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Influence of network-topologies on equilibrium in continuous double-auctions.

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Statuatory Declaration

I declare that I have developed and written the enclosed work completely by myself, and have not used sources or means without declaration in the text. Any thoughts from others or literal quotations are clearly marked. This Master Thesis was not used in the same or in a similar version to achieve an academic degree nor has it been published elsewhere.

Widmung

Ich widme diese Arbeit meinen beiden liebevollen Eltern, die den verlorenen Sohn nach 11 Jahren in Wien wie selbstverständlich wieder mit offenen Armen zu Hause in Vorarlberg aufgenommen haben und ihm so ein entspanntes Masterstudium ermöglichten und ihm dadurch halfen ein völlig neues Kapitel in seinem Leben aufzuschlagen.

Contents

Abstract

In the paper of [BSV13] a model for endogenous leverage in a continuous double-auction is introduced and it is shown under which circumstances holdings and trading prices approach an equilibrium. One main criteria is the trading network the agents use where Breuer et al. examine only two topologies and report that the prices come to an equilibrium only in the case of a fully connected network. They leave the question open on how the model behaves with different kind of networks and which network topology exactly allows an equilibrium to be reached for further research. This thesis builds upon this model and gives a hypothesis for a necessary condition a network must satisfy to allow the model to approach an equilibrium. Then a few network-topologies are examined in regard of their ability to allow equilibria to be reached or not through computer-driven simulation. As will be shown in this thesis through validation by computer-driven simulation the hypothesis turns out to be correct only after extending the simulation-model by an additional market. This result raises questions this thesis tries to answer about market-mechanisms and market-types when agents don't trade in a fully informed network.

Chapter 1

Introduction

TODO: überarbeiten, passt so noch nicht In 2008 the so called "Subprime Mortgage Crisis" struck the world. It was caused by declining house prices which rose during the US Housing Market Bubble in 2006 to an all-time high. Borrowers used their asset as collateral for the mortgage which constantly increased in value which guaranteed them a low payment-rate because the rate was coupled to the value of the asset. Banks granted "subprime" mortgages to more and more highly risky borrowers. In 2007 borrowers started to default which led to falling prices as the banks reclaimed the collateral and wanted to sell it again on the market to compensate for the loss. This led to a flood of assets which led to a decline of housing prices overall. As the prices fell dramatically the payment-rates rose dramatically to compensate for the cheaper asset. This in turn resulted in even more borrowers going default which resulted in a dramatic downward spiral. Even worse the banks were selling these collateralized products between each other and even insured themself against defaults of borrowers which led to an even more dramatic kick-back.

This mechanism of borrowing money to buy goods which in turn act as a security for the borrowed money is called leverage which was determined as the primary driving force behind systemic risk in the aftermath of the "Subprime Mortgage Crisis". See Chapter 2.1 "Leverage and Systemic Risk" for a more in-depth discussion.

Up until 2010 leverage was always exogenous in the literature on collateralized credit but recently Geanakoplos and Zame (TODO: cite) proposed theories which endogenized leverage within a general equilibrium framework.

[BSV13] developed a simulation on top of the model of Geanakoplos in which zero-intelligence agents trade assets and loans in a continuous double auction. They wanted to better understand the dynamic of such a theoretical

process and how prices develop instead of being predicted through an equilibrium theory. They TODO: zitierne "ask whether the competitive theory of trade in leveraged assets has decriptive and predictive power in a double auction environment."

4 contributions: 1. double auctions for leveraged assets is new 2. details of institutional specification matter a lot 3. limits of the endogenous leverage model 4.

They could show that in their simulation trading prices and wealth-distribution approach the theoretical equilibrium of Geanakoplos. In their simulation only a fully connected network and a hub-network of agents was investigated where the equilibrium was only reached in the case of the fully connected network. See Chapter 3. "The Leverage Cycle" for a thorough description of the simulation-model of [BSV13].

This thesis investigates more topologies of networks and their states of equilibrium. Furthermore it presents a hypothesis about the necessary property a topology of a network must satisfy to reach the theoretical equilibrium predicted in the theory of Geanakoplos. Interestingly it is shown experimentally that the hypothesis alone does not guarantee the reach of the theoretical equilibrium but further mechanisms needs to be implemented. See Chapter 4 "Hypothesis" and Chapter 6 "Results" for an in-depth explanation of both the hypothesis and why it does not hold and needs to be extended by means of an additional market-mechanism.

For experimental investigation a software was built for this thesis which implemented the exact simulation model of [BSV13] but extended it further to be applicable to arbitrary topologies. See Chapter 5 "Implementation" on details of the software.

In Chapter 2 "Theory" the theoretical background involved with this thesis is presented. First Leverage and systemic risk and its implications are discussed. Then an introduction into the mechanics of Continuous Double Auction as market-mechanisms and equilibrium theory in economics is given. Finally an overview of abstract networks, network-generating algorithms and and their properties is given.

In Chapter 3 "The Leverage Cycle" the theoretical model [BSV13] built their simulation upon is discussed in-depth.

In Chapter 4 "Topologies and Hypothesis" all topologies which are investigated are introduced and the conjecture about the type of topology necessary to reach the theoretical equilibirum is presented and discussed whether the given topologies could ever approach it or not.

Chapter 5 "Implementation" gives an in-depth explanation of the implementation of the computer-driven simulation presented in [BSV13] including a description of the architecture, implementation of the markets and trading mechanisms.

Chapter 6 "Results" shows the results of simulations of all implemented topologies.

Chapter 7 "Interpretation and Discussions" connects the content of the previous chapters to show that the initial hypothesis of Chapter 4 does not satisfy the equilibrium and shows how it can be reached by introducing an additional market. Then results of simulations with this market are given and discussed where will be shown that using the additional market an equilibrium will be reached but that it is different from the theoretical predictions.

In Chapter 8 "Conclusions" a short sum-up of the thesis and questions left for further research are presented.

Chapter 2

Theory

TODO: der theorie-teil. Soll in die verwendete Theorie des Hauptteils einführen und darauf hinweisen, aber nicht völlig trocken und losgelöst vom hauptteil sein. Soll immer den kontext des hauptteils berücksichtigen und schon gewisse anwendungsfälle vorwegnehmen.

2.1 Systemic risk and Leverage

Both are tightly coupled in a way that leverage increases systemic risk dramatically as was the case in the "Subprime Mortgage Crisis".

Systemic Risk

WIKI: It refers to the risks imposed by interlinkages and interdependencies in a system or market, where the failure of a single entity or cluster of entities can cause a cascading failure, which could potentially bankrupt or bring down the entire system or market.
[Bor10]

Leverage

WIKI: In finance, leverage (sometimes referred to as gearing in the United Kingdom and Australia) is any technique to multiply gains and losses.
Accounting Leverage Notational Leverage Economic Leverage

2.2 Continuous Double Auction

Paper: gode and sunders auszüge aus dem Breuer et al. Paper und Everything you wanted to know about Continous Double-Auctions

2.3 Equilibrium Theory

theoretisches: utiliy-funktionen und clearing preis in der simulation: ungeklärt, immer individuell, "steckenbleiben" vs. gleichgewicht, am ende an theoretischem gleichgewicht orientiert

2.4 Complex Networks

small-world power-law distribution generation algorithms dient hauptsächlich zur kategorisierung

TODO: In "State of the art" an overview of abstract networks and their properties is given. Also network-generating algorithms are presented and discussed. Because continuous double-auctions are the type of market which is used for matchings a short introduction is given on this topic too.

TODO: ziel hier eine theoretische übersicht über netzwerk-theorie zu geben wobei hauptaugenmerk auf die entwicklungen der letzten jahre (scale-free, small-world, ...)

Regular Graphs: [AlB99, vgl.] [New03, vgl.]

Random Graphs: but since then, most large scale networks with no apparent design principle were described as random graphs introduced by two Hungarian mathematicians Paul Erdos and Alfred Renyi [ER59, vgl.] [ER60, vgl.] Have small-world properties.

Small World Graphs or Average Path Length: Stanley Milgram [TM69] [Mil67] [Kle00]

Clustering Coefficient or Transitivity [WS98]

Degree Distribution [AlB02] Generally, it was believed that the degree distribution in most networks follows a Poisson distribution but in reality, real world networks have a highly skewed degree distribution following power-laws. Power-laws are expressions of the form y / x, where is a constant, x and y are the measures of interest [152].

Small World and Scale Free Network: A small world network as deined by Watts and Strogatz [WS98], is a network with high clustering coeffcient and small average path length. A scale free network as deined by Barabasi and Albert [AlB02], is a network where the degree distribution follows a power law.

Complex Networks: are Small-World and/or Scale-Free [BW00] [ASBS00] [Kle02] http://www.cs.princeton.edu/ chazelle/courses/BIB/big-world.htm Mathematical stuff [New06] [ACL01] [EMB02] [GP04]

2.5   Network-Generating Algorithms

- fully connected - ascending connected - ascending connected with shortcuts
- hubs - erdos-renyi - barbasi-albert - watts-strogatz

Chapter 3

The Leverage Cycle

Definition des Modells Märkte, Marktmechanismen, clearing, utiliy funktionen,.... alles theoretisch, um des dann in implementierung praktisch zu zeigen Bestehende Resultate mit Bezug auf paper Fully-Connected: prozess und endverteilung, erreicht theoretisches Gleichgewicht approximativ

Chapter 4

Hypothesis

Eigentliche Fragestellung: Wie wichtig ist die Vollvernetzung? Allgemeine Netzwerkstrukturen untersuchen aber mit hauptaugenmerk auf Ascending-Connected d.h. reicht ascending-connected aus?

hypothese vorstellen: jedes paar von agenten muss über einen kantenzug erreichbar sein, in dem der optimismusfaktor von agent zu agent monoton wächst.

"If there exists a path from each agent to every other agent in which each node has an monoton increasing optimism factor then equilibrium is reached."

Chapter 5

Implementation

5.1 Functionality

5.1.1 Inspection

5.1.2 Replications

5.1.3 Experiments

GUI

Command-Line

5.2 Architecture

5.2.1 Frontend

5.2.2 Controller

5.2.3 Backend

5.3 Agents

5.3.1 Utility

5.4 Markets

5.4.1 Asset/Cash

5.4.2 Loan/Cash

5.4.3 Asset/Loan

5.5 Simulation

5.5.1 Sweeping and Matching

5.6 Performance improvement

Chapter 6

Results

In this Chapter the results of the experiments are given. Each topology-type introduced in appendix A was simulated where in this chapter only fully-connected and Ascending-Connected topologies are handled as the Ascending-Connected topology - both without and with importance sampling - is the most minimal network which satisfies the requirements for the hypothesis. The results for the other topologies can be found in appendix B.

Note: The numbers in tables resemble always a median-value with the standard-deviation given in parentheses.

6.1   Replicating theoretical equilibrium

As a point-of-reference and as an experimental proof for the correctness of the implementation of the simulation the results of a replication of the theoretical equilibrium and the equilibrium found in [BSV13] are given. Because equilibrium differs across the number of agents and the type of loan traded to be comparable the same amount of agents and the same loan-type has to be used in the experiments which is 1000 Agents and a 0.5 loan because [BSV13] report their equilibria only for a count of 1000 Agents and loans between 0.1 to 0.5.

Table 1: Theoretical Equilibrium for 1000 Agents

Asset-Price p	0.715
Loan-Price q	0.374
Marginal Buyer i0	0.583
Marginal Seller i1	0.802

Table 2: Equilibrium in [BSV13] for 1000 Agents and 0.5 loan

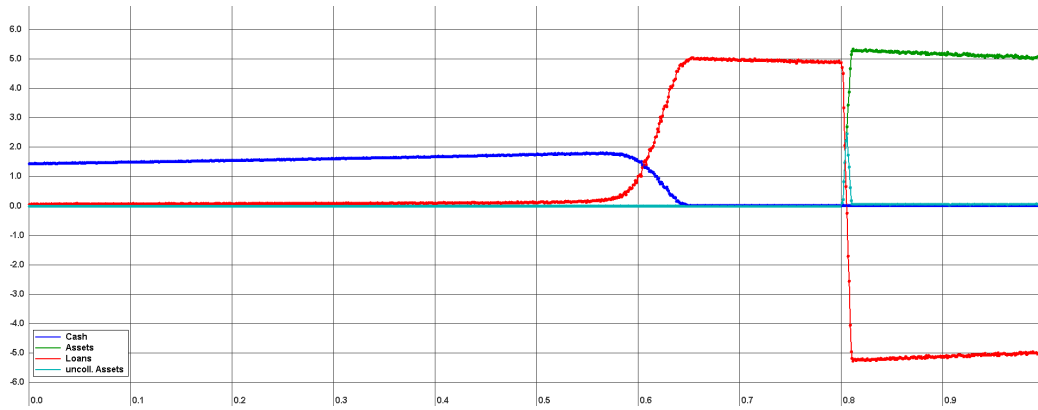| Asset-Price p | 0.716 |
Loan-Price q	0.375
Marginal Buyer i0	0.583
Marginal Seller i1	0.801
Pessimist Wealth	1.716
Medianist Wealth	4.578
Optimist Wealth	5.032

Figure 1: Wealth-Distribution of thesis-implementation of Fully-Connected topology

Table 3: Equilibrium of thesis-implementation

Asset-Price p	0.700 (0.005)
Loan-Price q	0.389 (0.002)
Marginal Buyer i0	0.616 (0.004)
Marginal Buyer i1	0.805 (0.001)
Pessimist Wealth	1.582 (0.01)
Medianist Wealth	4.578 (0.031)
Optimist Wealth	5.105 (0.025)

TODO: difference to breuer TODO: difference to theoretical equilibrium

6.2 Experiments configuration

In the following experiments 100 Agents were used, all markets (Asset/Cash, Loan/Cash, Asset/Loan) were enabled, as loan-type 0.5 was selected and the

Table 4: Performance of thesis-implementation with 1000 Agents and 0.5 loan

Successful TX	19,300.04 (101.68)
Total TX	29,606.82 (2938.82)
Failed TX	10,306.78 (2914.11)

number of replications run was 50. A replication was terminated after 1000 failed transactions in a row. Note that if trading is not possible any more before 1000 failed transactions have been reached in a row, the simulation is halted and thus it is possible that it terminates earlier as can be seen for the Ascending-Connected Importance Sampling topology.

[BSV13] showed that equilibrium can be reached already with 30 agents so this was the minimum number of agents to start with but for a smoother visual result 100 were chosen. Also one simulation-run takes not too much time with 100 as compared to the 1000 agents thus it is a very good match between visual accurateness and processing-power requirements.

The 0.5 loan was selected because its a risky one which is important as riskless loans (facevalue ¡= 0.2) the results are indifferent and wont show the characteristic progression.

Obviously the whole simulation-process is a random-process with an equilibrium (different for each topology) as the fixed-point solution thus one needs replications to reduce noise. The number of 50 replications was chosen because it is a good match between processing-power requirements and overall reduction of noise - increasing the number e.g. to 100 or 200 would not result in much better results but would need much longer to run. All facts can be seen and derived when using 50 replications thus for all figures 50 replications were used unless stated otherwise e.g. a single run.

Table 5: Configuration for all experiments

Agent-Count	100
Loan-Type	0.5
Replication-Count	50
Terminate after	1000 failed successive Transactions

Table 6: Theoretical Equilibrium for 100 Agents

Asset-Price p	0.716
Loan-Price q	0.384
Marginal Buyer i0	0.584
Marginal Seller i1	0.801

6.3 Fully-Connected Topology

This topology serves as the major point-of-reference for the other experiments as it reaches the theoretical equilibrium for 1000 agents as demonstrated.
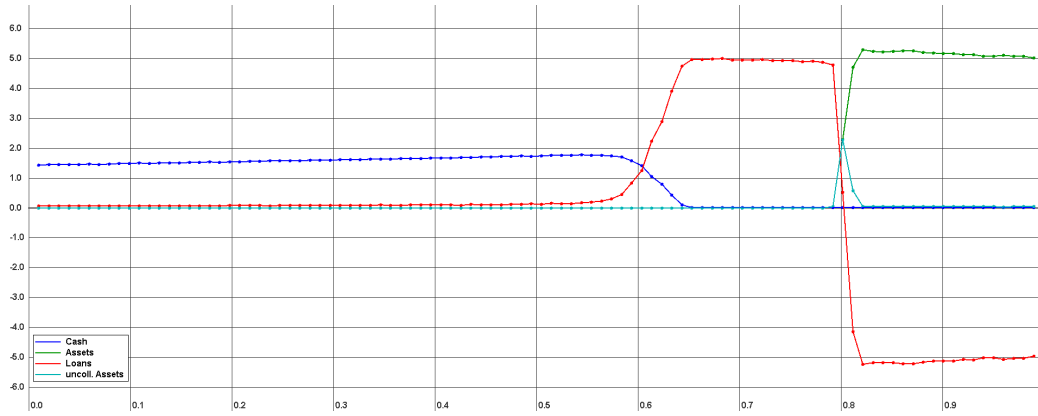

Figure 2: Wealth-Distribution of Fully-Connected topology

Table 7: Equilibrium of Fully-Connected topology

Asset-Price	0.689 (0.010)
Loan-Price	0.384 (0.004)
i0 (Marginal Buyer)	0.603 (0.007)
i1 (Marginal Seller)	0.803 (0.003)
Pessimist Wealth	1.597 (0.015)
Medianist Wealth	4.565 (0.113)
Optimist Wealth	5.021 (0.064)

Table 8: Performance of Fully-Connected topology

Successful TX	1916.14 (31.42)
Total TX	6364.8 (1679.21)
Failed TX	4448.66 (1668.93)

6.4 Ascending-Connected Topology
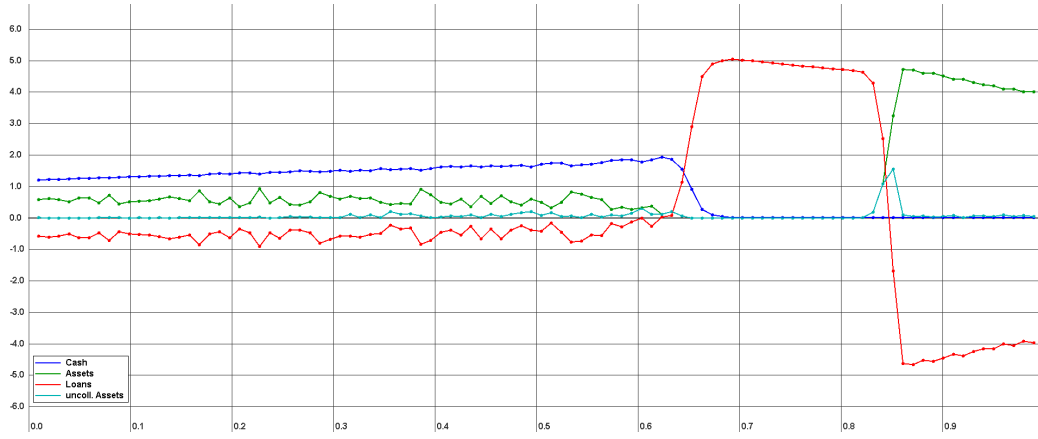
Figure 3: Wealth-Distribution of Ascending-Connected topology

Table 9: Equilibrium of Ascending-Connected topology

Asset-Price	0.711 (0.016)
Loan-Price	0.391 (0.005)
i0 (Marginal Buyer)	0.646 (0.012)
i1 (Marginal Seller)	0.850 (0.008)
Pessimist Wealth	1.166 (0.072)
Medianist Wealth	1.869 (0.243)
Optimist Wealth	4.307 (0.070)

Table 10: Performance of Ascending-Connected topology

Successful TX	36,940.96 (1948.69)
Total TX	38,117.04 (1934.06)
Failed TX	1176.08 (98.01)

TODO: move to interpretation: As can be clearly seen the equilibrium is

fundamentally different than the fully-connected one and thus the hypothesis is not satisfied.

6.4.1 Ascending-Connected Importance Sampling
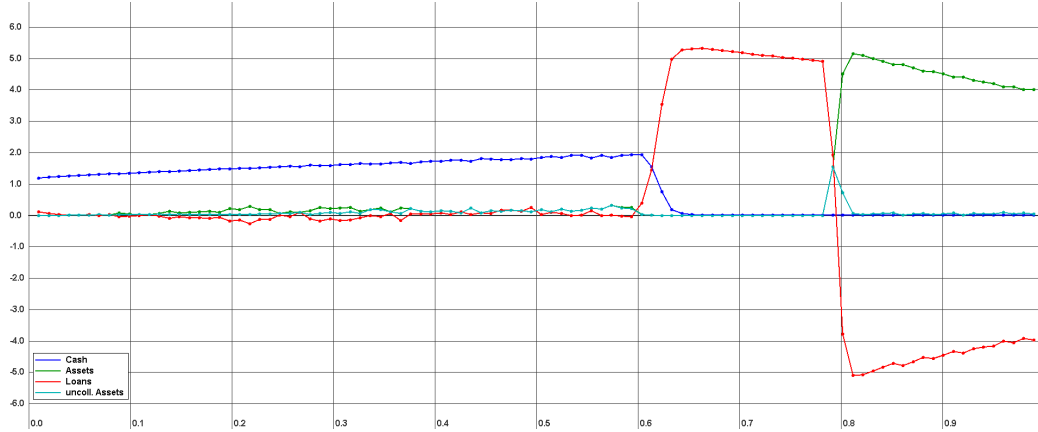
Figure 4: Wealth-Distribution of Ascending-Connected Importance Sampling topology

Table 11: Equilibrium of Ascending-Connected Importance Sampling topology

Asset-Price	0.691 (0.009)
Loan-Price	0.383 (0.004)
i0 (Marginal Buyer)	0.614 (0.009)
i1 (Marginal Seller)	0.799 (0.006)
Pessimist Wealth	1.497 (0.072)
Medianist Wealth	3.934 (0.505)
Optimist Wealth	4.519 (0.051)

Table 12: Performance of Ascending-Connected Importance Sampling topology

Successful TX	49,881.6 (1733.33)
Total TX	49,882.6 (1733.33)
Failed TX	1.0 (0.00)

Note that in this case the matching-probabilities are such that upon the first failed transaction the equilibrium has reached as no agent can trade with each other anymore which results in just on single failed transaction.

TODO: move to interpretation: As can be clearly seen the equilibrium is fundamentally different than the fully-connected one and thus the hypothesis is not satisfied.

Chapter 7

Interpretation

In this chapter interpretation of the results of Chapter 6 "Results" are given and discussed. Only the Ascending-Connected topology is handled - both without and with importance sampling - because it is the most minimal network which satisfies the requirements for the hypothesis. The Hub-, Scale-Free and Small-World Topologies are handled in appendix C as they turn out to fall far from satisfying the hypothesis because and almost all of them do not satisfy the hypothesis. Special treatment is given to Erdos-Renyi and Watts-Strogatz as they satisfy the hypothesis when using specific parameters for their generating algorithms.

Validating the Hypothesis When looking at the results of ascending-connected topology with and without importance sampling from Chapter 6 "Results" of figure 4 and 3 and comparing it with the results of the fully-connected topology of figure 2 it becomes immediately clear that the equilibrium is different from the equilibrium of the fully-connected network and thus theoretical equilibrium is not reached in the case of ascending-connected topology neither with or without importance sampling. Although the visual results come quite close to the fully-connected one - there is a clear distinction between pessimists, medianists and optimists and the wealth-distribution looks about the same as in fully-connected - there remain artefacts in the range of the pessimists. Thus the hypothesis is proven wrong by experiment.

Analysing pessimists artefacts Obviously the artefacts in the range of the pessimists indicate a miss-allocation of wealth, which are in fact collateralized assets. Pessimists, as noted in Chapter 3 "The Leverage Cycle", are maximally short on assets and bonds and hold only cash, thus it is clearly a miss-allocation. As will be shown it comes from the fact that the pessimists want to sell but no neighbour is able to buy any more - a scenario which

is not possible in fully-connected topology and is thus unique to ascending-connected networks with or without importance sampling.

Dynamics of a single run To better understand how such artefacts arise one needs to investigate the dynamics of a single run of ascending-connected topology.

The wealth stabilizes from both the left and the right end of the optimism-scale towards the i1-point where medianists become optimists - around this point the last trades will happen.

Pessimists try to sell all their assets against cash to the neighbour with higher optimism-factor.

Optimists try to buy as much assets as they can get from the neighbour with lower optimism-factor. In the beginning they use cash and after they've ran out of cash they buy assets against bonds.

The medianists serve as connection between the pessimists and optimists, serving in transferring the assets to the optimists by buying from lower optimism and selling to higher ones either through asset against cash or asset against bond.

Thus the assets move from the pessimists through the ascending chain of optimism to the optimists as no direct connection between these two groups exists with the medianists in between. Thus waves of uncollateralized assets can be seen moving from pessimists to optimists.

It is important to understand that all agents despite their optimism factor make offers on all markets if they are able to, e.g. cash, collateral or bond constraints satisfied. This implies that pessimists trade bonds as well as assets against bonds - the agents are not defined exogenous as pessimists/medianist/optimists but this property is emerges during the simulation.

Thus pessimists gain wealth in collateralized assets which can be seen by the green spikes with the same amount of negative bonds as those assets are bought against bond - see Chapter **??**. Of course they try to sell it to neighbours with higher optimism factor but this is only possible if these neighbours are able to buy which they can if they hold enough bonds to buy the offered asset for the offered amount of bonds.

Whether an agent has enough wealth to buy from a seller is more or less random and depends on its trading history. Matching happens randomly and thus it is possible that the neighbourhood of a seller "dries up" as the potential buyers sold all their good to the next agent with higher optimism

factor and become thus unable to buy from the potential seller because they have no more bonds to buy assets against bonds. In such a case a potential pessimist seller of collateralized assets is then cut from its environment and becomes unable to trade any more resulting in a miss-allocation in collateralized assets.

It is also possible for a group of agents to get cut from its environment through this random trading-process. In this case this "island" of agents still trades between each other resulting in the uncollateraliziing of assets which immediately are traded towards optimists but as soon as a point is reached where no buyer is available with enough bonds to buy collateralized assets this island is also incapable of trading any more resulting in an island of miss-allocated wealth.

An important fact to notice is that the artefacts must not necessarily show up. It is possible for a single run to finish without these artefacts showing up. This is due to the random-process of sweeping and matching and thus the artefacts are subject to this random process too. Importance sampling elevates this problem a bit as it allows for more trades as the matching probabilities are very much increased but fails in the end for the same reason as the simulation without it - the artefacts are just "smaller" but show up almost always.

Extending the Hypothesis   After it has become clear that the hypothesis is wrong the question arises what needs to be done to correct the it. It is clear that a mechanism needs to be found which prevents or resolves the arising of the artefacts within the pessimist wealth-range. Obviously two solutions are available:

Approaching fully connectedness
Re-enabling inactive pessimists to trade

Approaching fully connectedness   Increasing the connectedness of the topology increases the probability of global-optimal trades and allows more agents to trade between each other and thus the probability of resolving islands or artefacts of wealth miss-allocation is increased with the density of connectedness. The experiments of ascending-connected topology with shortcuts were desigend to get an understanding how the simulation behaves with increasing connectedness and also how the two types fully and regular of connectedness influence the results. It seems that full shortcuts seem to help dramatically although the number of full shortcuts seem to be dependent

on the number of agents. TODO: verweis auf results und interpretation appendixe

Of course in real environments approaching fully connectedness is not always possible and thus only the other mechanism is left as an option to resolve the artefacts.

Re-Enabling trading   Another way to look at the arising of the artefacts is because of suboptimal trades. [BSV13] were confronted with this circumstance when they introduced the "Asset against Bond" Market where they found that the equilibrium was fundamentally different from theoretical one because agents were trapped in suboptimal trades and couldn't reverse their decisions made earlier. As a solution they introduced the "Bonds-Pledgeability" (BP) mechanism which allows to trade bonds in both ways instead of only gathering them and not being able to sell them - see **??** for a more in-depth discussion of the BP-Mechanism.

Thus if those artifacts are treated as suboptimal trades one needs to introduce a mechanism similar to BP to allow the reversibility of suboptimal trades in the context of collateralized assets. The only possibility without altering the network-topology is to re-enable the pessimists to trade their collateralized assets against cash as all pessimists hold cash and are thus able to buy and sell collateralized assets against cash. This new mechanism is expected to repair the miss-allocated wealth and to restore the validity of the previously disproved hypothesis.

See Chapter **??** "A new Market" for the implementation and discussion of the results of this new mechanism.

Chapter 8

A new Market

In the thesis-software it is possible to run an ascending-connected topology until no more trades are possible and then activate the new market. the collateralized assets are bought and sold by the pessimists and move up the optimism-scale until they end up in the optimists.

neuer markt: Collateralisierte ASsets gegen Cash Neuer Markt: Collateralisierte Assets gegen Cash Käufer: bekommt Asset und Loan, zahlt Cash Verkäufer: bekommt Cash und gibt Asset und Loan limit-preis für diesen markt? Fläche unter P/M/O gleich bei Fully-Connected und Ascending-Connected obwohl unterschiedliche

Erweiterung mit zweiter Hypothese zweite Hypothese: erreicht ascending-connected nun mit dem neuen markt das gleichgewicht? ergebnis der zweiten hypothese: funktioniert

8.1   Results with the new market

As experiment-configuration the same as given in Chapter 5 "Results" is used.

Table 13: Configuration for all experiments

Agent-Count	100
Bond-Type	0.5
Replication-Count	50
Terminate after	1000 failed successive Transactions

8.1.1   Fully-Connected
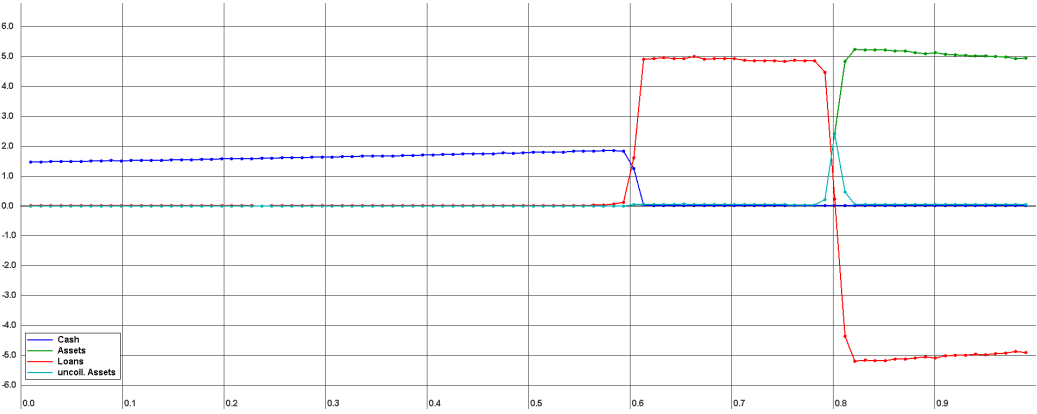
Figure 5: Wealth-Distribution of Fully-Connected topology
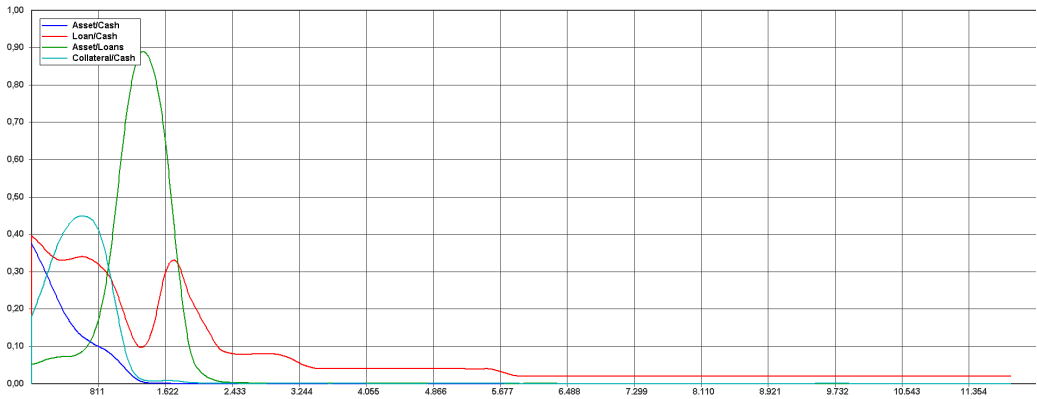
Figure 6: Market-activity over time of Fully-Connected topology
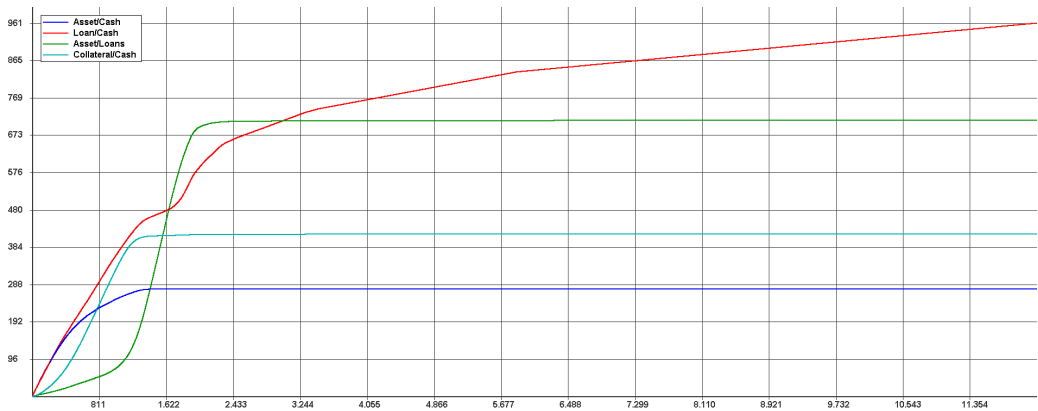
Figure 7: Market-activity accumulated of Fully-Connected topology

8.1.2 Ascending-Connected
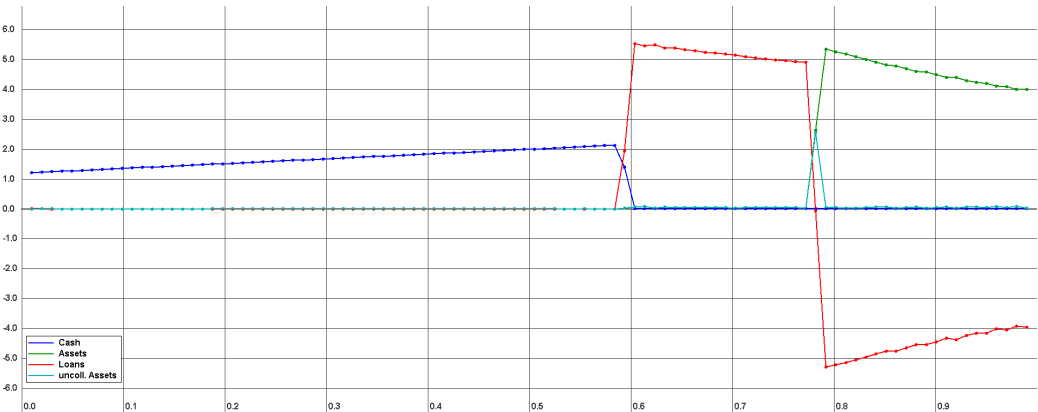
Figure 8: Wealth-Distribution of Ascending-Connected topology
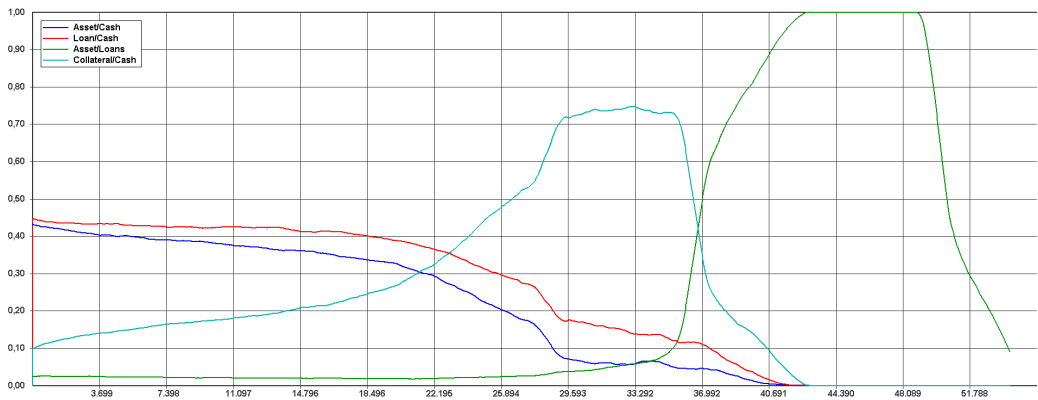
Figure 9: Market-activity over time of Ascending-Connected topology
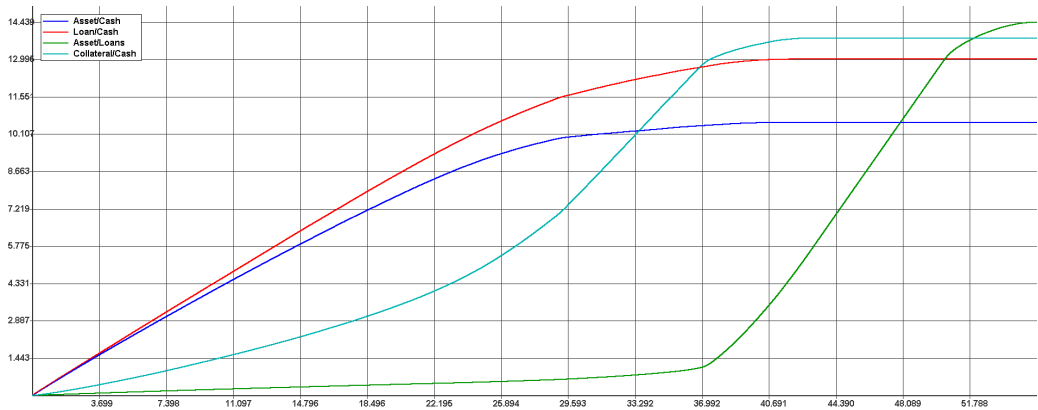
Figure 10: Market-activity accumulated of Ascending-Connected topology

Chapter 9

Conclusion, Summary and further Research

importance-sampling allgemein experimentelle simulationen mit echten menschen: einschränken der handelsbeziehungen wie lokal bzw. global muss die vernetzung sein (ascending-connected full shortcuts) beweisbarkeit der ascending-connected (MIT/OHNE neuem Markt)

9.1 Conclusion

9.2 Summary

9.3 Further Research

Appendix A

Topologies

All topologies are demonstrated with 30 Agents only for better visibility and übersicht of edges. All topologies have connected-component of 1 (TODO: warum) except Erdos-Renyi can produce connected-component ¿ 1.

A.1 Fully-Connected
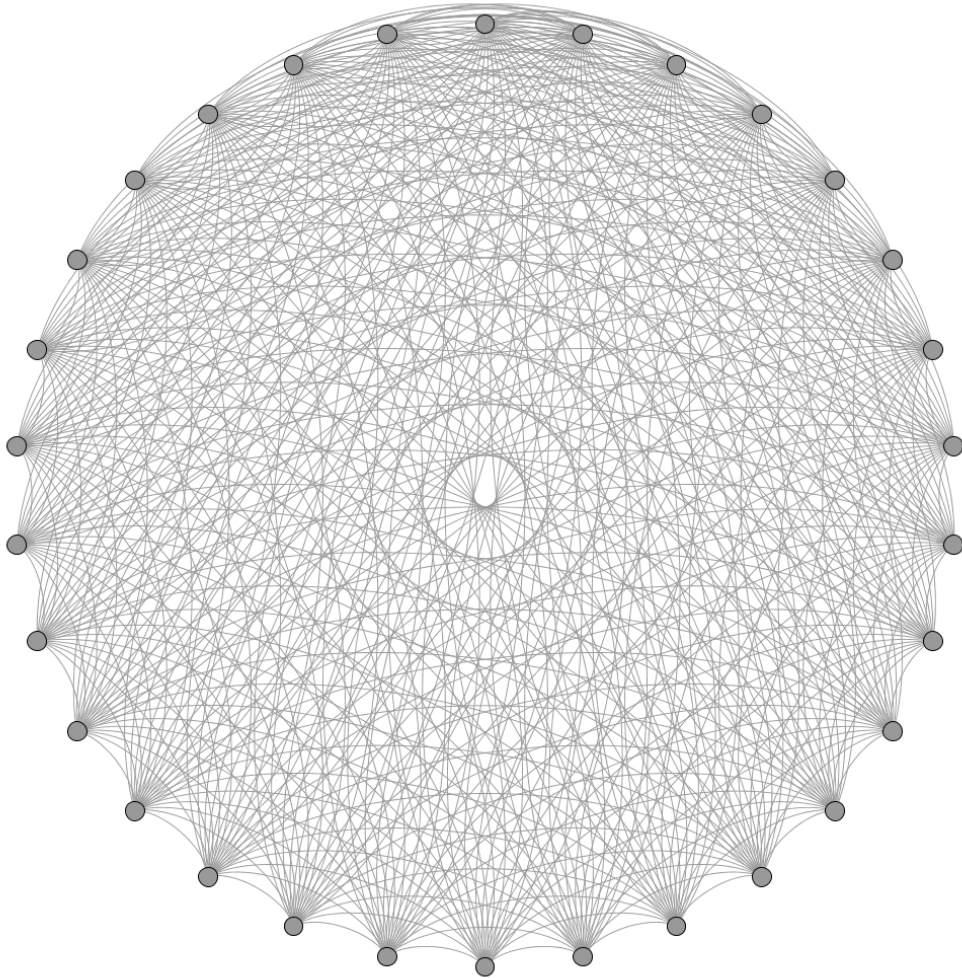
Figure 11: Fully-Connected topology

Table 14: Network metrics Fully-Connected topology

Avg. degree	29
Avg. path-length	1
Avg. clustering coefficient	1
Network diameter	1
Graph density	1

A.2 Half-Fully Connected
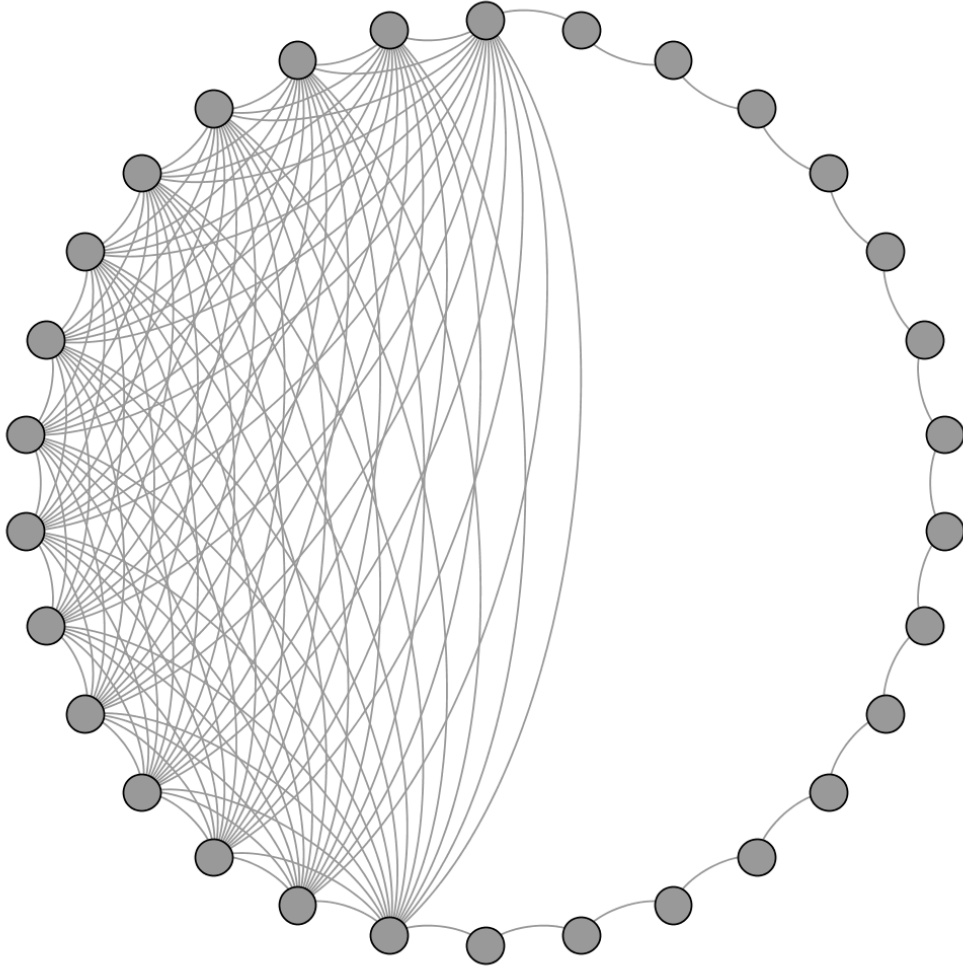
Figure 12: Half Fully-Connected topology

Table 15: Network metrics Half Fully-Connected topology

Avg. degree	8.067
Avg. path-length	4.007
Avg. clustering coefficient	0.491
Network diameter	9
Graph density	0.278

A.3 Ascending-Connected
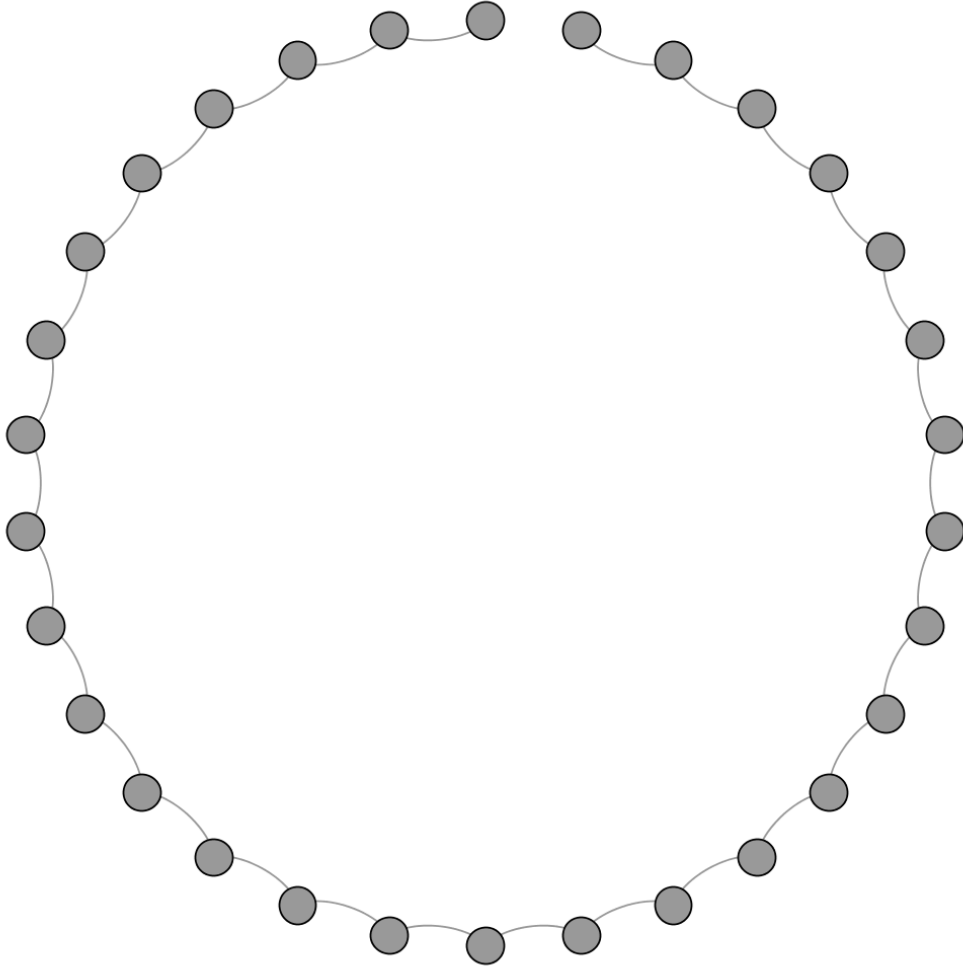
Figure 13: Ascending-Connected topology

Table 16: Network metrics Ascending-Connected topology

Avg. degree	1.933
Avg. path-length	10.33
Avg. clustering coefficient	0
Network diameter	29
Graph density	0.067

A.4 Ascending-Connected with short-cuts

A.4.1 Full short-cuts
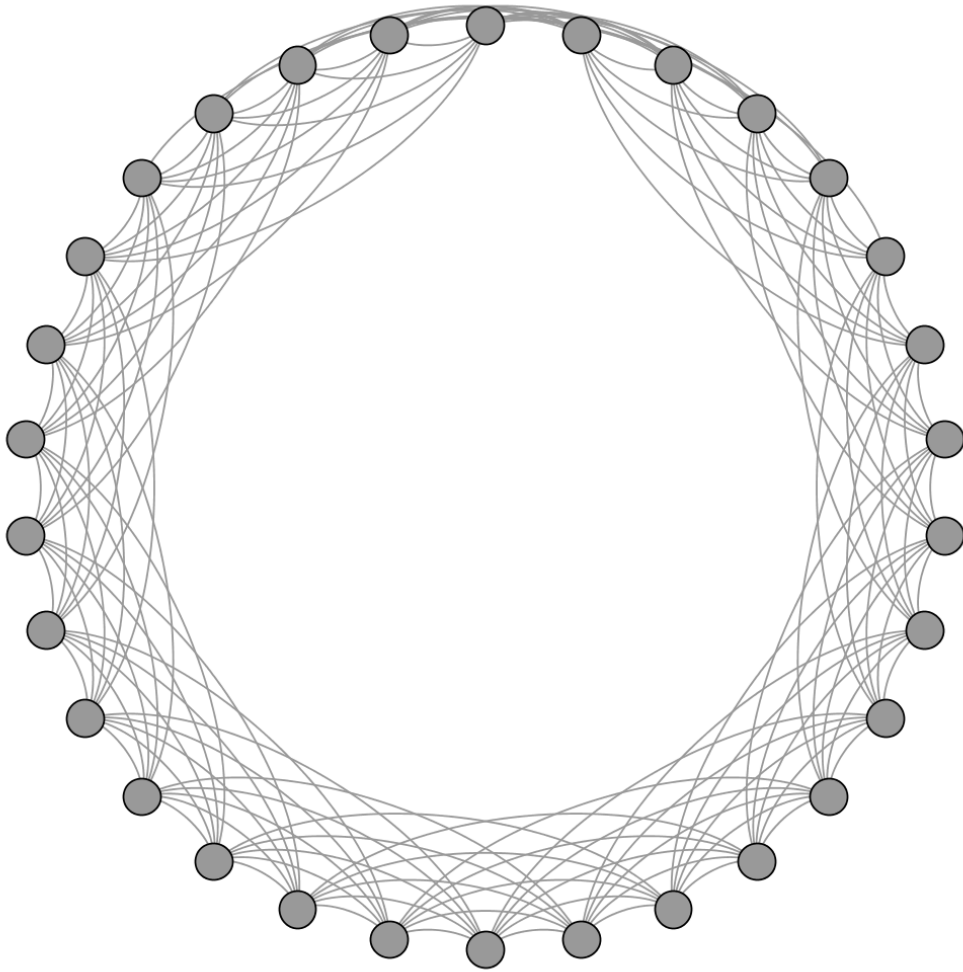
Figure 14: Ascending-Connected 5 full short-cuts topology

Table 17: Network metrics Ascending-Connected 5 full short-cuts topology

Avg. degree	10
Avg. path-length	1.966
Avg. clustering coefficient	0.667
Network diameter	3
Graph density	0.345

A.4.2 Regular short-cuts
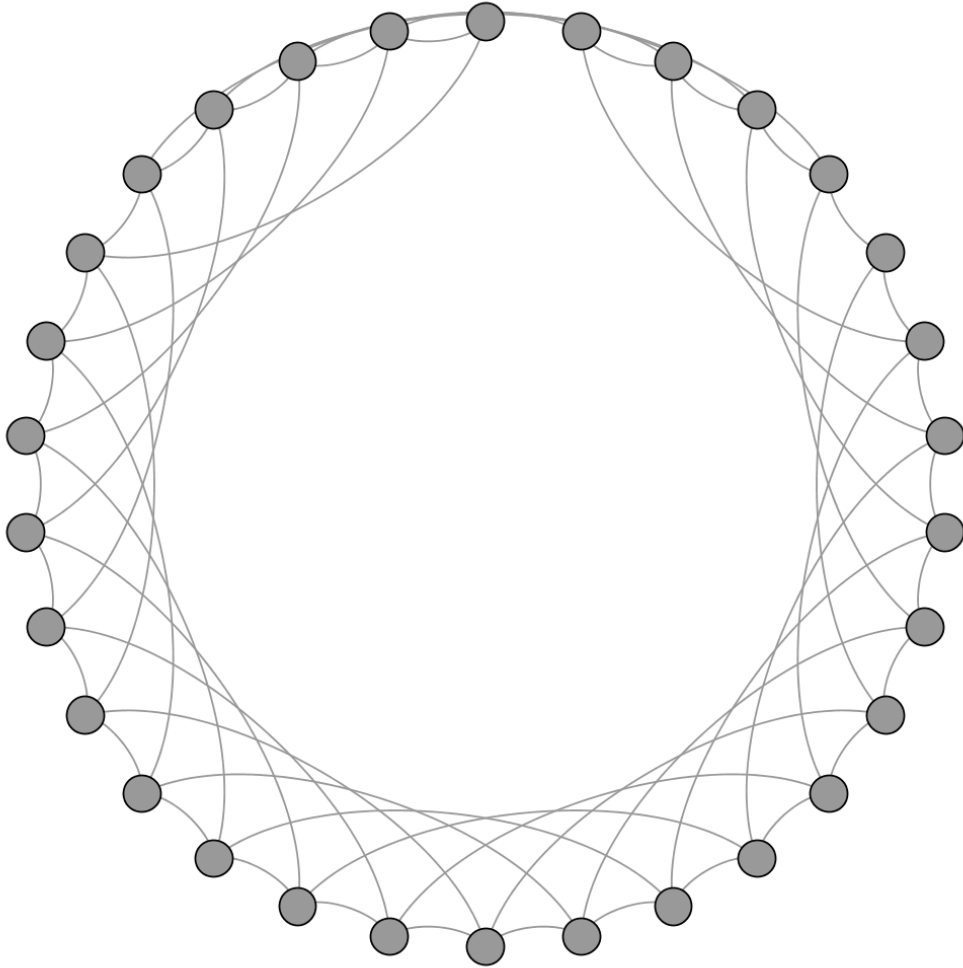
Figure 15: Ascending-Connected 5 regular short-cuts topology

Table 18: Network metrics Ascending-Connected 5 regular short-cuts topology

Avg. degree	3.867
Avg. path-length	2.839
Avg. clustering coefficient	0
Network diameter	6
Graph density	0.133

A.4.3 Random short-cuts
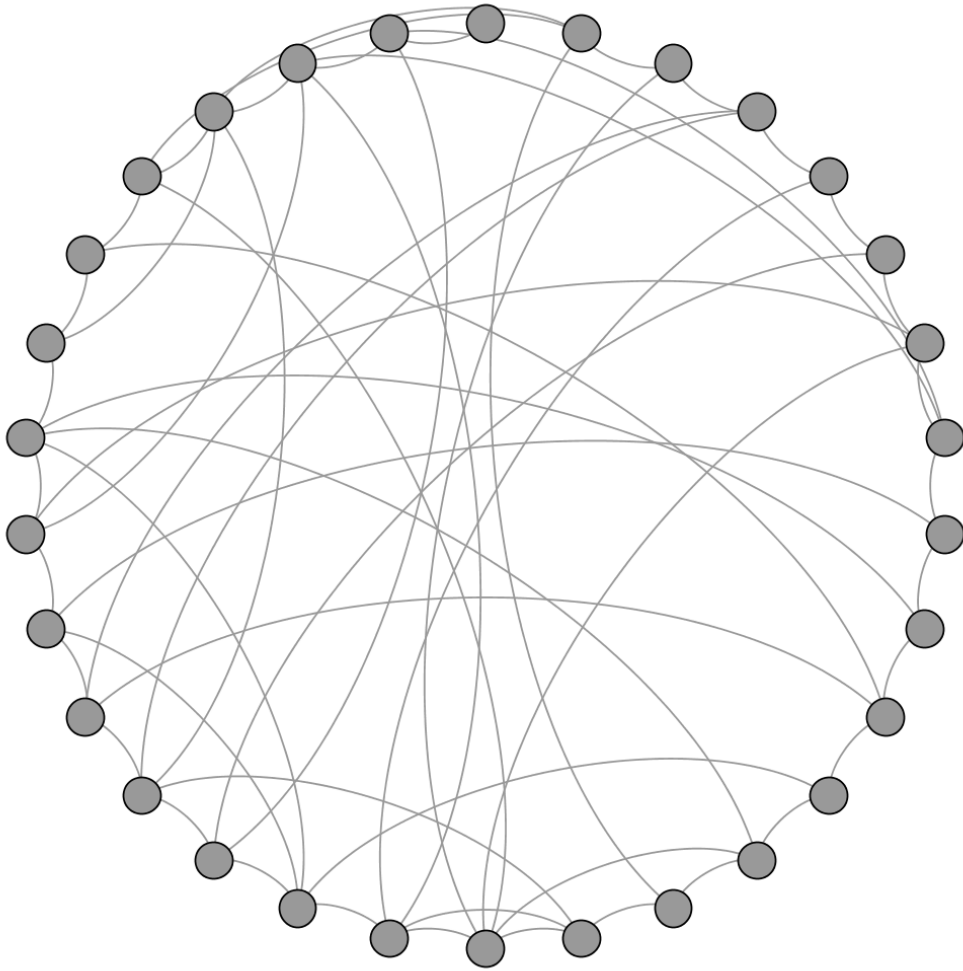
Figure 16: Ascending-Connected random short-cuts probability 1.0 topology

Table 19: Network metrics Ascending-Connected random short-cuts topology

Avg. degree	3.867
Avg. path-length	2.506
Avg. clustering coefficient	0.056
Network diameter	5
Graph density	0.133

A.5 Hub-based topologies

A.5.1 3 Hubs
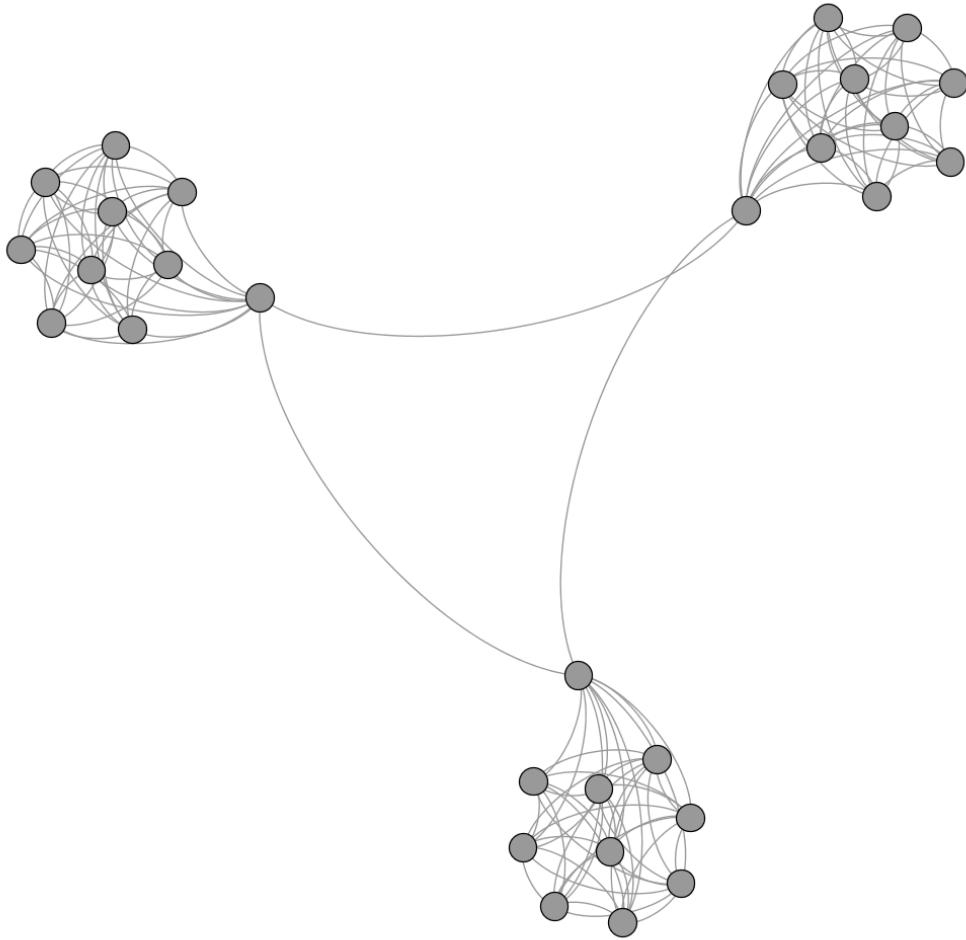
Figure 17: 3 Hubs topology

Table 20: Network metrics 3 Hubs topology

Avg. degree	9.2
Avg. path-length	2.241
Avg. clustering coefficient	0.976
Network diameter	3
Graph density	0.371

A.5.2   3 Median Hubs
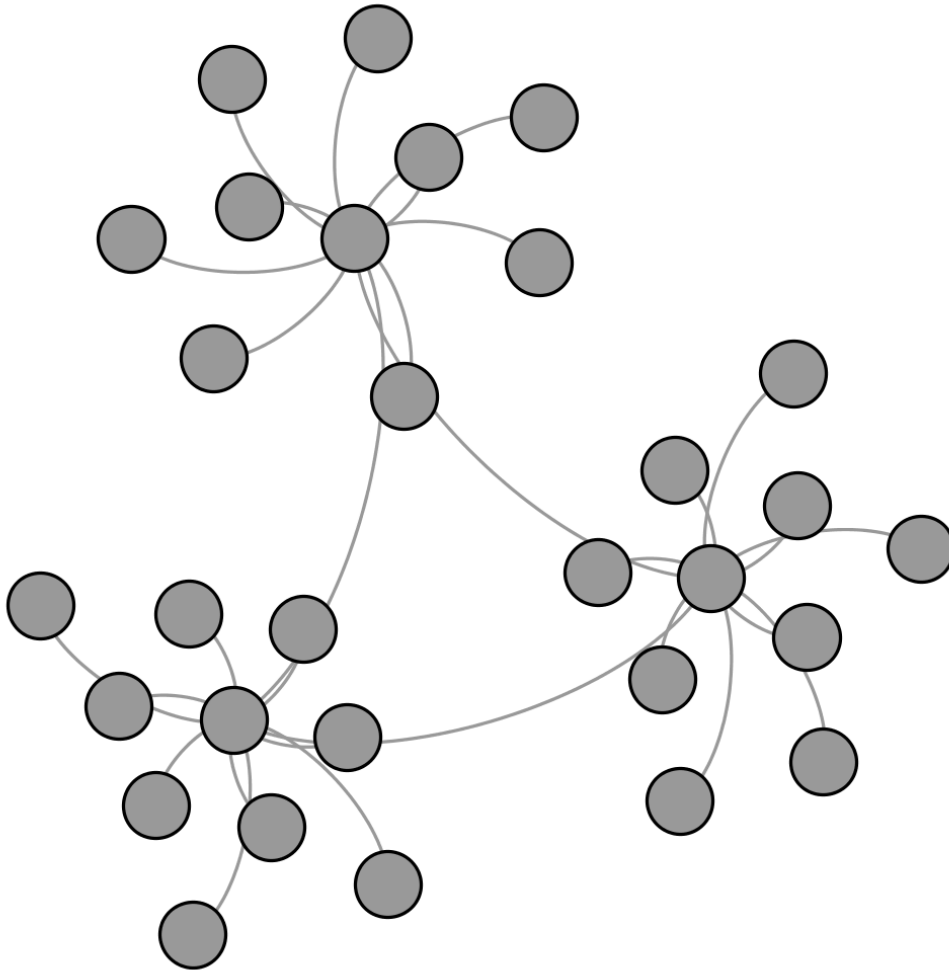
Figure 18: 3 Median Hub topology

Table 21: Network metrics 3 Median Hub topology

Avg. degree	2
Avg. path-length	2.49
Avg. clustering coefficient	0.018
Network diameter	3
Graph density	0.069

A.5.3   Median Hub
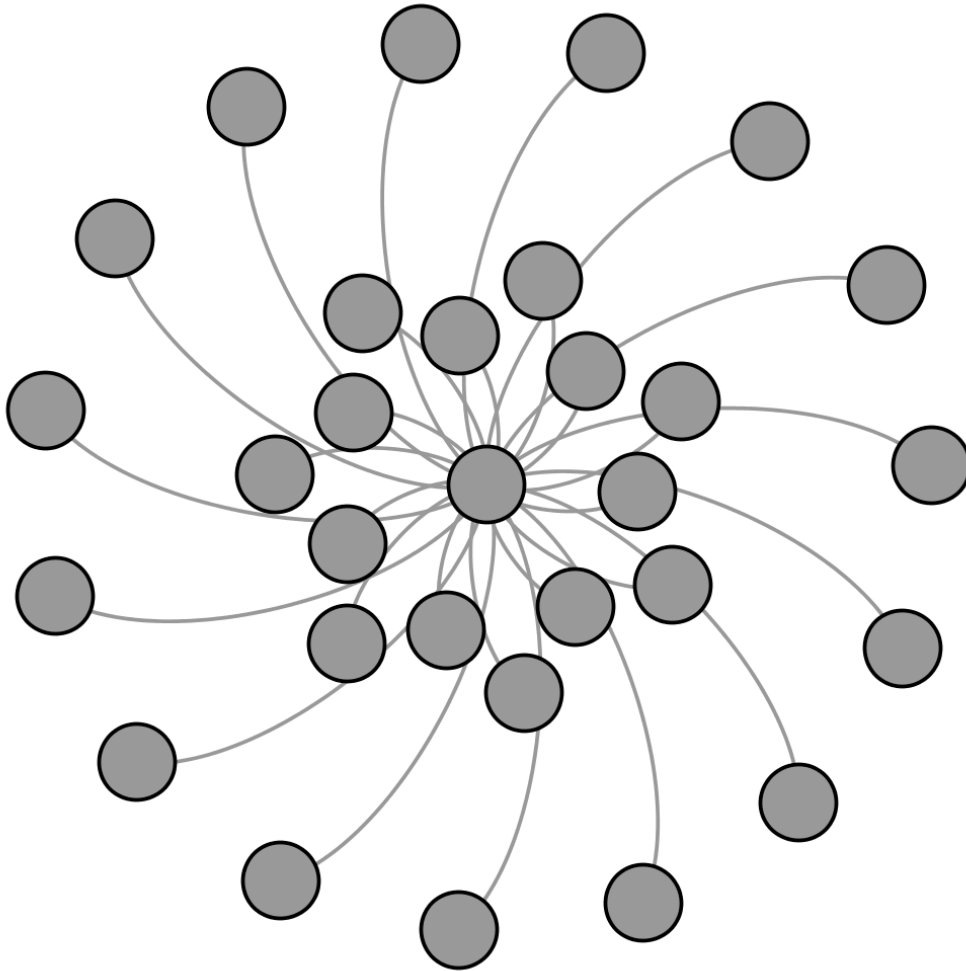
Figure 19: Median Hub topology

Table 22: Network metrics Median Hub topology

Avg. degree	1.933
Avg. path-length	1.933
Avg. clustering coefficient	0
Network diameter	2
Graph density	0.067

A.5.4 Maximum Hub

Looks the same as 1 Median Hub but all edges are connected to the agent with the highest optimism-value. Has thus also the same metrics as the optimism-values have no functional influence on the metrics.

A.6 Small-World and Scale-Free topologies

A.6.1 Erods-Renyi
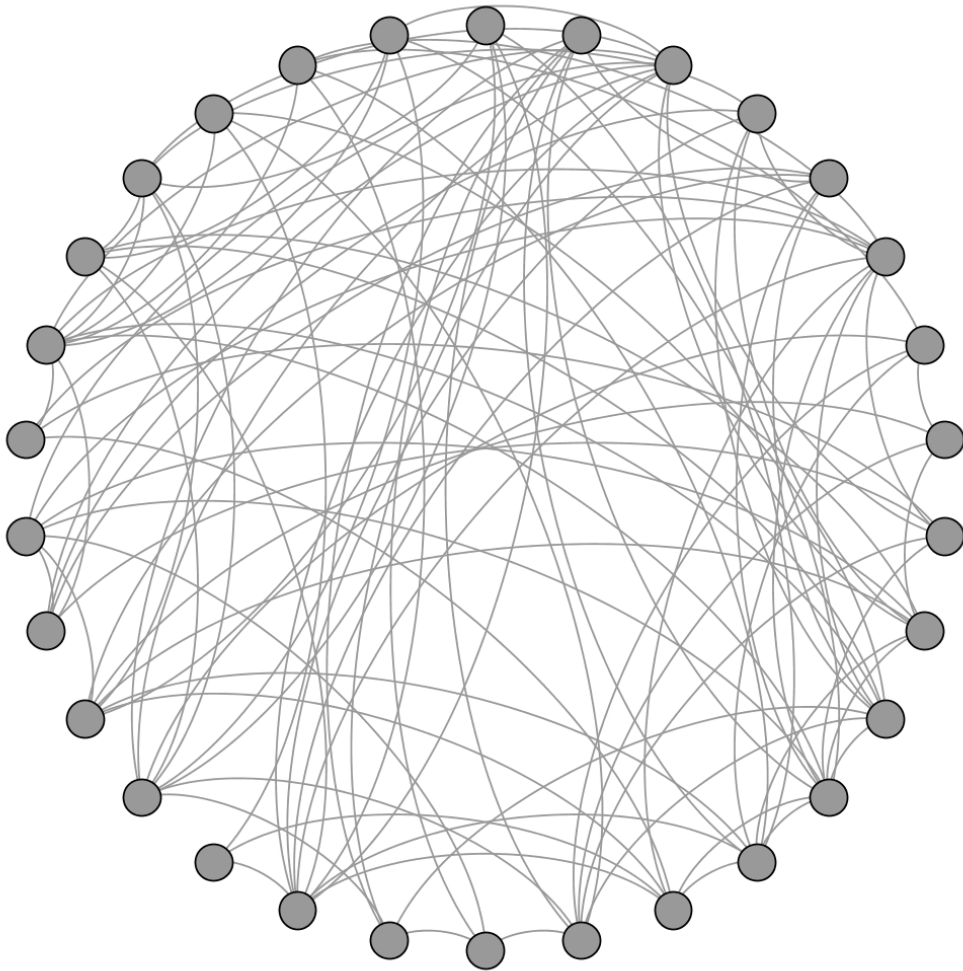
Figure 20: Erdos-Renyi topology with inclusion-probability of 0.2

Table 23: Network metrics Erdosy-Renyi 0.2

Avg. degree	6.8
Avg. path-length	1.913
Avg. clustering coefficient	0.266
Network diameter	3
Graph density	0.234
Connected component	1
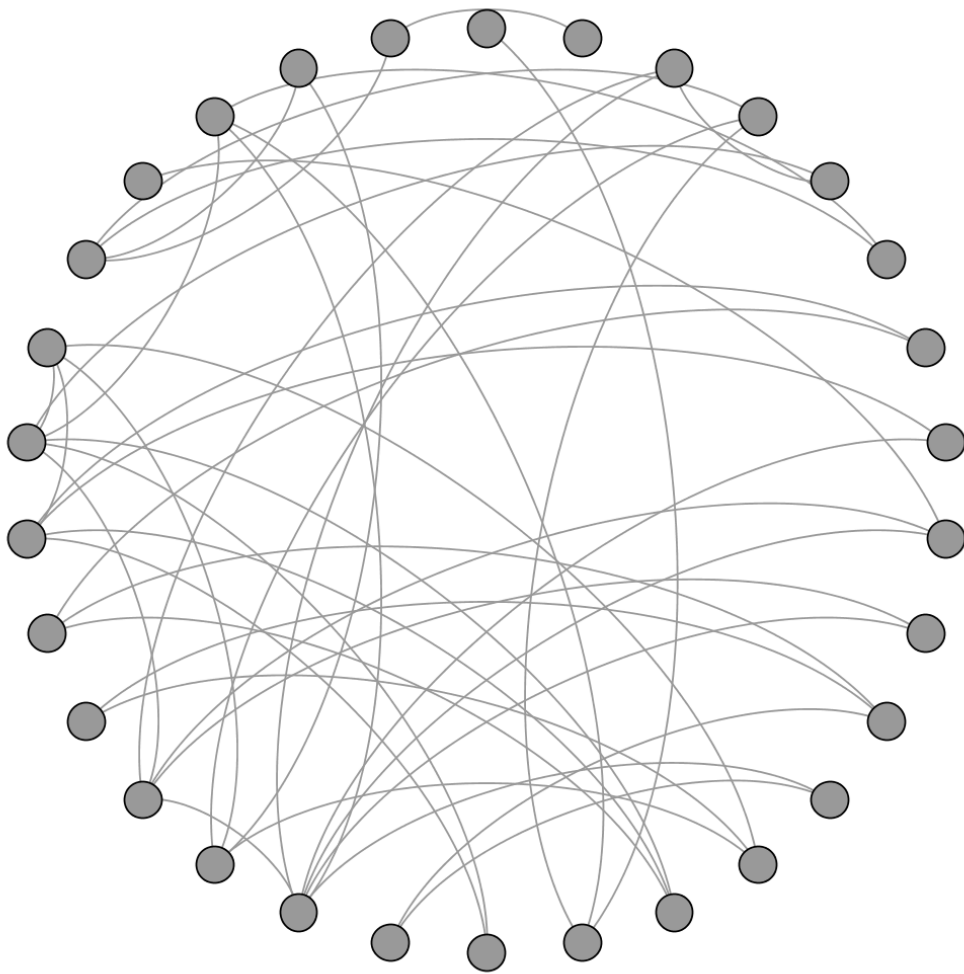
Figure 21: Erdos-Renyi topology with inclusion-probability of 0.1

Table 24: Network metrics Erdosy-Renyi 0.1

Avg. degree	2.933
Avg. path-length	3.262
Avg. clustering coefficient	0.103
Network diameter	7
Graph density	0.101
Connected component	1
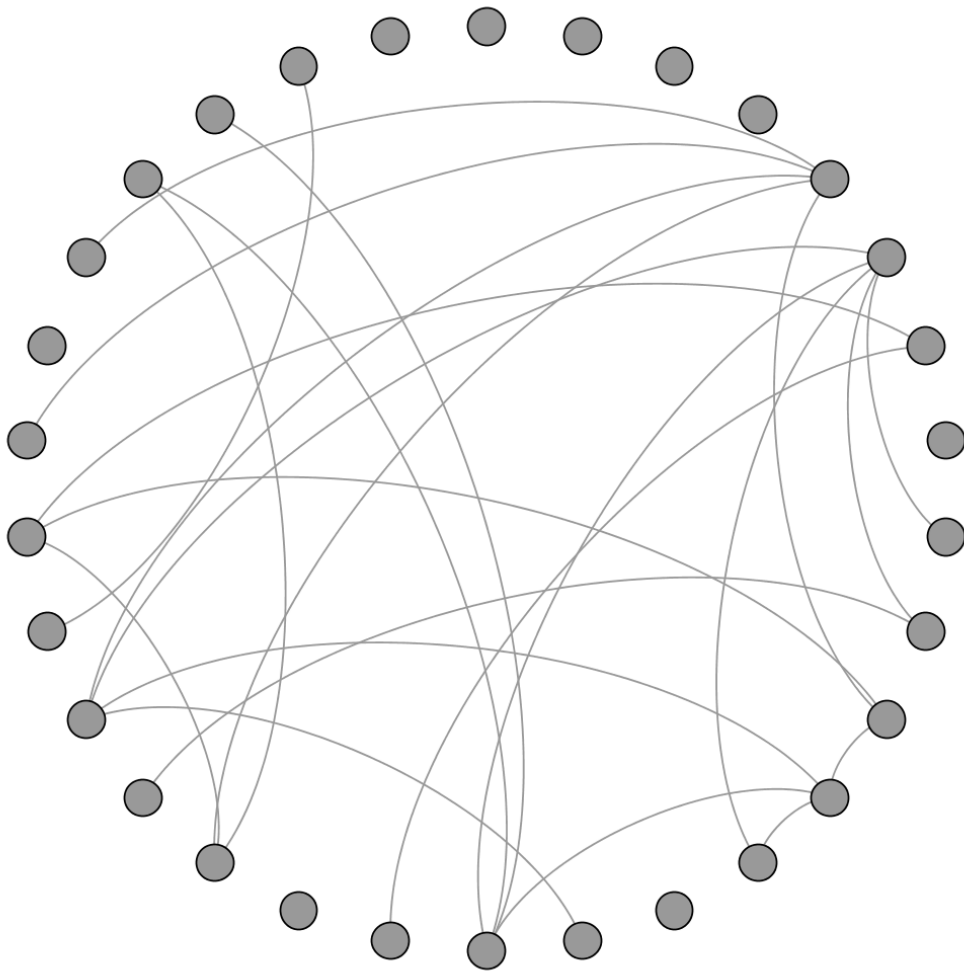
Figure 22: Erdos-Renyi topology with inclusion-probability of 0.05

Table 25: Network metrics Erdosy-Renyi 0.05

Avg. degree	1.6
Avg. path-length	3.052
Avg. clustering coefficient	0
Network diameter	8
Graph density	0.055
Connected component	11

A.6.2 Barbasi-Albert
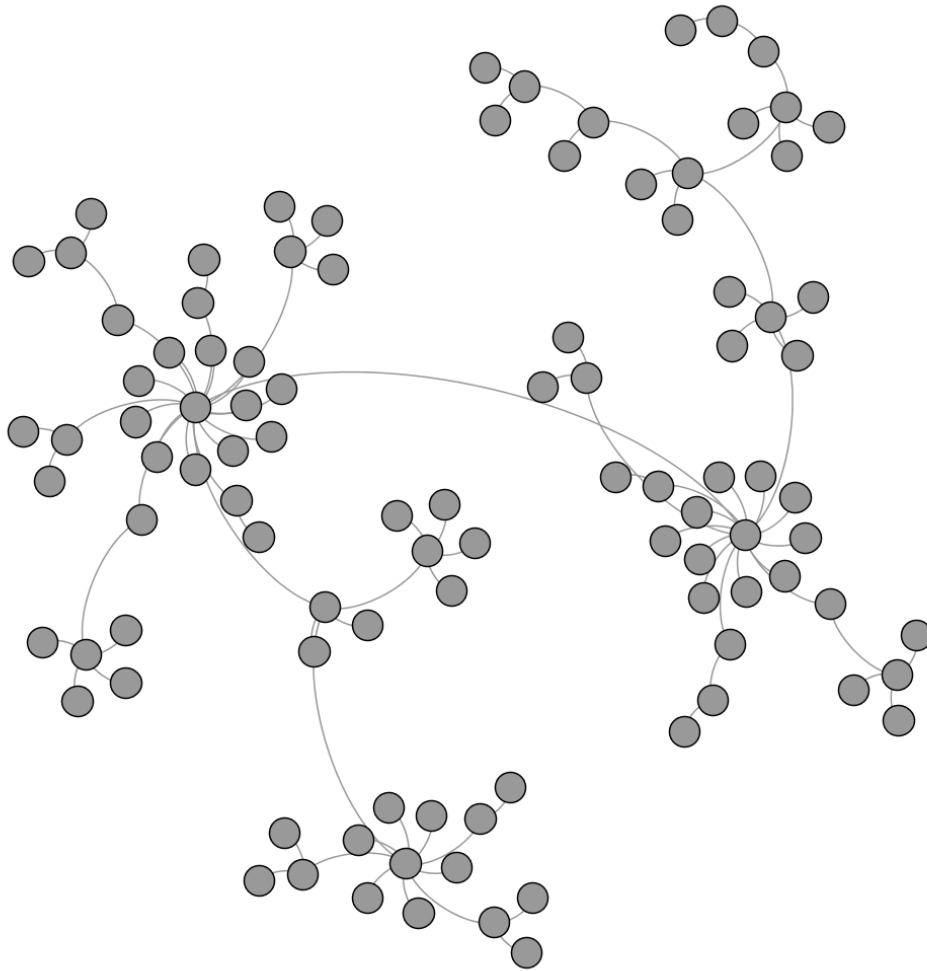
Figure 23: Barbasi-Albert topology with m0=3, m=1

Table 26: Network metrics Barbasi-Albert m0=3, m=1

Avg. degree	1.98
Avg. path-length	4.684
Avg. clustering coefficient	0
Network diameter	11
Graph density	0.02
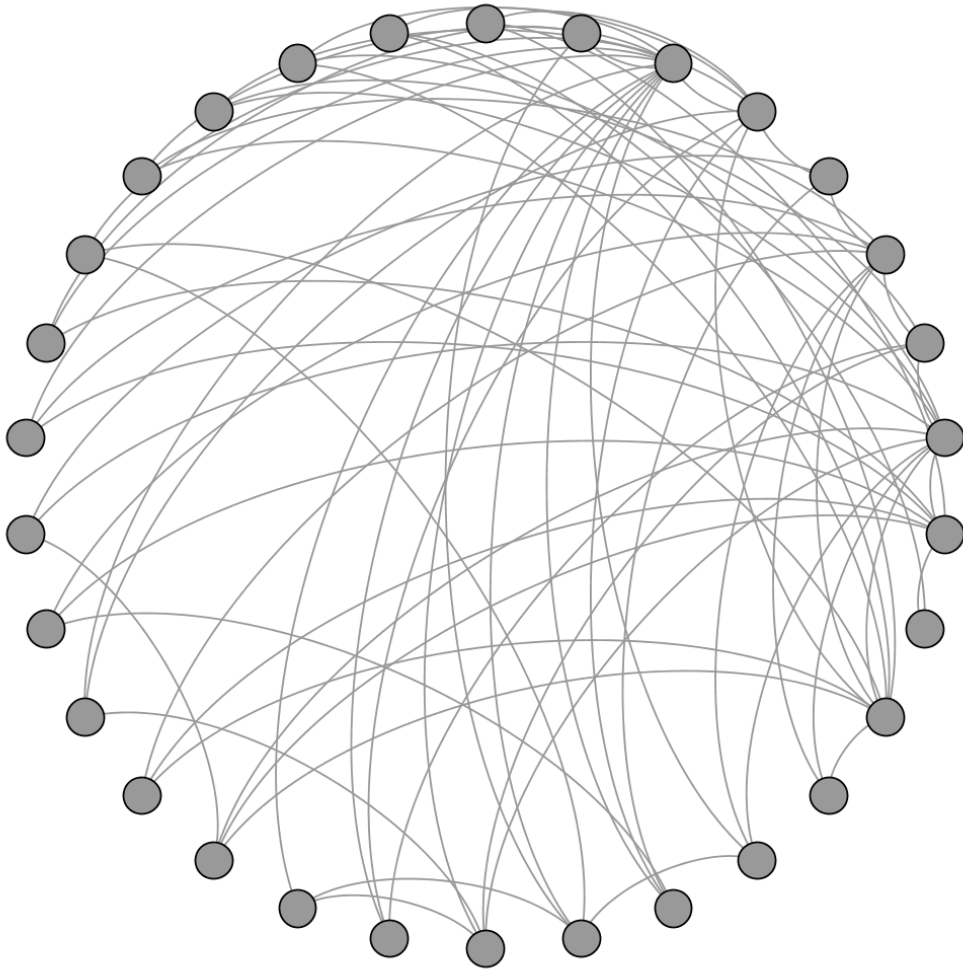
Figure 24: Barbasi-Albert topology with m0=9, m=3

Table 27: Network metrics Barbasi-Albert m0=9, m=3

Avg. degree	4.733
Avg. path-length	2.11
Avg. clustering coefficient	0.279
Network diameter	4
Graph density	0.163

A.6.3 Watts-Strogatz

Two params: k and p Creates N nodes and connects each to k neighbours and rewires each then existing edge with a probability of 0.2 to another node with lower id (younger).
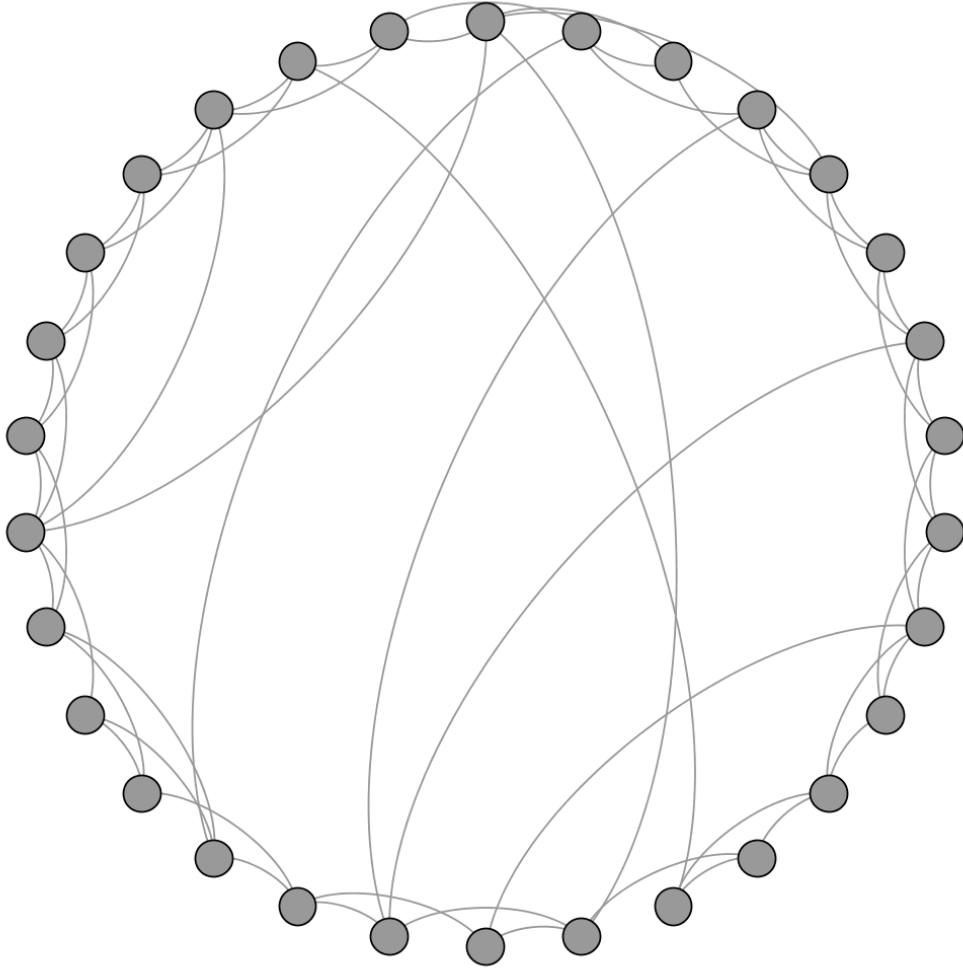
Figure 25: Watts-Strogatz topology with k=2, p=0.2

Table 28: Network metrics Watts-Strogatz k=2, p=0.2

Avg. degree	4
Avg. path-length	2.883
Avg. clustering coefficient	0.259
Network diameter	6
Graph density	0.138
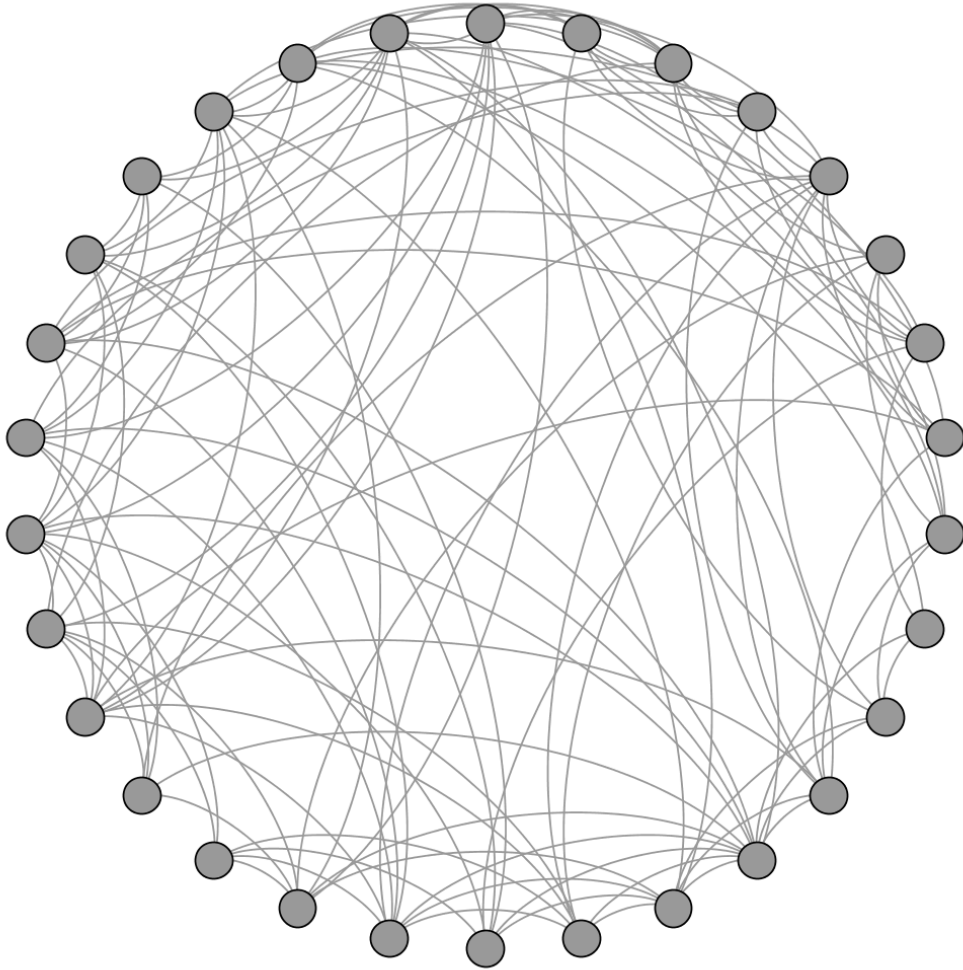
Figure 26: Watts-Strogatz topology with k=4, p=0.5

Table 29: Network metrics Watts-Strogatz k=4, p=0.5

Avg. degree	8
Avg. path-length	1.823
Avg. clustering coefficient	0.241
Network diameter	3
Graph density	0.276

Appendix B

Visual Results for Hub-Based, Scale-Free and Small-World Topologies

B.1   Half-Fully Connected
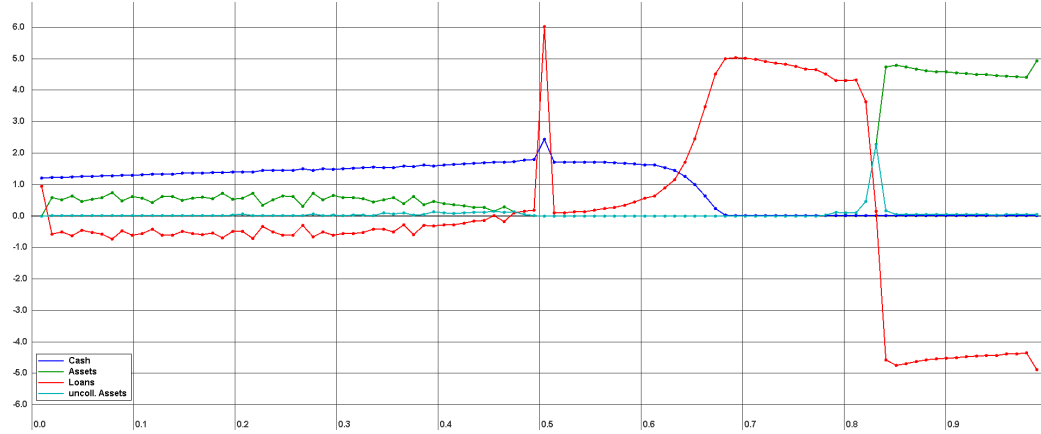
Figure 27: Wealth-Distribution of Half-Fully Connected topology

Table 30: Equilibrium of Half-Fully Connected topology

Asset-Price	0.651 (0.027)
Loan-Price	0.362 (0.013)
i0 (Marginal Buyer)	0.640 (0.015)
i1 (Marginal Seller)	0.833 (0.09)
Pessimist Wealth	1.22 (0.096)
Medianist Wealth	2.258 (0.409)
Optimist Wealth	4.526 (0.071)

Table 31: Performance of Half-Fully Connected topology

Successful TX	14,218.9 (4621.74)
Total TX	15,253.02 (4633.44)
Failed TX	1034.12 (22.99)

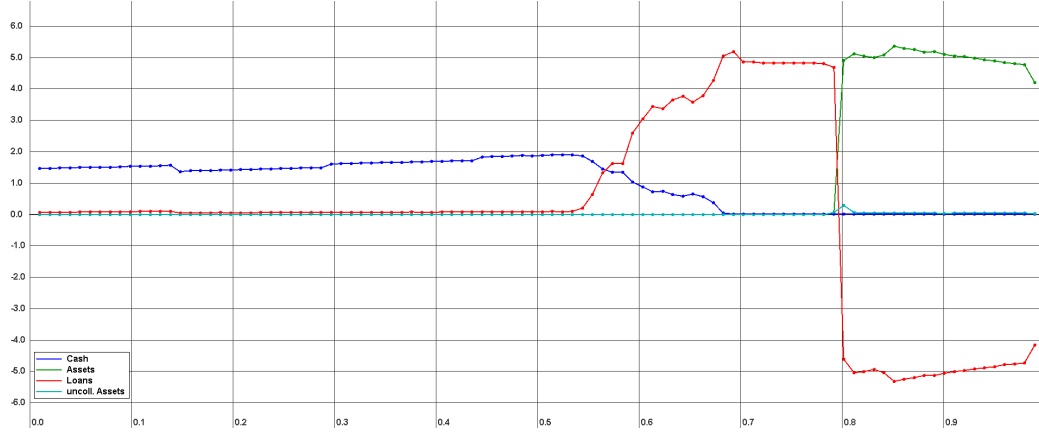## B.2 Ascending-Connected with short-cuts

B.2.1 Random short-cuts
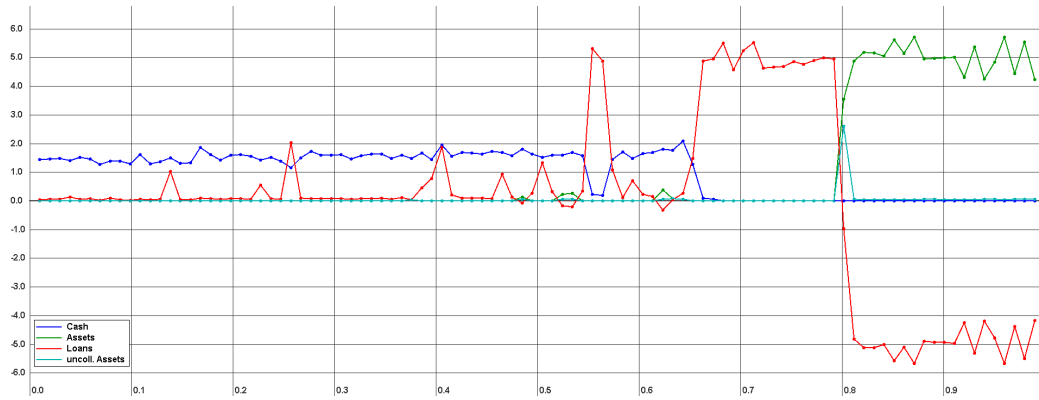
Figure 28: Wealth-Distribution of Ascending-Connected random short-cuts topology

Table 32: Equilibrium of Ascending-Connected random short-cuts topology

Asset-Price	0.731 (0.019)
Loan-Price	0.393 (0.009)
i0 (Marginal Buyer)	0.649 (0.005)
i1 (Marginal Seller)	0.804 (0.004)
Pessimist Wealth	1.441 (0.03)
Medianist Wealth	4.282 (0.278)
Optimist Wealth	4.974 (0.038)

Table 33: Performance of Ascending-Connected random short-cuts topology

Successful TX	8314.78 (229.85)
Total TX	9496.84 (228.23)
Failed TX	1182.06 (29.23)

B.2.2   2 short-cuts
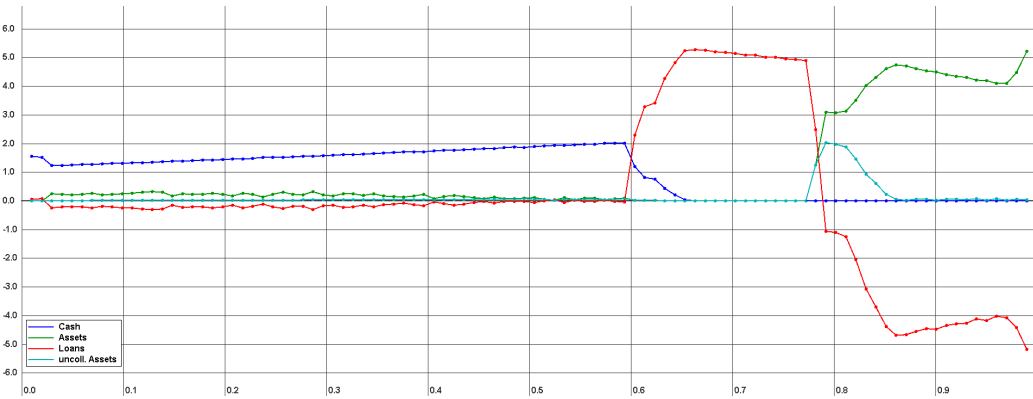
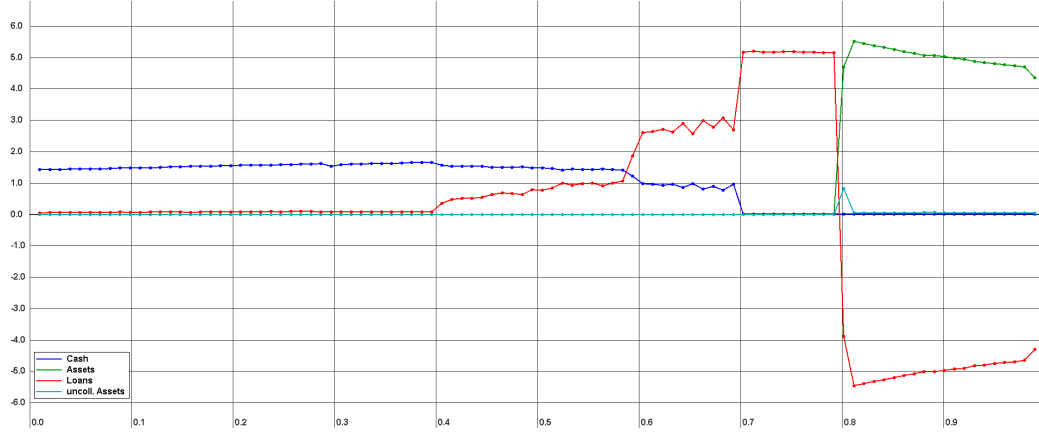
Figure 29: Wealth-Distribution of Ascending-Connected 2 short-cuts topology

Table 34: Equilibrium of Ascending-Connected 2 short-cuts topology

Asset-Price	0.662 (0.024)
Loan-Price	0.376 (0.006)
i0 (Marginal Buyer)	0.608 (0.018)
i1 (Marginal Seller)	0.805 (0.028)
Pessimist Wealth	1.441 (0.21)
Medianist Wealth	3.978 (1.442)
Optimist Wealth	4.514 (0.063)

Table 35: Performance of Ascending-Connected random short-cuts topology

Successful TX	37,093.64 (12,864.4)
Total TX	38,115.54 (12,851.53)
Failed TX	1021. (18.85)

B.2.3   5 full short-cuts
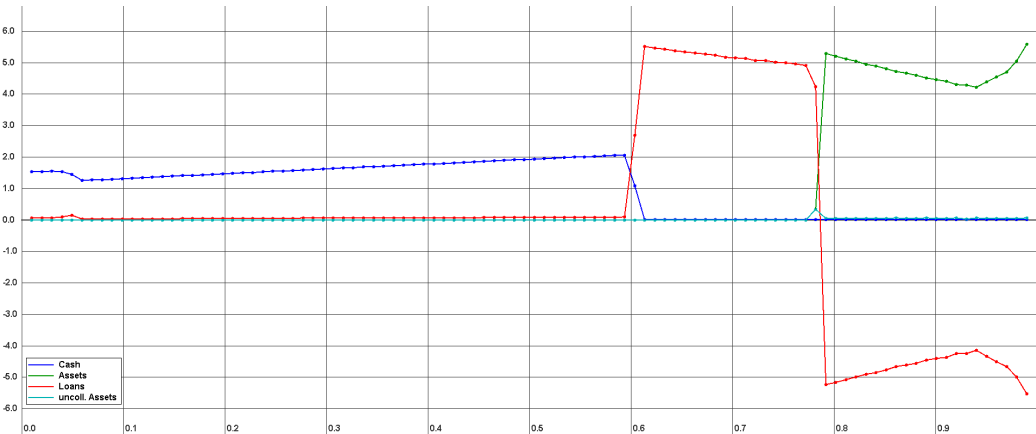
Figure 30: Wealth-Distribution of Ascending-Connected 5 full short-cuts topology

TODO: move to interpretation: As can be clearly seen 5 full shortcuts seem to be already enough to solve the inefficiencies seen in Ascending-Connected with/without Importance Sampling.

Table 36: Equilibrium of Ascending-Connected 5 full short-cuts

Asset-Price	0.656 (0.019)
Loan-Price	0.371 (0.003)
i0 (Marginal Buyer)	0.594 (0.0)
i1 (Marginal Seller)	0.792 (0.0)
Pessimist Wealth	1.649 (0.002)
Medianist Wealth	5.013 (0.018)
Optimist Wealth	4.746 (0.011)

Table 37: Performance of Ascending-Connected 5 full short-cuts topology

Successful TX	16,971.34 (228.0)
Total TX	17,998.26 (225.23)
Failed TX	1026.92 (22.68)

B.2.4 15 full short-cuts
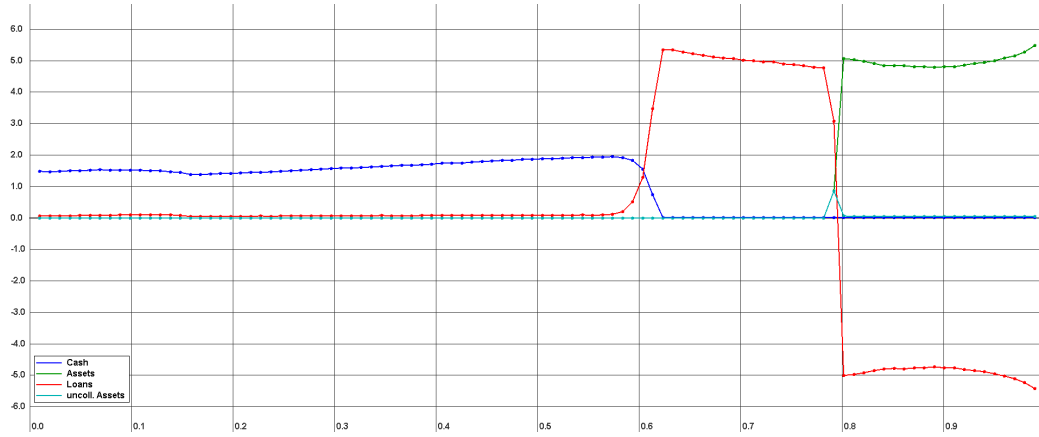
Figure 31: Wealth-Distribution of Ascending-Connected 15 full short-cuts topology

Table 39: Performance of Ascending-Connected 15 full short-cuts topology

Successful TX	4498.08 (58.67)
Total TX	5522.860 (64.72)
Failed TX	1024.78 (17.3)

Table 38: Equilibrium of Ascending-Connected 15 full short-cuts topology

Asset-Price	0.658 (0.024)
Loan-Price	0.366 (0.009)
i0 (Marginal Buyer)	0.601 (0.004)
i1 (Marginal Seller)	0.802 (0.0)
Pessimist Wealth	1.649 (0.004)
Medianist Wealth	4.811 (0.092)
Optimist Wealth	4.957 (0.021)

B.2.5 30 full short-cuts
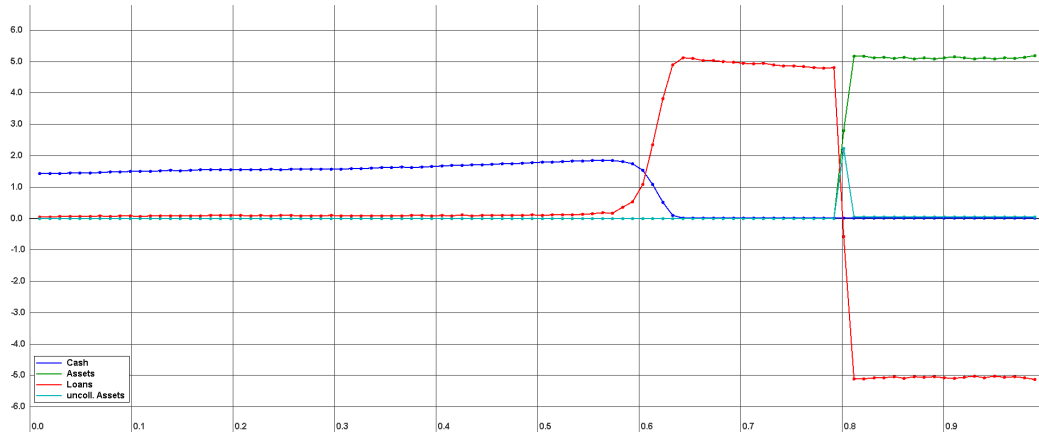
Figure 32: Wealth-Distribution of Ascending-Connected 30 full short-cuts topology

Table 40: Equilibrium of Ascending-Connected 30 full short-cuts topology

Asset-Price	0.681 (0.012)
Loan-Price	0.378 (0.006)
i0 (Marginal Buyer)	0.603 (0.006)
i1 (Marginal Seller)	0.802 (0.1)
Pessimist Wealth	1.649 (0.009)
Medianist Wealth	4.702 (0.112)
Optimist Wealth	5.004 (0.025)

Table 41: Performance of Ascending-Connected 30 full short-cuts topology

Successful TX	2211.08 (35.88)
Total TX	3225.76 (40.18)
Failed TX	1014.68 (10.55)

B.2.6   5 regular short-cuts
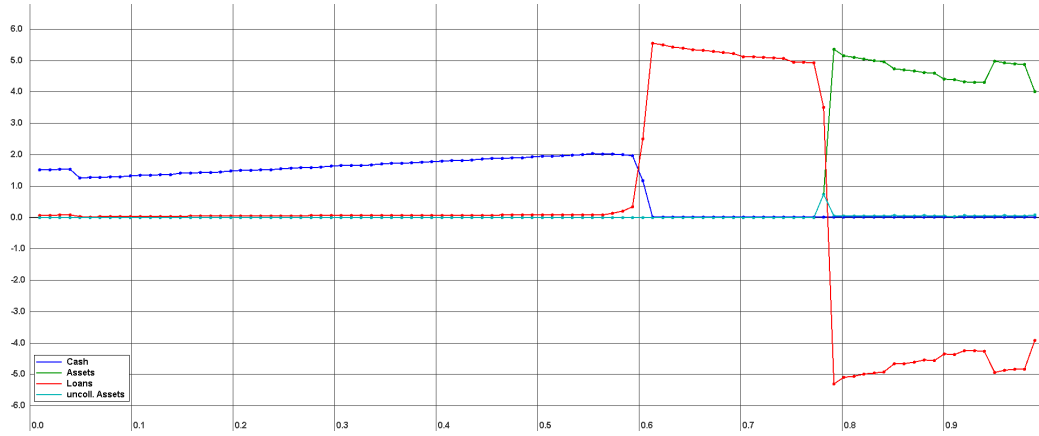
Figure 33: Wealth-Distribution of Ascending-Connected 5 regular short-cuts topology

Table 42: Equilibrium of Ascending-Connected 5 regular short-cuts topology

Asset-Price	0.665 (0.016)
Loan-Price	0.364 (0.007)
i0 (Marginal Buyer)	0.595 (0.003)
i1 (Marginal Seller)	0.792 (0.0)
Pessimist Wealth	1.649 (0.003)
Medianist Wealth	4.991 (0.045)
Optimist Wealth	4.727 (0.011)

Table 43: Performance of Ascending-Connected 5 regular short-cuts topology

Successful TX	14,570.44 (157.61)
Total TX	15,634.68 (166.21)
Failed TX	1064.24 (29.88)

B.2.7   15 regular short-cuts

Figure 34: Wealth-Distribution of Ascending-Connected 15 regular short-cuts topology

Table 44: Equilibrium Ascending-Connected 15 regular short-cuts topology

Asset-Price	0.705 (0.020)
Loan-Price	0.357 (0.018)
i0 (Marginal Buyer)	0.586 (0.023)
i1 (Marginal Seller)	0.802 (0.0)
Pessimist Wealth	1.649 (0.051)
Medianist Wealth	4.146 (0.101)
Optimist Wealth	4.997 (0.007)

Table 45: Performance of Ascending-Connected 15 regular short-cuts topology

Successful TX	4373.28 (50.13)
Total TX	5502.52 (52.11)
Failed TX	1129.24 (19.2)

B.2.8   30 regular short-cuts

Figure 35: Wealth-Distribution of Ascending-Connected 30 regular short-cuts topology

Table 46: Equilibrium of Ascending-Connected 30 regular short-cuts topology

Asset-Price	0.710 (0.021)
Loan-Price	0.398 (0.008)
i0 (Marginal Buyer)	0.589 (0.021)
i1 (Marginal Seller)	0.802 (0.0)
Pessimist Wealth	1.479 (0.049)
Medianist Wealth	3.713 (0.125)
Optimist Wealth	5.0 (0.0)

Table 47: Performance of Ascending-Connected 30 regular short-cuts topology

Successful TX	5427.02 (90.82)
Total TX	6566.06 (96.04)
Failed TX	1139.04 (27.74)

B.3   Hub-Based topologies

The Hub-Based Topologies fail to come even close to equilibrium due to reasons given in Chapter "Topologies and Hypothesis". This can be seen also

very clearly in the visual results and thus no performance- and equilibrium-tables are listed as they would not make any sense.

B.3.1   3-Hubs
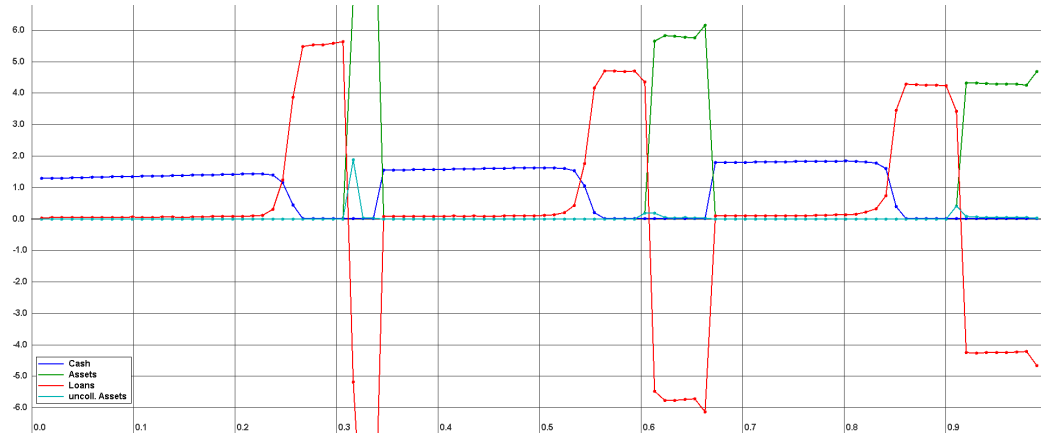
Figure 36: Wealth-Distribution of 3-Hubs topology

B.3.2   1-Median Hub
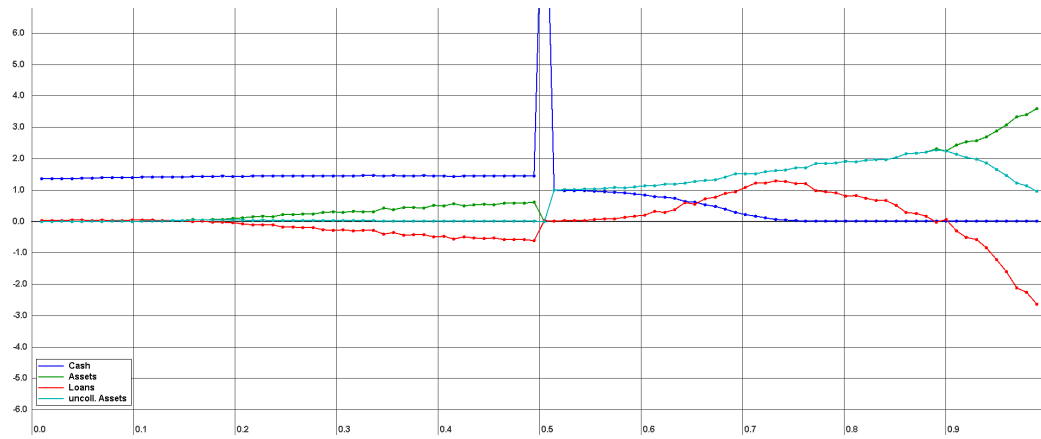
Figure 37: Wealth-Distribution of 1 Median-Hub topology

B.3.3   3-Median Hubs
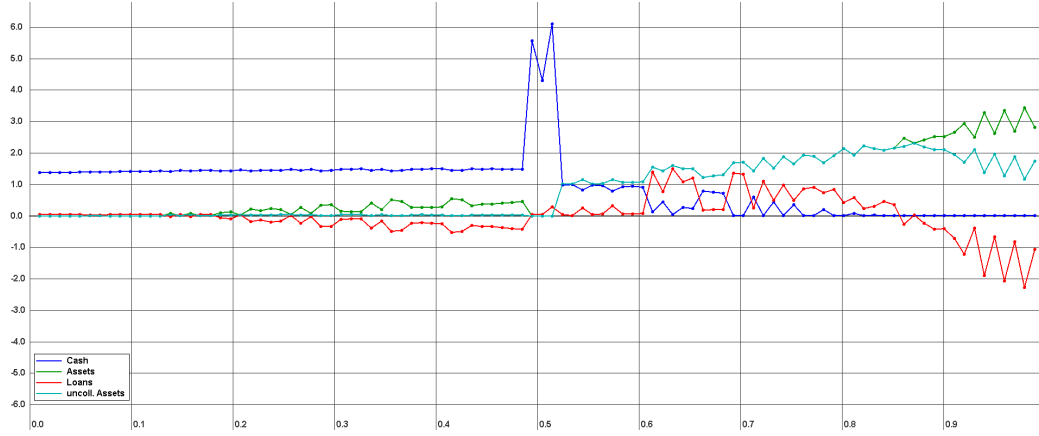
Figure 38: Wealth-Distribution of 3 Median-Hubs topology

B.3.4   Maximum Hub
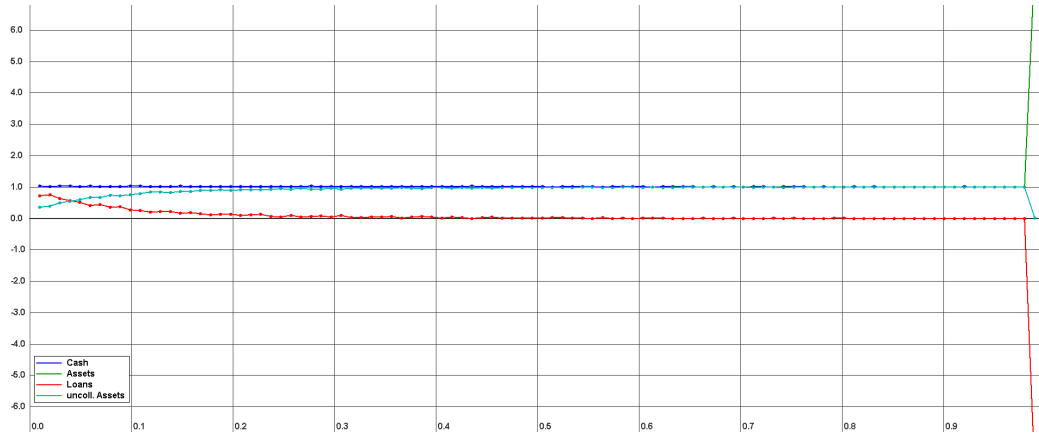
Figure 39: Wealth-Distribution of Maximum-Hub topology

B.4   Scale-Free and Small-World topologies

This topologies fail to come even close to equilibrium too due to reasons given in Chapter "Topologies and Hypothesis". This can be seen also very clearly in the visual results and thus no performance- and equilibrium-tables are listed as they would not make any sense.

B.4.1 Erdos-Renyi
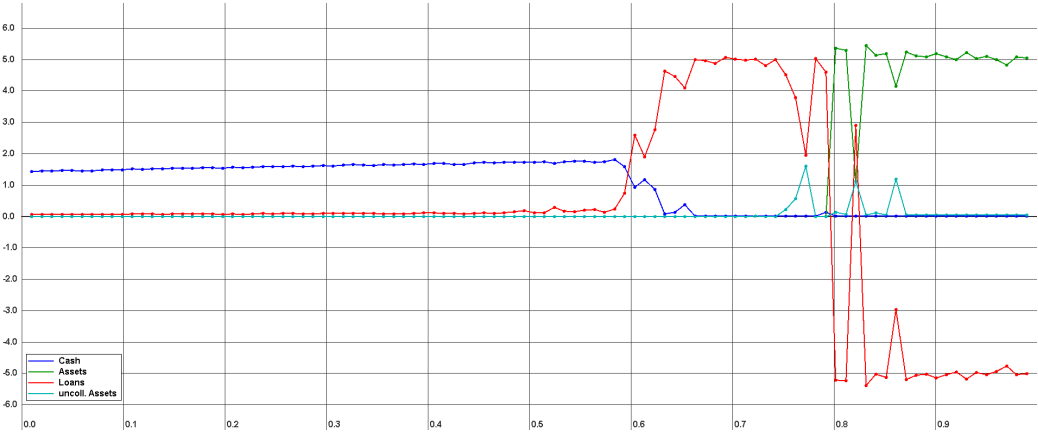
Figure 40: Wealth-Distribution of Erdos-Renyi 0.2 topology

need to show network too because random ?
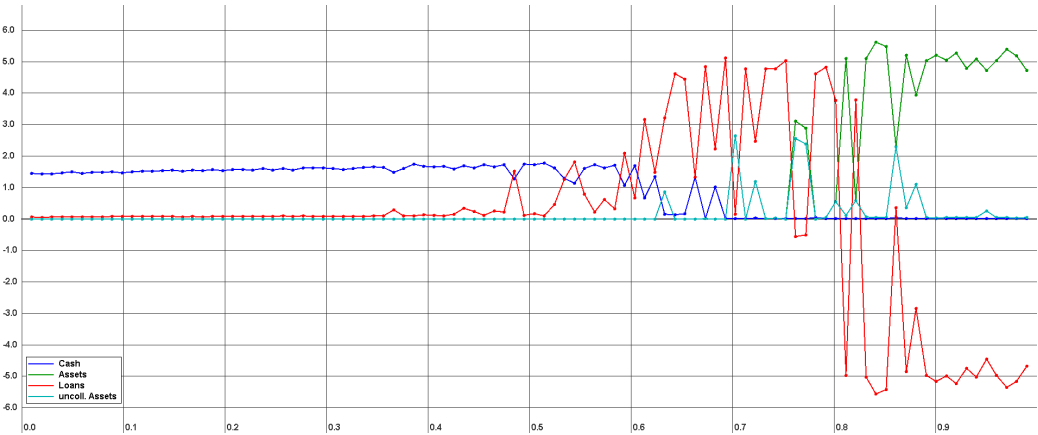
Figure 41: Wealth-Distribution of Erdos-Renyi 0.1 topology

need to show network too because random ?
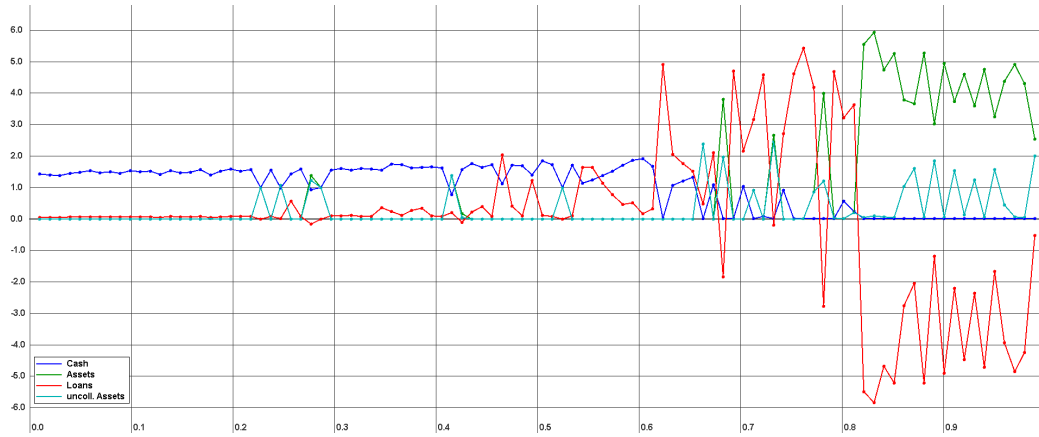
Figure 42: Wealth-Distribution of Erdos-Renyi 0.05 topology

need to show network too because random ?

B.4.2   Barbasi-Albert
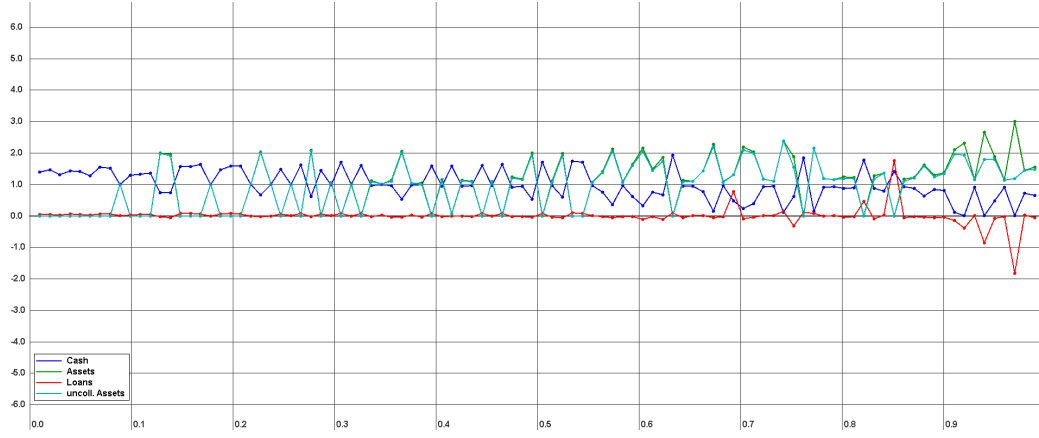
Figure 43: Wealth-Distribution of Barbasi-Albert m0=3, m=1 topology

need to show network too because random ?

Figure 44: Wealth-Distribution of Barbasi-Albert m0=9, m=3 topology

need to show network too because random ?

B.4.3 Watts-Strogatz
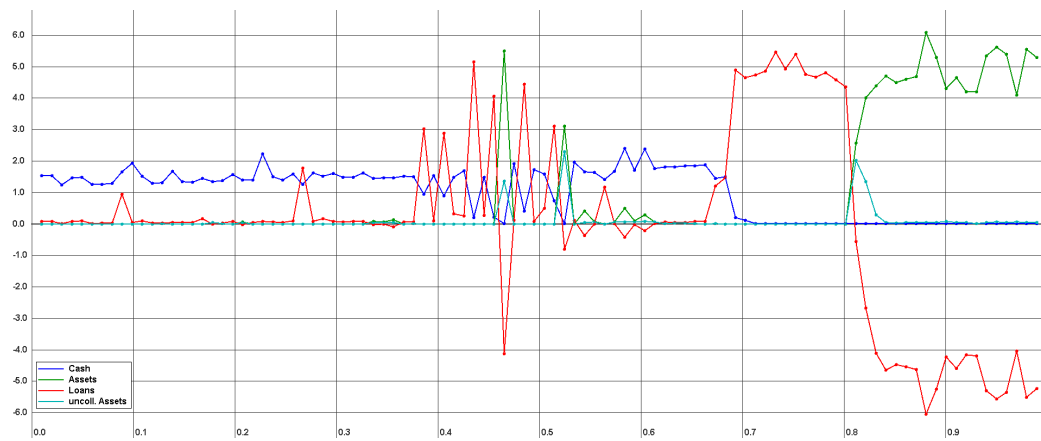
Figure 45: Wealth-Distribution of Watts-Strogatz k=2, b=0.2 topology

need to show network too because random ?

Appendix C

Interpretation of results of Hub-, Scale-Free and Small-World Topologies

C.1 Hub Topologies

C.1.1 3-Hubs

C.1.2 1-Median Hub

C.1.3 3-Median Hubs

C.1.4 Maximum Hub

C.2 Scale-Free and Small-World topologies

C.2.1 Erdos-Renyi

todo: kann hypothese erfüllen mit entsprechender parameterisierung

C.2.2 Barbasi-Albert

C.2.3 Watts-Strogatz

todo: kann hypothese erfüllen mit entsprechender parameterisierung

Figures

List of Tables

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