan market is hardly active but contributes enough to create the miss-allocation of the collateralized assets in the pessimists-range already.

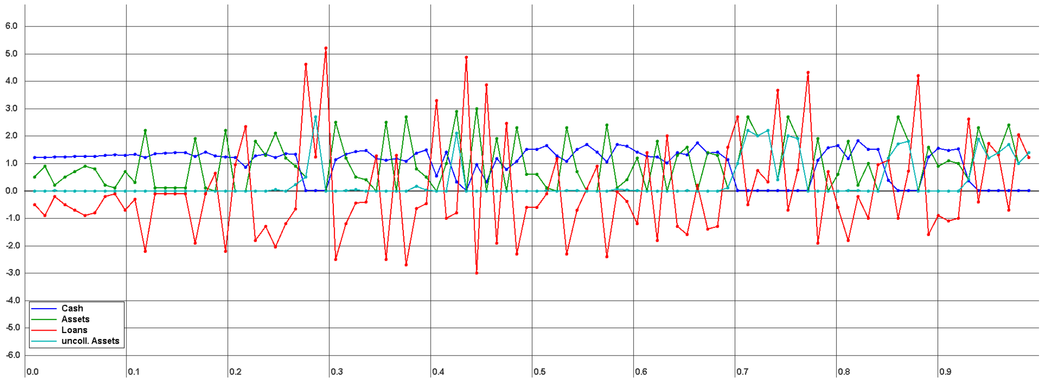

Figure 16: Wealth-Distribution of Ascending-Connected topology during Stage 1

Stage 2 The pessimists which hold collateralized assets try to trade them up to the optimists which looks like waves when visualisizing it in the thesis-software. The optimists are now about to emerge as most of them are maximally short on cash and hold either free or collateralized assets. The medianists are still not visible yet.

The Asset/Cash market seems to go down in the long term while the Loan/Cash and Asset/Loan markets seems to increase. This is because fewer and fewer assets can be traded against cash because the optimists are already very low on cash thus the Asset/Loan market is naturally increasing as they can trade only on this market any more.

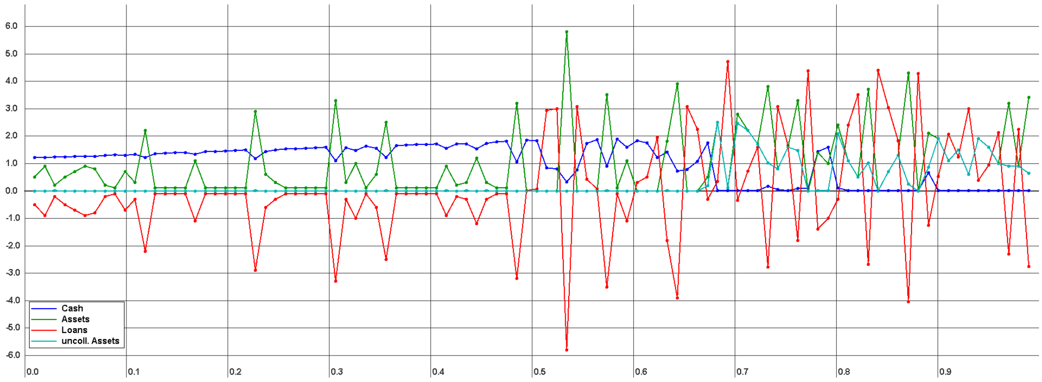

Figure 17: Wealth-Distribution of Ascending-Connected topology during Stage 2

Stage 3 The pessimists lie dormant and are completely inactive. The medianists begin to show up holding only bonds and the real optimists begin to crystallize holding only collateralized assets. These two frontiers move towards each other as only collateralized assets can be traded any more as both medianists and optimists hold only collateralized assets and bonds.

The Asset/Cash market lies dormant because the pessimists are no more able to trade and the optimists are maximally low on cash. The Loan/-Cash market is inactive too whereas the Asset/Loan market takes over and dominates 100% as only collateralized assets are traded any more as stated above.

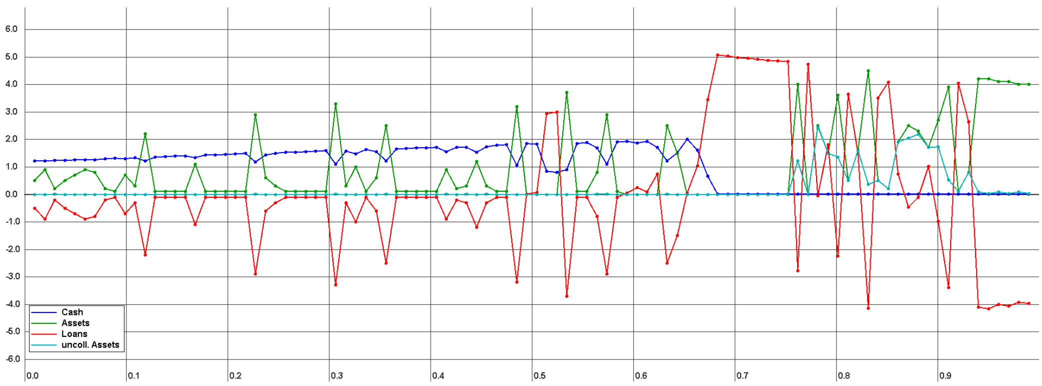

Figure 18: Wealth-Distribution of Ascending-Connected topology during Stage 3

Deriving the emerging of the artefacts The wealth stabilizes from both the left and the right end of the optimism-scale towards the i2-point where medianists become optimists - around this point the last trades will happen.

Pessimists try to sell all their assets against cash to the neighbour with higher optimism-factor.

Optimists try to buy as much assets as they can get from the neighbour with lower optimism-factor. In the beginning they use cash and after they've run out of cash they buy assets against bonds.

The medianists serve as connection between the pessimists and optimists transferring the assets to the optimists by buying from agents with lower optimism and selling to higher ones either through asset against cash or asset against bond.

Thus the assets move from the pessimists through the ascending chain of optimism to the optimists as no direct connection between these two groups exists with the medianists in between. Thus waves of uncollateralized assets can be seen moving from pessimists to optimists.

It is important to understand that all agents despite their optimism factor make offers on all markets if they are able to and their cash, collateral or bond constraints are satisfied. This implies that pessimists trade bonds as well as assets against bonds although they turn out to be pessimists. Note that the agents are not defined exogenous as pessimists/medianist/optimists but this property emerges during the simulation.

Thus pessimists gain wealth in collateralized assets too which can be seen by the green spikes with the same amount of negative bonds as those assets are bought against bond. Of course they try to sell it to neighbours with higher optimism factor but this is only possible if these neighbours are able to buy which they can only if they hold enough positive bonds to buy the offered asset for the offered amount of bonds.

Whether an agent has enough wealth to buy from a seller is more or less random and depends on its trading history. Matching happens randomly and thus it is possible that the neighbourhood of a seller "dries up" as the potential buyers sold all their goods to the next agent with higher optimism factor and become thus unable to buy from the potential seller because they have no more positive bonds to buy assets against bonds. In such a case a potential pessimist seller of collateralized assets is then cut from its environment and becomes unable to trade any more resulting in a miss-allocation in collateralized assets.

It is also possible for a group of agents to get cut from its environment through this random trading-process. In this case the agents within this "island" still trade between each other resulting in the uncollateralizing of assets which immediately are traded towards optimists but as soon as a point is reached where no buyer is available with enough positive bonds to buy collateralized assets this island is also incapable of trading any more resulting in an island of miss-allocated wealth.

An important fact to notice is that the artefacts must not necessarily show up. It is possible for a single run to finish without these artefacts showing up. This is due to the random-process of sweeping and matching and thus the artefacts are subject to this random process too as noted in section 5.8. Importance sampling elevates this problem a bit as it allows for more trades as the matching probabilities are very much increased but fails in the end for the same reason as the simulation without it - the artefacts are just "smaller" but show up almost always.

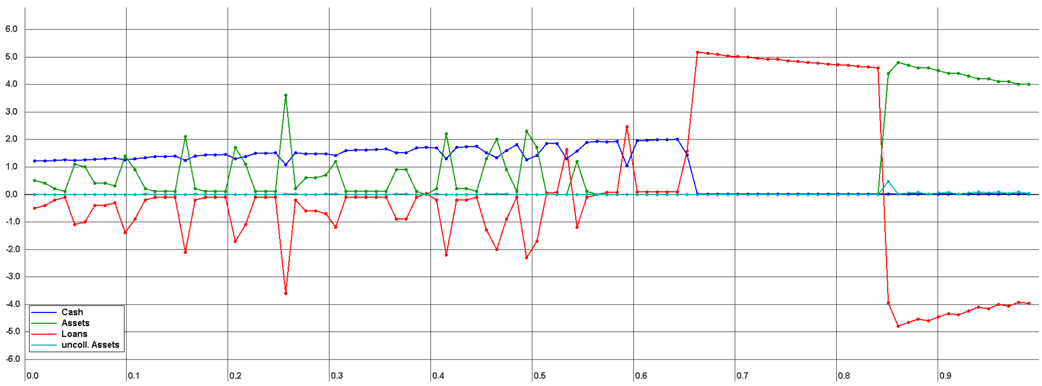

Figure 19: Final wealth-distribution of Ascending-Connected topology after a single run

7.3 Extending the Hypothesis

After it has become clear that the hypothesis is wrong the question arises what needs to be done to correct it. It is clear that a mechanism needs to be found which prevents or resolves the arising of the artefacts within the pessimist wealth-range. Obviously two solutions are available.

7.3.1 Approaching fully connectedness

Increasing the connectedness of the topology increases the probability of global-optimal trades and allows more agents to trade between each other and thus the probability of resolving islands or artefacts of wealth miss-allocation is increased with the density of connectedness. The experiments

of ascending-connected topology with short-cuts were desigend to develop an understanding how the simulation behaves with increasing connectedness and also how the two types of fully- and regular-connectedness influence the results. It seems that full short-cuts seem to help dramatically in reducing the miss-allocations where the number of full short-cuts seems to be dependent on the number of agents which this thesis leaves for further research. See section B.2 for a short overview of the results and interpretation of short-cut based Ascending-Connected topologies.
Of course in real environments approaching fully connectedness is not always possible and thus only the other mechanism is left as an option to resolve the artefacts.

7.3.2 Re-Enabling trading

Another way to look at the arising of the artefacts is in identifying them as suboptimal trades. Breuer et al. (2015) were confronted with this circumstance when they introduced the Asset/Bond Market where they found that the equilibrium was fundamentally different from the theoretical one because agents were trapped in suboptimal trades and couldn't reverse their decisions made earlier. The trades where suboptimal because each agent assigns depending on its optimism factor a different bond-value to each asset. As a solution they introduced the "Bonds-Pledgeability" (BP) mechanism which allows to trade bonds in both ways instead of only gathering them and not being able to sell them - see chapter 3 for a more in-depth discussion of the BP-Mechanism.

Thus if those artifacts are treated as suboptimal trades one needs to introduce a mechanism similar to BP to allow the reversibility of suboptimal trades in the context of collateralized assets. The only possibility without altering the network-topology is to re-enable the pessimists to trade their collateralized assets against cash as all pessimists hold cash and are thus able to buy and sell collateralized assets against cash. This new mechanism is expected to repair the miss-allocated wealth and to restore the validity of the previously disproved hypothesis.

See Chapter 8 for the implementation and results of this new mechanism.

Chapter 8

A new Market

As already introduced in section 7.3.2 a new market is necessary to repair the miss-allocation of collateralized assets in the range of the pessimist agents by enabling the agents to trade collateralized assets against cash.

8.1 Definition

8.1.1 Products

Collateralized assets are traded against cash. The buyer gets a specific amount of collateralized assets for a given amount of cash where the seller gives away the specific amount of collateralized assets and gets the given amount of cash.

8.1.2 Price-Range

As within all other three markets the price-ranges of the offers must be defined. Note that all prices must be obviously in the unit of cash according to the previously defined products.

minimum When calculating the minimum price of a collateralized asset - that is how much is the collateralized asset minimally worth - it is important to include the collateral-aspect of the asset. Thus one starts with the minimum asset-price in cash which is the down-price pD and subtracts the minimum amount of cash which is bound through a bond as collateral which is pD. This value is a constant for all agents.

$$\textit{min collateralized asset-price} = pD - pD = 0 \tag{8.1}$$

maximum To calculate the maximum price of a collateralized asset - that is how much is the collateralized asset maximally worth - one needs to include the collateral-aspect of the asset too. Equal to calculating the minimum one starts now with the maximum asset-price in cash which is the up-price pU and subtracts the maximum possible amount of cash which is bound through a bond as collateral which is the face-value V. This value is a constant for all agents.

$$\textit{max collateralized asset-price} = pU - V \tag{8.2}$$

limit Applying the same rules as in minimum and maximum to the limit price calculation one needs to subract the limit-price of loans from the limit-price of asset to receive the limit-price of a collateralized asset. This value is individual for each agent as the limit-prices differ across the agents both for assets and loans.

$$\textit{limit-price of collateralized asset} = \textit{limit-price of asset} - \textit{limit-price of loan} \tag{8.3}$$

8.1.3 Bid-Offering

The way bid-offers are generated is very similar to the Bond/Cash market. Bid offers are generated only when the agent has any cash holdings. The price is drawn randomly between the minimum price and the limit-price because when buying one wants to buy below the expected value to make a profit. As amount one TRADING-UNIT of an asset is selected - in the thesis-implementation 0.1 - but if there is not enough cash left to buy one TRADING-UNIT of assets then the amount of assets is selected which can be bought with the remaining cash holdings.

Table 29: Bid-Offering parameters

Pre-Condition	$\textit{cash holdings} > 0$
Asset-Price	random($\textit{min coll. asset-price}, \textit{limit-price of coll. asset}$)
Asset-Amount	$\min(\frac{\textit{cash holdings}}{\textit{Asset-Price}}, \textit{TRADING-UNIT})$

8.1.4 Ask-Offering

The way ask-offers are generated is very similar to the Bond/Cash market. Ask offers are generated only when the agent has any collateralized assets.

The price is drawn randomly between the limit-price and maximum price because when selling one wants to sell above the expected value to make a profit. As amount one TRADING-UNIT of an asset is selected - in the thesis-implementation 0.1 - but if there are fewer collateralized assets left then the remaining amount of collateral is selected. See Chapter 5 for the equation of collateral holdings.

Table 30: Ask-Offering parameters

Pre-Condition	*collateralized assets* > 0
Asset-Price	random(*limit-price of coll. asset*, *max coll. asset-price*)
Asset-Amount	min(*collateralized assets*, *TRADING-UNIT*)

8.1.5 Match

Below the wealth-exchange table is given in case of a match between two agents on the new market. Note that the wealth is increased/decreased as given by the +/- signs.

Table 31: Wealth-Exchange on match

	Seller	Buyer
Loan Given	+ matching-amount	N/A
Loans Taken	N/A	- matching-amount
Assets holdings	- matching-amount	+ matching-amount
Cash holdings	+ matching-price	- matching-price

8.2 Results

Of most importance are the results of the simulation when using the new market. The plain results are given in this section where the interpretation of the results are given in the following section.

As experiment-configuration the same is used as given in Chapter 6 except that the new market is now activated too.

Table 32: Configuration for all experiments

Agent-Count	100
Bond-Type	0.5
Replication-Count	50
Matching-Round	max. 500 offering-rounds
Terminate after	1000 failed successive matching-rounds

8.2.1 Fully-Connected topology

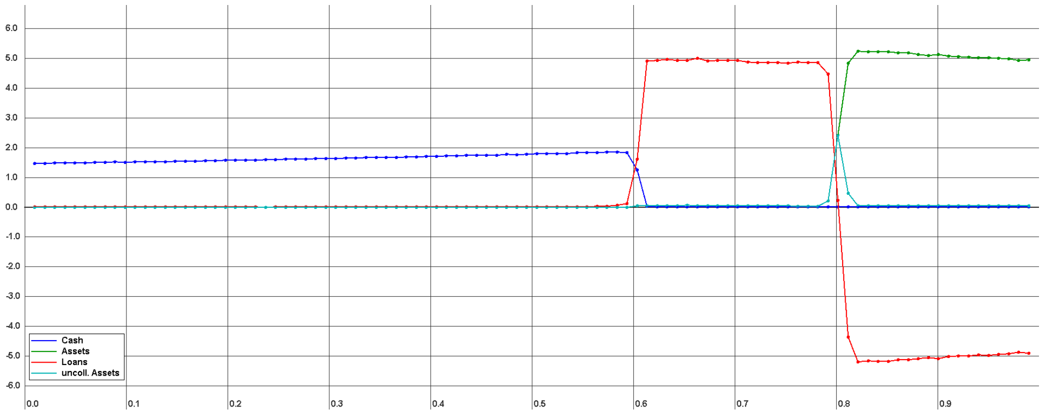

Figure 20: Wealth-Distribution of Fully-Connected topology with Collateral/Cash market

Table 33: Equilibrium of Fully-Connected topology with Collateral/Cash market

Asset-Price p	0.688 (0.008)
Bond-Price q	0.381 (0.002)
Marginal Agent i1	0.597 (0.005)
Marginal Agent i2	0.803 (0.003)
Pessimist Wealth	1.597 (0.009)
Medianist Wealth	4.76 (0.1)
Optimist Wealth	4.963 (0.052)

Table 34: Performance of Fully-Connected topology with Collateral/Cash market

Successful matching-rounds	1916.14 (31.42)
Failed matching-rounds	4448.66 (1668.93)
Total matching-rounds	6364.8 (1679.21)
Ratio successful/total	0.3
Ratio failed/total	0.7

Table 35: Difference of Fully-Connected topology to theoretical equilibrium as given in Table 17 of chapter 6

	Result	Reference	difference to Reference
Asset-Price p	0.688	0.717	-4.0%
Bond-Price q	0.381	0.375	+1.6%
Marginal Agent i1	0.597	0.584	+2.2%
Marginal Agent i2	0.802	0.803	+0.1%

Table 36: Difference of Fully-Connected topology to equilibrium without Collateral/Cash market as given in Table 18 of chapter 6

	Result	Reference	difference to Reference
Asset-Price p	0.688 (0.008)	0.689 (0.01)	-0.1% (-20%)
Bond-Price q	0.381 (0.002)	0.384 (0.004)	-0.7% (-50%)
Marginal Agent i1	0.597 (0.005)	0.603 (0.007)	-1.0% (-28%)
Marginal Agent i2	0.803 (0.003)	0.803 (0.003)	0.0% (0%)
Pessimist Wealth	1.597 (0.009)	1.597 (0.015)	0.0% (-40%)
Medianist Wealth	4.76 (0.1)	4.565 (0.113)	+4.2% (-11%)
Optimist Wealth	4.963 (0.052)	5.021 (0.064)	-1.1% (-19%)

8.2.2 Ascending-Connected topology

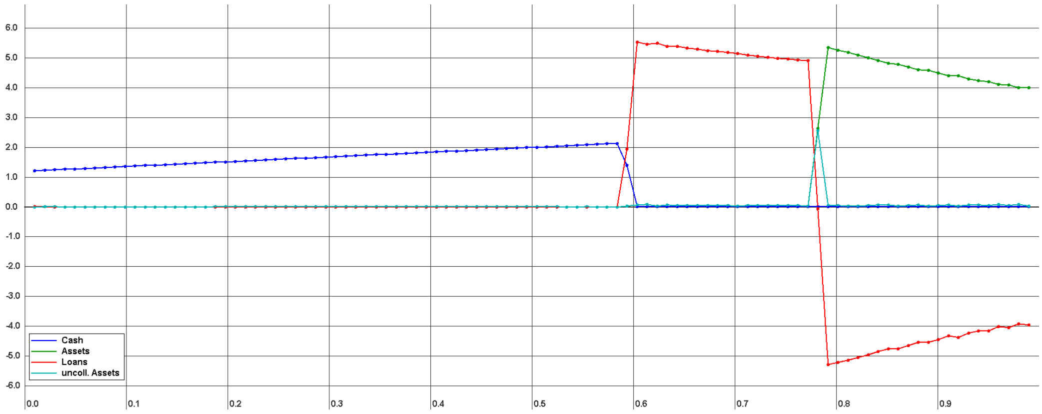

Figure 21: Wealth-Distribution of Ascending-Connected topology with Collateral/Cash market

Table 37: Equilibrium of Ascending-Connected topology

Asset-Price p	0.713 (0.013)
Bond-Price q	0.383 (0.005)
Marginal Agent i1	0.584 (0.0)
Marginal Agent i2	0.782 (0.0)
Pessimist Wealth	1.671 (0.0)
Medianist Wealth	5.032 (0.013)
Optimist Wealth	4.508 (0.006)

Table 38: Performance of Ascending-Connected topology

Successful matching-rounds	51,838.74 (1613.36)
Failed matching-rounds	1124.76 (28.31)
Total matching-rounds	52,963.5 (1612.31)
Ratio successful/total	0.98
Ratio failed/total	0.02

Table 39: Difference of Ascending-Connected topology to theoretical equilibrium as given in Table 17 of chapter 6

	Result	Reference	difference to Reference
Asset-Price p	0.713	0.717	-0.5%
Bond-Price q	0.383	0.375	+2.1%
Marginal Agent i1	0.584	0.584	0.0%
Marginal Agent i2	0.782	0.802	-2.5%

Table 40: Difference of Ascending-Connected topology to equilibrium without Collateral/Cash market as given in Table 21 of chapter 6

	Result	Reference	difference to Reference
Asset-Price p	0.713 (0.013)	0.711 (0.016)	+0.3% (19%)
Bond-Price q	0.383 (0.005)	0.391 (0.005)	-2.0% (0%)
Marginal Agent i1	0.584 (0.0)	0.646 (0.012)	-9.6% (-100%)
Marginal Agent i2	0.782 (0.0)	0.85 (0.008)	-8.0% (-100%)
Pessimist Wealth	1.671 (0.0)	1.166 (0.072)	+43.3% (-100%)
Medianist Wealth	5.032 (0.013)	1.869 (0.243)	+169.2% (95%)
Optimist Wealth	4.508 (0.006)	4.307 (0.07)	+4.6% (-91%)

Table 41: Difference of Ascending-Connected to equilibrium of fully-connected topology with Collateral/Cash market as given above

	Result	Reference	difference to Reference
Asset-Price p	0.713 (0.013)	0.688 (0.008)	+3.6% (+62%)
Bond-Price q	0.383 (0.005)	0.381 (0.002)	+0.5% (+150%)
Marginal Agent i1	0.584 (0.0)	0.597 (0.005)	-2.2% (-100%)
Marginal Agent i2	0.782 (0.0)	0.803 (0.003)	-2.6% (-100%)
Pessimist Wealth	1.671 (0.0)	1.597 (0.009)	+4.6% (-100%)
Medianist Wealth	5.032 (0.013)	4.76 (0.1)	+5.7% (-87%)
Optimist Wealth	4.508 (0.006)	4.963 (0.052)	-9.2% (88%)

Table 42: Difference of Ascending-Connected to equilibrium of fully-connected without Collateral/Cash market as given in Table 18 of chapter 6

	Result	Reference	difference to Reference
Asset-Price p	0.713 (0.013)	0.689 (0.01)	+3.5% (+30%)
Bond-Price q	0.383 (0.005)	0.384 (0.004)	-0.3% (+25%)
Marginal Agent i1	0.584 (0.0)	0.603 (0.007)	-3.2% (-100%)
Marginal Agent i2	0.782 (0.0)	0.803 (0.003)	-2.6% (-100%)
Pessimist Wealth	1.671 (0.0)	1.597 (0.015)	+4.6% (-100%)
Medianist Wealth	5.032 (0.013)	4.565 (0.113)	+10.23% (88%)
Optimist Wealth	4.508 (0.006)	5.021 (0.064)	-10.22% (91%)

8.3 Interpretation of results

When interpreting the results the following questions must be answered:

- Does the Fully-Connected topology reach the theoretical equilibrium as well?
- Does the new market repair the miss-allocation of wealth in the pessimists-range?
- If not why? If yes, does the Ascending-Connected topology approach theoretical equilibrium now?

Does the Fully-Connected topology reach the theoretical equilibrium as well? Yes it does. Both visual and statistical results show that it reaches the theoretical equilibrium. The medianist wealth is slightly higher with the new market but that difference, as well as the variations in the other variables are not statistically significant.

Does the new market repair the miss-allocation of wealth in the pessimists-range? Yes it does. The visual results are clear with no miss-allocations showing up within 50 replications. If there would have been miss-allocations within any replication they would have shown up in the final result.

If yes, does the Ascending-Connected topology approach theoretical equilibrium now? The miss-allocations are repaired but it does does not approach theoretical equilibrium. Both visual and statistical results show that it misses to reach the theoretical and Fully-Connected topology with or without new market equilibrium. Because of the way the new market works the wealth-distributions in medianists and pure optimists show a different shape than in Fully-Connected and thus the theoretical equilibrium is not reached. The reason for the different shape of the wealth-distributions is rooted in the way the market-dynamics work which is discussed in the following section.

8.4 Simulation and Market dynamics

When implementing a new market the market-dynamics are of very importance and thus the following questions must be answered.

- Can the trading stages 1-4 be identified too as given in Breuer et al. (2015)?
- How does trading progresses with this new market? Is it the same as without the new market?
- How does the new market resolve the miss-allocation (with and without deferred activation)?
- When and how much is each market active?
- How do the market-activities change when a new market is introduced?

To answer these questions one must look closely at the market-dynamics. There are trading stages to be identified but due to the new market and the different topology they are expected to be quite different from the ones found in Breuer et al. (2015). The method used to find these stages is through observation of a single run and refining and validating the derived facts over many additional runs. Note that replications provide no real value here as one needs to look very carefully into the dynamics of single runs instead of the mean of multiple runs.

8.4.1 Fully-Connected with new Market

4 Stages were identified which are quite different from the ones given in Breuer et al. (2015) as the new market makes quite a big impact.

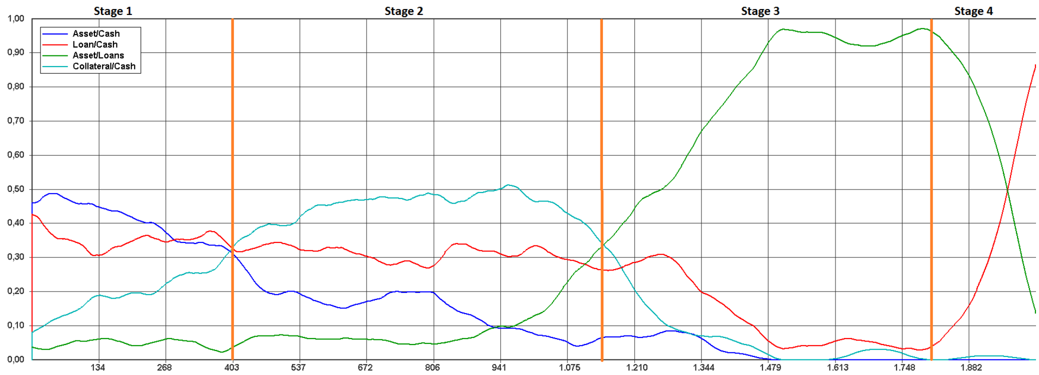

Figure 22: Market-activity stages of Fully-Connected topology with Collateral/Cash market

Stage 1 In this stage the pessimists become visible rapidly as they sell their assets and increase their cash wealth. One can get also a sense of the more optimistic range of agents as they gather assets both free and collateralized. The medianists are not visible yet.

The Asset/Cash market dominates but goes down slowly as fewer and fewer pessimists trade assets against cash compared to the very beginning. The Bond/Cash market fluctuates around the same point as loans are traded more or less equally the same. The Collateral/Cash market begins quite low and raises as more and more collateralized assets are created by pessimists and need to be sold again using the new market. The Asset/Bond market is hardly active as there is no strong need for its features yet.

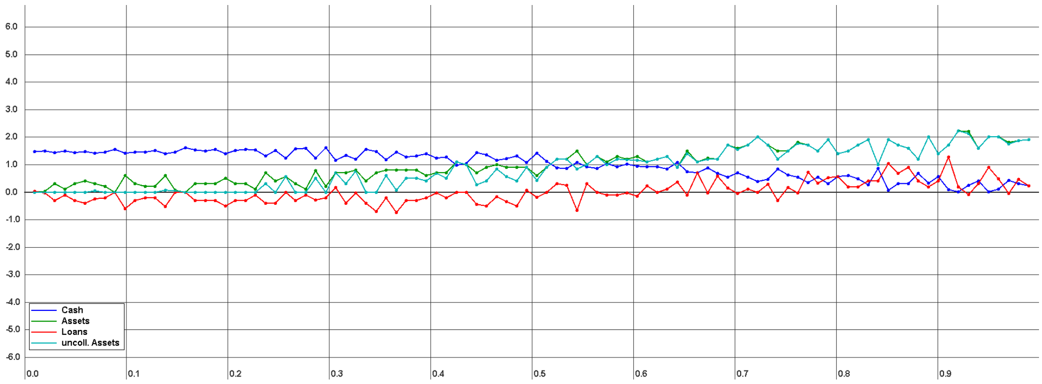

Figure 23: Wealth-Distribution of Fully-Connected topology with Collateral/Cash market during Stage 1

Stage 2 The collateralized assets are traded from the pessimists towards the optimists and the optimists crystallize themselves even more but no medianists are visible yet.

The Asset/Cash market continues to go down as the cash holdings of the pessimists begin to decline. The Bond/Cash market fluctuates around the same point as before. Now the Collateral/Cash market becomes very active as more and more collateralized assets need to be traded towards the optimists. At the beginning of this stage the Asset/Bond market is hardly active but raises fast towards the end of it as the optimists are then out of cash and need to distribute the collateralized assets between each other and the yet to come medianists.

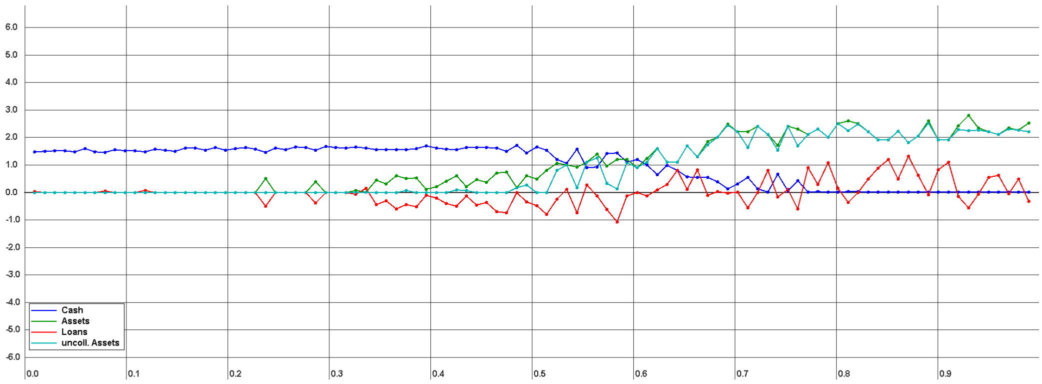

Figure 24: Wealth-Distribution of Fully-Connected topology with Collateral/Cash market during Stage 2

Stage 3 The pessimists have gone inactive as they hold no more wealth they can trade. The i1-point is emerging and the medianists and pure optimists begin to show up.

Because the pessimists are inactive now and hold no more assets the Asset/Cash and Collateral/Cash markets go down and decline completely. The Bond/Cash market goes down but does not decline as bonds are still traded because of the emerging of medianists. The medianists and pure optimists which are emerging have no other possibility than to trade on the Asset/Bond market to further distribute their collateralized assets among each other which is the reason for the raise of the Asset/Bond market above all others and its heavy domination. Despite the heavy domination of the Asset/Bond market still a few bonds are traded against cash in the range of the medianists.

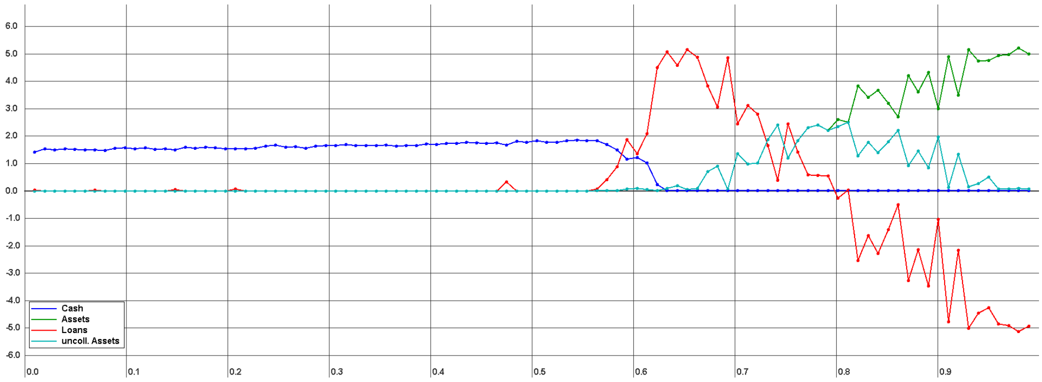

Figure 25: Wealth-Distribution of Fully-Connected topology with Collateral/Cash market during Stage 3

Stage 4 Finally the i2-point has emerged and both i1 and i2 are finalizing. The only active agents remaining are around these two points where the pure optimists trade with the medianists to finalize i2 and the medianists with the next closest pessimists to finalize i1 where the very last transactions occur around i1.

The Asset/Bond market goes down until i2 has finalized as the collateralized asset allocation has nearly reached its equilibrium. The Bond/Cash market goes up as agents around i1 are still refining the point as the equilibrium of the medianists is not reached yet. Thus bonds are traded against cash as i1 is the connecting point between pessimists with cash and medianists with bonds which enables the Bond/Cash market to trade again.

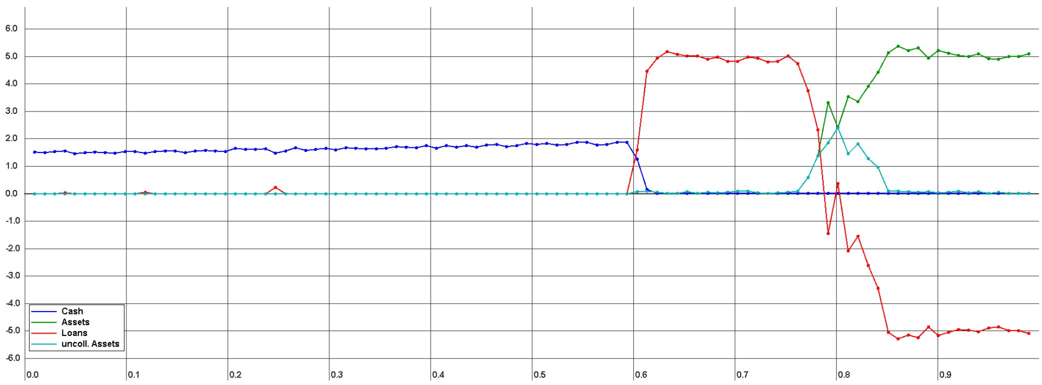

Figure 26: Wealth-Distribution of Fully-Connected topology with Collateral/Cash market during Stage 4

8.4.2 Deferred new market enabling

Using the thesis-software it is possible to start a simulation-run on Ascending-Connected topology without the Collateral/Cash market and enabling it after 1,000 successive failed matching-rounds which gives interesting hints about how the spikes of collateralized assets in the pessimists-range are resolved and distributed over the already existing pure optimists.

Of course there are the same 3 stages to be found as described already in section 7.2.1 whereas the deferred enabling of the Collateral/Cash market adds 2 new stages.

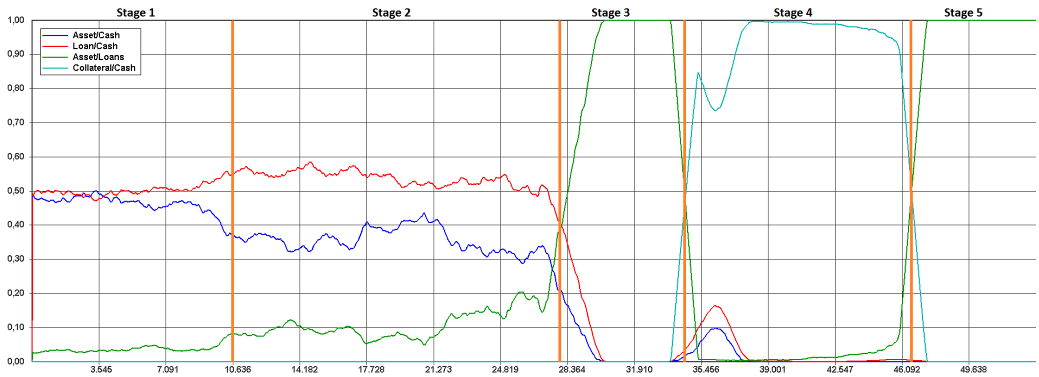

Figure 27: Market-activity stages of Ascending-Connected topology with deferred activated Collateral/Cash market

Stage 4 The collateralized assets are traded against cash and sum up at the i1-point where the first agent has no more cash.

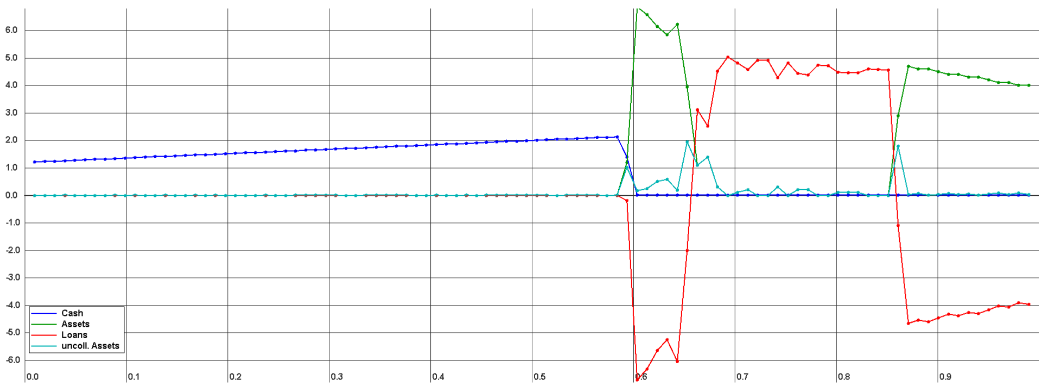

Figure 28: Wealth-Distribution of Ascending-Connected topology with deferred activated Collateral/Cash market during Stage 4

Stage 5 Now the Asset/Bond market becomes 100% dominant again and the collateralized assets are traded through the medianists towards the pure optimists as they have no more cash and can only trade anymore on this remaining active market.

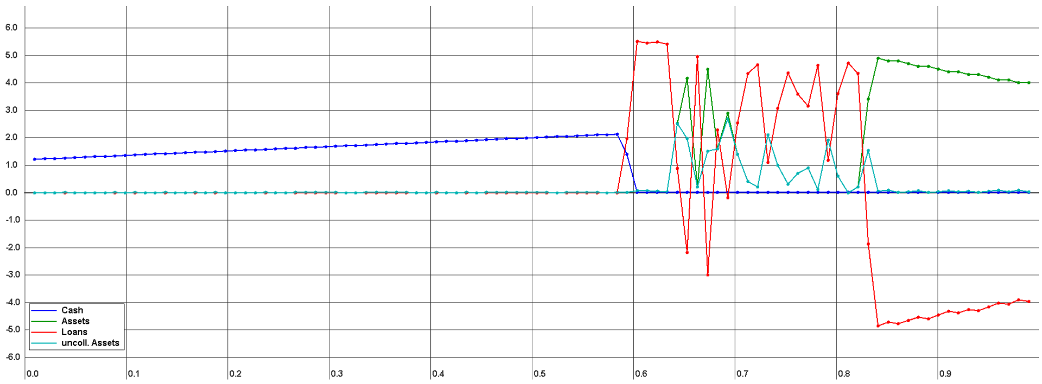

Figure 29: Wealth-Distribution of Ascending-Connected topology with deferred activated Collateral/Cash market during Stage 5

8.4.3 Ascending-Connected with new Market

4 stages were identified where only 3 of them can be seen in the Market-Dynamics diagram as stage 3 and 4 have indifferent market-activities.

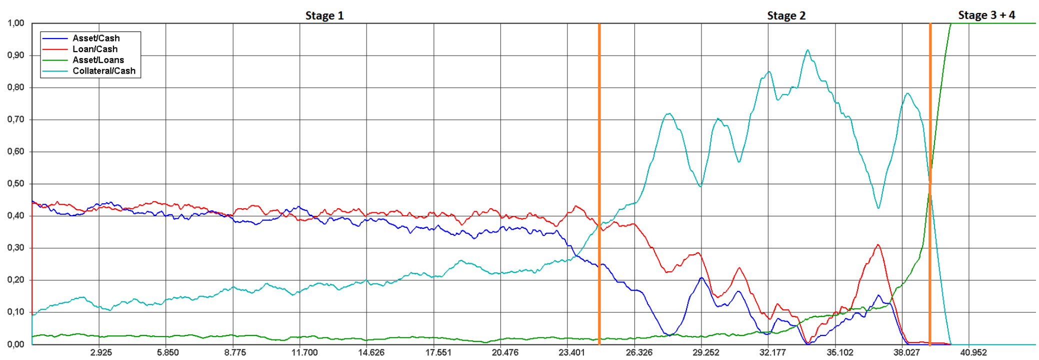

Figure 30: Market-activity stages of Ascending-Connected topology with Collateral/Cash market

Stage 1 Pessimists and optimists begin to emerge where the pessimists are gathering cash and collateralized assets and the optimists are gathering free assets against cash and a few collateralized assets. There are no medianists visible yet.

The Asset/Cash and Bond/Cash markets start around 40% slightly decreasing because the pessimists are slowly going low on assets. The Collateral/Cash market starts around 10% with increasing tendency because the amount of collateralized assets which move towards the optimists increases. The Asset/Bond market is hardly active as its features are not yet very necessary.

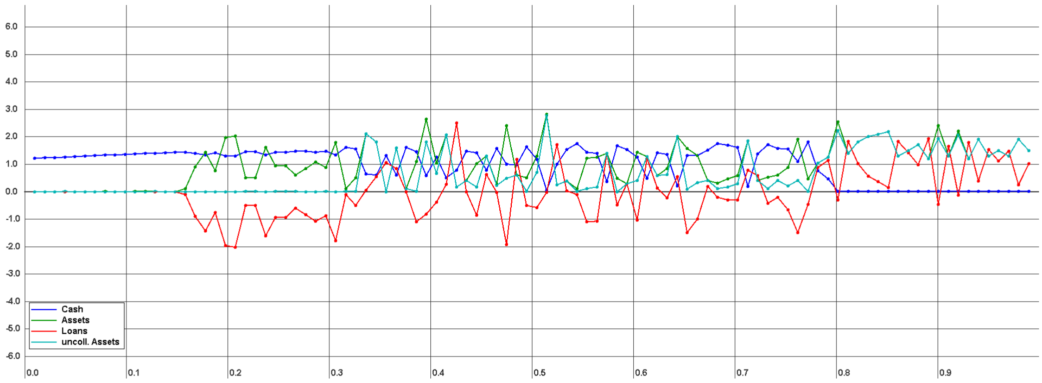

Figure 31: Wealth-Distribution of Ascending-Connected topology with Collateral/Cash market during Stage 1

Stage 2 In the pessimist-range large amount of collateralized assets have gathered which are traded now towards the optimists as the pessimists try to get maximally short on any assets and maximally plus on cash. Those collateralized assets are traded towards the i1-point - that is the first agent who holds no more cash - which can be seen by a spike in the wealth-distribution of figure 32 around 0.65. There are no medianists visible yet.

The Collateral/Cash market raises above the Asset/Cash and Bond/-Cash markets and either fluctuates or stays quite constant. In figure 30 the fluctuating variant is shown. If the market fluctuates the Asset/Cash and Bond/Cash fluctuate inverse in that if Collateral/Cash market raises they go down and vice versa. If the Collateral/Cash market stays quite constant in this stage it raises above 90% and Asset/Cash and Bond/Cash markets decline to 0%. Why the Collateral/Cash market either fluctuates or stays constant is not clear but is most probably dependent on the allocation of collateralized assets in the pessimists-range. The Asset/Bond market becomes a bit more active as more collateralized assets are traded.

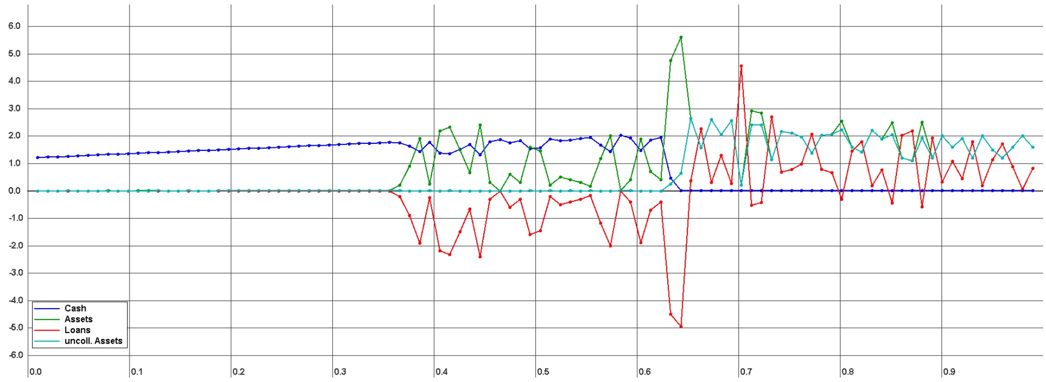

Figure 32: Wealth-Distribution of Ascending-Connected topology with Collateral/Cash market during Stage 2

Stage 3 Pessimists are now final and won't change any more. The optimists-range is now clearly visible and holds a large amount of collateralized assets from the pessimists which needs now to be traded and distributed to the remaining optimists. Medianists are still not visible yet.

Because the pessimists are no more able to trade and the optimists hold no more cash the activity of the Collateral/Cash market drops rapidly and Asset/Bond market raises to 100% activity as only collateralized-assets are tradeable any more.

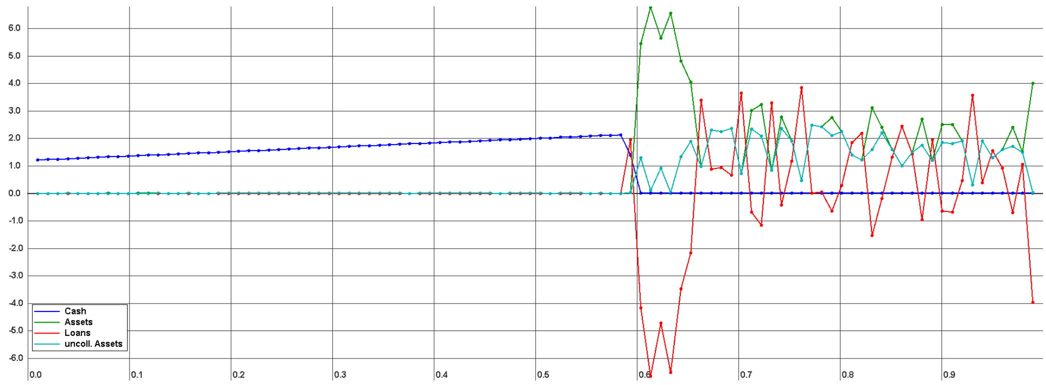

Figure 33: Wealth-Distribution of Ascending-Connected topology with Collateral/Cash market during Stage 3

Stage 4 Medianists begin to show up and to distinguish themselves from the pure optimists. This is no more visible on the market-dynamics as only Asset/Bonds are traded any more and thus only this market is active any more.

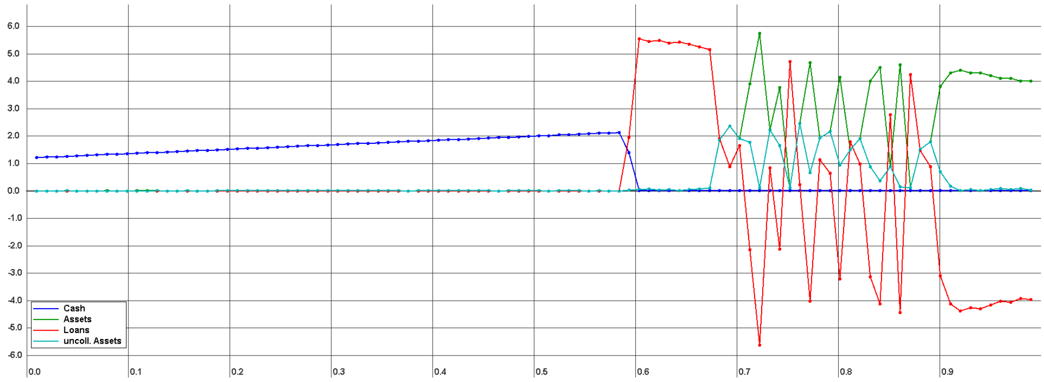

Figure 34: Wealth-Distribution of Ascending-Connected topology with Collateral/Cash market during Stage 4

After these observations made one can answer the questions.

Can the trading stages 1-4 be identified too as given in Breuer et al. (2015)? There are 4 stages in the case of both Fully-Connected and Ascending-Connected topology with the new market which are not the ones given in Breuer et al. (2015) but show up by pure chance and depend also a bit on the point-of-view on how to separate the stages from each other.

How does trading progress with this new market? Is it the same as without the new market? The progression of the trading is obviously very different with the new market as compared to the market-activities without as the usage of the new market changes the dynamics completely.

How does the new market resolve the miss-allocation (with and without deferred activation)? It becomes active during the formation of the pessimists agents as they gather collateralized assets wealth which must be traded towards the optimists. The collateralized assets are traded from neighbour to neighbour until they reach the optimists-region.

When and how much is each market active? This can be seen clearly in the market-activity diagrams.

How do the market-activities change when a new market is introduced? They have less share on the overall activity and thus the new market is quite a heavy competitor in the overall share. The Asset/Bond market though is still the market on which the final trades occur.

8.5 Conclusions on new Market

The equilibrium of the Ascending-Connected topology with the new market is different than the Fully-Connected one which reaches the theoretical equilibrium. Thus the hypothesis is still wrong because it predicted the Ascending-Connected topology to reach the theoretical equilibrium. This thesis can only speculate on the reason for this it is most probably rooted in the fundamental different trading dynamics of Ascending-Connected topology compared to Fully-Connected as can be seen in the market-dynamics. This thesis leaves the question of market-dynamics open for further research.

Chapter 9

Conclusions

The motivation for this thesis was to investigate the influence of various types of network-topologies in continuous double-auctions on the convergence towards theoretical equilibrium. It builds upon findings and results of Breuer et al. (2015) and an equilibrium framework developed by Geanakoplos (2009). As a starting point a hypothesis was formulated which gives properties a network must exhibit to reach the theoretical equilibrium defined in the model. Interpreting the results of the thesis-software it turned out that the hypothesis is not feasible for the model of Breuer et al. (2015) as the Ascending-Connected topology - the most minimal topology which satisfies the properties postulated by the hypothesis - leads to serious miss-allocation of wealth in the range of the pessimist agents. The hope then was to be able to repair the miss-allocation by extending the original model by introducing a new market and that then theoretical equilibrium will be reached. This new market on which collateral can be traded against cash is able to remove the allocation inefficiencies but the equilibrium is still different from the theoretical one thus the hypothesis turns out to be invalid and infeasible for the extended model too.

The major conclusion on the findings of the thesis is that equilibrium seems only to be possible in a fully-connected trading-network. Thus as soon as a market-institution exhibits signs of restricting trades between agents to a reduced neighbourhood then it will lead to miss-allocation and fail reaching equilibrium and thus will be unfair. The introduction of a new market solves the miss-allocations in theory but to the best knowledge of the author of this thesis such a market does not exist yet and it is open to question whether such a market can be put into practice in the real world and whether agents accept its products and the impact it has on the real trading world.

Further Research

Importance-Sampling

Importance sampling was used to increase the matching-probabilities for the special case of an Ascending-Connected network which led to a dramatic performance enhancement as much less matching-rounds were required to reach the point where no trading was possible any more. Further research could investigate a more general mechanisms of increasing the matching-probabilities independent of the topology without introducing a bias. One way could be through sampling the price-ranges in different intervals or learning and adapting them on-the-fly during a simulation run.

In-depth analysis of market-activities

The market-activities as presented in chapter 7 and to a greater extend in chapter 8 are only covered by intuitive description what is going on and why - no theory about market-activity is presented. Further research could dedicate its attention to develop a serious theory why the activities behave as they do, how they interact between each other and how they influence equilibrium. Furthermore it could establish connections to market-activity theory in economics to validate the simulation-results against theoretical frameworks.

Experiments with real subjects

Experimental economics has a long history and allows to verify theoretical equilibrium frameworks or theories. The author of this thesis is part of a group which has conducted a pilot experiment with real subjects on the subject of trading collateralized assets based on the model of Breuer et al. (2015). This was to best knowledge the first time that an experiment in this field was ever undertaken. Although it was only a pilot it gave already valuable insights on the models of Breuer et al. (2015) and Geanakoplos (2009) but more experiments need to be carried out to be able to give serious results. Based upon the results and findings of this thesis the already existing trading software could be extended furthermore to restrict the trading between agents to a specific topology e.g. Ascending-Connected. It would be of much interest whether intelligent human traders are able to resolve the miss-allocations produced by the Ascending-Connected topology without the use of the new market and if they can approach theoretical equilibrium.

Equilibrium theory vs. equilibrium process

It was noted on multiple occasions in this thesis that there is a fundamental difference between the equilibrium theory upon the model of the thesis is built and the equilibrium process implemented in the thesis-software. Further research could look for a formal definition of equilibrium in such a process and combines it with the given equilibrium theory of complex systems in general and economics in particular to find a definition for the equilibrium of such a trading process.

Appendix A

Topologies

In this chapter all simulated topologies are visualized and explained shortly. All topologies are demonstrated with N = 30 Agents instead of 100 for better visibility and clarity of the connections and nodes. All topologies have one connected component because otherwise equilibrium cannot be reached. Note that Erdos-Renyi could produce more than one connected component depending on the parameters to create it. The agents are always arranged in clockwise increasing optimism factor unless stated otherwise. All connections are undirected and there are no self-loops.

A.1 Metrics

For each network the following metrics are given:

Average degree Gives a measure of how many neighbours a vertex has on average.

Average path-length Gives a measure of how many edges a path between two vertices has on average.

Network diameter Is the longest shortest path between any two vertices.

Graph density Is the ratio of the number of edges to the number of possible edges.

Edge count Gives a formula to calculate the exact number of edges. Is omitted for the complex networks as there is no exact estimation possible.

Connected component Gives the number of components found in this network. All non-complex networks have always only 1 connected-component where the complex networks could result in more than 1.

A.2 Fully-Connected

Each agent is connected with each other agent. Included as major point-of-reference as Breuer et al. (2015) developed their model and equilibrium for fully-connected networks.

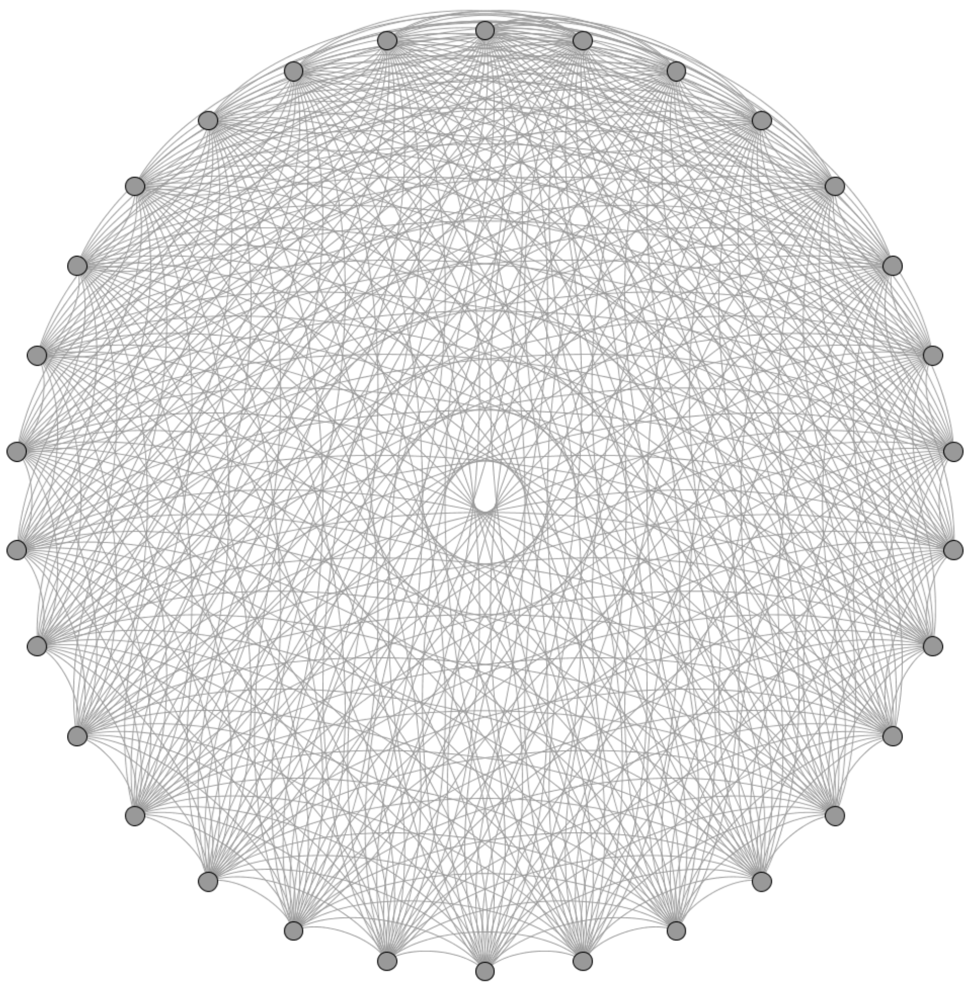

Figure 35: Fully-Connected topology

Table 43: Network metrics Fully-Connected topology

Avg. degree	29
Avg. path-length	1
Network diameter	1
Graph density	1
Edge count	$\frac{N(N-1)}{2}$

A.3 Half-Fully Connected

Agents with optimism-factor 0.5 to 1.0 are fully-connected and the others are connected to the agent with the next higher optimism-factor. The agents with highest and lowest optimism-factor are connected too, creating a closed circle. Included to investigate the influence of isolated agents.

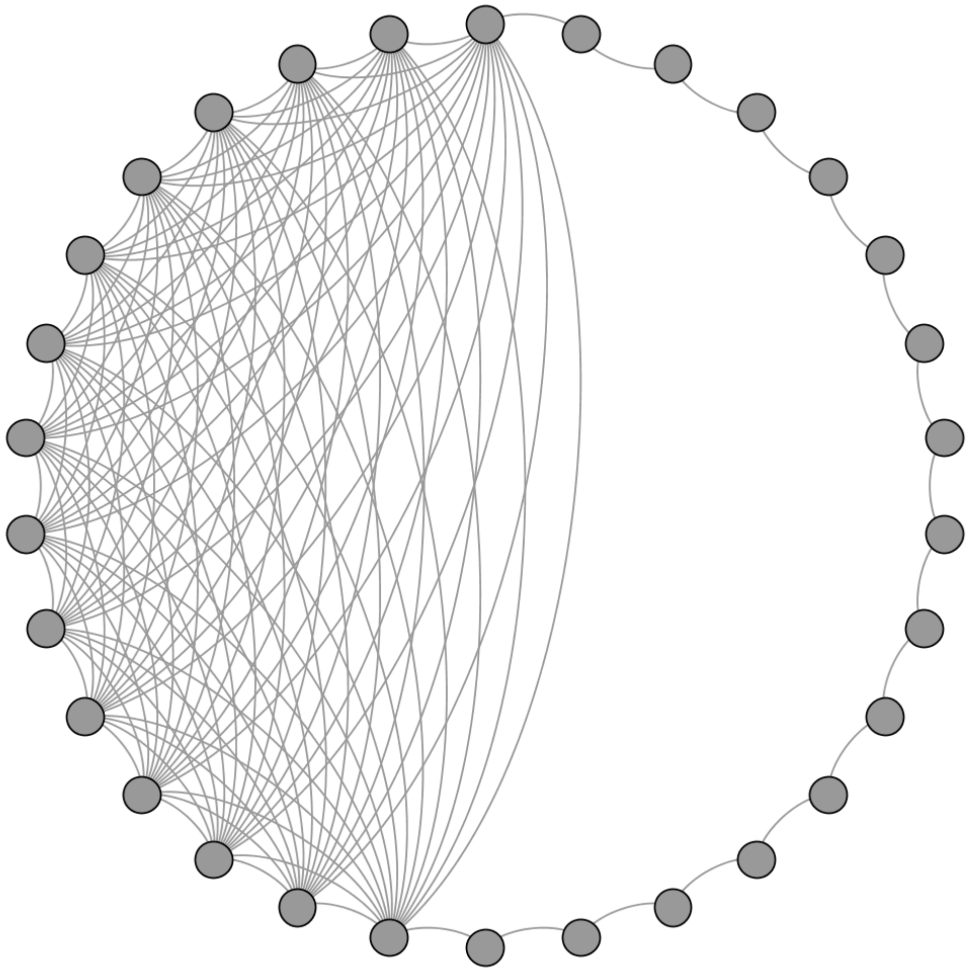

Figure 36: Half Fully-Connected topology

Table 44: Network metrics Half Fully-Connected topology

Avg. degree	8.067
Avg. path-length	4.007
Network diameter	9
Graph density	0.278
Edge count	$\frac{\frac{N}{2}(\frac{N}{2}-1)}{2}+\frac{N}{2}+1$

A.4 Ascending-Connected

Each agent is connected to the agent with the next higher optimism-factor. The agents with highest and lowest optimism-factor are not connected thus this network is not a closed circle. Included because it is the most minimal network which satisfies the theory and is therefore the major network of interest throughout the thesis.

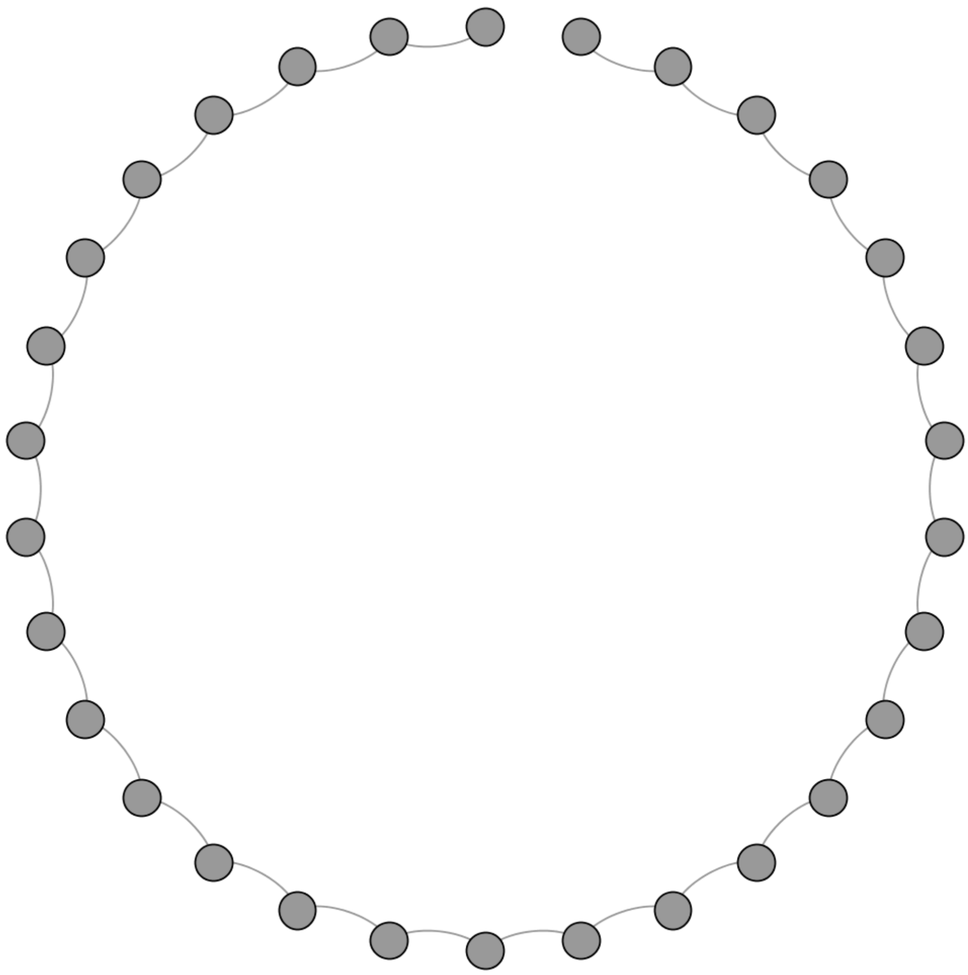

Figure 37: Ascending-Connected topology

Table 45: Network metrics Ascending-Connected topology

Avg. degree	1.933
Avg. path-length	10.33
Network diameter	29
Graph density	0.067
Edge count	$N - 1$

A.5 Ascending-Connected with short-cuts

A.5.1 Full short-cuts

Each agent is connected to the K next neighbours in the clockwise arrangement. Thus agent N - K + 1 is connected to K - 1 higher optimism-agents and wraps around to the agent with lowest optimism-factor. Included to analyse the influence of increasing connectivity.

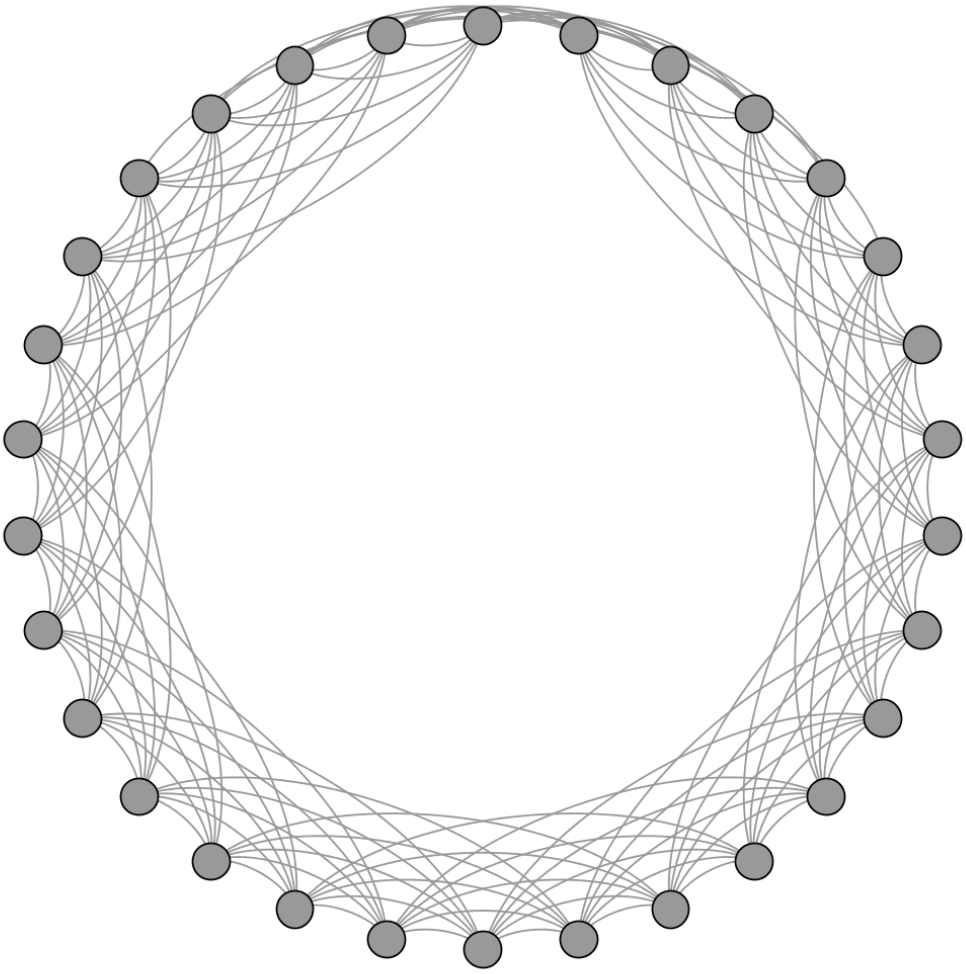

Figure 38: Ascending-Connected 5 full short-cuts topology

Table 46: Network metrics Ascending-Connected 5 full short-cuts topology

Avg. degree	10
Avg. path-length	1.966
Network diameter	3
Graph density	0.345
Edge count	NK

A.5.2 Regular short-cuts

The topology starts ascending-connected and each agent is additionally connected to one next neighbour in the clockwise arrangement where the distance to the next neighbour is K agents. Included to analyse the influence of trading-links to agents with much higher optimism-factor.

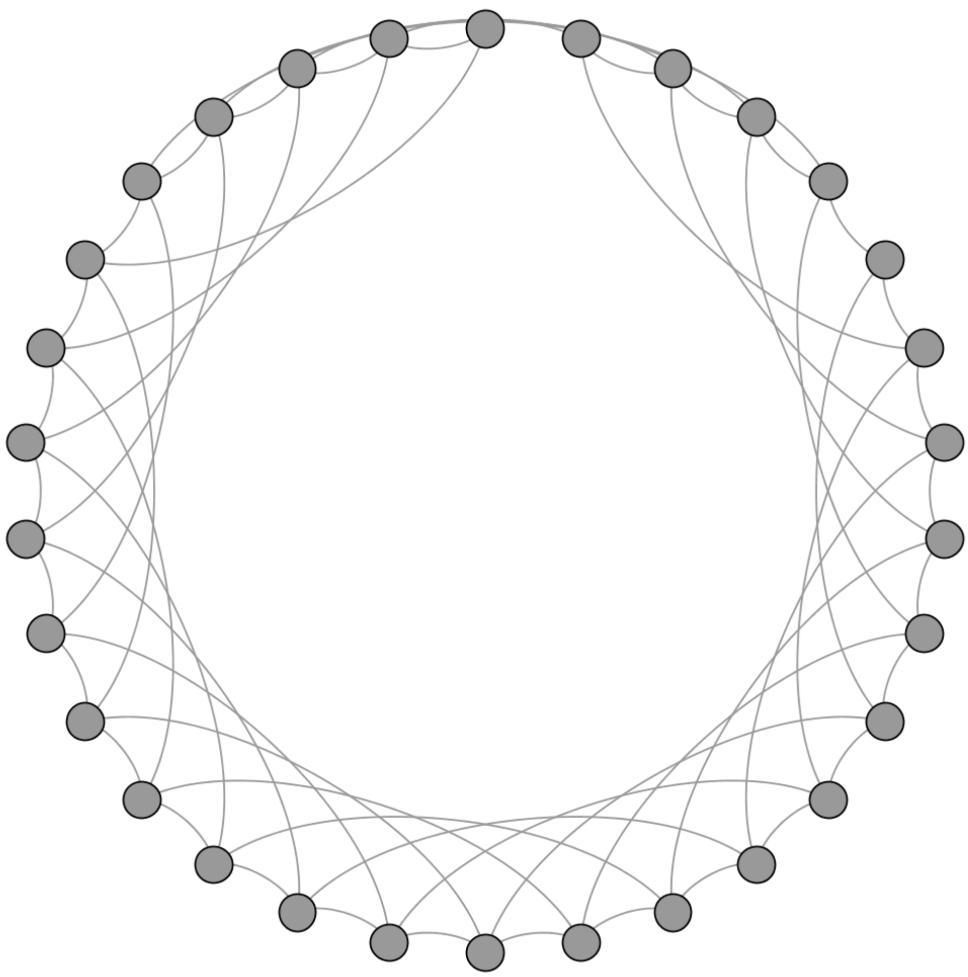

Figure 39: Ascending-Connected 5 regular short-cuts topology

Table 47: Network metrics Ascending-Connected 5 regular short-cuts topology

Avg. degree	3.867
Avg. path-length	2.839
Network diameter	6
Graph density	0.133
Edge count	$2N$

A.5.3 Random short-cuts

Starting with an ascending-connected network this topology adds one additional short-cut from each agent to another random agent with a given probability where a probability of 0.0 results in only the ascending-connectedness and a probability of 1.0 in each agent having an additional random short-cut. Self-loops and multi-edges are not allowed. Included to analyse the influence of randomness in ascending-connected topologies.

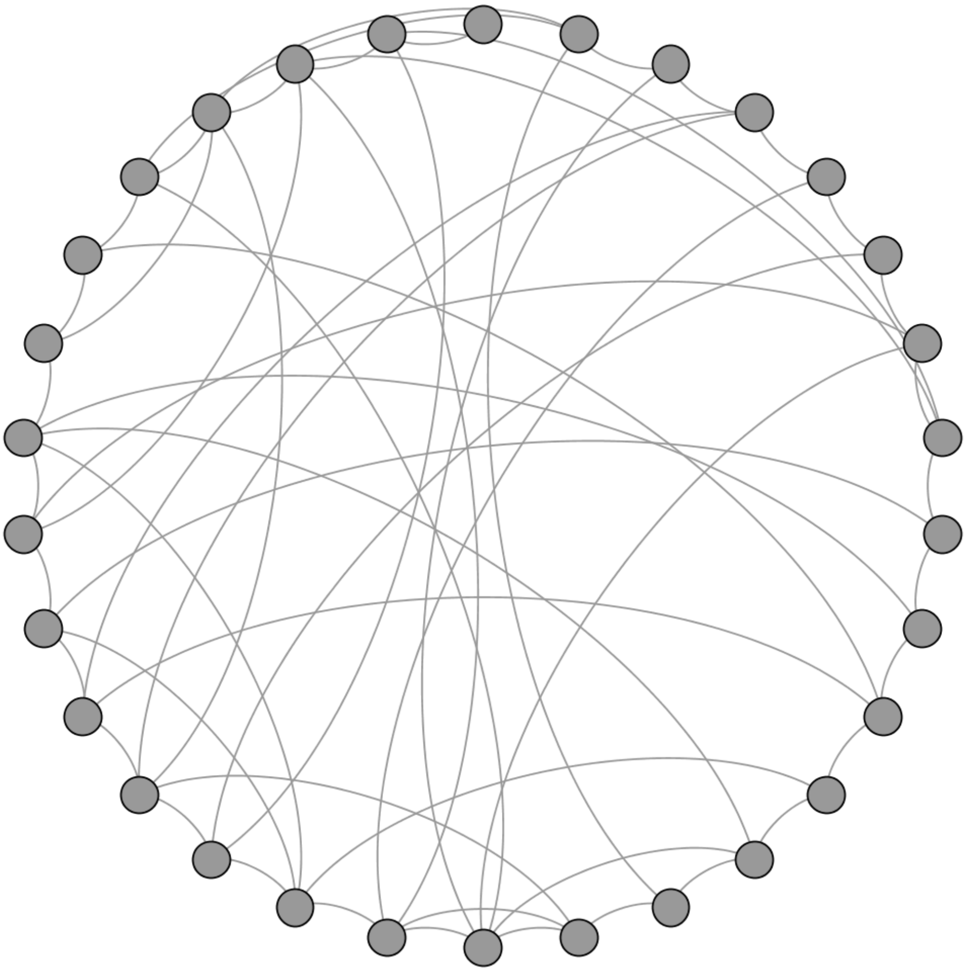

Figure 40: Ascending-Connected random short-cuts probability 1.0 topology

Table 48: Network metrics Ascending-Connected random short-cuts topology

Avg. degree	3.867
Avg. path-length	2.506
Network diameter	5
Graph density	0.133
Edge count	$\min N, \max 2N$

A.6 Hub-based topologies

A.6.1 3 Hubs

The agents are separated into 3 groups based upon their optimism-factor. Group 1 ranges from 0.0 to 0.33, group 2 from 0.33 to 0.66 and group 3 from 0.66 to 1.0. All agents within a group are fully-connected where the groups are interconnected between each other through a hub which is the agent with the highest optimism-factor of each group that is: 0.33, 0.66 and 1.0. Included as a point-of-reference as this topology was discussed too in Breuer et al. (2015).

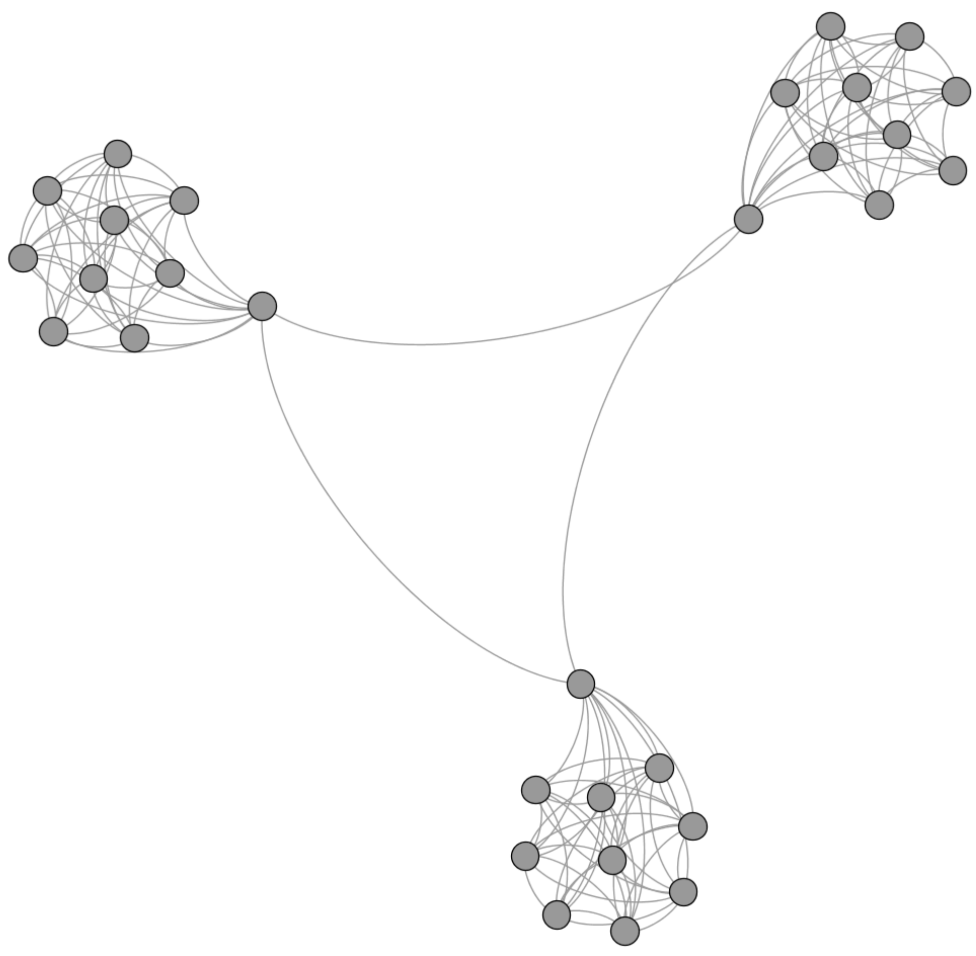

Figure 41: 3 Hubs topology

Table 49: Network metrics 3 Hubs topology

Avg. degree	9.2
Avg. path-length	2.241
Network diameter	3
Graph density	0.371
Edge count	$3(\frac{\frac{N}{3}(\frac{N}{3}-1)}{2})+3$

A.6.2 3 Median Hubs

There are 3 agents which act as median hubs which are the agent with the median optimism-factor and the next lower and higher ones. All three are connected to each other where the rest of the agents are randomly connected to one hub so that each hub has the same amount of agents. Included to see what happens if all agents can only trade through median agents which are the most active ones in the Fully-Connected topology.

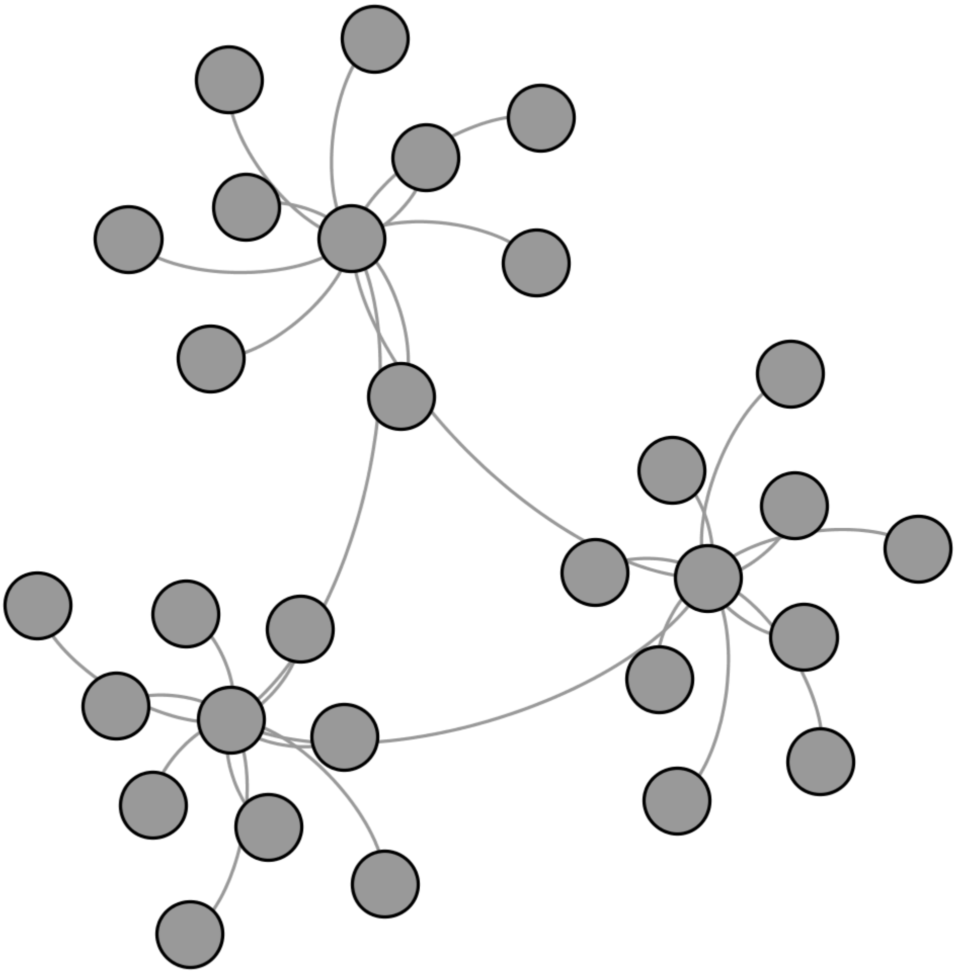

Figure 42: 3 Median Hub topology

Table 50: Network metrics 3 Median Hub topology

Avg. degree	2
Avg. path-length	2.49
Network diameter	3
Graph density	0.069
Edge count	N

A.6.3 Median Hub

All agents are connected to the agent with the median optimism-factor. Included for the same reason as in 3 median hubs but with just one median hub.

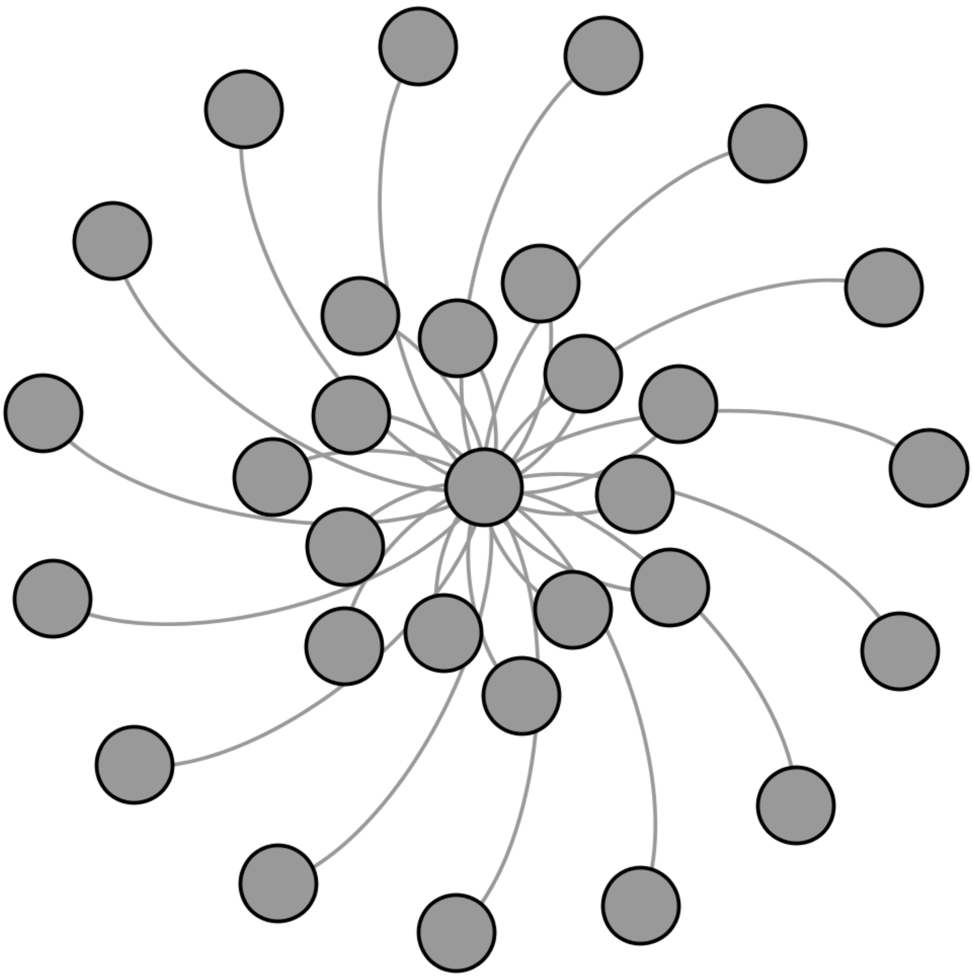

Figure 43: Median Hub topology

Table 51: Network metrics Median Hub topology

Avg. degree	1.933
Avg. path-length	1.933
Network diameter	2
Graph density	0.067
Edge count	$N-1$

A.6.4 Maximum Hub

Looks the same as 1 Median Hub but all edges are connected to the agent with the highest optimism-value. Has thus also the same metrics as the optimism-values have no functional influence on the metrics. Included just out of curiosity and has no real value as it is obviously clear that equilibrium is impossible to be reached in this case.

A.7 Complex network-topologies

A.7.1 Erods-Renyi

See section 2.4 for how this topology is created. Included to investigate the influence of randomness, small-world and scale-free effects upon equilibrium.

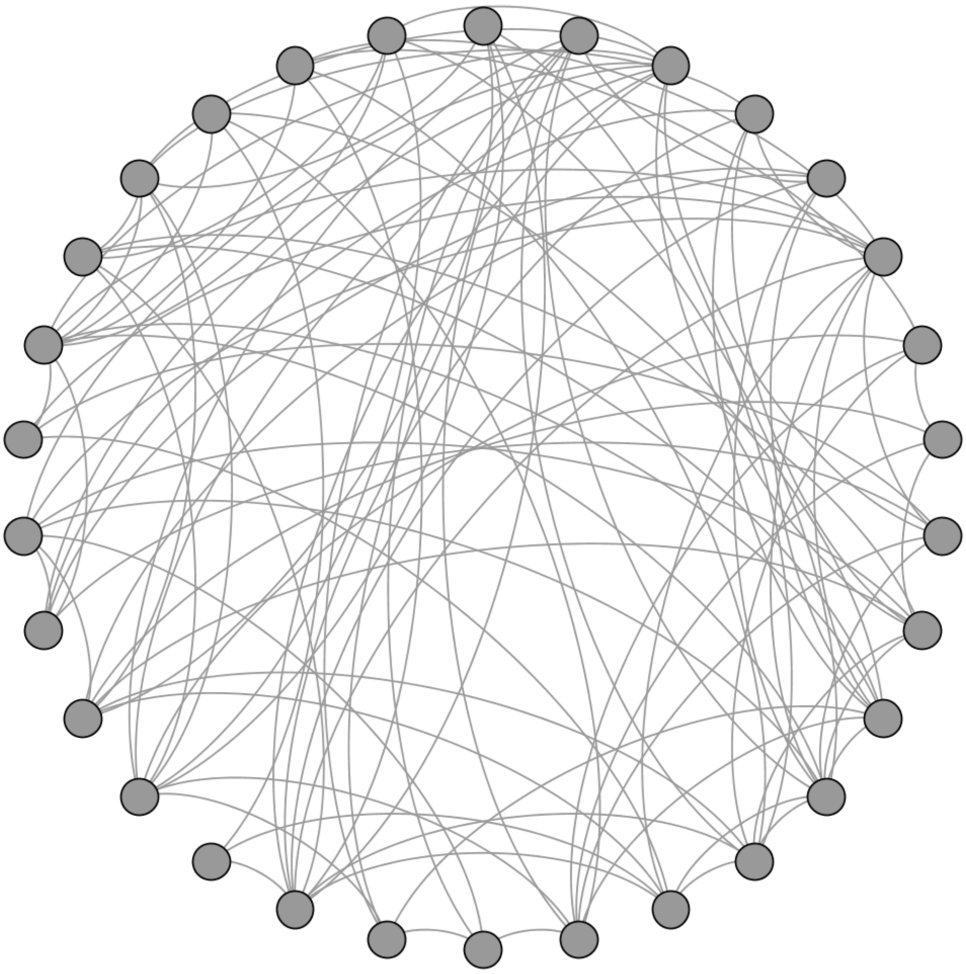

Figure 44: Erdos-Renyi topology with inclusion-probability of 0.2

Table 52: Network metrics Erdosy-Renyi 0.2

Avg. degree	6.8
Avg. path-length	1.913
Network diameter	3
Graph density	0.234
Connected component	1

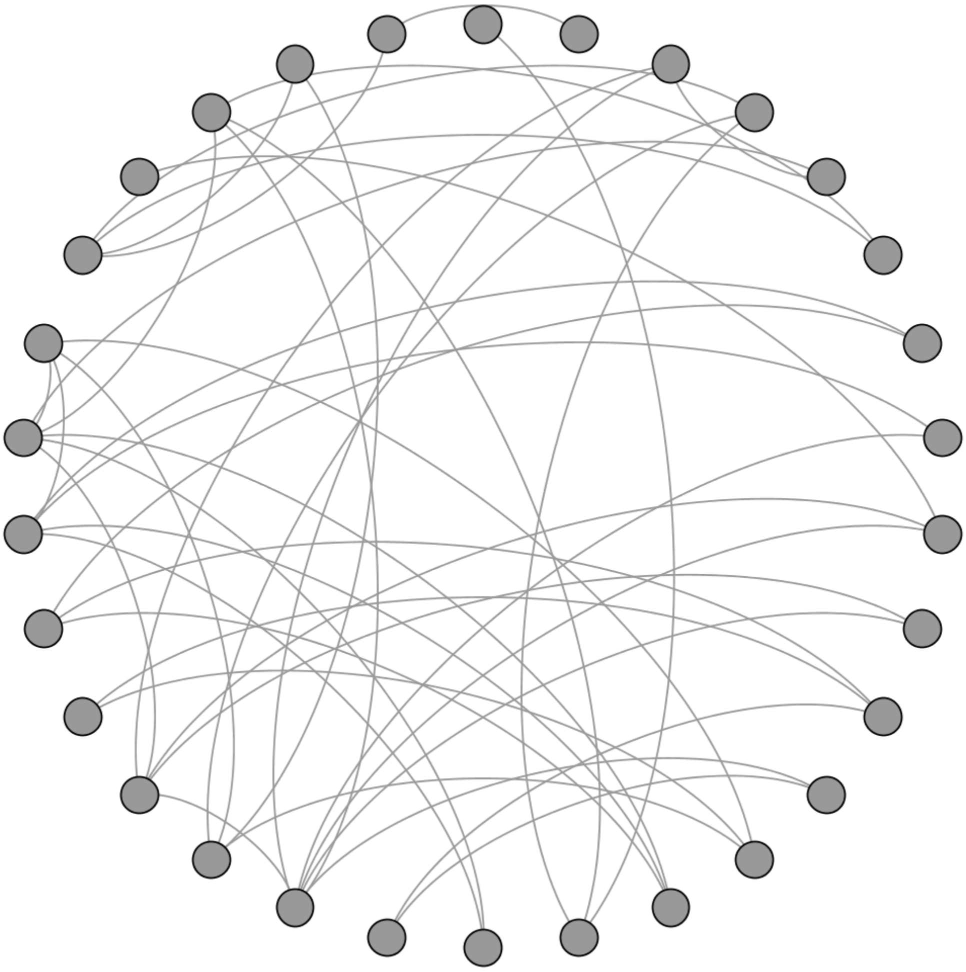

Figure 45: Erdos-Renyi topology with inclusion-probability of 0.1

Table 53: Network metrics Erdosy-Renyi 0.1

Avg. degree	2.933
Avg. path-length	3.262
Network diameter	7
Graph density	0.101
Connected component	1

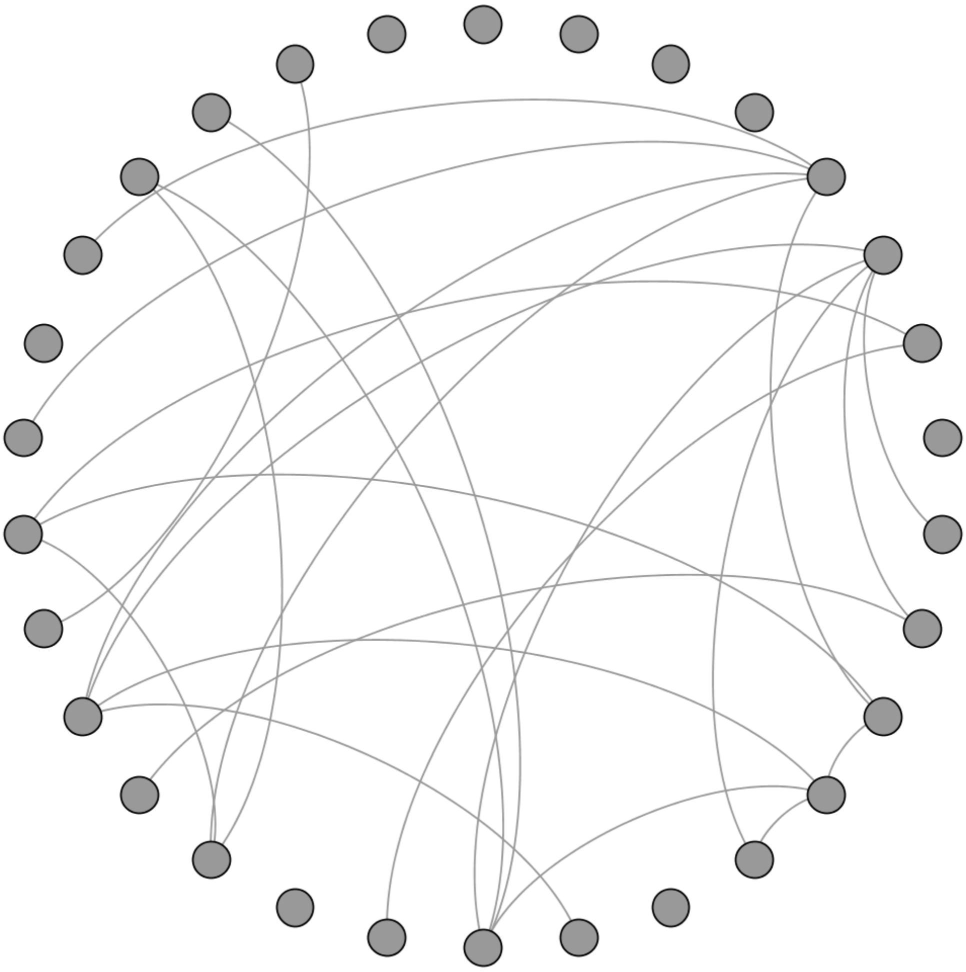

Figure 46: Erdos-Renyi topology with inclusion-probability of 0.05

Table 54: Network metrics Erdosy-Renyi 0.05

Avg. degree	1.6
Avg. path-length	3.052
Network diameter	8
Graph density	0.055
Connected component	11

A.7.2 Barbasi-Albert

See section 2.4 for how this topology is created. Included to investigate the influence of randomness, small-world and scale-free effects upon equilibrium.

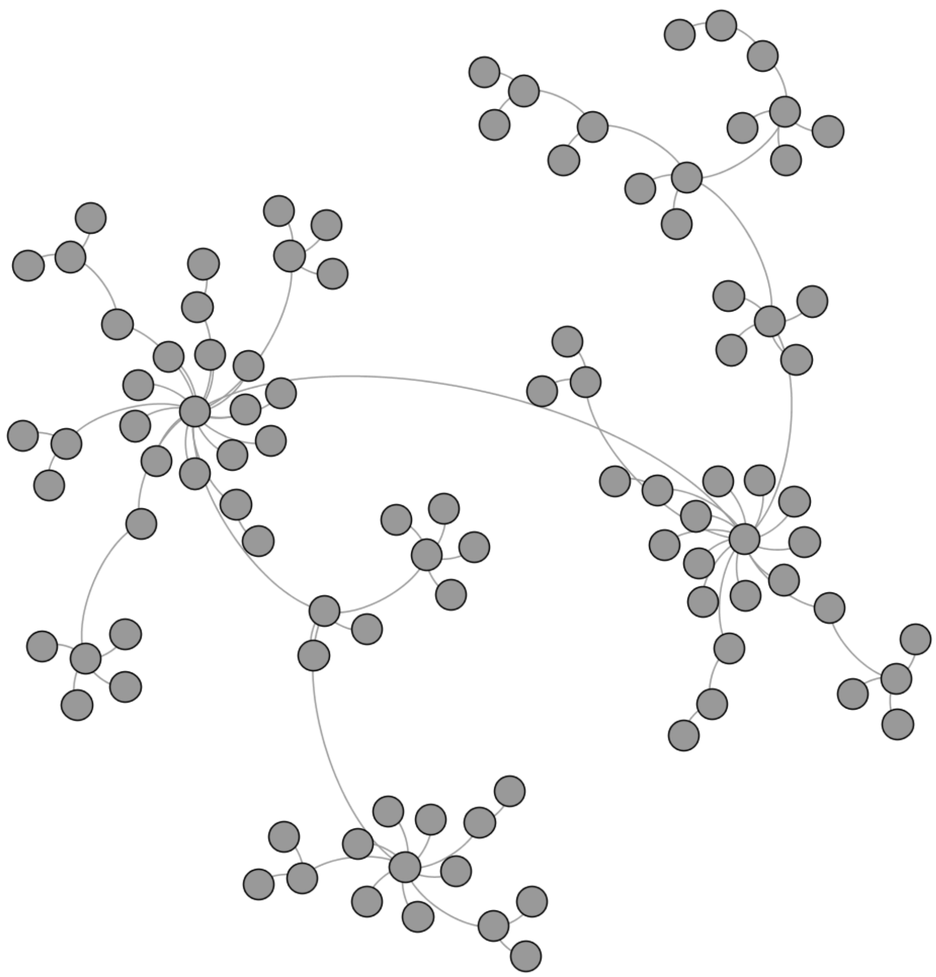

Figure 47: Barbasi-Albert topology with m0=3, m=1

Table 55: Network metrics Barbasi-Albert m0=3, m=1

Avg. degree	1.98
Avg. path-length	4.684
Network diameter	11
Graph density	0.02

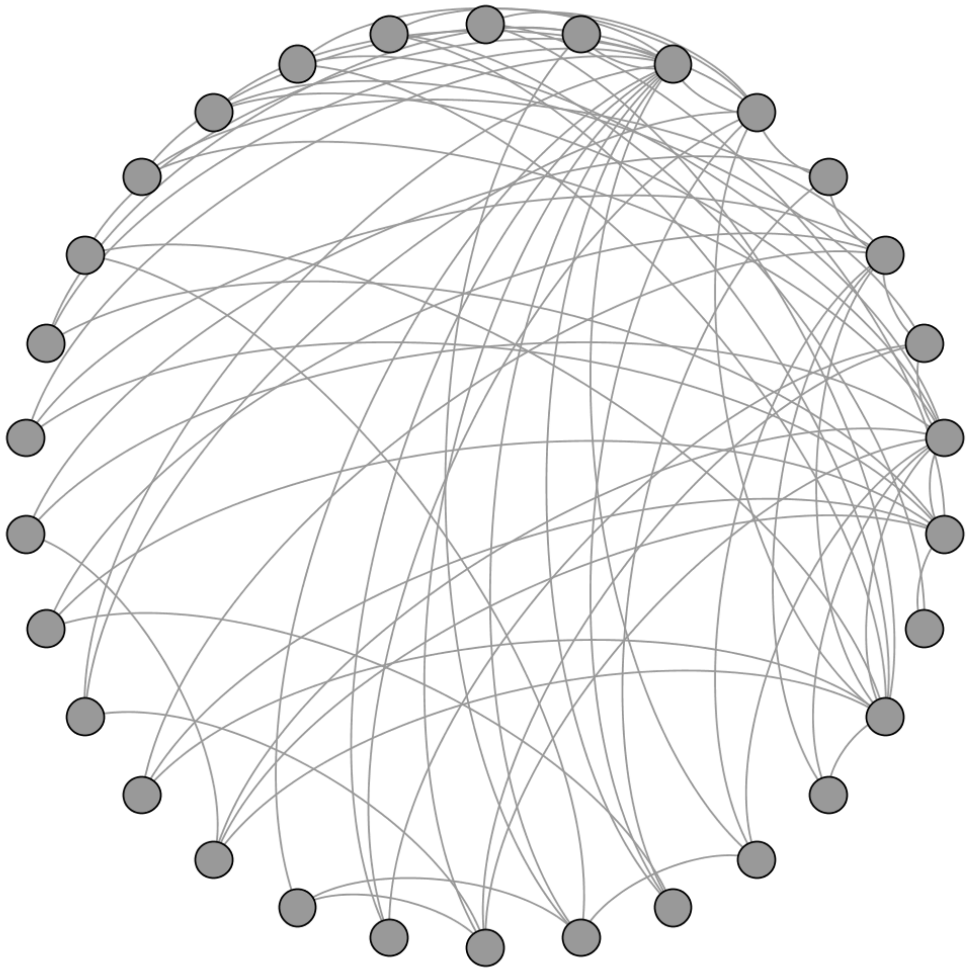

Figure 48: Barbasi-Albert topology with m0=9, m=3

Table 56: Network metrics Barbasi-Albert m0=9, m=3

Avg. degree	4.733
Avg. path-length	2.11
Network diameter	4
Graph density	0.163

A.7.3 Watts-Strogatz

See section 2.4 for how this topology is created. Included to investigate the influence of randomness, small-world and scale-free effects upon equilibrium.

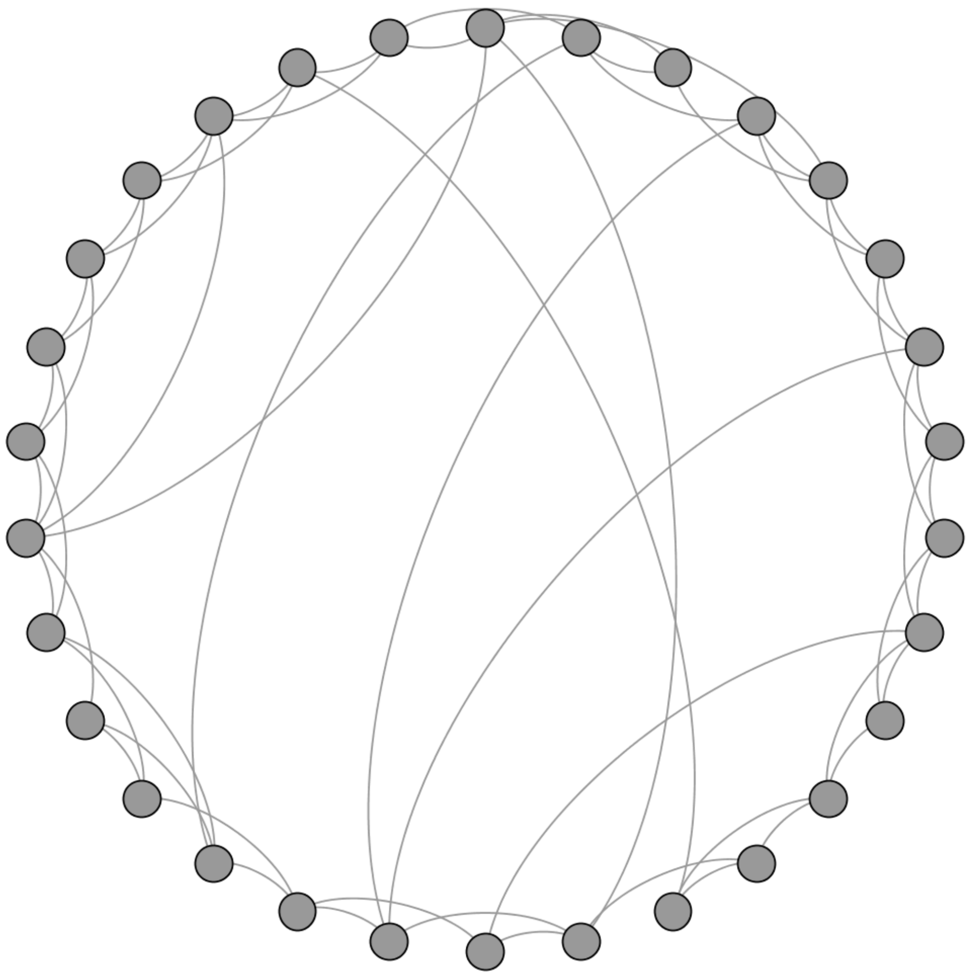

Figure 49: Watts-Strogatz topology with k=2, p=0.2

Table 57: Network metrics Watts-Strogatz k=2, p=0.2

Avg. degree	4
Avg. path-length	2.883
Network diameter	6
Graph density	0.138

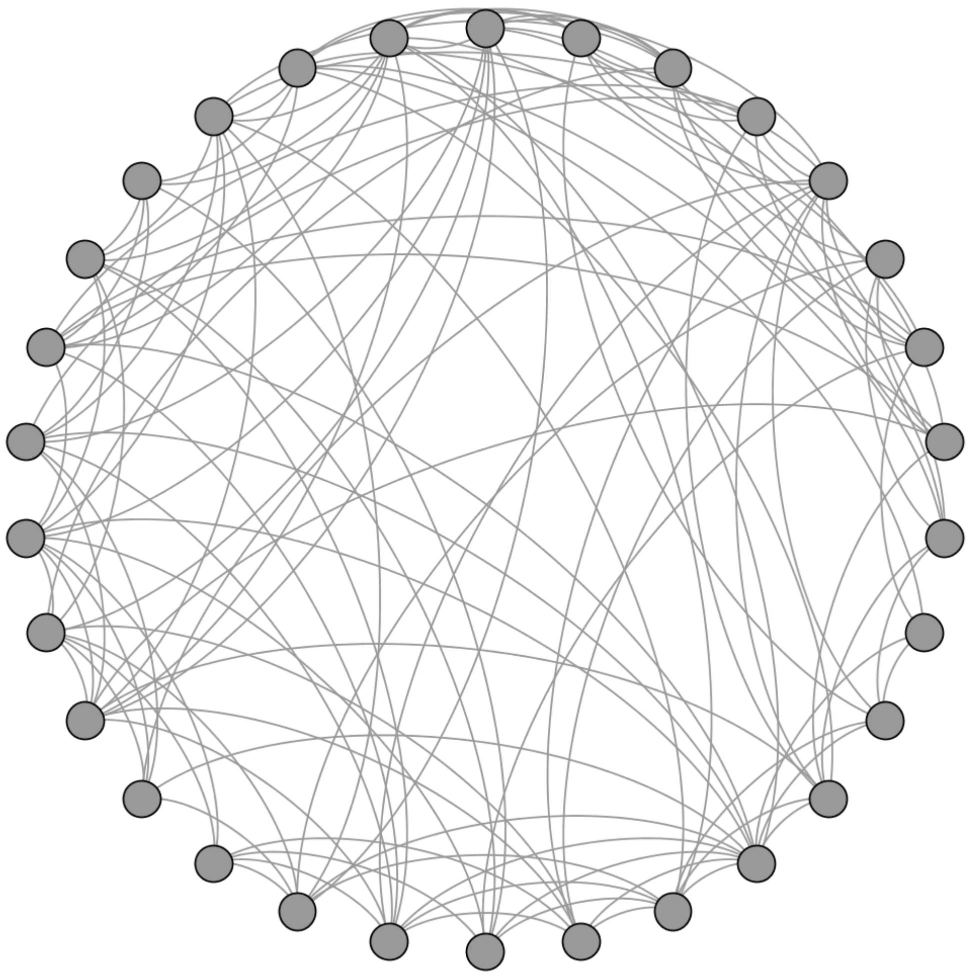

Figure 50: Watts-Strogatz topology with k=4, p=0.5

Table 58: Network metrics Watts-Strogatz k=4, p=0.5

Avg. degree	8
Avg. path-length	1.823
Network diameter	3
Graph density	0.276

Appendix B

Results for Hub-Based, Scale-Free and Small-World

B.1 Half-Fully Connected

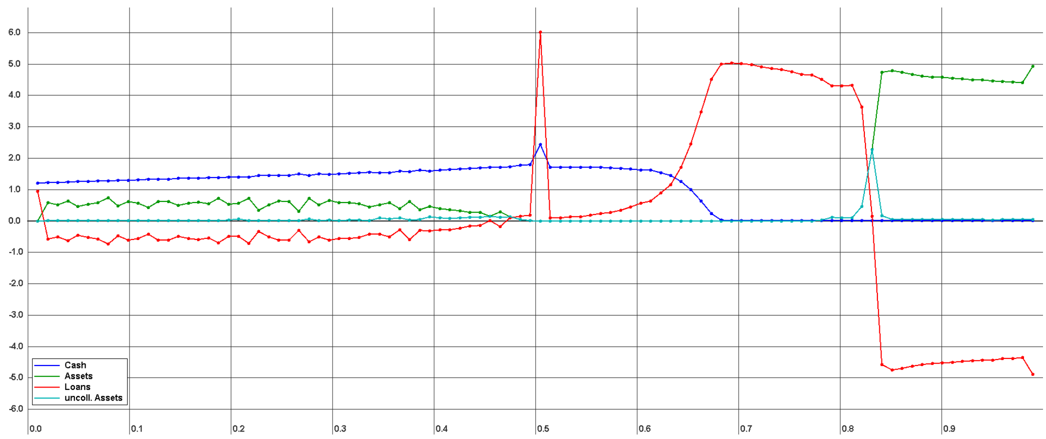

Figure 51: Wealth-Distribution of Half-Fully Connected topology

Table 59: Equilibrium of Half-Fully Connected topology

Asset-Price p	0.651 (0.027)
Bond-Price q	0.362 (0.013)
Marginal agent i1	0.640 (0.015)
Marginal agent i2	0.833 (0.09)
Pessimist Wealth	1.22 (0.096)
Medianist Wealth	2.258 (0.409)
Optimist Wealth	4.526 (0.071)

Table 60: Performance of Half-Fully Connected topology

Successful matching-rounds	14,218.9 (4621.74)
Failed matching-rounds	1034.12 (22.99)
Total matching-rounds	15,253.02 (4633.44)
Ratio successful/total	0.93
Ratio failed/total	0.07

The equilibrium is clearly distinct from the theoretical and Fully-Connected one as miss-allocation can be found within the pessimists-range. Also the i1- and i2-points and the wealth-distributions differ both numerically and visually.

B.2 Ascending-Connected with short-cuts

B.2.1 Random short-cuts

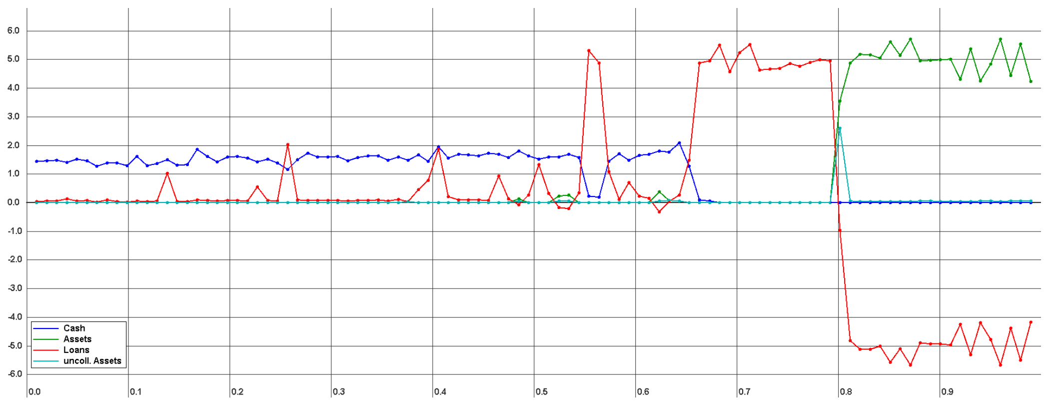

Figure 52: Wealth-Distribution of Ascending-Connected random short-cuts topology

Table 61: Equilibrium of Ascending-Connected random short-cuts topology

Asset-Price p	0.731 (0.019)
Bond-Price q	0.393 (0.009)
Marginal agent i1	0.649 (0.005)
Marginal agent i2	0.804 (0.004)
Pessimist Wealth	1.441 (0.03)
Medianist Wealth	4.282 (0.278)
Optimist Wealth	4.974 (0.038)

Table 62: Performance of Ascending-Connected random short-cuts topology

Successful matching-rounds	8314.78 (229.85)
Failed matching-rounds	1182.06 (29.23)
Total matching-rounds	9496.84 (228.23)
Ratio successful/total	0.87
Ratio failed/total	0.13

Random short-cuts seem to reduce the miss-allocation of pessimists-wealth a bit but lead to a fundamental different equilibrium than the theoretical or fully-connected one as can clearly be seen both visually and numerically.

B.2.2 2 short-cuts

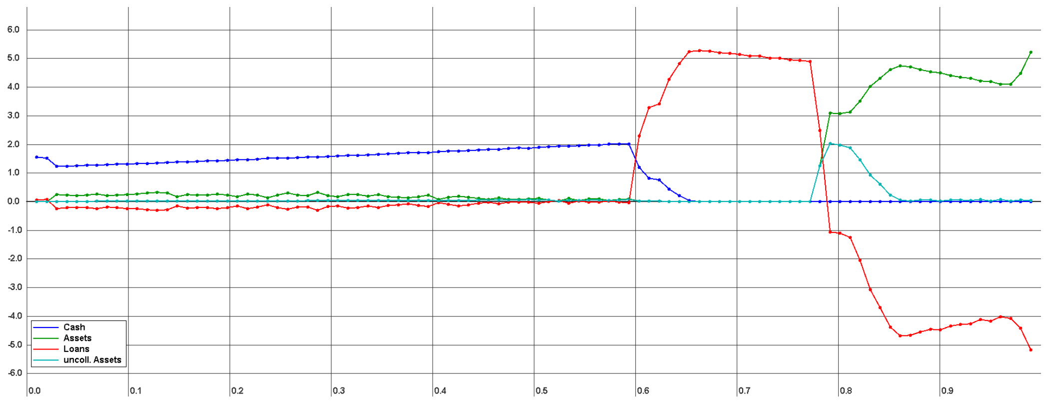

Figure 53: Wealth-Distribution of Ascending-Connected 2 short-cuts topology

Table 63: Equilibrium of Ascending-Connected 2 short-cuts topology

Asset-Price p	0.662 (0.024)
Bond-Price q	0.376 (0.006)
Marginal agent i1	0.608 (0.018)
Marginal agent i2	0.805 (0.028)
Pessimist Wealth	1.441 (0.21)
Medianist Wealth	3.978 (1.442)
Optimist Wealth	4.514 (0.063)

Table 64: Performance of Ascending-Connected random short-cuts topology

Successful matching-rounds	37,093.64 (12,864.4)
Failed matching-rounds	1021. (18.85)
Total matching-rounds	38,115.54 (12,851.53)
Ratio successful/total	0.97
Ratio failed/total	0.03

This topology reduces the miss-allocation in the pessimists-range dramatically but doesn't solve it yet. Unfortunately it leads to a dramatically different wealth-distribution within the medianists and optimist.

B.2.3 5 full short-cuts

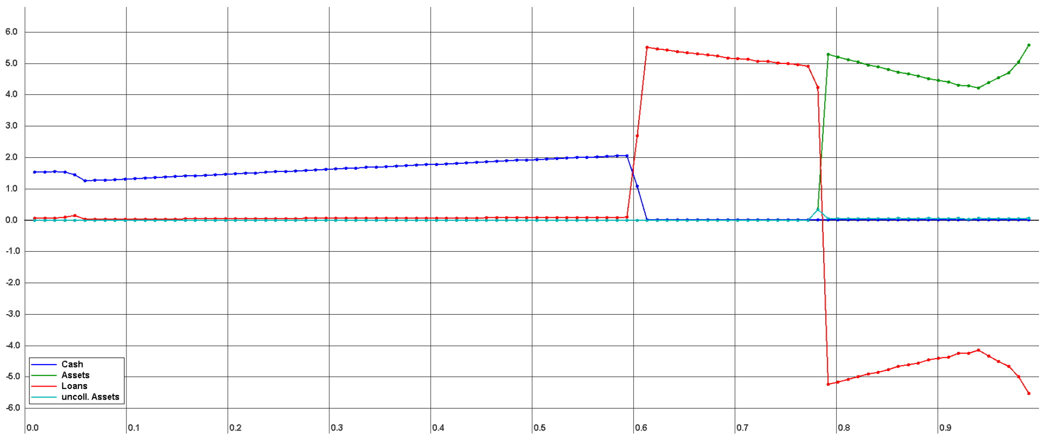

Figure 54: Wealth-Distribution of Ascending-Connected 5 full short-cuts topology

Table 65: Equilibrium of Ascending-Connected 5 full short-cuts

Asset-Price p	0.656 (0.019)
Bond-Price q	0.371 (0.003)
Marginal agent i1	0.594 (0.0)
Marginal agent i2	0.792 (0.0)
Pessimist Wealth	1.649 (0.002)
Medianist Wealth	5.013 (0.018)
Optimist Wealth	4.746 (0.011)

Table 66: Performance of Ascending-Connected 5 full short-cuts topology

Successful matching-rounds	16,971.34 (228.0)
Failed matching-rounds	1026.92 (22.68)
Total matching-rounds	17,998.26 (225.23)
Ratio successful/total	0.94
Ratio failed/total	0.06

As can be clearly seen this topology seems to be able to solve miss-allocations in the pessimists-range seen in Ascending-Connected topology but is still different than the theoretical and Fully-Connected equilibrium.

B.2.4 15 full short-cuts

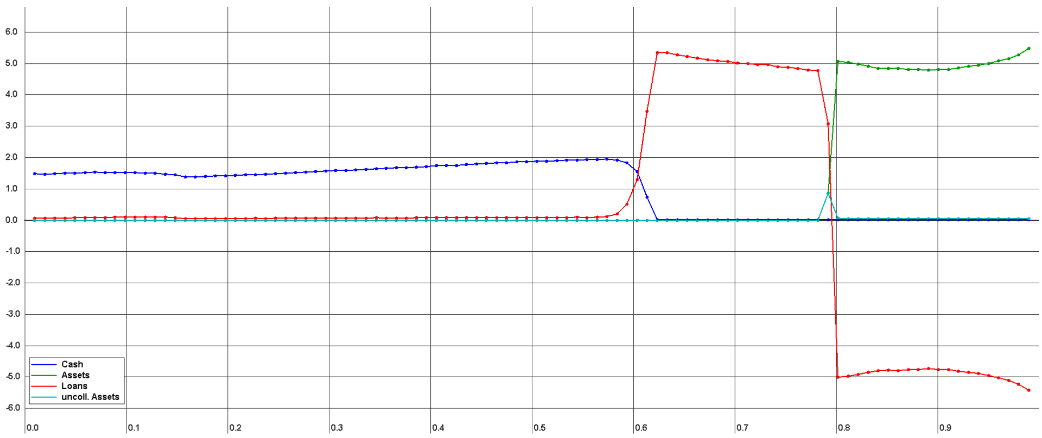

Figure 55: Wealth-Distribution of Ascending-Connected 15 full short-cuts topology

Table 67: Equilibrium of Ascending-Connected 15 full short-cuts topology

Asset-Price p	0.658 (0.024)
Bond-Price q	0.366 (0.009)
Marginal agent i1	0.601 (0.004)
Marginal agent i2	0.802 (0.0)
Pessimist Wealth	1.649 (0.004)
Medianist Wealth	4.811 (0.092)
Optimist Wealth	4.957 (0.021)

Table 68: Performance of Ascending-Connected 15 full short-cuts topology

Successful matching-rounds	4498.08 (58.67)
Failed matching-rounds	1024.78 (17.3)
Total matching-rounds	5522.860 (64.72)
Ratio successful/total	0.81
Ratio failed/total	0.19

This topology comes very close to the theoretical equilibrium but is still a bit different as can be seen in the curved wealth-distributions of the pure optimists.

B.2.5 30 full short-cuts

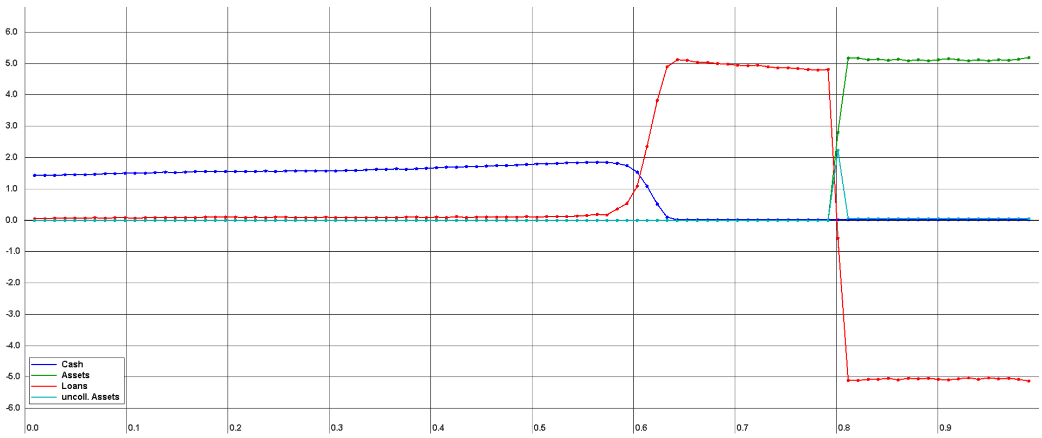

Figure 56: Wealth-Distribution of Ascending-Connected 30 full short-cuts topology

Table 69: Equilibrium of Ascending-Connected 30 full short-cuts topology

Asset-Price p	0.681 (0.012)
Bond-Price q	0.378 (0.006)
Marginal agent i1	0.603 (0.006)
Marginal agent i2	0.802 (0.1)
Pessimist Wealth	1.649 (0.009)
Medianist Wealth	4.702 (0.112)
Optimist Wealth	5.004 (0.025)

Table 70: Performance of Ascending-Connected 30 full short-cuts topology

Successful matching-rounds	2211.08 (35.88)
Failed matching-rounds	1014.68 (10.55)
Total matching-rounds	3225.76 (40.18)
Ratio successful/total	0.68
Ratio failed/total	0.32

This topology is very close to the theoretical and Fully-Connected equilibrium although it differs in asset-price p and in the wealth-distributions. Of course with 30 fully short-cuts in a network of 100 agents one is already very close to fully connectedness.

B.2.6 5 regular short-cuts

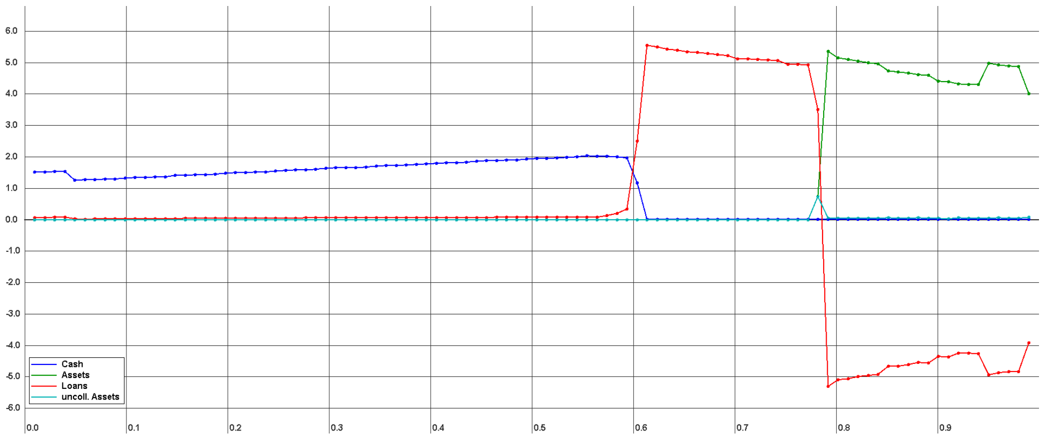

Figure 57: Wealth-Distribution of Ascending-Connected 5 regular short-cuts topology

Table 71: Equilibrium of Ascending-Connected 5 regular short-cuts topology

Asset-Price p	0.665 (0.016)
Bond-Price q	0.364 (0.007)
Marginal agent i1	0.595 (0.003)
Marginal agent i2	0.792 (0.0)
Pessimist Wealth	1.649 (0.003)
Medianist Wealth	4.991 (0.045)
Optimist Wealth	4.727 (0.011)

Table 72: Performance of Ascending-Connected 5 regular short-cuts topology

Successful matching-rounds	14,570.44 (157.61)
Failed matching-rounds	1064.24 (29.88)
Total matching-rounds	15,634.68 (166.21)
Ratio successful/total	0.93
Ratio failed/total	0.07

As can be seen in the visual results this topology shows a different equilibrium than the theoretical and Fully-Connected one.

B.2.7 15 regular short-cuts

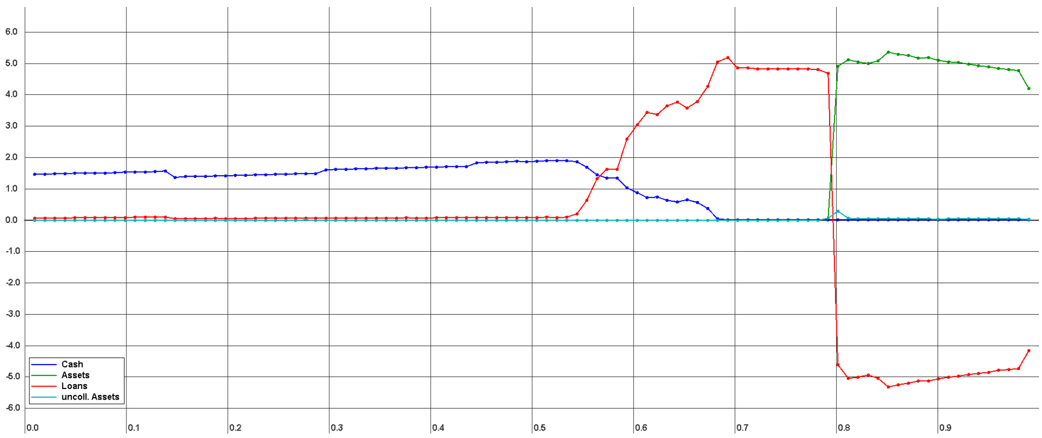

Figure 58: Wealth-Distribution of Ascending-Connected 15 regular short-cuts topology

Table 73: Equilibrium Ascending-Connected 15 regular short-cuts topology

Asset-Price p	0.705 (0.020)
Bond-Price q	0.357 (0.018)
Marginal agent i1	0.586 (0.023)
Marginal agent i2	0.802 (0.0)
Pessimist Wealth	1.649 (0.051)
Medianist Wealth	4.146 (0.101)
Optimist Wealth	4.997 (0.007)

Table 74: Performance of Ascending-Connected 15 regular short-cuts topology

Successful matching-rounds	4373.28 (50.13)
Failed matching-rounds	1129.24 (19.2)
Total matching-rounds	5502.52 (52.11)
Ratio successful/total	0.79
Ratio failed/total	0.21

The equilibrium of this topology is falls very far from the theoretical and Fully-Connected one.

B.2.8 30 regular short-cuts

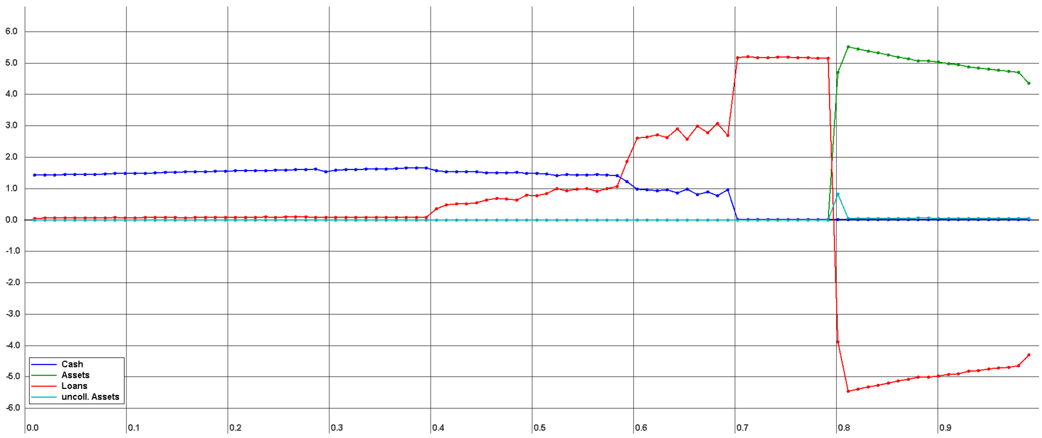

Figure 59: Wealth-Distribution of Ascending-Connected 30 regular short-cuts topology

Table 75: Equilibrium of Ascending-Connected 30 regular short-cuts topology

Asset-Price p	0.710 (0.021)
Bond-Price q	0.398 (0.008)
Marginal agent i1	0.589 (0.021)
Marginal agent i2	0.802 (0.0)
Pessimist Wealth	1.479 (0.049)
Medianist Wealth	3.713 (0.125)
Optimist Wealth	5.0 (0.0)

Table 76: Performance of Ascending-Connected 30 regular short-cuts topology

Successful matching-rounds	5427.02 (90.82)
Failed matching-rounds	1139.04 (27.74)
Total matching-rounds	6566.06 (96.04)
Ratio successful/total	0.82
Ratio failed/total	0.18

The equilibrium of this topology is falls very far from the theoretical and Fully-Connected one.

B.3 Hub-Based topologies

The Hub-Based Topologies fail to come even close to equilibrium due to reasons given in chapter 4 "Hypothesis". This can be seen also very clearly in the visual results and thus no performance- and equilibrium-tables are listed as they would not make any sense.

B.3.1 3-Hubs

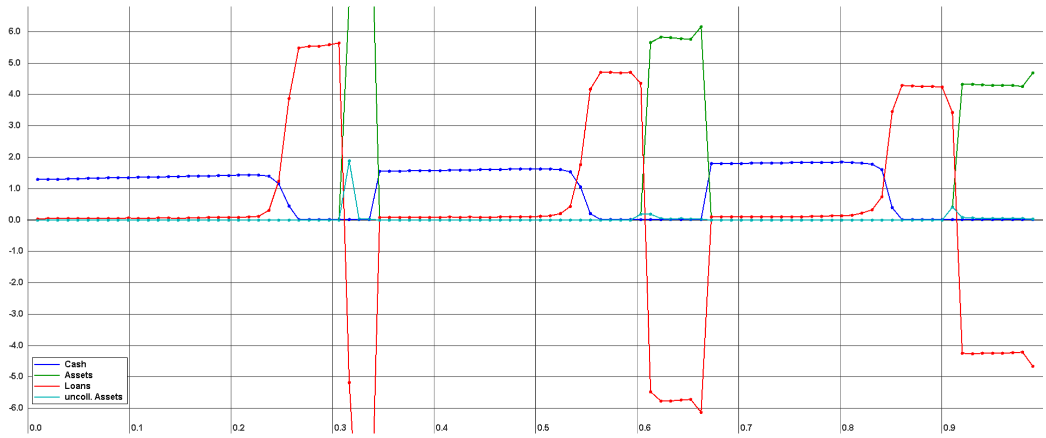

Figure 60: Wealth-Distribution of 3-Hubs topology

B.3.2 1-Median Hub

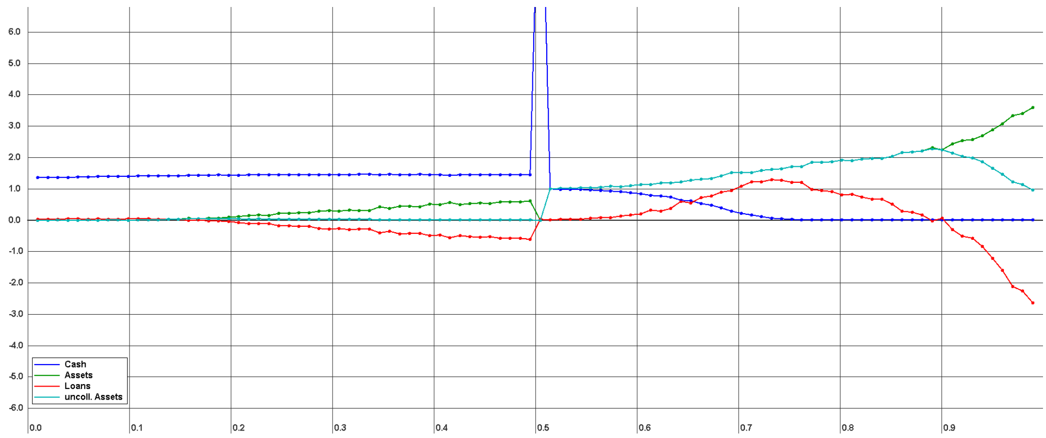

Figure 61: Wealth-Distribution of 1 Median-Hub topology

B.3.3 3-Median Hubs

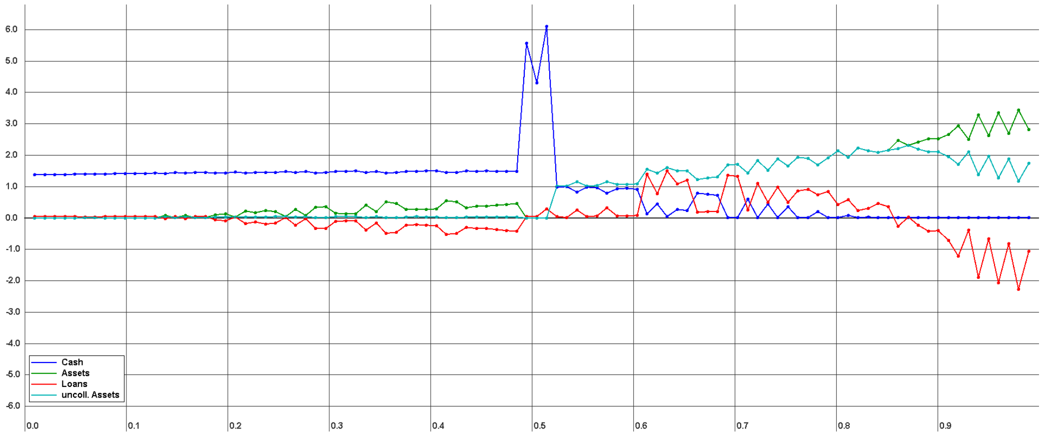

Figure 62: Wealth-Distribution of 3 Median-Hubs topology

B.3.4 Maximum Hub

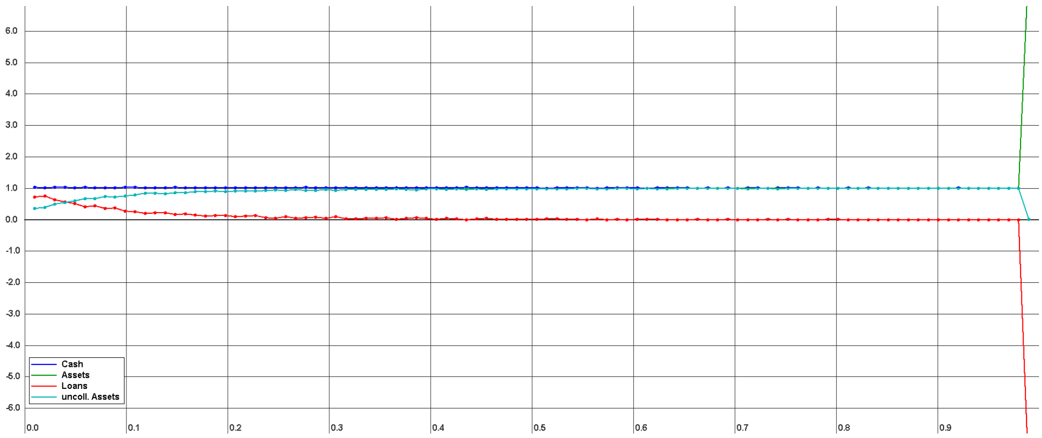

Figure 63: Wealth-Distribution of Maximum-Hub topology

B.4 Scale-Free and Small-World topologies

This topologies fail to come even close to equilibrium too due to reasons given in chapter 4 "Hypothesis". This can be seen also very clearly in the visual results and thus no performance- and equilibrium-tables are listed as they would not make any sense.

B.4.1 Erdos-Renyi

Note that with the correct parametrization this topology could satisfy the hypothesis by pure chance. The result would be a pure random network as an Ascending-Connected topology with random short-cuts but as already showed above this Ascending-Connected random short-cuts network fails from producing the theoretical and Fully-Connected equilibrium.

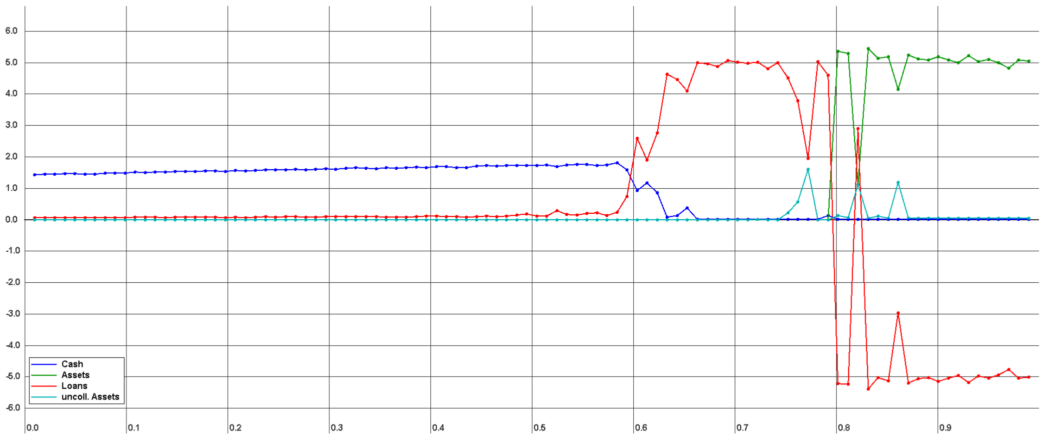

Figure 64: Wealth-Distribution of Erdos-Renyi 0.2 topology

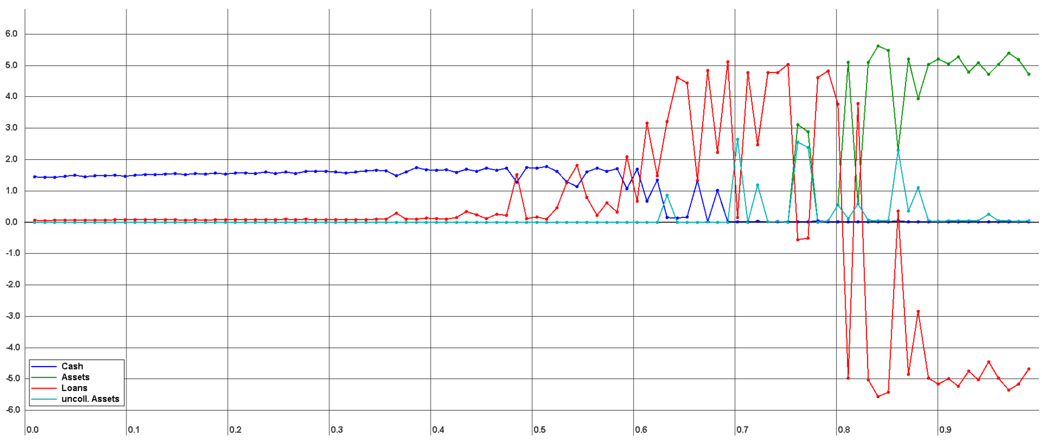

Figure 65: Wealth-Distribution of Erdos-Renyi 0.1 topology

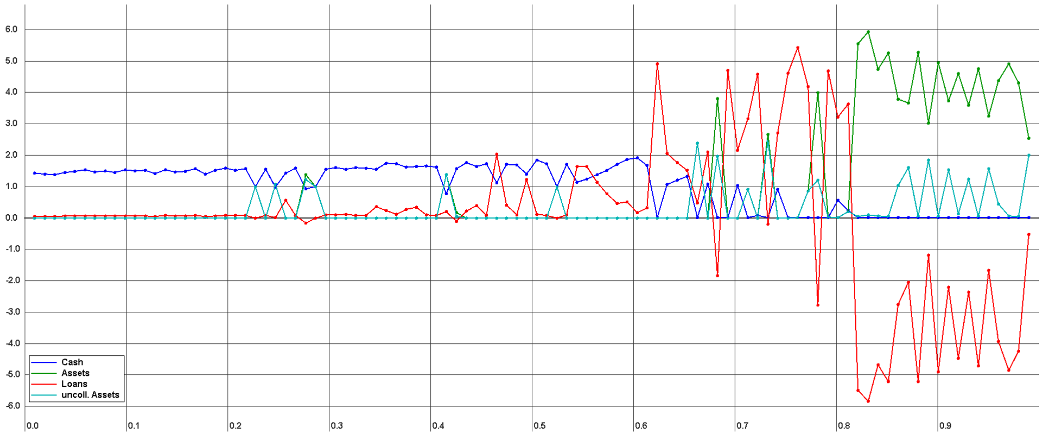

Figure 66: Wealth-Distribution of Erdos-Renyi 0.05 topology

B.4.2 Barbasi-Albert

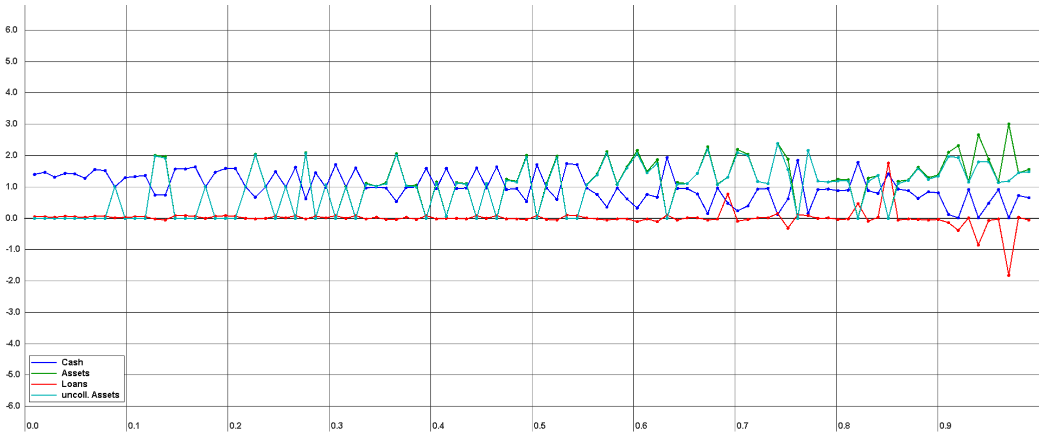

Figure 67: Wealth-Distribution of Barbasi-Albert m0=3, m=1 topology

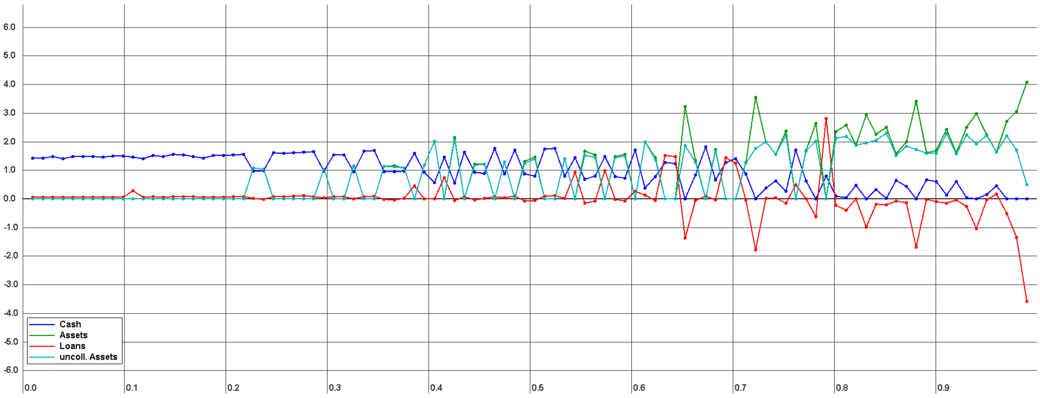

Figure 68: Wealth-Distribution of Barbasi-Albert m0=9, m=3 topology

B.4.3 Watts-Strogatz

Note that with the correct parametrization this topology could satisfy the hypothesis by pure chance too. The result would be a pure random network as an Ascending-Connected topology with random short-cuts but as already showed above this Ascending-Connected random short-cuts network fails from producing the theoretical and Fully-Connected equilibrium.

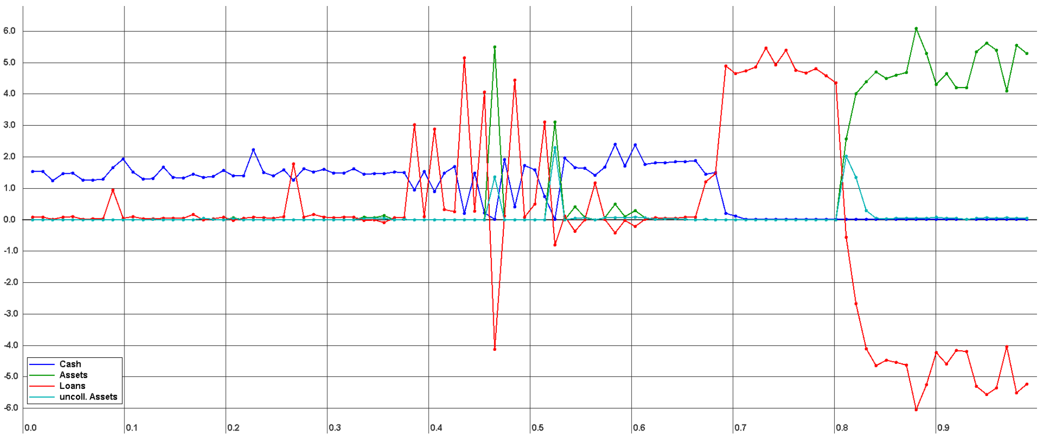

Figure 69: Wealth-Distribution of Watts-Strogatz k=2, b=0.2 topology

Appendix C

Increasing matching-probabilities

This appendix gives the formulas for increasing the matching-probabilities in Ascending-Connected topology. This technique was not developed by the author of the thesis but by the supervisor Mr. Hans-Joachim Vollbrecht and is included for completeness.

The idea is to adjust the price-ranges by a specific constant for each agent so that the shape of the matching-probabilities is kept but the absolute value of a match is dramatically increased where the edges as seen in 9 are at 1. So the process is to calculate the constants for each market for each agent and then in a next step adjust the price-ranges.

C.1 Calculating constants

The constants are named *ca* for the Asset/Cash market, *cl* for the Bond/Cash market and *cal* for the Asset/Bond market. The number of agents is denoted by R and the face-value of the used bond is V.

C.1.1 Asset/Cash market

$$E_i^{asset} = 0.8\frac{i}{R+1} + 0.2 \tag{C.1}$$

$$ca_{i+1} = \frac{(R-i)(i+2)(\frac{0.8}{R+1} + ca_i)}{(R+1-i)(i+1)} - \frac{0.8}{R+1} \tag{C.2}$$

C.1.2 Bond/Cash market

$$E_i^V = 0.2 + \frac{i}{R+i}(V-0.2) \tag{C.3}$$

$$cl_{i+1} = \frac{(R-i)(i+2)(\frac{V-0.2}{R+1}+cl_i)}{(R+1-i)(i+1)} - \frac{V-0.2}{R+1} \tag{C.4}$$

C.1.3 Asset/Bond market

$$E_i^{asset,V} = \frac{E_i^{asset}}{E_i^V} \tag{C.5}$$

$$d_{i,V} = E_{i+1}^{asset,V} - E_i^{asset,V} \tag{C.6}$$

$$cal_{i+1} = \frac{d_{i,V}(5.0 - E_{i+1}^{asset,V})(E_{i+2}^{asset,V} - \frac{0.2}{V})(d_{i,V} + cal_i)}{d_{i+1,V}(5 - E_i^{asset,V}(E_{i+1}^{asset,V} - \frac{0.2}{V})} - d_{i+1,V} \tag{C.7}$$

C.1.4 Adjusting the ranges

Using the previously calculated constants *ca, cl, cal* for each agent on each market now the minimum and maximum values of the price-ranges of each agent are adjusted using the constants to increase the matching-probability. Note that only the min- and max-values are changed but the limit-price has obviously to be left untouched.

Ask-offering ranges

$$min\ asset\text{-}price\ agent_i = limit\text{-}price\ asset\ agent_i \tag{C.8}$$

$$max\ asset\text{-}price\ agent_i = limit\text{-}price\ asset\ agent_{i+1} + ca_{i+1} \tag{C.9}$$

$$min\ bond\text{-}price\ agent_i = limit\text{-}price\ bond\ agent_i \tag{C.10}$$

$$max\ bond\text{-}price\ agent_i = limit\text{-}price\ bond\ agent_{i+1} + cl_{i+1} \tag{C.11}$$

$$min\ asset/bond\text{-}price\ agent_i = \frac{limit\text{-}price\ asset\ agent_i}{limit\text{-}price\ bond\ agent_i} \tag{C.12}$$

$$max\ asset/bond\text{-}price\ agent_i = \frac{limit\text{-}price\ asset\ agent_{i+1}}{limit\text{-}price\ bond\ agent_{i+1}} + cal_{i+1} \tag{C.13}$$

Bid-offering ranges

$$\textit{min asset-price agent}_i = \textit{limit-price asset agent}_{i-1} \tag{C.14}$$

$$\textit{max asset-price agent}_i = \textit{limit-price asset agent}_i \tag{C.15}$$

$$\textit{min bond-price agent}_i = \textit{limit-price bond agent}_{i-1} \tag{C.16}$$

$$\textit{max bond-price agent}_i = \textit{limit-price bond agent}_i \tag{C.17}$$

$$\textit{min asset/bond-price agent}_i = \frac{\textit{limit-price asset agent}_{i-1}}{\textit{limit-price bond agent}_{i-1}} \tag{C.18}$$

$$\textit{max asset/bond-price agent}_i = \frac{limit - priceassetagent_i}{limit - pricebondagent_i} \tag{C.19}$$

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Glossary

allocative efficiency Is a state in which all products match the consumers preferences. In other words it means that goods are distributed optimally between all consumers reflecting the wants and tastes of the agents involved. If allocative efficiency is at 100% then it is not possible any more to increase the utilities of *both* traders in a case of a trade thus traders are no more willing to trade. In other words if two agents trade at an allocative efficiency of 100% then one agent will decrease its utility and will lose.. 12

auction Is a market institution in which messages from traders include some price information.. 9

buyer A trader who is willing to buy a given amount of good for a given price.. 9

clearing Is the process of finding a price in which all demands are matched to the given supplies thus clearing the market by leaving no unmatched demands or supplies.. 9

good A generic object which is traded between agents. Can be an asset, food, gold,.... 9

limit-price Is the private price a trader assigns to a good they want to exchange. This private price is different from the price in the offering and is higher in case of the buyer and lower in the case of the seller.. 9

market institution Defines how exchange between traders takes place by defining rules what traders can do.. 9

numeraire A generic form of money.. 9

offer A tuple of price and quantity on a given market which signals the willingness to trade by these given quantities.. 9

offer-book Keeps all offers made by the traders.. 9

round In each round all traders have the opportunity to place an offer. At the end of each round matching is applied and if a successful match is found the unmatched offers are deleted from the offer-book.. 9, 11

seller A trader who is willing to sell a given amount of good for a given price.. 9

transaction-price Is the price upon a buyer and a seller agree when trading with each other.. 11

zero-intelligence agents Place offers strictly in a range which increases their utility and do not learn. They are completely deterministic in a way that they never change their behaviour.. 10

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