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Influence of network-topologies on equilibrium in continuous double-auctions

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Statuatory Declaration

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Widmung

Ich widme diese Arbeit meinen beiden liebevollen Eltern, die den verlorenen Sohn nach 11 Jahren in Wien wie selbstverständlich wieder mit offenen Armen zu Hause in Vorarlberg aufgenommen haben und ihm so das Masterstudium ermöglichten und ihm dadurch halfen ein völlig neues Kapitel in seinem Leben aufzuschlagen.

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Abstract

In the paper of Breuer et al. (2015) a model for endogenous leverage in a continuous double-auction is introduced and it is shown under which circumstances holdings and trading prices approach an equilibrium. A main criteria is the trading network the agents use where the authors examine only two topologies and report that the prices come to an equilibrium only in the case of a fully connected network. They leave the question open on how the model behaves with different kinds of networks and which network topology exactly allows an equilibrium to be reached for further research. This thesis builds upon this model and gives a hypothesis for the necessary property a network must satisfy to allow the model to approach theoretical equilibrium as reported in Breuer et al. (2015). Then a few network-topologies are examined in regard of their ability to allow equilibria to be reached or not through computer-driven simulation, where it turns out that although the model is extended by an additional market the hypothesis is infeasible.

Chapter 1

Introduction

In 2008 the so called *Subprime Mortgage Crisis* struck the world. It was caused by declining house prices which rose to an all-time high during the US Housing Market Bubble in 2006. Borrowers used their assets as collateral for the mortgage which constantly increased in value - this guaranteed them a low payment-rate because the rate was coupled to the value of the asset. Banks granted highly risky *subprime* mortgages to more and more borrowers which had a very low credit-rating. The banks didn't care about this fact because in the worst-case of a default of the borrower they would receive the very valuable asset which would gain in value even more. In 2007 borrowers started to default which led to falling prices as the banks reclaimed the collateral and wanted to sell it again on the market to compensate for the loss. This led to a flood of assets on the market which led to a decline of housing prices overall. As the prices fell, the payment-rates rose dramatically to compensate for the cheaper asset. This in turn resulted in even more borrowers defaulting because of margin requirements which resulted in a dramatic downward spiral. Even worse the banks were selling these collateralized products to each other and even insured themselves against defaults of borrowers which led to an even more extreme kick-back. BBC (2007), StockMarketInvestors (2007) The primary driving force behind systemic risk in the aftermath of the *Subprime Mortgage Crisis* was identified as leverage, which is the mechanism of borrowing money to buy goods which in turn act as security for the borrowed money.

1.1 Motivation

In the classic economics literature leverage was always an exogenous parameter in the works on collateralized credit but recently Geanakoplos (2009) and

Geanakoplos and Zame (2014) proposed an equilibrium framework which endogenized leverage. Breuer et al. (2015) built upon these findings and developed a simulation on top of the equilibrium frameworks in which zero-intelligence agents trade assets and bonds in a continuous double-auction. Breuer et al. asked

> ... whether the competitive theory of trade in leveraged assets has descriptive and predictive power in a double auction environment.

and wanted to better understand the dynamic of such an equilibrium process, how prices develop and whether they approach the equilibrium predicted in the framework or not. They made three contributions:

1. Continuous double-auction for leveraged assets is new.
2. Institutional specifications matter a lot.
3. Robustness tests were conducted to show under which circumstances equilibrium cannot be reached.

The authors could show that in their simulation trading prices and wealth-distribution approaches the theoretical equilibrium of Geanakoplos (2009). They investigated a fully connected network and a hub-network of agents where the equilibrium was only reached in the case of the fully connected network.

1.2 Objectives

This thesis investigates additional topologies of networks and their convergence towards theoretical equilibrium. Furthermore it presents a hypothesis defining the properties a network topology must satisfy to reach the same equilibrium as in Breuer et al. (2015). For experimental investigation software was built for this thesis which implemented the exact simulation model of Breuer et al. (2015) but extended it further to be applicable to arbitrary topologies.

1.3 Struture

The thesis starts with chapter 2 where the theoretical background involved is handled. In chapter 3 a short overview of the model which is used in the simulation is presented. In chapter 4 the hypothesis (which is the motivation for this thesis) is derived and then in chapter 5, an in-depth explanation of the implementation of the computer-driven simulation built for this thesis is given. Chapter 6 validates the previously described thesis-software against the results found in Breuer et al. (2015) and reports the results for the additional network-topologies. Chapter 7 connects the content of the previous chapters to verify whether the hypothesis of chapter 4 is valid or not. It turns out that the hypothesis is not valid and needs an adjustment in the form of a new market which is introduced and discussed with additional results in chapter 8. Chapter 9 provides conclusions on the findings of the thesis and outlines topics for further research.

Chapter 2

Theory

2.1 Equilibrium

Equilibrium is a fundamental property in all kinds of complex systems and indicates a state where the change-rates of all time-dependent variables are 0 and stay at 0 from some point t onwards - in other words the system has come to a halt and does not change any more over time. This thesis investigates whether the simulation-model presented in chapter 3 reaches equilibrium for a given configuration or not. Because the model upon which the simulation is based is rooted in economics a short definition of equilibrium in economics is necessary.

2.1.1 Economics

Equilibrium theory in economics is the theory of finding prices which will clear markets. Clearing is the process of finding a price in which all demands are matched to the given supplies thus clearing the market by leaving no unmatched demands or supplies. In other words it tries to find a price which satisfies all offers. Thus equilibrium in economics is reached when supply equals demand.

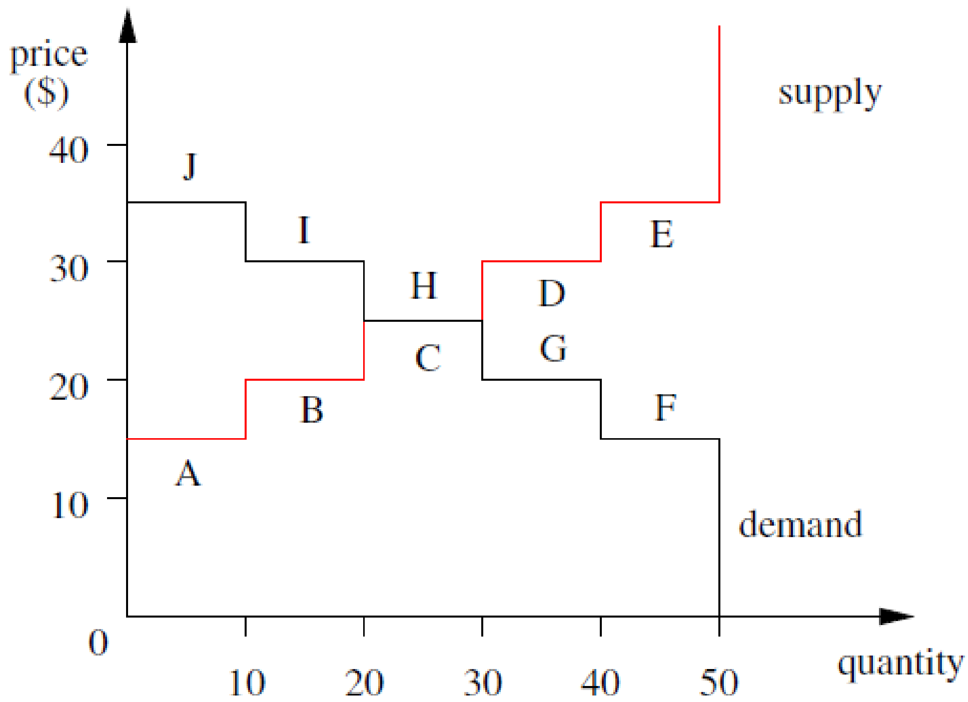

Figure 1: Illustrative supply and demand curves. Parsons et al. (2006)

In figure 1 the equilibrium price which clears the market is at \$25 because this price satisfies the constraint buyer-price $\geq$ seller-price for all traders, thus all traders match and the market is cleared. The constraint buyer-price $\geq$ seller-price is a fundamental law of economics and imposes an ordering on the prices of the market. It is rooted in the fact that a buyer values the goods it wants to buy more than the seller and has to pay at least the amount of money the seller wants for it, or more.

2.1.2 Equilibrium theory vs. dynamic process

It is important to note that equilibrium theory in economics is inherently non-dynamic and models no dynamic process of any kind over time. It is just a framework by providing formulae for calculating equilibrium prices but does not describe the dynamic process of how these equilibrium prices are settled. However the simulation in this thesis and the model by Breuer et al. (2015) upon which it is built as defined in chapter 3 is a dynamic process which tries to approach the equilibrium predicted by the economics theory through a trading-process over time as defined in the following section and in chapter 3. Thus both parts are necessary: the equilibrium theory to predict the theoretical equilibrium prices and the dynamic simulation-process in investigating whether the equilibrium prices can actually be reached or not and how these prices are settled over time. The fundamental difference is in equilibrium theory a price is searched which clears the market whereas in the dynamic trading process no prices are set in the beginning but they develop and clear the markets over time.

2.2 Continuous Double Auction

The continuous double-auction (CDA) is a type of auction upon which the model of Breuer et al. (2015) presented in chapter 3 and thus the thesis-software is based. The reasons why they chose the continuous double-auction as the auction-mechanism is:

> "Experimental economists believe that the continuous double auction is a trading institution that comes close to an environment which abstract equilibrium theories of competitive trading try to describe. It is an institution that allows for competitive bidding and trade on both sides of the market over time. One of the discoveries of experimental economists is that in many experiments double auctions converge to states where trading activity comes to a halt. In these final states prices and allocations often are similar to what equilibrium theory predicts." Breuer et al. (2015)

2.2.1 Definition

To explain the details of a *continuous* double-auction one has to start with the double-auction (DA) alone. Generally speaking the DA is a market institution which defines rules how traders can exchange goods for some numeraire between each other. It is an auction process that coordinates messages between traders which includes some price information. Thus a DA is a multilateral process in which during multiple rounds, traders can enter offers into an offer-book and accept offers made by others. Traders can be distinguished between seller and buyer and send their messages/place their offers in a given price-range according to their limit-price. Depending on the type of the DA, at some point a clearing of the market happens leading to the actual exchange between the traders and a change in the allocation of their goods and cash. It is important to note that the *double* in DA means that a trade is always just between two parties: the seller and the buyer - there are e.g. no intermediaries. Parsons et al. (2006)

2.2.2 Characterization

It is important to note that the *double auction* does not exist as a single concept, as there are many variants which can all be differentiated in the process by which traders place their offers and when and how the auction clears the market. According to Parsons et al. (2006) the following questions must be answered when characterizing a double-auction instance:

- When does the clearing happen? Is it periodic or continuous?
- When do offers of traders arrive? Do offers arrive over time?
- What information is available to each trader about current offers and other traders?
- How are unmatched offers treated? What happens to unmatched bids and asks when a match occurs?
- How are the trades priced? Are trades priced using discriminatory or uniform pricing and how are the uniform or discriminatory prices determined?
- Are there one or multiple trading-periods? Is the market one-shot or repeated?

When does the clearing happen? Is it periodic or continuous? Clearing happens at the end of a round where the first match of two random traders on a random market is searched. It is continuous because traders agree on each others' offers and exchange the traded goods immediately. In a periodic DA, clearing happens at discrete time slots during the trading-process after multiple rounds.

When do offers of traders arrive? Do offers arrive over time? Traders place their offers simultaneously at the beginning of each round. Offers do not arrive over time as time advances only from round to round and is not modelled explicitly as a time-flow.

What information is available to each trader about current offers and other traders? None. The traders are not able to look into the offer-book or to communicate with other traders. They act only as zero-intelligence agents as introduced in section 2.2.4.

How are unmatched offers treated? What happens to unmatched bids and asks when a match occurs? They are deleted. Prices are placed randomly as will be seen in 3 and if they haven't matched in the current round they won't match in the future ones which results in them all being deleted from the offer-book.

How are the trades priced? Are trades priced using discriminatory or uniform pricing and how are the uniform or discriminatory prices determined? Discriminatory pricing is used. In uniform pricing one price is chosen and applied to all trades which clears the market where in discriminatory pricing the prices are determined individually for each trade. The transaction-price, where the buyer and seller meet, is the half-way price between the offers of both. Another possibility for the transaction-price as reported in Gode and Sunder (1993) is to select the price of the offer which was placed first.

Are there one or multiple trading-periods? Is the market one-shot or repeated? A repeated DA comprises of multiple trading-periods where traders are endowed with new allocations and may or may not keep their final allocations after each period. This is not the case in this thesis where only one trading-period is simulated thus the market is one-shot. Note that this is not to be confused with a round as there are many rounds within one trading-period.

2.2.3 The continuous double-auction process

The following points summarise the workings of the instance of CDA used in this thesis:

- Endow all traders with initial goods and numeraire.
- Open all markets.
- Execute rounds as long as traders are able to trade.
- In each round every trader is allowed to place one buy and one sell offer on all opened markets.
- After all offers have been placed the auction searches for matches on the markets.
- During matching the first match between random traders on a random market is searched where buyer-price $\geq$ seller-price.

- On a match the offered amount is transferred and both traders meet at the half-way price.
- Upon a match all the other offers on all markets are deleted and a new round starts.

2.2.4 Zero-intelligence agents

The traders in this thesis are modelled as zero-intelligence agents as introduced by Gode and Sunder (1993). These are traders which place offers strictly in a range which increases their utility and can neither learn nor adapt to the behaviour of other agents or changing conditions on the market. They are completely stable in a way that they never change their behaviour.

When using zero-intelligence agents the question about the allocative efficiency of the market must be raised: "Are zero-intelligence agents able to achieve or come close to 100% allocative efficiency?". According to Gode and Sunder (1993) this is the case if:

> Imposing a budget constraint [...] is sufficient to raise the allocative efficiency of these auctions close to 100 percent. Allocative efficiency of a double auction derives largely from its structure, independent of traders' motivation, intelligence, or learning.

Because the model used in this thesis which is presented in chapter 3 is "imposing a budget constraint", the potential of coming close to 100% allocative efficiency is given. Results given in chapter 6 show that this is really the case.

2.3 Leverage

Leverage in economics is a major part in the model of this thesis as defined in chapter 3 and thus a short introduction and overview of its meaning and implications is given.

2.3.1 Definition

Leverage is "any technique to multiply gains and losses" as defined by Brigham (2012). There are different types of leverage where in this context only the so called *financial leverage* is of interest. In leverage, money is borrowed to buy some goods where the leverage is the ratio of the total debt to the traders equity. The greater the debt or the smaller the equity of the trader the higher the leverage.

$$leverage = \frac{total\ debt}{trader\ equity} \tag{2.1}$$

As an example an agent wants to buy a house for 500€ but has only 50€ in cash. Thus the agent borrows 450€ to finance the house creating a leverage of $\frac{500}{50} = 10$.

2.3.2 Dangers

In the example above, leverage not only multiplies gains but also losses. If house prices rise by 25% this will result in a gain of 125€ or 250% return on the agents investment when selling the house. However if house prices decline by 15% this will result in a loss of 75€ or 150% loss on the agents investment - in this case the agent has lost more money than it initially invested. The leveraged loss can become a serious issue in financed housing because of margin requirements towards the lender. The margin requirement is the minimum margin the house-buyer has to maintain independent of the gains or losses of the house. So if the house-price drops to 475€, the net value of the margin is 25€ and the agent needs to bring in 25€ to meet the margin requirements. If the losses are higher or the agent is not able to compensate them the house has to be sold.

2.4 Complex Networks

Networks play an important role in trading-processes and in the context of double auctions as they define which trader is able to interact with others thus influencing the process fundamentally. This thesis lays its main focus on the influence of network-topologies on the equilibrium found in continuous double-auction. The networks define the neighbourhood between agents and determine which pair of agents can trade with each other. All graph-related definitions in the following sections are provided through Drmota et al. (2007).

A network is a graph $G = (V,E)$ which has a finite set of vertices $V = V(G)$ and a finite set of edges $E = E(G)$. The vertices represent the agents and the edges connecting them represent the neighbourhood between agents or the knowledge of each other. Two agents know each other and can trade with each other if there exists an edge between them. In this context only undirected graphs consisting of undirected edges $e \in E(G)$, $e = v_1, v_2$ between two vertices v_1, $v_2 \in V(G)$ without multi- and self-edges are of interest.

- Undirected: if one agent v_1 knows another agent v_2 then v_2 knows v_1 too - trading is always possible in both directions.
- No multi-edges: one neighbourhood connection is enough as edges have no additional properties or weights.
- No self-edges: agents are not allowed to trade with themselves.

This thesis also investigates the equilibrium in so called *complex networks* which are a special kind of random network which could exhibit small-world properties and could follow a power-law distribution which are discussed below. In the following sections a short overview of the development of network-topologies and recent findings in this research-field is given and the complex networks used in this thesis are discussed. See appendix A for a complete catalogue of network-topologies investigated in this thesis - the complex ones are:

- Erdos-Renyi
- Barbasi-Albert
- Watts-Strogatz

The main sources for the following sections is the paper Newman (2003) and the books Jackson (2008) and Easley and Kleinberg (2010). Note that in this thesis only static networks are of interest thus no models of network-growth or processes in and on networks are discussed.

2.4.1 Overview

Since the first proof in network-theory by Euler in 1735 the analysis of networks has had a long tradition. Up until a few years ago the analysis of small graphs and properties of individual vertices or edges dominated the field of network-research, but in recent years the focus shifted towards large-scale statistical properties. This transition of focus was made possible by the availability of an ever increasing amount of processing power through computers which allow the investigation of networks with millions of vertices.

2.4.2 Random graphs

One of the simplest network models studied was the random graph which was investigated first by Erdős and Renyi (1959). The motivation behind random graphs is to assume a random-process of network formation and to study the resulting networks. Such networks are constructed by having N unconnected vertices and then adding at random each possible edge with a given probability p where the distribution of p follows a specific model, e.g. uniform, poisson, gaussian and so on. Thus random graphs can be reduced to a "binomial model of link formation" Jackson (2008) where out of all the possible random graphs with N nodes one graph is selected with a probability of

$$p^m (1-p)\frac{N(N-1)}{2} - m \tag{2.2}$$

as reported in Jackson (2008).

2.4.3 Small-World effects

In the 1960s Stanley Milgram conducted an experiment in which he demonstrated that a letter can reach any destination person by an average of just 6 intermediate steps in between. In his papers on this experiment Travers and Milgram (1969) and Milgram (1967), he termed this phenomenon the *small-world effect*. Although the results of his work have been questioned - more specifically that the world is really a small one e.g. by Kleinfeld (2002) - it had big influences within the network-research community and led to the development of the small-world property. It is of great importance e.g. in social networks because this property implies that information spreads very quickly in the network as it needs very few steps to reach all nodes. This can also be applied to trading networks where it enables traders to trade goods within very few intermediary steps to traders which value the goods the most.

To calculate whether a network has the small-world property, one starts with the formula given by Newman (2003) for calculating the average path length in a network

$$\ell = \frac{1}{\frac{1}{2}N(N+1)} \sum_{i \geq j} d_{ij} \tag{2.3}$$

where d_{ij} is the shortest distance from vertex i to vertex j. Networks then exhibit the small-world property if ℓ scales at max logarithmically with network size of mean degree.

2.4.4 Scale-free networks

Albert and lászló Barabási (2002) found that real-world networks in contrast to random graphs are non-random which led to the discovery of the phenomena of scale-free networks which are networks whose vertex-degree distribution follows a power-law as defined below.

Degree distribution

In an undirected Graph G the edges adjacent to $v \in V(G)$

$$\Gamma(v) = w \in V(G)|vw \in E(G) \tag{2.4}$$

are the neighbours. The quantity of the neighbours of $v \in V(G)$

$$d(v) = |\Gamma(v)| = |w \in V(G)|vw \in E(G)| \tag{2.5}$$

is the degree of a vertex $v \in V(G)$. When regarding the adjacent-matrix of a graph the degree of a vertex $v_i \in V(G)$ is given by

$$d(v_i) = \sum_{j=1}^{n} a_{ij} \tag{2.6}$$

Having defined the degree of a vertex one can calculate the degree distribution of of a given network simply by counting the number of nodes which have a given degree k:

$$P_{deg}(k) = \textit{fraction of nodes in the graph with degree } k \tag{2.7}$$

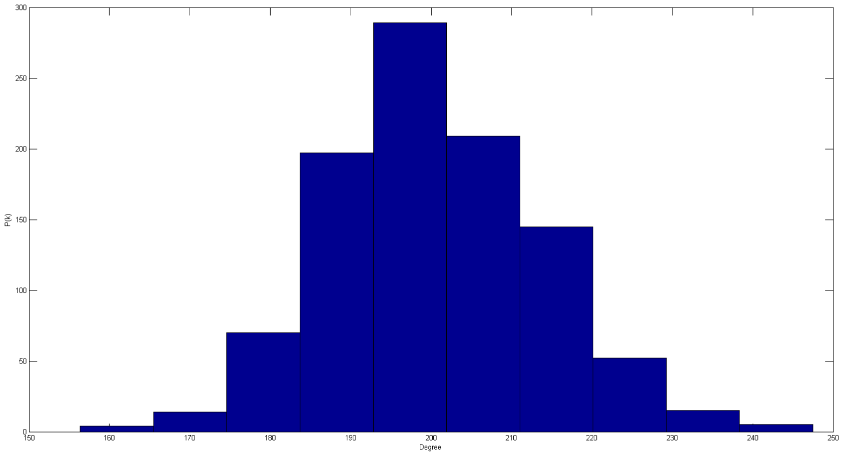

Figure 2: Histogram of the degree-distribution of a random Erdos-Renyi network with 1000 agents generated by the thesis-software. The average degree is 200.44.

Power-law distribution

In figure 3 the degree-distribution of a scale-free network is shown.

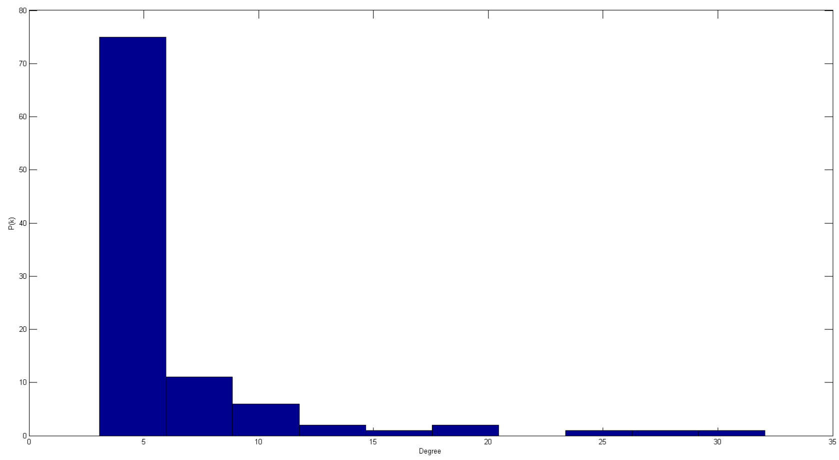

Figure 3: Histogram of the degree-distribution of a Barbasi-Albert network with 100 agents generated by the thesis-software. The average degree is 4.1. Note that the Barbasi-Albert model creates scale-free networks.

What is striking about figure 3 is the long right tail. This shows that there exist few vertices in this network which have a very high degree far beyond the average degree. This means that many other vertices are connected to them - they act as a kind of hub. Most of the vertices though have a low degree around 5 which in combination with the few high-degree vertices results in the long tail of this distribution which follows a power-law

$$P_{deg}(k) \sim k^{-\gamma} \tag{2.8}$$

Note that due to the power-law the distribution of such networks remain unchanged when scaling k with a given factor a. Thus those networks are called *scale-free*.

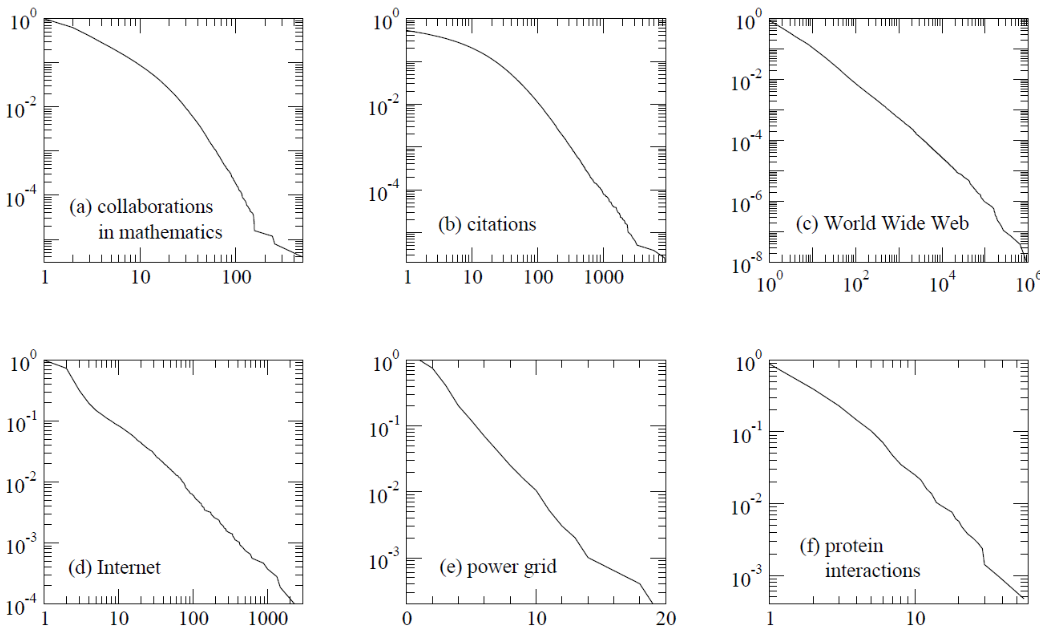

Figure 4: Examples for degree-distributions of real-world networks. Note that a straight line in a log-log plot is evidence of a power-law distribution thus only (c), (d) and (f) appear to have power-law distributions. Newman (2003)

The strengths of scale-free networks is their resilience against *random* removal of vertices.

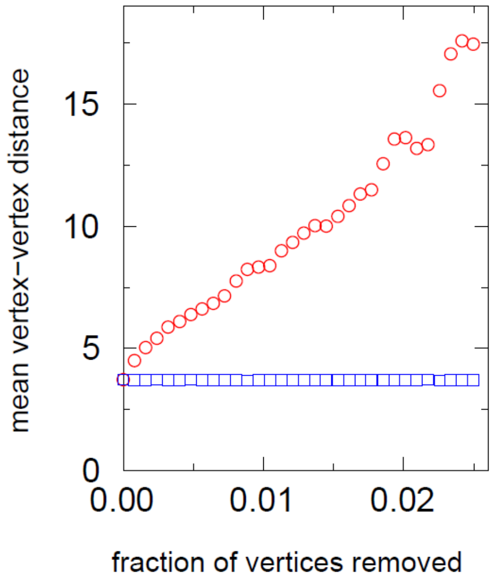

Figure 5: Random removal of vertices increases the average path length very slightly. Removing vertices with highest degree first increases the average path length rapidly. Newman (2003)

This resilience can be of benefit in a scale-free trading network where the inability of a random trader to trade because it has no more goods or cash won't impair the overall trading ability. On the other hand if an important trader which acts as a hub becomes unable to trade then the overall trading process may be affected.

2.4.5 Complex Network examples

To summarize complex networks are random networks that could have small-world properties and could be scale-free depending on the algorithms used to create them. There exist a few models to create complex networks where three well known models are implemented in this thesis. See appendix A for examples of the following networks created by the thesis-software with varying parametrization.

Erdos-Renyi

The Erdos-Renyi model generates a pure random-graph which exhibits *no* small-world properties. It starts with N vertices and selects one out of all the possible graphs with n vertices by random iteration over all possible edges and including each with a given probability p. Erdős and Renyi (1959) and Erdős and Renyi (1960)

Watts-Strogatz

The Watts-Strogath model generates a random-graph which exhibits small-world properties. It starts with N unconnected vertices and creates a regular lattice where each vertex is connected to K neighbours with $\frac{K}{2}$ on each side. It then iterates through all vertices and and rewires each edge which is connected to a vertex already visited with a given probability b to another vertex from all the possible vertices - self-loops and link-duplication is not allowed. Watts and Strogatz (1998)

Barbasi-Albert

The Barbasi-Albert model generates a random-graph which exhibits small-world and scale-free properties which is achieved through preferential attachment. To create a network with N vertices one starts with m_0 vertices and adds $N - m_0$ vertices. Each new vertex is connected to m existing vertices where the probability to be connected to a given vertex is proportional to its degree. When choosing m = 1 then one creates a hub-structure. Albert and lászló Barabási (2002)

Chapter 3

The Leverage Cycle

In this chapter the model for the simulation is given. All the following chapters build upon this model where the thesis-software is an implementation of it.

3.1 Geanakoplos

The model of Breuer et al. (2015) which is discussed in the next section is based upon the works of John Geanakoplos article Geanakoplos (2009) "The Leverage Cycle", thus for a better understanding a short overview of the innovations and influences found therein is given.

The work of Geanakoplos focuses on asset-pricing, the influence of leverage on asset-prices and how leverage affects crises. He claims that because of leverage during boom times the asset-prices are too high due to massive leverage and during bad times the asset-prices are too low due to a massive drop in leverage. That is what he terms "the leverage cycle". Further he predicts that leverage cycles will occur despite people remembering past ones unless the central bank tries to stop those cycles by regulating leverage. Geanakoplos proposes a theory of equilibrium leverage and asset pricing which gives a central bank a tool for regulating leverage during boom times to prevent asset-prices skyrocketing and reinforce leverage in down times to lift the asset-prices which are too low to a reasonable level.

3.1.1 The natural buyer

For Geanakoplos all crises start with bad news which are then the reason why asset-prices drop below a price which is lower than everyone expected. He introduces the so-called "natural buyer" which is an agent who values the asset more than the public. This can be because the agent is less risk averse, gets more utility out of it and uses the asset more efficiently. In the end the details do not matter, the natural buyer is just more optimistic than the public. To prevent a too specific distinction between the natural buyer and the public Geanakoplos introduces a range of optimism $h \in H = [0..1]$ in which all agents are ordered by their optimism h where the extreme pessimists reside at the lower end at 0 and the extreme optimists at the upper end at 1. Each agent assigns the probability that good news will occur according to its optimism h where the extreme optimist thinks that good news will happen for sure and the extreme pessimist thinks that it will never occurs - thus the more optimistic an agent, the more a natural buyer it is. If the natural buyers drop out of the business then the asset-prices drop as the natural buyers are the only ones willing to drive asset-prices up through leverage because they value the asset-prices the highest. Thus the natural buyers buy as many assets they can both by cash, through borrowing and using the assets as security thus creating leverage. Because of these mechanics Geanakoplos emphasises that it is very important to note who lost money in a crisis - the public or the natural buyers whereas a loss for the natural buyers is the real catastrophe as no one is willing to drive up the asset-prices any more.

3.1.2 Two-period economy

Geanakoplos then introduces a two-period economy. In the first period each agent of the previously mentioned continuum H is endowed with one consumption good C and one asset Y and can then trade with each other. The second period can be one of two states: U(p) and D(own) where in the up-state the asset Y is worth 1.0 and in the down-state only 0.2. The agents differ only in their optimism h by which they assign the probability that the up-state will happen tomorrow in the second period. Consequently they trade on the market according to their utility-function which depends on their optimism h. The following formula gives the limit-price of an agent according to Geanakoplos. It defines how much an agent values the asset - obviously the more optimistic the higher the price.

$$\textit{limit-price} = h + (1 - h)\ 0.2 \tag{3.1}$$

Now if the *limit-price* is larger than some offered price p then the agent is going to buy the asset for the offered price p as the agent values it more, thus when buying the asset the agent will make an expected profit. If the *price* is less than the offered price p the agent is going to sell the asset as the value the agent assigns to it is lower than the offered price p, thus the agent can make an expected profit in selling it.

3.1.3 Loan market

Geanakoplos introduces a loan market where agents can lend and borrow money through loans in order to further buy assets after they have run out of cash. A loan can be sold and bought for $j = 0.2$ and needs to be paid back at the beginning of the second state. Because lenders worry about default, each loan needs to be backed up by an asset as security.

In the up-state the borrower will pay back $j = 0.2$ and in the down-state the borrower will pay back either $j = 0.2$ or the asset which is worth of 0.2 in the down-state. Thus a loan which is bought for $j = 0.2$ and pays back the same amount is a risk-less loan as the lender can not lose money because independent of the occurring state always $j = 0.2$ will be given back.

Geanakoplos then predicts the so called *marginal buyer* around $h = 0.69$. All agents with $h < 0.69$ are pessimists and sell their assets. All agents above $h > 0.69$ are optimists and buy all the assets the pessimists sell, either through cash or by borrowing money from the pessimists through loans and using the borrowed money to buy further assets which then in turn act as security - the leverage is endogenous.

Geanakoplos then introduces loans with $j > 0.2$ where in the up-state they promise their initial value j and in the down-state they deliver only 0.2 - the collateral assets which is worth 0.2 in the down-state. Thus loans with $j > 0.2$ are risky loans because a lender can lose money depending on the occurring state. If a lender granted a bond of type $j = 0.5$ and the down-state will occur the borrower will either return 0.2 cash or the security-asset which is now only worth 0.2 - the lender has lost 0.3 cash.

In the classic equilibrium theory as outlined in chapter 2 the only equilibrating variables are prices. Geanakoplos argues that the problem with the classic model is that for determining the equilibrium of loans one needs two variables: the promise j and the collateral requirement which is impossible to solve with just one equation. The solution of Geanakoplos to modelling collateral is to

> ...think of many loans, not one loan. Conceptually we must replace the notion of contracts as promises with the notion of contracts as ordered pairs of promises and collateral. Each ordered pair-contract will trade in a separate market with its own price.

$$Contract_j = (Promise_j, Collateral_j) = (A_j, C_j) \tag{3.2}$$

He then shows that if there exist markets for all type of bonds which include the risk-less bond $j = 0.2$ then only the risk-less bond will be traded. The case with only risky bonds available are excluded by him by assumption.

Note that this is only a small part of the quite involved economic theory. Geanakoplos does not stop at this point but this overview is already enough to understand the basic influences found in the work of Breuer et al. (2015).

3.2 Breuer et al.

As already outlined the model of Breuer is heavily influenced by the work of Geanakoplos with the major difference that it is not a pure static equilibrium theory but is a simulation-process which approaches the equilibrium iteratively over time. Also a major achievement is that not only assets and bonds are traded against cash but the model has been extended by an additional market which allows collateralized assets to be traded. According to Breuer et al. (2015) this is the first time that the trading of leveraged assets was investigated in a continuous double-auction environment. It is also of great importance to note that although the up- and down-states are part of this model, they are actually never realized and act only as a model - thus only the first period is simulated. Furthermore agents are not an infinite continuum but finite entities because the equilibrium-solving is done as an iterative simulation-process in software and thus finite agents are required.

The major differences to the approach of Geanakoplos are:

1. Collateralized assets are traded in addition to the other markets.
2. Up- and down-states are never realized but only the first period is simulated.
3. Equilibrium in the case of only a risky-bond available is treated.
4. There is a finite number of agents as opposed to a continuum.
5. It is an auction-process over time which iteratively approaches theoretical equilibrium where Geanakoplos is a static equilibrium theory. The mechanism used in this auction is a continuous double-auction as introduced in chapter 2

In the following sections some details which are different to the model of Geanakoplos or need more explanation are discussed.

3.2.1 States

Both the up- and the down-state are the same as in Geanakoplos where the up-state is denoted with pU and the down-state is denoted as pD and assets are worth 1 in pU and 0.2 in pD. Agents are endowed with 1 unit of cash and 1 unit of assets today and are then able to trade between each other. The tomorrow-state will not be drawn - agents trade only today.

3.2.2 Markets and limit-functions

Asset/Cash market The asset market is the same as in Geanakoplos (2009) where assets are just bought and sold against cash.

$$limit_{asset} = h + (1 - h)\, pD \tag{3.3}$$

Bond/Cash market The bond market acts the same way as described in Geanakoplos (2009). Collateral acts as enforcement of financial promises and thus for a given amount of loans the same amount of assets must be held as securities. A loan can be bought for a given price which is the face-value V. This face-value has to be paid by the borrower in up-state whereas only pD has to be paid in down-state. Again note that up- and down-states are never realized but influence the utility-functions. Risk-less as well as risky bonds are available in the model of Breuer et al. (2015) where the risky bonds are those with face-value $V > 0.2$. Although Breuer et al. allowed more than one bond-type simultaneously in their model, this is not implemented in the thesis-software as it is not the primary focus of this work and would have required substantial changes in the software.

$$limit_{bond} = hV + (1 - h)\, pD \tag{3.4}$$

Asset/Bond market The Asset/Bond market trades assets against bonds thus the utility-function is just the ratio of the Asset/Cash utility to the Bond/Cash utility which gives the amount of bonds one asset is worth for a given optimism h.

$$limit_{asset/bond} = \frac{limit_{asset}}{limit_{bond}} \tag{3.5}$$

3.2.3 Agent utilitiy

The utility of an agent is a generic measure to calculate the trading-preferences of the agent. In the case of this thesis it calculates the potential earning of a given configuration of goods by multiplying the holdings of a good with its limit-function. In equilibrium theory a configuration is found which clears the markets.

Agents can hold cash, assets and bonds thus for each of these goods a separate utility-function exists and thus the total utility of an agent is given as the sum of all separate utilities.

$$u_{agent} = u_{cash} + u_{asset} + u_{bond} \tag{3.6}$$

Cash utility

$s_{asset} \ldots$ *selling amount asset*
$g_{bond} \ldots$ *giving amount bond*
$b_{asset} \ldots$ *buying amount asset*
$t_{bond} \ldots$ *taking amount bond*
$p_{asset} \ldots$ *price asset*
$p_{bond} \ldots$ *price bond*
$h_{cash} \ldots$ *holdings cash*

$$u_{cash} = (s_{asset} - b_{asset})p_{asset} + (g_{bond} - t_{bond})p_{bond} + h_{cash} \tag{3.7}$$

Asset utility

$h_{asset} \ldots$ *holdings asset*

$$u_{asset} = h_{asset} * limit_{asset} \tag{3.8}$$

Bond utility

$h_{bond} \ldots$ *holdings bond*

$$u_{bond} = h_{bond} * limit_{bond} \tag{3.9}$$

3.2.4 Collateralized asset market

One of the major inventions of the work of Breuer et al. is the introduction of a market for collateralized assets which has never been studied in continuous double-auction simulations so far. This market enables an agent which is out of cash but high on assets to buy additional assets by selling bonds and thus borrowing money which the agent uses to buy the desired assets and in return using them as security for collateral constraints. When implementing this mechanism Breuer et al. had to overcome two major difficulties.

1. Coordination of asset and bond markets - the buying of an asset and the selling of a bond needs to be coordinated across both markets and must happen at the same time.
2. Reversibility of suboptimal trades - earlier trades could have been suboptimal for an agent because it couldn't fully anticipate the behaviour of other agents and thus needs to get out of old trades. Technically speaking this would require freeing collateralized assets by unlocking them and transferring them in to the state of a real asset - no longer collateralized, thus being completely owned by the agent.

Breuer et al. proposed solutions to these two difficulties:

ABM mechanism The solution to the coordination of the asset and bond markets would be to condition a buy offer of an asset to a sell offer of a bond. Breuer et al. reported that "Separate utility improvement in each of the coupled trades is more restrictive than a net sum utility improvement of all coupled trades." which prevents theoretical equilibrium to be reached. Thus they define the market to trade assets directly against bonds thus reducing the involved agents from three to two and removing the coordination-problem because only one product with one price is traded. This resolves the problem with the restrictiveness of utility in the case of two products.

Bond pledgeability The problem with the reversibility of suboptimal trades was solved in allowing the uncollateralization of an asset by buying a bond. Breuer et al. called this mechanism "bond pledgeability" (BP) and showed that without this mechanism the simulation never converges towards the theoretical equilibrium. See chapter 5 "Implementation" for details on the implementation of this mechanism.

3.2.5 Auction Mechanism

The auction mechanism used is a continuous double-auction on all markets open at the same time with a finite number of agents.

Bidding To prevent a bias one agent is picked at random and then submits offerings on all markets while respecting the following constraints.

- If the agent has no more assets it can't sell them either through cash or bonds.
- The agent cannot buy more assets or bonds than it owns cash.
- When placing sell-offers of bonds or assets there must not remain bonds which have no collateral as security.

Matching Again to prevent a bias pick one market at random and pick at random the buy or sell offers on this market and compare them with the offers of the previously selected random agent. A match occurs only if:

$$\textit{buy-price} \geq \textit{sell-price} \tag{3.10}$$

In this case the offers of all other agents which have not matched are deleted from the offering-book and the matching-price is calculated at the half-way price:

$$\textit{matching-price} = \frac{\textit{buyer-price} + \textit{seller-price}}{2} \tag{3.11}$$

If no match occurs with the current random agent pick another agent at random and continue with submitting its offers on all markets.

3.2.6 Equilibrium

Breuer et al. reported equilibria for prices and allocations both of bonds and assets where the equilibria are fundamentally different whether a risk-free bond is available or not.

Risk-free bond If a risk-free bond with a face-value of $V \leq 0.2$ is available then the agents are divided into two subgroups by i^*:

1. Agents with $0 < i \leq i^*$ are pessimists and hold only cash or the risk-free bond with highest face-value.
2. Agents with $i^* < i \leq 1$ are optimists and are maximally short in risk-free bonds with highest face-value and hold only assets.

Below the formulas reported in Breuer et al. (2015) are given for calculating i^*, the asset-price p and the bond-price q in equilibrium.

$$i^* = \frac{p - 0.2}{0.8} \tag{3.12}$$

$$p = \frac{1 + q - i^*}{i^*} \tag{3.13}$$

$$q = 0.2 \tag{3.14}$$

Risky bond When only a risky bond with face-value $V > 0.2$ is available then the agents divide into three instead of two subgroups separated by i_1 and i_2:

1. Agents with $0 < i \leq i_1$ are pessimists and hold only cash.
2. Agents with $i_1 < i \leq i_2$ are median agents and hold only bonds with the lowest face-value.
3. Agents with $i_2 < i \leq 1$ are optimists and hold only assets and are maximally short in risky bonds with the lowest face-value.

Below the formulas reported in Breuer et al. (2015) are given for calculating i_1, i_2, the asset-price p and the bond-price q in equilibrium. Note that in this thesis equilibria are always calculated for a risky bond with a face-value of $V = 0.5$.

$$i_1 = \frac{q - 0.2}{V - 0.2} \tag{3.15}$$

$$i_2 = \frac{0.2(p - q)}{0.8q - (V - 0.2)p} \tag{3.16}$$

$$p = \frac{1}{i_1} - 1 \tag{3.17}$$

$$q = p\frac{i_2 - i1}{1 - i_1} \tag{3.18}$$

Note that this case is not discussed in Geanakoplos (2009) where it is excluded by assumption.

Calculating theoretical Equilibrium

Theoretical equilibrium can be calculated through the previously given equations for an infinite number of agents. In the simulation a finite set of agents is used for which the theoretical equilibrium must found in order to compare the results of the simulation with the theoretical equilibrium. For this purpose Breuer et al. (2015) developed an algorithm in MATLAB which searches the finite solution-space for the given equilibrium. Mr. Martin Jandacka wrote a short, unpublished documentation on the approach for risky bonds which is summarized here.

For given asset prices q and bond-prices q each agent optimises its expected utility. As can be seen in the section 3.2.2 the utility-functions are linear which makes this optimization problem a linear one which can be solved through Linear Programming (LP). Thus the two agents *i1* and *i2* are searched where *i1* marks the end of the pessimists and *i2* the beginning of the optimists. This is done by iterating through all possible combinations of *i1* and *i2* and checking if they generate equilibrium on the market or not. The time dependence is $O(N^2)$ where N is the amount of agents.

3.2.7 Endogenous leverage

Endogenous leverage is the central topic of the models both of Geanakoplos and of Breuer et al. Because it may not seem immediately clear where and how leverage is endogenous in the model of Breuer et al., this section outlines where this is the case and how it is implemented.

In the work of Breuer et al. (2015) it is noted that

> In this theory the amount that can be borrowed against a particular asset to purchase it is determined in the market.

and furthermore

> Leverage, the percentage of the value of the real asset that can be borrowed to purchase it, is determined by contract selection through the market. Leverage is endogenous.

Contract selection amounts to the selection of the bond-types used by the agents to finance their trades. Geanakoplos and Breuer report that if multiple bonds are available which include risk-free bonds the agents will select the risk-free bond with the highest face-value which is 0.2. The reason is that buyers which are more optimistic towards the up-state expect that they would have to pay the face-value so they try to select a bond with the lowest possible face-value the sellers would accept. The sellers which are more pessimistic towards the up-state expect that they will more likely get back the down-state value of 0.2 and don't want to trade above this value as they expect to lose money in this case. So buyers and sellers select the risk-free bond with face-value of 0.2 endogenously through the mechanics of the model and not by parameters which are set exogenous by an experimenter. Thus Leverage is regarded as endogenous, coming from within the simulation-model itself.

Note that the prices of assets and debts are distorted through leverage because optimists value the goods more and are willing to drive the prices up through the use of leverage. This was a major finding by both Geanakoplos and Breuer.

3.2.8 Equilibrium of trading-process

As already noted the model given above is a dynamic process which approaches an equilibrium over time. The equilibrium is established if the system does not change any more over time: the process has come to a halt and all time dependent variables stay constant. Due to its design the process of this model will always come to a halt at some point - and will thus have some equilibrium - which is the case when all traders have become unable to trade:

- Due to collateral constraints.
- They cannot place utility-increasing offers any more - utility-reduction is not allowed in this model.

It is of great importance to note that if the process has come to a halt *some* equilibrium has been established but the established equilibrium *must not* necessarily be the theoretical one as given by the equations in section 3.2.6.

Chapter 4

Hypothesis

In this chapter the question of the importance of fully-connectedness for reaching the equilibrium is raised where the question is whether it is really necessary to have a fully-connected network to reach equilibrium or not. We challenge this and claim that a much lower connectivity with a special property is sufficient. First the motivation is presented and the claims behind it are proven mathematically. Then the property is introduced and it is proven that it is necessary to reach theoretical equilibrium. Whether it is sufficient is tested by computer-driven simulation where the results are given in chapter 6.

The initial hypothesis presented in this chapter was conjectured first by the supervisor of this thesis Mr. Hans-Joachim Vollbrecht.

4.1 Motivation

The motivation behind the hypothesis is the fact that according to the double-auction definition - see chapter 2 - for a match to happen the buyer-price must be larger or equal to the seller-price. This can only be the case if the buyer has a higher optimism-factor than the seller because only then the limit-price of the buyer will be larger than the one of the seller. For a match to occur the limit-price of the buyer has to be strictly larger than the one of the seller as shown below.

In figure 6 the price-ranges of both a seller and buyer are given where limit(s) and limit(b) denote the limit-price of the seller and buyer respectively, which are determined by their optimism-factor h. The seller places its offerings in the price-range of [limit(s)..pU] as it wants to sell the goods above the expected price to make a profit. The buyer places its offerings in the price-range of [pD..limit(b)] as it wants to buy the goods below the expected price to make a profit. The resulting matching-range on which the prices can meet - again buyer-price $\geq$ seller-price - is marked by the red rectangle. It is easy to see that a match with these mechanics can occur only if the optimism-factor of the buyer limit(b) is strictly higher than the one of the seller limit(s).

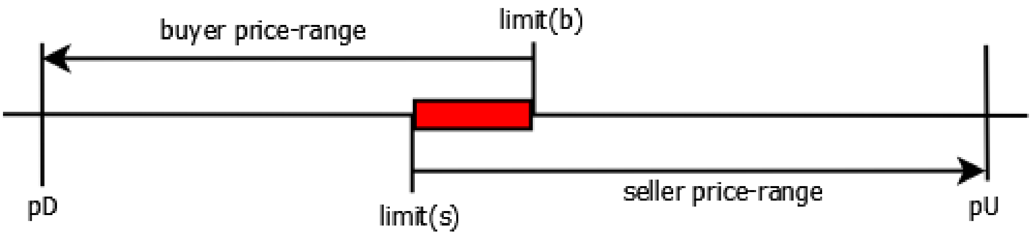

Figure 6: Matching of buyers and sellers price-ranges. The red rectangle marks the matching-range.

4.2 Proof buyer more optimistic than seller

Proving that the optimism of a buyer must be greater than or equal to the optimism of the seller $h_B \geq h_S$ by using the fact that the limit-price of the buyer must be greater than or equal to the limit-price of the seller $limit_B \geq limit_S$ and then showing that this can only be the case if $h_B \geq h_S$.

4.2.1 Asset/Cash

$limit_B = h_B\, pU + (1 - h_B)\, pD = h_B + \frac{1}{5} - \frac{h_B}{5}$
$limit_S = h_S\, pU + (1 - h_S)\, pD = h_S + \frac{1}{5} - \frac{h_S}{5}$

Proof.

$$\begin{aligned}
limit_S - limit_B &\leq 0 \\
(h_S + \frac{1}{5} - \frac{h_S}{5}) - (h_B + \frac{1}{5} - \frac{h_B}{5}) &\leq 0 \\
h_S + \frac{1}{5} - \frac{h_S}{5} - h_B - \frac{1}{5} + \frac{h_B}{5} &\leq 0 \\
5h_S + 1 - h_S - 5h_B - 1 + h_B &\leq 0 \\
4h_S - 4h_B &\leq 0 \\
h_S - h_B &\leq 0 \qquad \text{can only hold if } h_B \geq h_S
\end{aligned}$$

$\square$

4.2.2 Bond/Cash

$limit_B = h_B\,V + (1 - h_B)\,pD = h_B\,V + \frac{1}{5} - \frac{h_B}{5}$
$limit_S = h_S\,V + (1 - h_S)\,pD = h_S\,V + \frac{1}{5} - \frac{h_S}{5}$

Proof.

$$
\begin{aligned}
limit_S - limit_B &\leq 0 \\
(h_S\,V + \frac{1}{5} - \frac{h_S}{5}) - (h_B\,V + \frac{1}{5} - \frac{h_B}{5}) &\leq 0 \\
h_S\,V + \frac{1}{5} - \frac{h_S}{5} - h_B\,V - \frac{1}{5} + \frac{h_B}{5} &\leq 0 \\
5h_S\,V + 1 - h_S - 5h_B\,V - 1 + h_B &\leq 0 \\
5h_S\,V - 5h_B\,V - h_S + h_B &\leq 0 \\
5V(h_S - h_B) - (h_S - h_B) &\leq 0 \\
(h_S - h_B)(5V - 1) &\leq 0 \qquad | : (5V - 1) \Rightarrow \geq 0 \mid V\ [0..1] \\
h_S - h_B &\leq 0
\end{aligned}
$$

$\square$

4.2.3 Asset/Bond

$limit_B = \frac{h_B+\frac{1}{5}-\frac{h_B}{5}}{h_B V+\frac{1}{5}-\frac{h_B}{5}}$
$limit_S = \frac{h_S+\frac{1}{5}-\frac{h_S}{5}}{h_S V+\frac{1}{5}-\frac{h_S}{5}}$

Proof.

$$
\begin{aligned}
limit_S - limit_B \leq 0 \\
\frac{h_S + \frac{1}{5} - \frac{h_S}{5}}{h_S V + \frac{1}{5} - \frac{h_S}{5}} - \frac{h_B + \frac{1}{5} - \frac{h_B}{5}}{h_B V + \frac{1}{5} - \frac{h_B}{5}} \leq 0 \\
(h_S + \frac{1}{5} - \frac{h_S}{5})(h_B V + \frac{1}{5} - \frac{h_B}{5}) - (h_B + \frac{1}{5} - \frac{h_B}{5})(h_S V + \frac{1}{5} - \frac{h_S}{5}) \leq 0 \\
substituting\ S = \frac{1}{5} - \frac{h_S}{5}, B = \frac{1}{5} - \frac{h_B}{5} \\
(h_S + S)(h_B V + B) - (h_B + B)(h_S V + S) \leq 0 \\
h_S h_B V + h_S B + S h_B V + BS - (h_B h_S V + h_B S + B h_S V + BS) \leq 0 \\
h_S h_B V + h_S B + S h_B V + BS - h_B h_S V - h_B S - B h_S V - BS \leq 0 \\
h_S B + S h_B V - h_B S - B h_S V \leq 0 \\
h_S B - h_B S + V(h_B S - h_S B) \leq 0 \\
h_S(\frac{1}{5} - \frac{h_B}{5}) - h_B(\frac{1}{5} - \frac{h_S}{5}) + V(h_B(\frac{1}{5} - \frac{h_S}{5}) - h_S(\frac{1}{5} - \frac{h_B}{5})) \leq 0 \\
\frac{h_S}{5} - \frac{h_S h_B}{5} - \frac{h_B}{5} + \frac{h_B h_S}{5} + \frac{h_B V}{5} - \frac{h_B h_S V}{5} - \frac{h_S V}{5} + \frac{h_S h_B V}{5} \leq 0 \\
\frac{h_S}{5} - \frac{h_B}{5} + \frac{h_B V}{5} - \frac{h_S V}{5} \leq 0 \\
h_S - h_B + h_B V - h_S V \leq 0 \\
h_S(1 - V) + h_B(-1 + V) \leq 0 \\
| : (1 - V) \Rightarrow \geq 0 \mid V\ [0..1] \\
h_S + h_B \frac{-(1 - V)}{(1 - V)} \leq 0 \\
h_S - h_B \leq 0
\end{aligned}
$$

□

4.3 Proof of monotony of limit-functions

The property that the optimism-factor of the buyer has to be greater than or equal to the one of the seller is not enough for a match to occur. Additionally the limit-function must be monotonically increasing in the defined range [0..1] of the optimism-factor h because if it is not then no matching would be possible. It is proven by showing that $limit' > 0$ in the range of $h = [0..1]$.

4.3.1 Asset/Cash market

Proof.

$$\begin{aligned}
limit_{asset} &= h\,pU + (1-h)pD && \text{pU} = 1,\ \text{pD} = 0.2 \\
&= h + (1-h)0.2 \\
&= h + \frac{1}{5} - \frac{h}{5} && \frac{\mathrm{d}}{\mathrm{d}h} \\
\frac{\mathrm{d}limit_{asset}}{\mathrm{d}h} &= 1 - \frac{1}{5} = \frac{4}{5}
\end{aligned}$$

Constant ⇒ limit-function is monotonically increasing over the real numbers. QED □

4.3.2 Bond/Cash market

Proof.

$$\begin{aligned}
limit_{bond} &= h\,V + (1-h)pD \\
&= h\,V + (1-h)pD \\
&= h\,V + pD - h\,pD && \frac{\mathrm{d}}{\mathrm{d}h} \\
\frac{\mathrm{d}limit_{bond}}{\mathrm{d}h} &= V - pD
\end{aligned}$$

V is a constant in range of [0..1] ⇒ limit-function is monotonically increasing were $V > pD$. QED □

Note that this implies that a bond with face-value $V = pD$ cannot be traded as the limit-function is constant for all h. This needs to be covered in the implementation.

4.3.3 Asset/Bond market

Proof.

$$
\begin{aligned}
limit_{asset/bond} &= \frac{h\,pU + (1-h)pD}{h\,V + (1-h)pD} && \text{pU} = 1,\ \text{pD} = 0.2 \\
&= \frac{h + \frac{1}{5} - \frac{h}{5}}{h\,V + \frac{1}{5} - \frac{h}{5}} && \frac{\mathrm{d}}{\mathrm{d}h} \\
\frac{\mathrm{d}limit_{asset/bond}}{\mathrm{d}h} &= -\frac{5(V-1)}{(h(5V-1)+1)^2} && \text{assume h and V in range [0..1]} \\
&\Rightarrow 5(V-1) \leq 0 \\
&\Rightarrow (h(5V-1)+1)^2 \geq 0 \\
&\Rightarrow -\frac{5(V-1)}{(h(5V-1)+1)^2} \geq 0 && \text{for h and V in range [0..1]}
\end{aligned}
$$

first derivation $> 0 \Rightarrow$ limit-function is monotony increasing if $h > 0$ and $V > 0$. QED □

Note that the limit-function is undefined in the case of h=0 which should be prevented in the implementation - see chapter 5.

4.4 Hypothesis

$$
\begin{aligned}
&\textit{fully-connectedness equilibrium} \iff \\
&\forall\ \textit{agent-pairs}\,(a_1, a_n)\ \exists\ \textit{path}\,P\ \{a_1, a_2, ..., a_{n-1}, a_n\} \mid h(a_i) < h(a_{i+1})
\end{aligned}
\tag{4.1}
$$

Conjecture 1. *If and only if for all agents exists a path between two agents in which each visited agent has a larger optimism factor than the previous one then the same equilibrium as in fully-connectedness will be reached.*

4.5 Proof of necessity

In this section it is proven that the given property is *necessary* to reach theoretical equilibrium but it is open to question whether it is *sufficient* or not. This will be investigated by computer-simulation.

4.5.1 Most minimal topology satisfying hypothesis

As proven above the buyer optimism must be larger than the seller optimism. Each agent acts both as a buyer and a seller thus the optimism-factor h is creating a total ordering on the agents where $h_i \leq h_{i+1}$. When now the agent with h_i is connected to h_{i+1} a Hamiltonian path is created which is per definition a spanning tree because it visits each vertex exactly once. If we now assign to each edge the weight of 1, it is clear to see that this graph must also be the minimal spanning tree. An example for such a graph is the Ascending-Connected topology - see appendix A - which is the major network of interest in this thesis (besides the fully-connected one) as it is the most minimal topology which satisfies the property of the hypothesis.

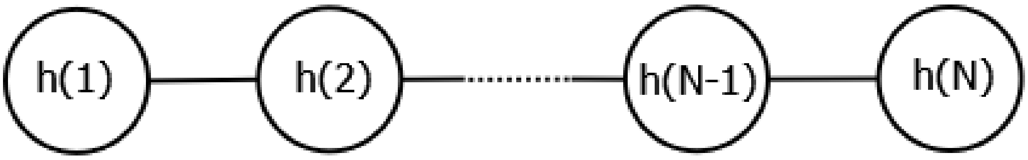

Figure 7: The minimal graph which satisfies the property of the hypothesis is a Hamiltonian path between the totally ordered agents.

4.5.2 Violation of property

To show that the property of the hypothesis is necessary to reach equilibrium it is now shown that a violation of the property must lead to a miss-allocation because of inability to trade, thus missing equilibrium.

A failure in reaching equilibrium is equivalent in miss-allocation of goods to agents who don't value the goods as much as others would. Trading with another agent could lead to an increased utility of both agents but trading is impossible due to a violation of the property through an invalid network. To see when and how miss-allocations could occur one must investigate the flow-direction of goods.

Asset/Cash In this market the buyer h_{i+1} gets assets and gives cash to the seller h_i. Thus the flow is as follows:

$$Asset : h_i \rightarrow h_{i+1}$$

$$Cash : h_i \leftarrow h_{i+1}$$

Bond/Cash In this market the buyer h_{i+1} grants a loan to the seller by giving cash to the seller h_i which takes a bond from the buyer. Thus the flow is as follows:

$$Bond : h_i \rightarrow h_{i+1}$$

$$Cash : h_i \leftarrow h_{i+1}$$

Asset/Bond In this market the buyer h_{i+1} gets assets and takes a bond from seller h_i. Thus the flow is as follows:

$$Asset : h_i \rightarrow h_{i+1}$$

$$Bond : h_i \leftarrow h_{i+1}$$

There are basically 3 violations possible within the most minimal network. Note that also combinations of those violations are possible but for simplicity each is treated separately.

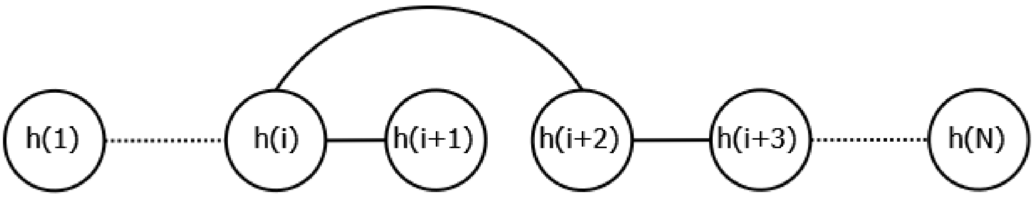

Figure 8: There is no connection between the seller h_{i+1} and the buyer h_{i+2} thus there exists no Hamlitonian path in the ordered agents which violates the property.

In figure 8 the seller h_{i+1} is unable to sell to h_i because of the lower optimism-factor and is unable to sell to h_{i+2} obviously because of a non-existent connection. Depending on whether this violation happens in the pessimists, medianists or optimists different miss-allocations will happen.

- Asset/Cash - h_{i+1} buys assets against cash but cannot sell them further up to h_{i+2} because of the missing link. This is a problem in the range of pessimists as they try to get minimal on assets and maximal on cash. For medianists this is a problem too as they also try to get minimal on assets but maximal on bonds. Optimists try to get as many assets as possible thus no miss-allocation will show up there.

- Bond/Cash market - h_{i+1} buys bonds but cannot sell them further up to h_{i+2} because of the missing link. This is a problem in the range of pessimists as they try to get minimal on bonds but maximal on assets. This is also a problem in the range of optimimsts.

- Asset/Bond market - h_{i+1} buys assets against bonds but cannot sell them further up to h_{i+2} because of the missing link. This is a problem in the range of pessimists and medianists as they try to get maximally short on assets.

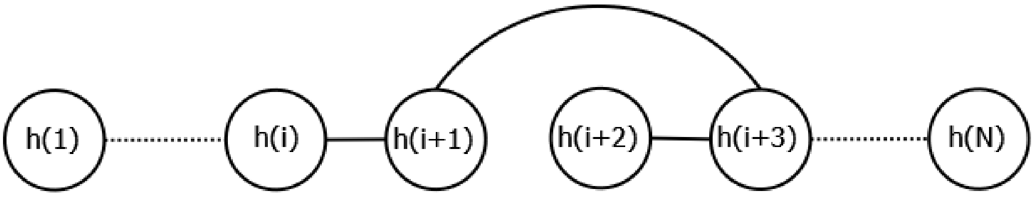

Figure 9: There is no connection between the buyer h_{i+2} and the seller h_{i+1} thus there exists no Hamlitonian path in the ordered agents which violates the property.

In figure 9 the buyer h_{i+2} is unable to buy from h_{i+1} because of a non-existent connection. Depending on whether this violation happens in the pessimists, medianists or optimists different miss-allocations happen.

- Asset/Cash - h_{i+2} sells assets up to h_{i+3} transforming to cash but can't use the cash to buy goods because of the missing link to seller h_{i+1}. This is a problem in the range of optimists as they try to maximise their assets and minimize their cash. Its also a problem in the range of medianists as they try to maximise their bonds and minimize their cash. Pessimists try to sell all assets and get as much cash as possible thus no miss-allocation will show up there.
- Bond/Cash market - h_{i+2} sells bonds up to h_{i+3} transforming to cash but can't use the cash because of the missing link to seller h_{i+1}. This is the same situation as in Asset/Cash market.
- Asset/Bond market - h_{i+2} sells assets against bonds up to h_{i+3} transforming assets to bonds but can't use the bonds to buy goods because of the missing link to seller h_{i+1}. This is the same situation as in Asset/Cash market.

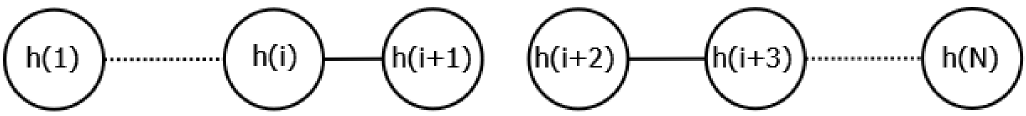

Figure 10: Graph is disconnected.

In figure 10 obviously the buyer h_{i+2} is unable to trade with the seller h_{i+1} because of a non-existent connection thus it is trivial to see that theoretical equilibrium cannot be reached.

4.6 Predictions

The following topologies found in appendix A satisfy the definition of the hypothesis:

- Fully-Connected
- Half-Fully connected
- Ascending-Connected
- Ascending-Connected with all kind of short-cuts
- Erdos-Renyi and Watts-Strogatz with the correct parametrization by pure chance.

If the hypothesis is valid and the property is sufficient these topologies will reach the equilibrium found in fully-connectedness. All other topologies do not satisfy the necessary property and are expected to clearly fail reaching the equilibrium of the Fully-Connected topology.

See chapter 6 and 7 where it is explained whether the property is sufficient or not.

Chapter 5

Implementation

For this thesis a software was written to be able to investigate the behaviour and results of the different types of networks and get visual and numerical results to be embedded in this written text. In this chapter the implementation-details of the software are discussed.

5.1 Requirements

The author of this thesis had access to the software of Breuer et al. (2015) which was written in C++ therefore the question is why a new software had to be written and why the original could not be used. The reason for it was that the original software supported only a very narrow feature-set focusing only on the numerical results of a fully-connected network. Thus a complete redevelopment in Java was the option used. The following requirements were identified:

- Java based.
- Comprehensive GUI functionality.
- Emulate the functionality of Breuer et al. (2015) and its results.
- Represent arbitrary networks in the simulation.
- Step-through forward and backwards in a simulation-run.
- Run replications of simulations.
- Store results of replications to be opened again for later usage.
- Command-line mode to run multiple experiments as replications.

Java based The original software was written in C++ which is very powerful and provides the highest speed if used correctly but comes with a very high responsibility regarding memory-management. Java offers a much more relaxed programming model regarding memory-management as it is garbage collected. This does not mean that the programmer can waste memory without giving thought to it but that one has not to put so much emphasis into it thus debugging is easier. The most compelling argument for Java are the vast libraries which are included in the JDK which are missing in standard C++. As complete GUI-functionality for the whole software is a requirement Java is the way to go. Although multi-purpose libraries like Boost and GUI-frameworks like Qt are available for C++, it takes quite some time to set them up correctly for the target platform one develops for which implies that in C++ the development would always have been for just one platform whereas Java runs on every platform without recompilation - if no platform-dependent stuff was used. As one will see later in the command-line mode feature this is a major requirement to make it practical. Thus the reasons for using Java were:

- Platform-independence which applies to 3rd party libraries too.
- GUI-framework provided by JDK.
- Relaxed memory-management which emphasises fast iterations and easy debugging.
- Support for smooth XML-Serialization.
- 3rd party libraries for network-modelling and -visualization.

GUI functionality All functionality should be accessible through a GUI where some features e.g. "Inspection" are only possible to use through a GUI.

Emulate Breuer et al. (2015) functionality Of course the whole software should be a super-set of the functionality of the one found in Breuer et al. (2015) so this was the point to start from.

Arbitrary networks It should be possible to restrict trading between the agents to arbitrary networks. As network-modelling and visualization library JUNG is used.

Step-through simulation The software should support going through a simulation-run step-by-step and storing all steps of the simulation in order to jump back and forth between them.

Real-time visualisation and information The original C++ software didn't provide real-time information about the current wealth-distribution and market-dynamics and provided the user just with the numerical results in the end through the means of a command-line output. For better understanding of dynamics of both wealth and markets a real-time visualisation of both are necessary together with extensive information on the current state like the offering-book, agent-information, network-activity, history of matches and the current equilibrium. Also the real-time visualization is necessary to provide this written part of the thesis with diagrams of various results and processes.

Replications Because the whole trading-process includes randomness, the results are subject to noise thus replications are an absolute must-have feature in order to give reliable results. Each replication is data-independent from all others thus it is a candidate for parallel programming to speed up the already very time-consuming process of running replications. According to the number of CPU-cores the software should spawn threads up to the number of cores and run replications in parallel thus speeding up by a considerable amount of time.

Store results When running a bunch of replications for a given set-up the results of it should be automatically stored as XML to be accessible for later inspection thus conserving state and eliminating the necessity to re-run time-consuming simulation-runs with a high number of replications.

Command-line mode As already described replications are required to be implemented for parallel processing where up to the number of CPU-cores replications can run in parallel. When running a vast number of replications one does not need GUI-functionality and most probably the machine is so occupied by the heavy work-load that a smooth usage of GUI would not be possible anymore. Thus replications should be runnable through a separate command-line mode of the thesis-software which reads information from a XML-File in which one can specify multiple simulation set-ups for which replications should be run. The command-line mode iterates through all configurations, runs the required replications and writes the result out for later inspection. Obviously the more CPU-cores the faster a simulation-run with e.g. 50 replications finishes. For this reason most of the final replication-runs were done on a 40-core machine of the FH Vorarlberg which runs on Linux on which the thesis-software runs without recompilation due to Java's platform-independence.

5.2 Functionality

In this section the functionality of the thesis-software is explained to get an understanding of the implemented features. All features are available through the GUI unless stated otherwise. For inspection, replications and experiments an individual tab is available inside the main window so they can be used in parallel. The features were implemented in combination with emulating the functionality of the model introduced in chapter 3 thus this thesis-software is a super-set of the software used in Breuer et al. (2015).

5.2.1 Inspection

Inspection allows for a given simulation-configuration to step through the whole process match-by-match, keep track of the successful matches and the wealth-distribution of the agents at this time and to jump back to each successful match. Through an offer-book the state and current offerings of all agents at a given time of the simulation (including past successful matches) can be inspected and compared by opening multiple offer-book windows. Furthermore this feature provides the user with statistics of successful, failed and total matches and the current equilibrium. A great deal of attention was paid to the implementation of the real-time visualization of the agents wealth-distribution and the market-activities as visualization is of great importance for a successful inspection and interpretation of a dynamic process. To be able to compare the visual results of 2 different simulation-configurations there exist 2 "Inspection" tabs in the main window.

5.2.2 Networks

This functionality allows the modelling of different kinds of network-topologies as described in appendix A to restrict trading only to this kind of neighbourhood. Obviously network-topologies are graphs and thus it would have been required to implement a graph-library but that was not implemented. Instead the graph-library JUNG was used as it supported all the required features like visualisation, neighbourhood- and path-queries.

5.2.3 Replications

To get robust results replications are used to reduce the influence of random noise. This feature allows a specific simulation configuration for a given number of replications to be run in parallel. The specific simulation-configuration serves as a template - especially the network-topology - and each replication does a clone of the network together with the agents to be able to run in parallel without the need of synchronization. An attempt was made to implement the network as a shared network between all replications but because the agents serve as nodes this was not feasible and would have required a very fundamental re-factoring of already existing functionality. Thus the higher requirements in memory and time for cloning was accepted for a more elegant solution. When replications are processed, running ones can be inspected and information can be queried using the GUI. It is possible to cancel one replication, cancel a whole task which reduces thread-load, inspect wealth-distribution, see the failed and successful matches and statistics of already finished replications. Whenever a replication is finished it is added to the pool of results and the median result is re-calculated and visualized and informations updated. When all replications have finished the results are written to a new XML-file in the results-folder of the software. This result-file can be opened using the experiments-feature which is described below. A simulation-configuration can be saved as a separate new experiment-file or added to an existing experiment-file - see the next section for experiments.

5.2.4 Experiments

The experiments-feature allows the opening of experiment-files and result-files of replication- and experiment-runs. An experiment-file is just a collection of simulation-configurations to be run for a given number of replications. When an experiment-file is opened all simulation-configurations are displayed in a list and each of them can be opened in a new replication-tab which in turn can be run as explained in the section 5.2.3. To be able to conveniently run all simulation-configurations of an experiment-file without needing user-interaction, a command-line feature was implemented.

Each replication-run of a given simulation-configuration in the experiment-file produces a result-file with the name provided in the configuration. These result-files are dubbed result-files of experiment-runs as previously stated but are the same as the result-file of a replication-run as discussed in the section 5.2.3. When opening a result-file a new tab is added to the main-window and the statistics of the run together with equilibrium-statistics are displayed. Furthermore the wealth-distribution and market-activities are visualized and it is possible to view the statistics of each individual replication. Furthermore the user can display the network-topology used for this simulation-configuration which is important as random networks are different when newly created.

5.2.5 Result-file

Because an experiment-run can take a lot of time to save the results for later investigation, all necessary data is automatically stored as XML in a result-file when a replication has finished. Note that an experiment will thus write the result-file multiple times if it has multiple replications. It contains the following information:

- Agents wealth-distribution - mean of final wealth-distribution of all agents over all replications.
- Starting-, ending-time and duration in seconds of experiment.
- Experiment-configuration.
- Mean and standard-deviation of marginal agents i1 and i2.
- Graph-structure - some topologies create random-graphs thus the structure is stored to allow an inspection.
- Mean of successful, failed and total matching-rounds.
- Mean duration of a replication.
- Mean of market-activity.
- Meta-information of each replication: equilibrium, starting- and ending-time, duration.

5.3 Architecture

In this section the basic architecture of the thesis-software is discussed. It is important to note that the software of this thesis is a fat-client and has its major emphasis not on the software-development aspect but on the visual- and numerical results where the accompanying software is just a tool for the means to calculate the required results. Thus the architecture is guided by a simple division of layers into front-end, controller and back-end resembling a MVC architecture. The front-end is responsible for input and output of the user through GUI or command-line. The controller-layer provides encapsulated chunks of functionality of the back-end to the front-end. It is necessary to abstract, encapsulate and combine the stateful nature of communication with the back-end into a separate layer instead of polluting the front-end with it and creating unnecessary dependencies. The back-end layer provides the functionality where the real work is done e.g. simulation is executed. Theoretically the dependencies are top-down where the front-end includes only controller-functionality, the controller includes only back-end functionality and the back-end has no dependencies to the preceding layers. In this thesis-software a more pragmatic approach was chosen so this dogma was not followed where over-engineering and over-complication would have resulted when sticking strictly to the separation of dependencies. Thus in very few cases the controller- and back-end layer include front-end layer functionality to create different types of network topologies in a more convenient way. Also the front-end accesses instances of pure back-end classes for graphical visualization and information purposes. Despite the seemingly flawed architecture the development-process has proven to be very smooth - expansions and refactorings went quite smoothly and always resulted in a better and clearer structure with reduced code-smell which is always a sign for a good architecture.

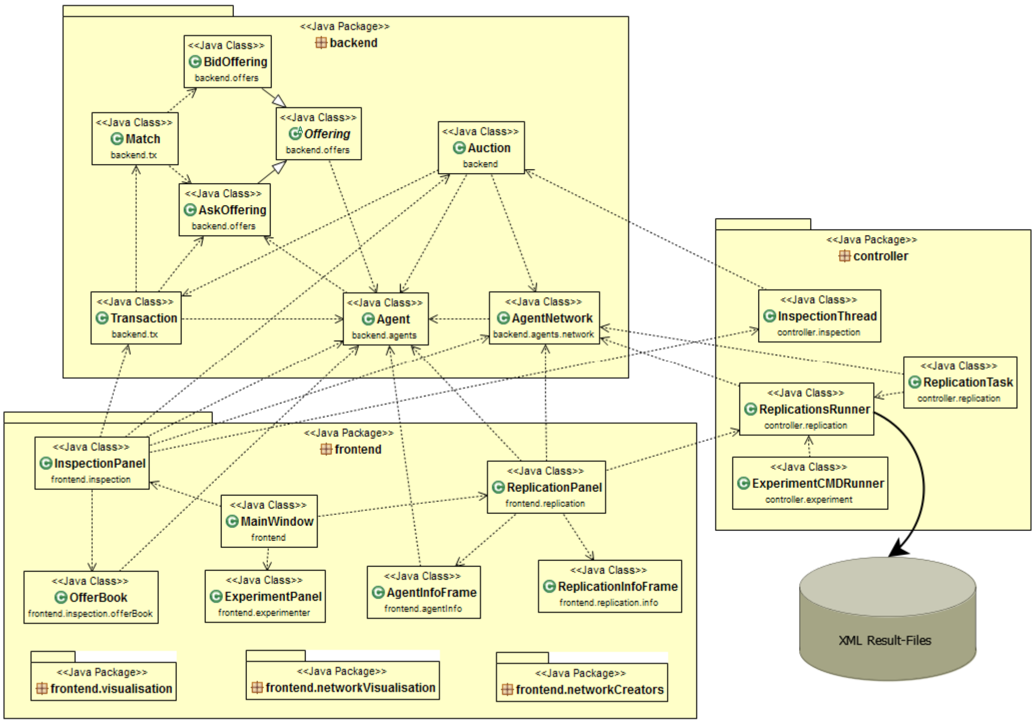

Figure 11: Package-diagram with most important classes and dependencies illustrating the architecture. (Note that the software contains many more classes and dependencies which are omitted for clarity reasons as this diagram should provide only a basic overview and would contain far too much information if everything would have been included.)

5.3.1 Frontend

The front-end contains the GUI functionality through which the user interacts with the program and the real-time visualization of wealth-distribution and market-activity. Because data like the offer-book, agent/replication/equilibrium-information and network-visualization is required in different contexts, much emphasis was put on massive re-use of GUI components through the use of sub-classing and aggregation already provided by SWING. Thus many small components were implemented and aggregated together to create bigger chunks of logical functionality. In the package-diagram 11 only the most important panels are shown. Those provide the main entry-point for the user to the given functionality as indicated by their names.

- InspectionPanel - Is the entry-point for the inspection-functionality. There exist 2 instances during run-time to be able to compare two inspection-runs.

- ReplicationPanel - Is the entry-point for the replication-functionality.
- ExperimentsPanel - Is the entry-point for the experiments-functionality. Results of replications and experiment-files can be loaded there.
- AgentInfoFrame - Displays the information of agents: current wealth, optimism and its price-ranges.
- OfferBook - Is only available in the inspection-functionality and allows inspection of the current offerings of each agent on all markets and the agent-information of each agent the same way as in *AgentInfoFrame*. Multiple instances can be created simultaneously by the user to compare the data of multiple agents.

5.3.2 Controller

The controller package is rather slim and contains the steering functionality to drive inspections, replications and experiments.

- InspectionThread - Inspections are driven by this class which allows the advancement of the simulation step-by-step using the consumer-producer paradigm through lock and wait functionality provided by Java through the use of its monitors.
- ReplicationsRunner and ReplicationTask - These classes encapsulate functionality to run a number of replications for a given simulation-configuration in parallel, calculating the current median for all important values on-line and writing results to xml-files.
- ExperimentCMDRunner - Experiments are executed from the command-line through the use of this class where it cycles through all simulation-configurations of a experiment-configuration and uses *ReplicationsRunner* to execute it in parallel.

5.3.3 Backend

The back-end contains the domain-specific functionality for the simulation. The important classes can be seen in the package-diagram 11 where each one is introduced briefly.

- Auction - holds the state and operations for a single simulation-run. This class does the sweeping and clearing discussed in section "Simulation", determines whether trading is still possible or not and calculates current equilibrium.

- Agent - encapsulates the functionality and state necessary for the agents in the simulation. See section 5.4 for a more in-depth discussion.
- AgentNetwork - holds the connections between the agents and provides functionality to query neighbourhood, paths and connections between agents and random and sequential in-order iterators over all agents.
- Offering - encapsulates the data necessary for offerings where there are two subclasses *BidOffering* and *AskOffering* to differentiate between the two types of offerings. See the section 5.5 on details of offerings.
- Match - provides the functionality to create a match out of given offers and returning an instance of *Match* through a factory-method. The *Match* instance holds the price, amount and the offers which have been matched. See section 5.7.1 for more details.
- Transaction - encapsulates functionality to search for a match within a given neighbourhood, is used by *Auction* class to sweep through the agents and is returned after a matching-round and provides necessary information about whether it was successful or not.

5.4 Agents

Although agents are used in this software it is not an agent-based simulation in the classical way as these agents are zero-intelligence ones. That is each agent makes bid- and ask-offers on all markets if the constraints allow it where the prices are selected from random ranges which improve the utility of the agent - that means it always makes offers which would result in a profit.

Each agent is characterized by its state where the main variables are:

- Id - the unique id of the agent in the range of the natural numbers in the range of [1..number of agents].
- Optimism-factor h - defines how optimistic the agent is in the range of [0..1]. The distribution of the optimism-factor among the agents is linearly ascending with the id and is defined through the following equation:

$$optimism\text{-}factor = \frac{id}{number\ of\ agents + 1} \tag{5.1}$$

  It is important that no agent has the optimism-factor of 0 because then the first derivative of the limit-function of the Asset/Bond market becomes undefined and is not monotony increasing if $h = 0$. This property is ensured by starting with id 1 and dividing by *number of agents* + 1.

- Cash holdings - the current cash holdings of the agent.
- Assets holdings - the current asset holdings of the agent including the assets granted as securities for giving loans.
- Loan given - the amount of loans bought from / granted to other agents. For a given amount of loans the equal amount of assets are granted as security to the buyer of the loan.
- Loan taken - the amount of loans sold to other agents. For a given amount of loans the equal amount of assets are granted as security to the buyer of the loan. Thus this variable decreases the amount of assets the agent can trade with as this amount of loans needs to be kept as security.

There are three important derived variables which are calculated from the previous ones:

- Collateralized assets - is the amount of assets which are bound through collateral obligations because of taken bonds.

$$\textit{collateralized assets} = max(0, \textit{loans taken} - \textit{loans given}) \tag{5.2}$$

- Free assets - is the amount of assets which are unbound and act not as security and are owned completely by the agent.

$$\textit{free assets} = \textit{assets holding} - \textit{collateralized assets} \tag{5.3}$$

- Loans - is the net number of loans and calculated through

$$\textit{loans} = \textit{loans given} - \textit{loans taken} \tag{5.4}$$

  Thus this value is positive if the agent has granted more loans to other agents than received and it is negative if the agent has received more loans from other agents than granted.

Note that all variables cannot be negative except loans.

5.5 Markets

In this section all markets and their implementations are described. Each market is completely characterised by the following points:

- Products - the products traded on the corresponding market.
- Price-ranges - the price-ranges in which an agent places profit-making offers on the market. A range is defined by a minimum and maximum where the limit-prices vary across this range. This implies that the most pessimistic agent is at the minimum end of the range and the most optimistic agent is at the maximum end of the range.
- Bid- and Ask-Offerings - define the pre-conditions for an agent to make offerings on the market and the amount and the prices generated during an offer. Note that in case of bid-offerings the price of the profit-making offer is drawn randomly between the minimum price and the limit-price because when buying, one wants to buy below the expected value to make a profit where in case of ask-offerings the price is drawn randomly between the limit-price and maximum price because when selling one wants to sell above the expected value to make a profit.
- Match-Table - In case of a match between two agents the wealth-exchange is declared where the wealth is increased/decreased as indicated by the +/- signs.

5.5.1 Asset/Cash

Products

Free assets are traded against cash. The buyer gets a specific amount of free assets for a given amount of cash where the seller gives away the specific amount of free assets and gets the given amount of cash.

Price-Range

minimum The minimum value of one asset in cash is the down-value pD tomorrow as defined in chapter 3. This value is obviously a constant for all agents.

$$min_{asset} = pD \tag{5.5}$$

maximum The maximum value of one asset in cash is the up-value pU tomorrow as defined in chapter 3. This value is obviously a constant for all agents.

$$max_{asset} = pU \tag{5.6}$$

limit The limit-price of an asset in cash depends on the optimism-factor h of the agent where the most optimistic agent assigns pU and the most pessimistic pD as the price.

$$limit_{asset} = h\ pU + (1.0 - h)\ pD \tag{5.7}$$

Bid-Offering

An amount of TRADING-UNIT of assets is selected but if there is not enough cash left to buy TRADING-UNIT of assets then no bid-offer is made.

Table 1: Bid-Offering parameters of Asset/Cash market

Pre-Condition	*cash holdings* > *price of TRADING-UNIT assets*
Asset-Price	random($min_{asset}, limit_{asset}$)
Asset-Amount	*TRADING-UNIT*

Ask-Offering

Ask offers are generated only when the agent has at least TRADING-UNIT of free assets left. As amount TRADING-UNIT assets is selected but if there are fewer free assets left then no offer is made.

Table 2: Ask-Offering parameters of Asset/Cash market

Pre-Condition	*free assets* > *TRADING-UNIT*
Asset-Price	random($limit_{asset}, max_{asset}$)
Asset-Amount	*TRADING-UNIT*

Match

Table 3: Wealth-Exchange during a match on Asset/Cash market

	Seller	Buyer
Assets holdings	- matching-amount	+ matching-amount
Cash holdings	+ matching-price	- matching-price

5.5.2 Bond/Cash

In chapter 4 it was proven that the limit-function of the Bond/Cash market is not monotony increasing if the face-value of the bond available is $V = pD$. This needs to be covered by the implementation through ensuring no Bond/-Cash market activity in this case as mathematically no match is possible as proven.

Products

Bonds are traded against cash. The buyer grants the seller a loan in buying a bond from the seller thus the buyer gets a given bond-amount from the seller and gives a given cash-amount to the seller. For a given amount of sold bonds the equal amount of assets needs to be held as security.

Price-Range

minimum The minimum value of one bond in cash is the down-value pD tomorrow as defined in chapter 3. This value is obviously a constant for all agents.

$$min_{bond} = pD \tag{5.8}$$

maximum The maximum value of one bond in cash is the face-value V tomorrow as defined in chapter 3. This value is obviously a constant for all agents.

$$max_{bond} = face\text{-}value\ \ V \tag{5.9}$$

limit The limit-price of a bond in cash depends on the optimism-factor h of the agent where the most optimistic agent assigns face-value V and the most pessimistic pD as the price. Thus applying linear interpolation one receives the following equation.

$$limit_{bond} = h\ V + (1.0 - h)\ pD \tag{5.10}$$

Bid-Offering

Bid offers are generated only when the agent has any cash holdings. An amount of TRADING-UNIT bonds is selected but if there is not enough cash left to buy TRADING-UNIT of bonds then the amount of bonds is selected which can be bought with the remaining cash holdings.

Table 4: Bid-Offering parameters of Bond/Cash market

Pre-Condition	$cash\ holdings > 0$
Bond-Price	random($min_{bond}, limit_{bond}$)
Bond-Amount	min($\frac{cash\ holdings}{Bond\text{-}Price}$, $TRADING\text{-}UNIT$)

Ask-Offering

Ask offers are generated only when the agent has a positive amount of loans. An amount of TRADING-UNIT bonds is selected but if there are fewer loans left then the remaining amount of loans is selected.

Table 5: Ask-Offering parameters of Bond/Cash market

Pre-Condition	$loans > 0$
Bond-Price	random($limit_{bond}, max_{bond}$)
Bond-Amount	min(*free assets*, $TRADING\text{-}UNIT$)

Match

Table 6: Wealth-Exchange during a match on Bond/Cash market

	Seller	Buyer
Loan Given	N/A	+ matching-amount
Loans Taken	+ matching-amount	N/A
Cash holdings	+ matching-price	- matching-price

5.5.3 Asset/Bond

Products

Assets are traded against bonds. The buyer gets a specific amount of free assets for a given amount of bonds where the seller gives away the specific amount of free assets and gets the given amount of bonds.

Price-Range

minimum The minimum value of one asset in bonds is defined as the ratio of the minimum asset-price in cash to the minimum bond-price in cash. This value is obviously a constant for all agents.

$$min_{asset/bond} = \frac{min_{asset}}{min_{bond}} = \frac{pD}{pD} = 1 \tag{5.11}$$

maximum The maximum value of one asset in bonds is defined in the ratio of the maximum asset-price in cash to the maximum bond-price in cash. This value is obviously a constant for all agents.

$$max_{asset/bond} = \frac{max_{asset}}{max_{bond}} = \frac{pU}{V} \tag{5.12}$$

limit The limit-price in bonds of an asset in bonds depends on the optimism-factor h and is just the ratio of the limit-price of the asset to the limit-price of the bond.

$$limit_{asset/bond} = \frac{limit_{asset}}{limit_{bond}} \tag{5.13}$$

Bid-Offering

Bid offers are generated only when the agent is not negative on free assets after the trade. An amount of TRADING-UNIT assets is selected. To calculate the free assets after a match the following steps are performed:

1. Start with current assets holdings.
2. Subtract current collateral obligations.
3. Subtract taken loans after trade.
4. Add assets bought through trade.

Table 7: Bid-Offering parameters of Asset/Bond market

Pre-Condition	*free assets >= 0 after trade*
Asset/Bond-Price	random($min_{asset/bond}$, $limit_{asset/bond}$)
Asset/Bond-Amount	*TRADING-UNIT*

Ask-Offering

Bid offers are generated only when the agent is not negative on free assets after the trade. An amount of TRADING-UNIT bonds is selected. To calculate the free assets after a match the following steps are performed:

1. Start with current assets holdings.
2. Subtract current collateral obligations.

3. Add given loans after trade.
4. Subtract assets sold through trade.

Table 8: Bid-Offering parameters of Asset/Bond market

Pre-Condition	*free assets* $>=$ *0 after trade*
Asset/Bond-Price	random$(limit_{asset/bond}, max_{asset/bond})$
Asset/Bond-Amount	*TRADING-UNIT*

Match

Table 9: Wealth-Exchange during a match on Asset/Bond market

	Seller	Buyer
Loan Given	+ matching-amount	N/A
Loans Taken	N/A	+ matching-amount
Asset holdings	- matching-price	+ matching-price

5.6 Bond pledgeability (BP) mechanism

Simply put without BP an agent can only increase the loans taken from other agents but never reduces them. Breuer et al. (2015) define the BP-mechanism technically as:

> This amounts to a collateral constraint requiring the negative *net* number of bonds (short minus long) to be less or equal the number of assets.

Thus to implement the BP-mechanism one must be able to differentiate between the amount of loans the agent has taken (short) and the amount of loans the agent has given (long) which is already done through the two loan-variables *loan given* and *loan taken.*

In the next step the equation of the collateralized assets must include both the given and taken loans - see equation 5.2. Without BP the given loans would be omitted and loan-obligations could only increase. This implies that the amount of free assets is affected too by BP because the variable *free assets* depends on the amount of collateralized assets.

Another implication of BP is that it enables agents with a positive net-amount of loans to place ask-offers on the Bond/Cash market although they have no more free assets - see section 5.5.3. The positive amount of loans is backed by the agents which have taken the loans by their securities thus one can view the positive amount of loans as an additional amount of assets which cannot be traded freely but are backing the loans. Thus it is allowed to sell a loan to another agent which is effectively just a transfer of the loan and its virtual security backup to the buyer.

5.7 Simulation

In this section a few details of the simulation-implementation are discussed as they are not so obvious but interesting and important enough to be mentioned.

As noted above in the section 5.5 bonds and assets are always traded in chunks of TRADING-UNITS which is 0.1 in case of assets and 0.2 in case of bonds which are the same values used in the implementation of Breuer et al. (2015). Note that it is nothing unusual to scale the initial wealth-endowments to 1.0 and trade fractions of them as in the end the absolute endowment does not matter but the distribution over the agents so a transformation to natural numbers would not gain anything. What is much more important is that the system conserves wealth: no wealth is lost and no wealth is created *ex nihilo*. This is guaranteed by the equations and pre-conditions found in the section 5.5.

5.7.1 Sweeping and Matching

Sweeping Sweeping is the process of searching a successful match by running through the agents and apply matching between their offers. Because offering-prices are created randomly in specific ranges it could happen that no offers match. To elevate this problem not only 1 but up to 500 sweeps are done when calculating the next successful match. The number of 500 was chosen empirically through experiments as it turned out to be a good compromise between responsiveness of the GUI and efficiency. It is important to note that if a user initiates to cancel the simulation the current matching-round needs to be finished finished thus the lower this value the quicker the response but the more failed matching-rounds will happen.

Matching Matching is the process of finding a match of a given agent in the sweeping-process with the offerings of its neighbourhood. Note that a match can only occur on one specific market between two agents thus satisfying only one buy- and one sell-offer.

Algorithm 1 Sweeping Pseudocode

clear best offerings of all agents
while *sweeps* < 500 **do**
    clear global offerings of previous sweep
    shuffle agents
    generate offerings for all agents
    for all shuffled agents **do**
        find match in neighbourhood
        if match found **then**
            execute match
            exit sweeping
    if no trading possible **then**
        exit sweeping

Algorithm 2 Matching Pseudocode

get best offerings of neighbourhood
for all best neighbourhood offerings **do**
    randomly check sell or buy offerings first
    if *buy-price* $\geq$ *sell-price* **then**
        $m \leftarrow$ **new Match**
        $m.price = \frac{\textit{buy-price}+\textit{sell-price}}{2}$
        $m.amount = \min(\textit{buy-amount}, \textit{sell-amount})$
        $m.normPrice = m.price * m.amount$
        exit matching

Note that as defined in the double-auction theory - see chapter 2 - the agents meet at the half-way price. This guarantees that not more than the previously defined amount is traded which again conserves the wealth. Care must be taken when calculating the real price of the match as all offering prices are for 1.0 Unit of the traded good. Thus to obtain the real matching-price the previously calculated half-way price needs to be scaled by the matching-amount to obtain the price for the given matching-amount. Also note that if one match is found on any market then the sweeping and matching is terminated and a successful match is returned thus the agents can place offers on all markets although the sum of the offers would be infeasible because only one offer will be satisfied and the other ones are thrown away.

5.8 Performance improvement

5.8.1 Local and Global Offer-book

The pseudo-code presented in algorithm **??** shows how to get the best offerings within a neighbourhood. One starts with the current agent and iterates over all neighbours and compares their offerings with the current best offerings. If any of the neighbour offerings is better than the current best then the current best is replaced by the one of the neighbour. It is easy to see that the calculation of the best neighbourhood can be quite costly the more neighbours an agent has. For the Ascending-Connected topology this is not a big deal but in the case of the fully-connected topology the impact is serious. Because fully-connected is an important topology as it serves as a benchmark for the others an optimization was implemented specially suited for this kind of topology: the introduction of a global offer-book. Instead of calculating the best offerings within the neighbourhood all offers are checked against each other in a global offering-list during the time of offering-creation. After all offers have been generated this list contains only the best sell and ask offerings for each market. The step of calculating the best neighbourhood can then be omitted in the case of fully-connectedness because it would be the same as the global best-offerings list thus saving a substantial amount of processing time.

5.8.2 Matching probabilities

General matching For a match to happen the buyer-price must be larger or equal to the seller-price. Thus as shown in chapter 4 a price higher than

Algorithm 3 Get Best Offerings of Neighbourhood Pseudocode

```
bestOfferings ← null
a ← current agent
for all neighbours of a do
    n ← next neighbour of a
    neighbourOfferings ← n.currentOfferings
    for all neighbourOfferings do
        nO ← next offering in neighbourOfferings
        for all bestOfferings do
            bO ← next offering in bestOfferings
            if nO.price dominates bO.price then
                bO ← nO
```

the seller-price can only be placed by a buyer which has a strictly larger optimism-factor than the seller because only then their offering-ranges have a chance to cover each other partly. Thus matching in general works between a seller with a lower optimism-factor and a buyer with a higher optimism-factor. As noted in chapter 3 agents place offering-prices randomly in their ranges which are specified in this chapter in the section 5.5 so matching is also subject to randomness. To investigate the matching-probabilities one must look at the coverage probabilities of the matching-ranges. The limit-price of bidders increases with rising optimism-factor and thus the width of the bidding-range decreases because bidders make their offers in the range of [pD..limit]. The situation is the inverse for askers as their limit-price increases too but the width of their asking-range increases because they place their offers in the range of [limit..pU]. Thus obviously the larger the difference in the optimism-factor between a seller and a buyer - again note that buyers optimism-factors are strictly larger than those of their corresponding sellers - the more likely a match will happen as both ranges increase and cover a greater total distance thus increasing the probability that prices will be drawn that match. The following formulae derive the probabilities that an asker and a bider with a given limit-price will match on the Asset/Cash market. First the coverage-range is calculated by subtracting the limit-price of the asker from the one of the bidder as the one of the bidder must be larger. Then the probabilities of the asker and of the bidder to fall into the range of the coverage-range is calculated. Finally the probability that both the asker and bidder range will match is calculated by multiplying the previously calculated probabilities. Note that a division by 2 needs to be done due to symmetry because it does not matter where in the range the prices will meet, but because the buy-price has to be greater than or equal to the seller-price only the half-range is available.

$$\begin{aligned} coverage\text{-}range &= limit\ bidder - limit\ asker \\ p_{asker}(match) &= \frac{coverage\text{-}range}{pU - limit\ asker} \\ p_{bidder}(match) &= \frac{coverage\text{-}range}{limit\ bider - pD} \\ p_{asker\&bidder}(match) &= \frac{p(asker) * p(bider)}{2} \end{aligned} \tag{5.14}$$

Matching in ascending-connectedness When considering the Ascending-Connected topology, matching-probabilities become quite an issue as each agent has only 2 neighbours where the one with lower optimism-factor is the seller and the one with higher optimism-factor is the buyer. Thus compared to fully-connectedness where agents - depending on their optimism-factor - have multiple buyers and sellers the matching-probabilities are likely to be very low. To get a better understanding of the matching-probabilities in ascending-connectedness three diagrams visualizing them are shown below. Note that for clarity only 30 agents are considered on the Asset/Cash market. The following figure 12 visualizes the probabilities of pairwise neighbours to fall into their coverage-range as calculated by the formulas given above.

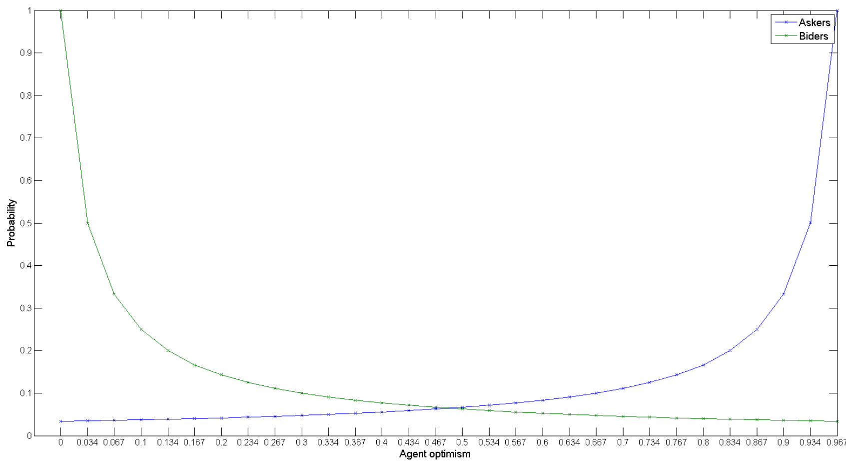

Figure 12: Single probabilities of falling into coverage-range of 30 agents in Ascending-Connected topology on Asset/Cash market

The first bidder with optimism-factor of 0.034 has a probability of 1.0 that the generated buy-prices fall into the range of its asker-neighbour with optimism-factor of 0. In turn the first asker with optimism-factor of 0 has a probability of 0.033 that its generated sell-prices fall into the range of its bid-neighbour with optimism-factor of 0.034. Thus the probabilities decrease for the bidders with increasing optimism-factor as their offering-range gets smaller and smaller where the probabilities increase for the askers with increasing optimism-factors as their offering-range gets wider and wider. This leads to the highest matching-probabilities at the edges and the lowest around the center.

The following figure 13 visualizes the combined probabilities that asker and bidder match their ranges.

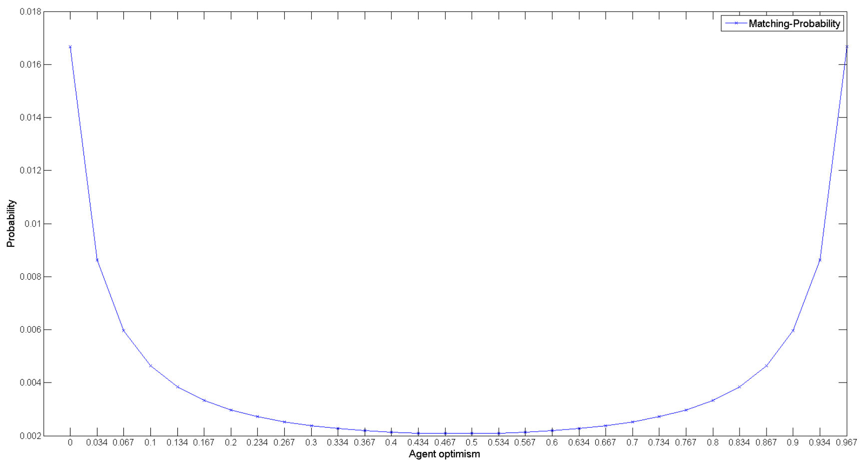

Figure 13: Pairwise matching-probabilities of 30 Agents in Ascending-Connected topology on Asset/Cash market

To see the influence of incresing number of agents, the following figure 14 visualizes the combined probabilities of 50 agents.

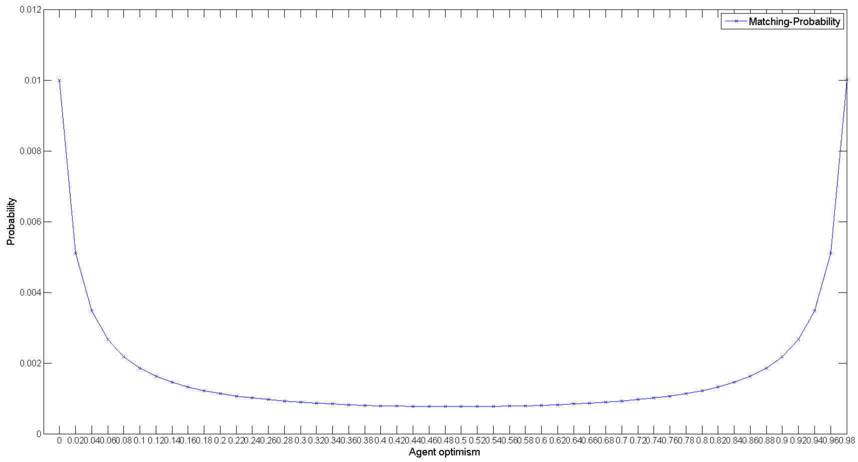

Figure 14: Pairwise matching-probabilities of 50 Agents in Ascending-Connected topology on Asset/Cash market

Looking at these diagrams one can derive the following facts about the matching-probabilities in Ascending-Connected topology:

1. Matching-probabilities are continuous and symmetric, are reduced towards the center of the optimism-scale and have their maximum at the very edges.

2. The more agents in the network the lower the matching-probabilities.

The lower matching-probabilities result in longer simulation-runs and more inefficiencies towards the end where free assets are kept by optimists instead of traded as the matching-probabilities decrease so much that no more trading will occur. To alleviate this problem one can adjust the matching-probabilities by transforming the price-ranges. It is of great importance that the shape of the matching-probabilities distribution must be exactly the same. Only the absolute values of the probabilities are increased where the edges are at the maximum of 1.0. This is mathematically quite involved and was developed by supervisor Mr. Vollbrecht. The formulae and explanations have been moved to appendix C. This variant is dubbed "Ascending-Connected topology with Importance Sampling". See chapter 6 for the performance and results of this variant.

Chapter 6

Results

In this chapter the results of the experiments are given. Each topology-type introduced in appendix A was simulated whereas in this chapter only Fully-Connected and Ascending-Connected topologies are handled as the Ascending-Connected topology - both with and without importance sampling - is the most minimal network which satisfies the requirements for the hypothesis. The results for the other topologies can be found in appendix B.

Note: The numbers in tables always represent a mean-value with the standard-deviation given in parentheses.

6.1 Validating thesis-software

As a point-of-reference and as an experimental proof for the correctness of the implementation of the thesis-software the result of a validation against the equilibrium found in Breuer et al. (2015) is given. Because equilibrium differs across the number of agents and the type of bond traded, to be comparable the same amount of agents and the same bond-type has to be used in the experiments. Thus 1,000 agents and a bond with face-value of 0.5 were chosen because Breuer et al. (2015) report their equilibria for this number of agents and bonds with face-value between 0.1 to 0.5.

6.1.1 References for 1,000 agents with 0.5 bond

Table 10: Equilibrium predictions in Breuer et al. (2015) for 1,000 agents and 0.5 bond

Asset-Price p	0.716
Bond-Price q	0.375
Marginal Agent i1	0.583
Marginal Agent i2	0.801
Pessimist Wealth	1.716
Medianist Wealth	4.578
Optimist Wealth	5.032

6.1.2 Thesis-software result for 1,000 agents with 0.5 bond

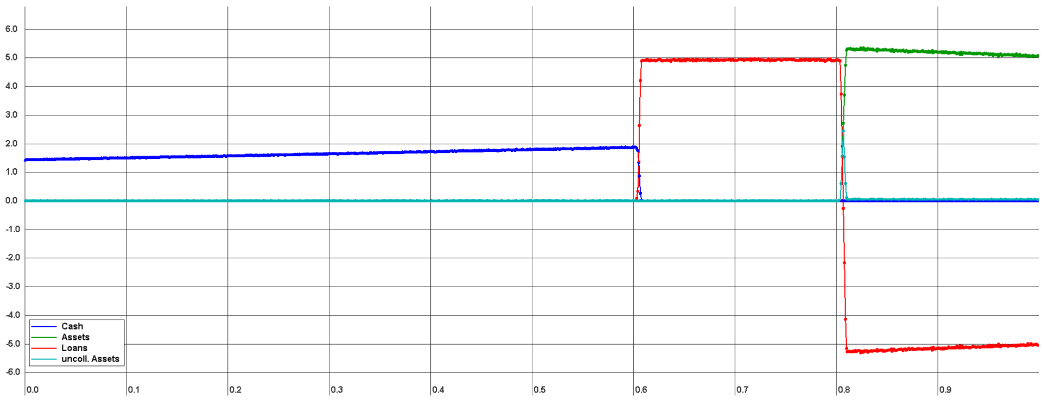

Figure 15: Wealth-Distribution of thesis-implementation of Fully-Connected topology for 1,000 agents and 0.5 bond

Table 11: Equilibrium of thesis-implementation for 1,000 agents and 0.5 bond

Asset-Price p	0.730 (0.011)
Bond-Price q	0.382 (0.000)
Marginal Agent i1	0.605 (0.001)
Marginal Agent i2	0.807 (0.001)
Pessimist Wealth	1.649 (0.001)
Medianist Wealth	4.895 (0.014)
Optimist Wealth	5.160 (0.018)

Table 12: Difference of Fully-Connected topology wealth-equilibrium as given in table 11 to wealth-equilibrium as predicted in Breuer et al. (2015) from table 10

	Result	Reference	difference to Reference
Asset-Price p	0.730	0.716	+2.0%
Bond-Price q	0.382	0.375	+1.9%
Marginal Agent i1	0.605	0.583	+3.8%
Marginal Agent i2	0.807	0.801	+0.7%
Pessimist Wealth	1.649	1.716	-3.9%
Medianist Wealth	4.895	4.578	+6.9%
Optimist Wealth	5.160	5.032	+2.5%

Although Marginal Agent i1 and Bond-Price q are quite different than from theoretical equilibrium and the pessimists wealth is 7.8% less than given in Breuer et al. (2015) these results are nonetheless accepted as reaching the equilibrium. The differences emerge from the reasons that the thesis-simulation runs were terminated earlier than in Breuer et al. (2015) which results in the i1 and i2 edges to be not as sharp as reported in Breuer et al. (2015). It would be necessary to run the simulation an order of magnitude longer as the matching probabilities are reduced rapidly when only direct neighbours are able to trade any more within a network of 1000 agents. See section 5.8 for details on matching-probabilities.

6.2 Performance and termination measurements

As noted in section 5.7.1 a matching-round performs up to 500 offering-rounds where during one round all agents make an offer to find a match. If a match occurs during one offering-round the current matching-round is terminated and marked as successful. If no match occurs during all 500 offering-rounds the current matching-round is terminated too but marked as failed. Thus the following terminology is defined:

Successful matching-round a match occurred within maximal 500 offering-rounds where in each offering-round all agents make an offer.

Failed matching-round no match occurred within 500 offering-rounds where in each offering-round all agents make an offer.

Termination criteria after 1,000 successive failed matching-rounds it is expected that no more trading will occur thus the simulation is terminated.

Table 13: Performance of thesis-implementation with 1,000 agents and 0.5 bond

Successful matching-rounds	18,167.86 (72.60)
Failed matching-rounds	757.44 (251.21)
Total matching-rounds	18,925.30 (268.15)
Ratio successful/total	0.96
Ratio failed/total	0.04

6.3 Experiments configuration

In the following experiments 100 agents were used, all markets (Asset/Cash, Bond/Cash, Asset/Bond) were enabled, a bond with face-value of 0.5 was selected and in each experiment 50 replications were run. A replication was terminated after 1,000 failed matching-rounds in a row. Note that if trading is not possible any more before this criteria is met the simulation is terminated and thus it is possible that it halts earlier as can be seen for the Ascending-Connected topology with Importance Sampling.

Breuer et al. (2015) showed that equilibrium can be reached already with 30 agents so this was the minimum number of agents to start with but for a smoother visual result 100 were chosen. Also one simulation-run does not take much time with 100 as compared to the 1,000 agents thus it is a very good match between visual accuracy and processing-power requirements.

As already described in section 5.7.1 the simulation is a random-process thus one needs replications to reduce noise. The number of 50 replications was chosen because it is a good match between processing-power requirements and overall reduction of noise. Thus increasing the number e.g. to 100 or 200 would not provide much better results - both visual and numerical - but would need much longer to run. All facts can already be seen and derived when using 50 replications thus for all figures this number was used unless stated otherwise e.g. a single run.

Table 14: Configuration for all experiments

Agent-Count	100
Bond-Type	0.5
Replication-Count	50
Matching-Round	max. 500 offering-rounds
Terminate after	1,000 failed successive matching-rounds

Table 15: Theoretical Equilibrium for 100 agents and 0.5 bond

Asset-Price p	0.717
Bond-Price q	0.375
Marginal Agent i1	0.584
Marginal Agent i2	0.802

6.4 Fully-Connected

This topology serves as the major point-of-reference for the other experiments as it reaches the theoretical equilibrium for 1,000 agents as demonstrated and explained.

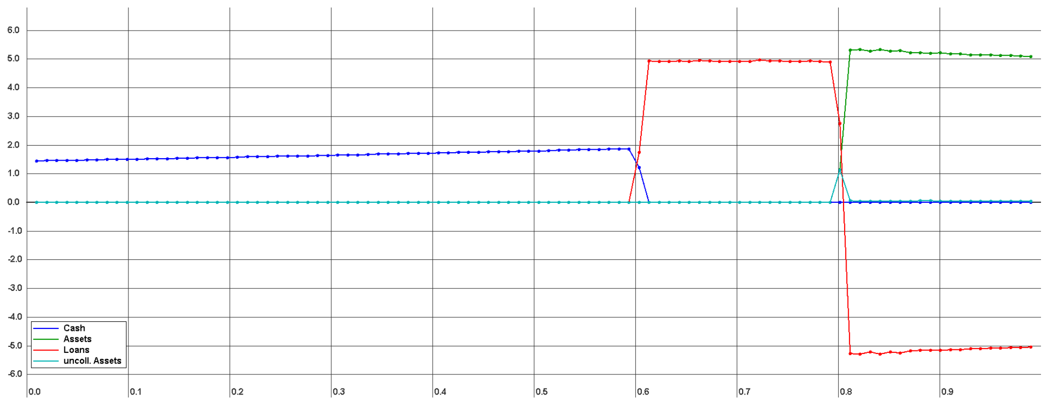

Figure 16: Wealth-distribution of Fully-Connected topology

Table 16: Equilibrium of Fully-Connected topology

Asset-Price p	0.717 (0.015)
Bond-Price q	0.382 (0.001)
Marginal Agent i1	0.596 (0.004)
Marginal Agent i2	0.810 (0.004)
Pessimist Wealth	1.646 (0.004)
Medianist Wealth	4.736 (0.089)
Optimist Wealth	5.179 (0.082)

Table 17: Performance of Fully-Connected topology

Successful matching-rounds	1,812.20 (22.76)
Failed matching-rounds	1,287.68 (129.55)
Total matching-rounds	3,099.88 (135.68)
Ratio successful/total	0.58
Ratio failed/total	0.42

Table 18: Difference to theoretical equilibrium

	Result	Reference	difference to Reference
Asset-Price p	0.717	0.717	0.0%
Bond-Price q	0.382	0.375	+1.9%
Marginal Agent i1	0.596	0.584	+2.0%
Marginal Agent i2	0.810	0.802	+1.0%

6.5 Ascending-Connected topology

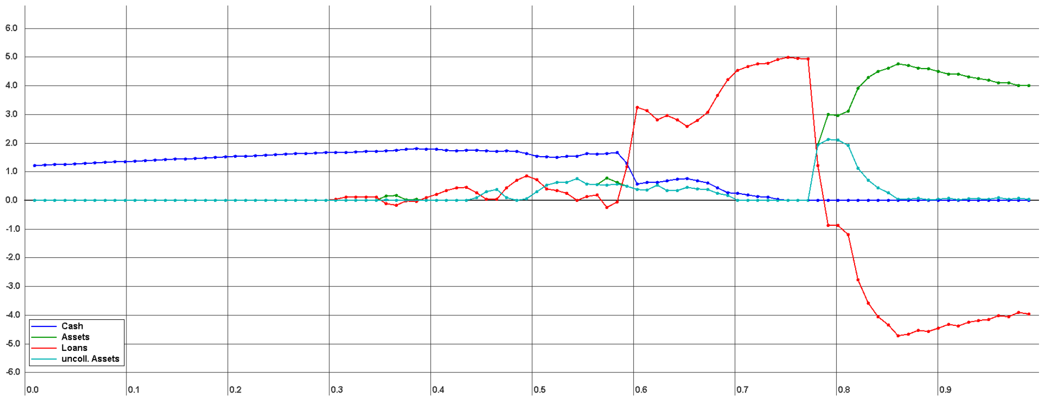

Figure 17: Wealth-distribution of Ascending-Connected topology

Table 19: Equilibrium of Ascending-Connected topology

Asset-Price p	0.698 (0.034)
Bond-Price q	0.375 (0.018)
Marginal Agent i1	0.629 (0.054)
Marginal Agent i2	0.675 (0.130)
Pessimist Wealth	1.545 (0.147)
Medianist Wealth	4.628 (0.632)
Optimist Wealth	3.428 (1.293)

Table 20: Performance of Ascending-Connected topology

Successful matching-rounds	47,021.48 (4,202.62)
Failed matching-rounds	1,001.06 (2.73)
Total matching-rounds	48,022.54 (4,201.62)
Ratio successful/total	0.98
Ratio failed/total	0.02

Table 21: Difference to theoretical equilibrium

	Result	Reference	difference to Reference
Asset-Price p	0.698	0.717	-2.6%
Bond-Price q	0.375	0.375	0.0%
Marginal Agent i1	0.629	0.584	+7.7%
Marginal Agent i2	0.675	0.802	-15.8%

Table 22: Difference to Fully-Connected topology equilibrium

	Result	Reference	difference to Reference
Asset-Price p	0.698 (0.034)	0.717 (0.015)	-2.6% (+126%)
Bond-Price q	0.375 (0.018)	0.382 (0.001)	-1.8% (+1700%)
Marginal Agent i1	0.675 (0.054)	0.596 (0.004)	+13.2% (+1250%)
Marginal Agent i2	0.850 (0.130)	0.810 (0.004)	+4.9% (+3150%)
Pessimist Wealth	1.545 (0.147)	1.646 (0.004)	-6.1% (+3575%)
Medianist Wealth	4.628 (0.632)	4.736 (0.089)	-2.2% (+610%)
Optimist Wealth	3.428 (1.293)	5.179 (0.082)	-33.8% (+1476%)

6.5.1 Ascending-Connected with Importance Sampling

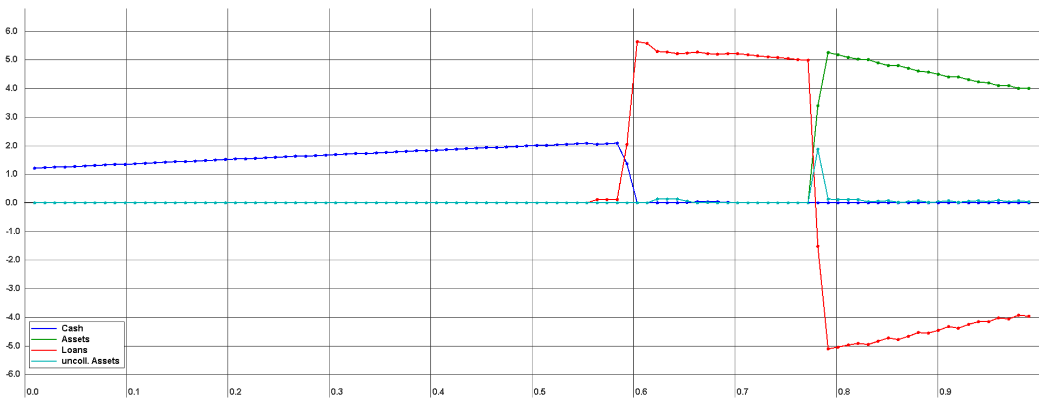

Figure 18: Wealth-distribution of Ascending-Connected toplogy with Importance Sampling

Table 23: Equilibrium of Ascending-Connected topology with Importance Sampling

Asset-Price p	0.710 (0.006)
Bond-Price q	0.379 (0.002)
Marginal Agent i1	0.586 (0.014)
Marginal Agent i2	0.779 (0.022)
Pessimist Wealth	1.666 (0.035)
Medianist Wealth	5.094 (0.109)
Optimist Wealth	4.501 (0.291)

Table 24: Performance of Ascending-Connected topology with Importance Sampling

Successful matching-rounds	50,989.30 (637.48)
Failed matching-rounds	1.00 (0.00)
Total matching-rounds	50,990.30 (637.48)
Ratio successful/total	1.00
Ratio failed/total	0.00

Note that in this case the matching-probabilities are such that upon the first failed matching-round the equilibrium is reached as no agent can trade with each other any more which results in just one single failed matching-round.

Table 25: Difference to theoretical equilibrium

	Result	Reference	difference to Reference
Asset-Price p	0.710	0.717	-0.9%
Bond-Price q	0.379	0.375	+1.0%
Marginal Agent i1	0.586	0.584	+0.3%
Marginal Agent i2	0.779	0.802	-2.8%

Table 26: Difference to Fully-Connected topology equilibrium

	Result	Reference	difference to Reference
Asset-Price p	0.710 (0.006)	0.717 (0.015)	-0.9% (-60%)
Bond-Price q	0.379 (0.002)	0.384 (0.001)	-1.3% (+100.0%)
Marginal Agent i1	0.586 (0.014)	0.603 (0.004)	-2.8% (+250%)
Marginal Agent i2	0.779 (0.022)	0.803 (0.004)	-2.9% (+450%)
Pessimist Wealth	1.666 (0.035)	1.646 (0.004)	+1.2% (+775%)
Medianist Wealth	5.094 (0.109)	4.736 (0.089)	+7.5% (+22%)
Optimist Wealth	4.501 (0.291)	5.179 (0.082)	-13.0% (+254%)

Chapter 7

Interpretation

In this chapter the interpretation of the results of chapter 6 are given and discussed where the central question is whether the hypothesis is sufficient that is if Ascending-Connected topology reaches the theoretical equilibrium. Thus only this topology is handled - both with and without importance sampling - because it is the most minimal network which satisfies the requirements for the hypothesis. The interpretations for the results of hub-, scale-free and small-world topologies are handled in appendix B but only to a minimal extent as they turn out to fall far from satisfying the hypothesis and the theoretical equilibrium because almost all of them do not meet the requirements although they do show interesting behaviour.

7.1 Validating the hypothesis

When comparing the results of Ascending-Connected topology of figure 17 with the results of the Fully-Connected topology of figure 16 it becomes immediately clear that the equilibrium is different from the one of the Fully-Connected network and thus theoretical equilibrium is not reached in the case of Ascending-Connected without importance sampling. Although the visual results come quite close to the Fully-Connected one where there is a clear distinction between pessimists, medianists and optimists there remain serious artefacts in the wealth-distribution. Thus the hypothesis is only necessary but not sufficient to reach theoretical equilibrium.

7.2 Analysing artefacts

Obviously the artefacts in the range of the pessimists and medianists and partly of the optimists indicate a miss-allocation of wealth. Pessimists, as noted in chapter 3 are maximally short on assets and bonds and hold only cash but the results violate this prediction as the pessimists still hold free- and collateralized assets and a few bonds. Medianists should hold only bonds but still have cash and free assets left and the optimists still have a big chunk of free assets left. As will be shown it comes from the fact that the pessimists want to sell but no neighbour is able to buy any more - a scenario which is not possible in Fully-Connected topology and is thus unique to Ascending-Connected networks with and without importance sampling. In fully-connected networks all agents are connected with each other thus if there is any agent who wants to sell some goods there will be statistically speaking not only 1 potential buyer as in ascending-connected networks but a much larger range, depending on the sellers optimism-factor and the number of agents. In ascending-connected networks each seller has only one buyer where this buyer is at the same time the seller for the next buyer and so on. This can lead to situations where one seller wants to sell goods but its buyer can't buy any more as it has already sold e.g. all assets in its role as seller and is thus unable to satisfy the sellers offer due to trading constraints. Even worse a buyer farther away could satisfy the sell-offer but is not directly connected to the seller so they can't trade, although both could potentially match.

7.2.1 Dynamics of a single run

To better understand how such artefacts arise one needs to investigate the dynamics of a single run of the Ascending-Connected topology. The tools used are both the *market-activity* and *wealth-distribution* diagrams where the former one shows for each point of time the relative activity strength of each market. Being active means a successful match on a given market which implies that in a successful matching-round only one market can be active as only one match on a specific market happens. Because of this a moving window of size 100 is used to create a moving-average filter over all active markets where the result is normalized and all market-activity sums to 1.0 at each point in time of the diagram. This allows for a very good visual analysis of distinct trading-stages because noise is reduced but the overall trend of a market can be still seen clearly.

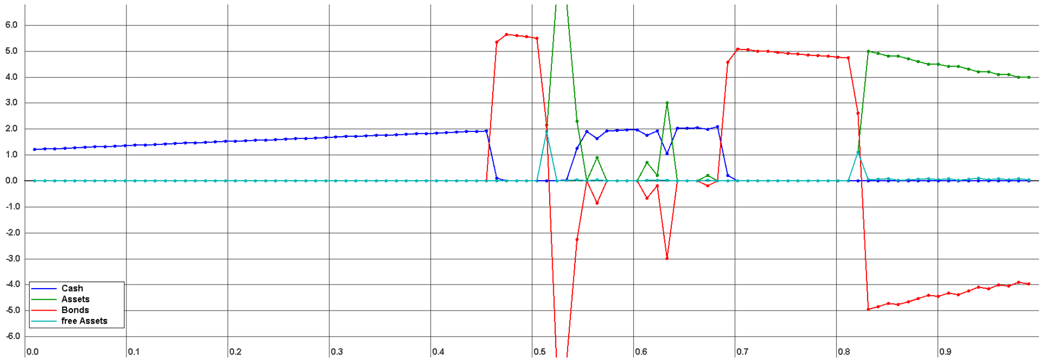

Figure 19: Final wealth-distribution after a single run of the Ascending-Connected topology. Note the artefacts in the range of the pessimists. The following wealth-distributions are all taken from various points in time of this single run.

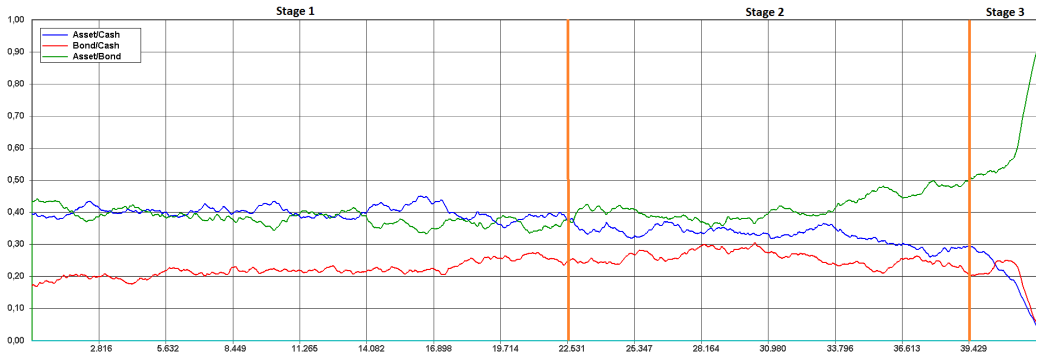

Figure 20: Market-activity stages of the single run of figure 19

3 trading-stages can be identified in the market-activity diagram 20.

Stage 1 The allocations are very random overall but pessimists are already emerging which can be seen at the very left end of the wealth-range. They sell their free assets against cash thus holding primarily cash. The optimists are beginning to emerge as well by buying as many assets as they can which can be seen at the very right end of the wealth-range. The medianists are far from showing up.

The Asset/Cash and Asset/Bond markets are very dominant in this stage as the pessimists try to get cash for their free assets and the optimists try to buy assets against cash and bond. The Bond/Cash market is not as active but nevertheless contributes as well because optimists use bonds to leverage their asset trades and pessimists try to get their bonds towards the optimists by using the BP-mechanism.

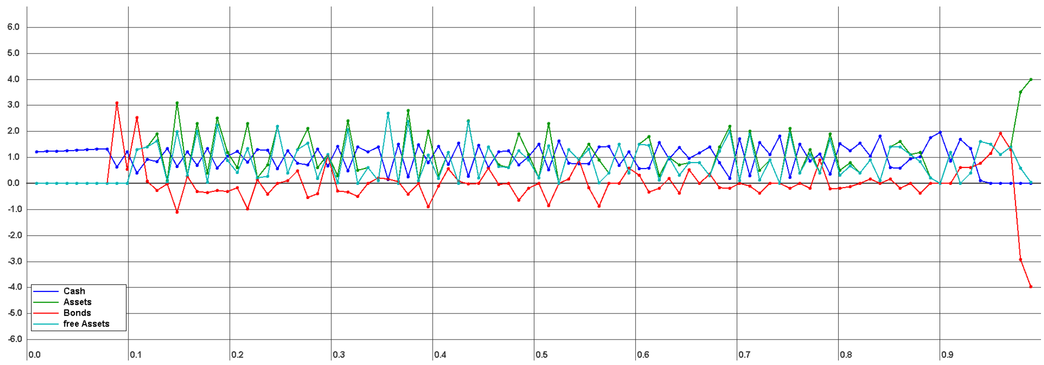

Figure 21: Wealth-Distribution of Ascending-Connected topology during Stage 1 of the single run of figure 19.

Stage 2 The pessimists are clearly visible and hold both bonds and collateralized assets which they try to trade up to the optimists. The optimists are also clearly visible as they are maximally short on cash and hold either free or collateralized assets. The medianists are still not visible yet.

The Asset/Cash market seems to go down in the long term while the Bond/Cash and Asset/Bond markets seems to increase. This is because fewer and fewer assets can be traded against cash because the optimists are already very low on cash thus the Asset/Bond market is naturally increasing as they can only trade only on this market anymore.

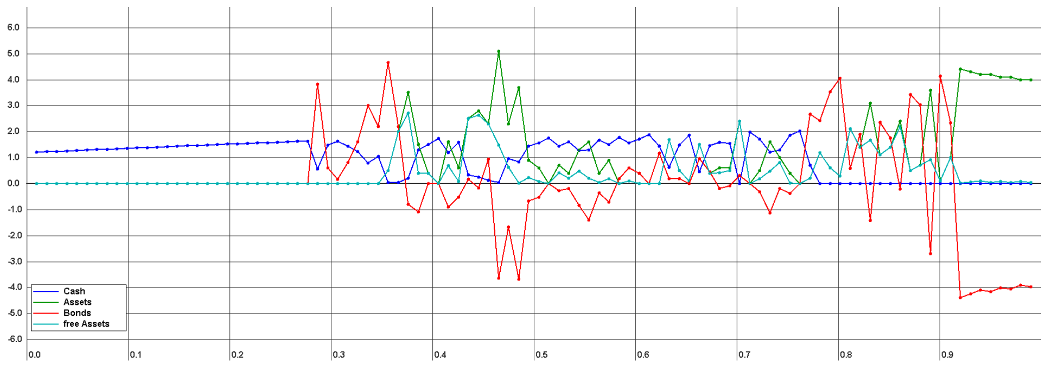

Figure 22: Wealth-Distribution of Ascending-Connected topology during Stage 2 of the single run of figure 19.

Stage 3 The pessimists lie dormant and are completely inactive. Although they still hold positive bonds and free- and collateralized assets they have become unable to trade them towards the optimists. The reason for this is investigated in section 7.3. The medianists begin to show up holding only bonds and free- and collateralized assets. The optimists try to get their hands on these free- and collateralized assets which they can only do by trading assets for bonds because they have no more cash. Thus these two frontiers move towards each other and will result in the final wealth-distribution found in figure 19.

The Asset/Cash and Bond/Cash markets decline completely because the pessimists are no longer able to trade as outlined above and the optimists and medianists are maximally low on cash thus being unable to make offers on these markets too. The Asset/Bond market takes over and dominates 100% as the medianists hold only free- and collateralized assets any more that the optimists want to buy which they can only do through this market by taking a bond for buying the asset.

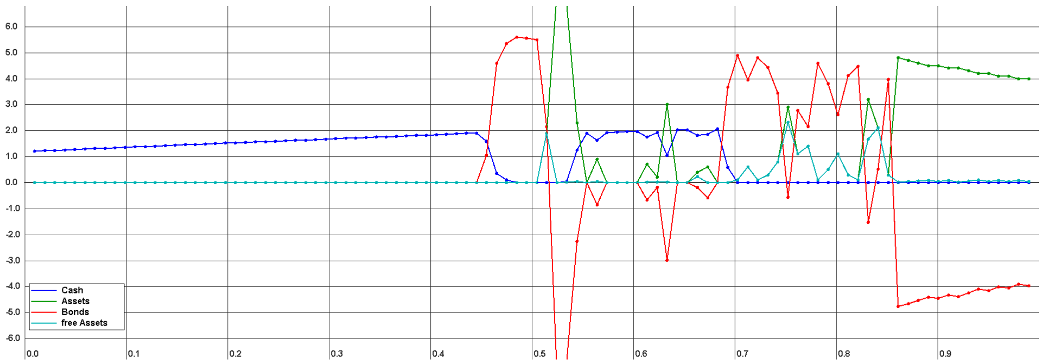

Figure 23: Wealth-Distribution of Ascending-Connected topology during Stage 3 of the single run of figure 19.

7.3 Deriving the emerging of the artefacts

7.3.1 Blocking-Situation prevents trading

As already outlined the artefacts arise due to a blocking situation where trading becomes and *remains* impossible between two neighbours. The seller places ask-offerings on a given market but due to trading constraints the only potential buyer which is the neighbour with the next higher optimism-factor can't place any potential matching bid-offering on the given market. In figure 19 this blocking-situation can be seen between the agent-pairs 55

(0.545) & 56 (0.554), 57 (0.564) & 58 (0.574), 62 (0.614) & 63 (0.624), 64 (0.634) & 65 (0.644) and 68 (0.673) & 69 (0.683). In all pairs the seller, which is the agent with the lower optimism-factor, holds only collateralized assets and thus places ask-offers only on the Asset/Bond market. The buyer, which is the agent with the higher optimism-factor, holds only cash and places bid-offers only on the Asset/Cash and Bond/Cash market but not on the Asset/Bond market - trading has become impossible between the two agents. It is interesting to see that the wealth-distribution in the range of 0.455 - 0.554 already shows basic similarities to the global distribution because all wealth is traded up towards the blocking-point creating the small sub-distribution of pessimists, medianists and optimists.

7.3.2 Situation can occur only on the Asset/Bond market.

An agent can hold a mix of cash, free assets and positive bonds or collateralized assets. To show that the situation can occur only on the Asset/Bond market one must investigate the combinations of allocations and possible trading options.

1. Seller has cash, buyer any goods
   Seller does not sell anything.

2. Seller has positive bonds, buyer has cash
   Buyer can buy from seller through the BP-mechanism.

3. Seller has positive bonds, buyer has free assets
   No direct trading with these goods but buyer will act as seller and can sell assets up to optimists on Asset/Cash market thus generating cash which creates then the same situation as in 2.

4. Seller has positive bonds, buyer has collateralized assets
   No trading with these goods but buyer will act as seller towards higher optimists on the Asset/Bond market creating uncollateralized assets which results in cash and thus enabling to buy the positive bonds which creates then the same situation as in 2.

5. Seller has free assets, buyer has cash
   Buyer can buy assets from seller against cash.

6. Seller has free assets, buyer has positive bonds
   Buyer can by the assets against bonds.

7. Seller has free assets, buyer has collateralized assets
   Buyer can buy free assets against bonds.

8. Seller has collateralized assets, buyer has positive bonds
   Buyer can buy assets against bonds.

9. Seller has collateralized assets, buyer has free assets
   No trading with these goods but buyer will act as seller and will sell assets up to optimists against cash or bonds, thus generating cash or positive bonds which is then same as in 8 or 10.

10. Seller: collateralized assets, buyer has cash
    No trading with these goods as the buyer has no possibility to buy the collateralized assets against cash. In the role of a seller it can't sell anything to transform cash to other goods which could potentially unlock this situation.

The critical situation is the one given in the last point. As long as an agent has a mix of goods allocated, then the blocking situation does not occur as trading is possible through a different channel than the last one. However if the allocations result in an extreme where the seller holds *only* collateralized assets and the buyer *only* cash then the blocking-situation occurs and is impossible to unlock any more.

7.3.3 Artefacts are random

An important fact to notice is that the artefacts must not necessarily show up. It is possible for a single run to finish without these artefacts showing up. This is due to the random-process of sweeping and matching and thus the artefacts are subject to this random process too as noted in section 5.8. When looking at the result of Ascending-Connected topology with importance sampling in figure 18 it seems that Importance sampling elevates this problem as no artefacts have shown up during an experiment with 50 replications. This however is no proof that importance sampling is the solution to this problem as the mechanism is still the same. We conjecture that due to the dramatically increased matching-probabilities the occurrence of such miss-allocations have become highly unlikely but are not guaranteed to never show up any more thus the importance sampling is not accepted as a remedy for the miss-allocations. In an extra experiment with 200 replications artefacts have shown up which proves that importance sampling is definitely not a remedy for the miss-allocations.

7.3.4 Formal proof using utility-functions

It is of great importance to note that an agent can never have a total wealth of 0 in this simulation. Per definition in chapter 3 each trade results in an increase of the utility. If the agents start with the basic endowment of 1 unit of cash and 1 unit of assets as given in chapter 3 then they will start already with a utility > 0. Thus it can never reach 0.

To prove that trading is no longer possible the utility-functions of the seller and buyer are used where the utility-function as already outlined in chapter 3 is:

$$\begin{aligned} utility &= u_{asset} + u_{bond} + u_{cash} \\ &= limit_{asset}\ holdings_{asset} + \\ &\quad limit_{bonds}\ holdings_{bonds} + \\ &\quad (sell_{asset} - buy_{asset})\ price_{asset} + \\ &\quad (give_{bonds} - take_{bonds})\ price_{bonds} + \\ &\quad holdings_{cash} \end{aligned}$$

Substituting allocations of seller with only collateralized assets and buyer with only cash to get the utility *before* the potential trade.

$$\begin{aligned} utility_{seller} = limit_{asset}\ holdings_{asset} - \\ limit_{bond}\ holdings_{bond} \end{aligned}$$

Note that $holdings_{asset} = holdings_{bond}$ because all assets are collateralized and thus for all taken bonds the equal amount of assets is held as security.

$$utility_{buyer} = holdings_{cash}$$

Regarding the utility-functions the following trades are theoretically possible:

- Buyer buys the assets only and leaves the bonds to the seller.
- Buyer buys the negative bonds only and leaves the assets to the seller.
- Buyer buys both the assets and the negative bonds from the seller.

Now each possibility is investigated in regard to the utility-functions and we demand as defined in chapter 3 that each trade results in a utility-gain thus $utiliy_{after} > utility_{before}$ must hold.

Buyer buys all the assets only for cash and leaves the bonds to the seller. The utilities *after* the trade are

$$\begin{aligned} utility_{buyer} = limit_{asset}\ holdings_{asset} - \\ buy_{asset}\ price_{asset} + \\ holdings_{cash} \end{aligned}$$

Proof.

$$\begin{aligned} utiliy_{after} > utility_{before} & \\ = limit_{asset}\ holdings_{asset} - & \\ buy_{asset}\ price_{asset} + & \\ holdings_{cash} > holdings_{cash} & \qquad | - holdings_{cash} \\ = limit_{asset}\ holdings_{asset} - & \\ buy_{asset}\ price_{asset} > 0 & \qquad | holdings_{asset} = buy_{asset} \\ = limit_{asset}\ buy_{asset} - & \\ buy_{asset}\ price_{asset} > 0 & \qquad | : buy_{asset} \\ = limit_{asset} - price_{asset} > 0 & \\ = limit_{asset} > price_{asset} & \end{aligned}$$

□

$$\begin{aligned} utility_{seller} = -limit_{bond}\ holdings_{bond} + \\ sell_{asset}\ price_{asset} \end{aligned}$$

Proof.

$$\begin{aligned} utiliy_{after} > utility_{before} & \\ = -limit_{bond}\ holdings_{bond} + & \\ sell_{asset}\ price_{asset} > & \\ limit_{asset}\ holdings_{asset} - & \\ limit_{bond}\ holdings_{bond} & \\ = sell_{asset}\ price_{asset} > & \\ limit_{asset}\ holdings_{asset} & \qquad | sell_{bond} = holdings_{asset} \\ = sell_{asset}\ price_{asset} > & \\ limit_{asset}\ sell_{asset} & \qquad | : sell_{asset} \\ = price_{asset} > limit_{asset} & \\ = limit_{asset} < price_{asset} & \end{aligned}$$

□

Thus it is proven that $limit_{buyer} > price_{asset} > limit_{seller}$ which is only possible if the buyer has a higher optimism-factor than the seller which was already proven in chapter 4. Unfortunately this trade is not allowed as it would violate the collateral constraint which enforces that for each taken bond the same amount of assets must be held as security.

Buyer buys the negative bonds only and leaves the assets to the seller. The utilities *after* the trade are

$$\begin{aligned} utility_{buyer} = -limit_{bond}\, holdings_{bond} - \\ buy_{bond}\, price_{bond} + \\ holdings_{cash} \end{aligned}$$

Proof.

$$\begin{aligned}
utiliy_{after} > utility_{before} & \\
= -limit_{bond}\, holdings_{bond} - & \\
buy_{bond}\, price_{bond} + & \\
holdings_{cash} > holdings_{cash} & \qquad | - holdings_{cash} \\
= -limit_{bond}\, holdings_{bond} - & \\
buy_{bond}\, price_{bond} > 0 & \qquad | \; holdings_{bond} = buy_{bond} \\
= -limit_{bond}\, buy_{bond} - & \\
buy_{bond}\, price_{bond} > 0 & \qquad | : buy_{bond} \\
= -limit_{bond} - price_{bond} > 0 & \qquad \text{infeasible as limit-price and price are} > 0
\end{aligned}$$

□

The utility-gain for the buyer is impossible in this trade thus it is rejected.

Buyer buys both the assets and the negative bonds from the seller.

$$\begin{aligned} utility_{buyer} = limit_{asset}\, holdings_{asset} - \\ limit_{bond}\, holdings_{bond} - \\ buy_{asset}\, price_{asset} - \\ buy_{bond}\, price_{bond} + \\ holdings_{cash} \end{aligned}$$

Proof.

$$
\begin{aligned}
utiliy_{after} &> utility_{before} \\
= limit_{asset}\ holdings_{asset} - \\
limit_{bond}\ holdings_{bond} - \\
buy_{asset}\ price_{asset} - \\
buy_{bond}\ price_{bond} + \\
holdings_{cash} &> holdings_{cash} && \mid - holdings_{cash} \\
= limit_{asset}\ holdings_{asset} - \\
limit_{bond}\ holdings_{bond} - \\
buy_{asset}\ price_{asset} - \\
buy_{bond}\ price_{bond} &> 0 && \mid \text{all holdings and buy are same} = \text{h} \\
= limit_{asset}\ h - \\
limit_{bond}\ h - \\
h\ price_{asset} - \\
h\ price_{bond} &> 0 && \mid : h \\
= limit_{asset} - limit_{bond} - \\
price_{asset} - price_{bond} &> 0
\end{aligned}
$$

□

This is feasible as in case of the buyer $limit_{asset} > price_{asset}$ and $limit_{bond} > price_{bond}$ because the buyers limit-price is always above the final price as proven in 4. Further $limit_{asset} > limit_{bond}$ holds.

$$
\begin{aligned}
utility_{seller} = sell_{asset}\ price_{asset} + \\
sell_{bond}\ price_{bond}
\end{aligned}
$$

Proof.

$$
\begin{aligned}
utiliy_{after} &> utility_{before} \\
= sell_{asset}\ price_{asset} + \\
sell_{bond}\ price_{bond} &> limit_{asset}\ holdings_{asset} - limit_{bond}\ holdings_{bond} \\
& \mid \text{all holdings and sell are same} = \text{h} \\
= h\ price_{asset} + \\
h\ price_{bond} &> limit_{asset}\ h - limit_{bond}\ h && \mid : h \\
= price_{asset} + price_{bond} &> limit_{asset} - limit_{bond}
\end{aligned}
$$

□

This is feasible as in case of seller $limit_{asset} < price_{asset}$ and $limit_{bond} < price_{bond}$ because the sellers limit-price is always below the final price as proven in 4. Further $limit_{asset} > limit_{bond}$ holds. Unfortunately this trade is not possible as it amounts to trading collateralized assets where no market exists for it.

7.4 Extending the Hypothesis

After it has become clear that the hypothesis is not sufficient the question arises what needs to be done to approach the theoretical equilibrium anyway. It is clear that a mechanism needs to be found which prevents or resolves the arising of the artefacts within wealth-distribution. Obviously two solutions are available.

7.4.1 Approaching fully connectedness

Increasing the connectedness of the topology allows more agents to trade between each other and thus the probability of resolving islands or artefacts of wealth miss-allocation is increased with the density of connectedness. The experiments of Ascending-Connected topology with short-cuts were designed to develop an understanding of how the simulation behaves with increasing connectedness and also how the two types of fully- and regular-connectedness influence the results. It seems that full short-cuts seem to help dramatically in reducing the miss-allocations where the number of full short-cuts seems to be dependent on the number of agents which this thesis leaves for further research. See section B.2 for a short overview of the results and interpretation of short-cut based Ascending-Connected topologies. Of course in real environments approaching fully connectedness is not always possible and thus only the other mechanism is left as an option to resolve the artefacts.

7.4.2 Re-Enabling trading

Another way to look at the arising of the artefacts is in identifying them as suboptimal trades. Breuer et al. (2015) were confronted with this circumstance when they introduced the Asset/Bond Market where they found that the equilibrium was fundamentally different from the theoretical one because agents were trapped in suboptimal trades and couldn't reverse their decisions made earlier. The trades where suboptimal because each agent assigns depending on its optimism factor a different bond-value to each asset. As a solution they introduced the "Bonds-Pledgeability" (BP) mechanism which allows trading of bonds in both ways instead of only gathering them and not being able to sell them - see chapter 3 for a more in-depth discussion of the BP-Mechanism.

Thus if those artefacts are treated as suboptimal trades where decisions of the traders lead to a specific blocking-situation in the context of collateralized assets as described above one needs to introduce a new mechanism to allow the reparation of such blocking-situations. The only possibility without altering the network-topology is to re-enable the pessimists to trade their collateralized assets against cash as all pessimists hold cash and are thus able to buy and sell collateralized assets against cash. This new mechanism is expected to repair the miss-allocated wealth and to restore the validity of the previously disproved hypothesis.

See chapter 8 for the implementation and results of this new mechanism.

Chapter 8

A new Market

As already introduced in section 7.4.2 a new market is necessary to repair the miss-allocation of collateralized assets in the range of the pessimist agents by enabling the agents to trade collateralized assets against cash.

The initial idea for this market was by the supervisor of this thesis Mr. Hans-Joachim Vollbrecht.

8.1 Definition

8.1.1 Products

Collateralized assets are traded against cash. The buyer gets a specific amount of collateralized assets for a given amount of cash where the seller gives away the specific amount of collateralized assets and gets the given amount of cash.

8.1.2 Price-Range

As with all other three markets the price-ranges of the offers must be defined. Note that all prices must obviously be in the unit of cash according to the previously defined products.

minimum When calculating the minimum price of a collateralized asset - that is how much is the collateralized asset is minimally worth - it is important to include the collateral-aspect of the asset. Thus one starts with the minimum Asset-Price in cash which is the down-price pD and subtracts the minimum amount of cash which is bound through a bond as collateral which is pD. This value is a constant for all agents.

$$min_{collateral} = pD - pD = 0 \tag{8.1}$$

maximum To calculate the maximum price of a collateralized asset - that is how much is the collateralized asset is maximally worth - one needs to include the collateral-aspect of the asset too. Equal to calculating the minimum, one starts now with the maximum Asset-Price in cash which is the up-price pU and subtracts the maximum possible amount of cash which is bound through a bond as collateral which is the face-value V. This value is a constant for all agents.

$$max_{collateral} = pU - V \tag{8.2}$$

limit Applying the same rules as in minimum and maximum to the limit price calculation one needs to subract the limit-price of bonds from the limit-price of assets to receive the limit-price of a collateralized asset. This value is individual for each agent as the limit-prices differ across the agents both for assets and bonds.

$$limit_{collateral} = limit_{asset} - limit_{bond} \tag{8.3}$$

8.1.3 Bid-Offering

The way bid-offers are generated is very similar to the Bond/Cash market. Bid offers are generated only when the agent has any cash holdings. The price is drawn randomly between the minimum price and the limit-price because when buying one wants to buy below the expected value to make a profit. An amount of TRADING-UNIT assets is selected but if there is not enough cash left to buy TRADING-UNIT of assets then the amount of assets that can be bought with the remaining cash holdings is selected.

Table 27: Bid-Offering parameters

Pre-Condition	$cash\ holdings > 0$
Asset-Price	random($min_{collateral}, limit_{collateral}$)
Asset-Amount	min($\frac{cash\ holdings}{Asset\text{-}Price}$, $TRADING\text{-}UNIT$)

8.1.4 Ask-Offering

The way ask-offers are generated is very similar to the Bond/Cash market. Ask offers are generated only when the agent has any collateralized assets.

The price is drawn randomly between the limit-price and maximum price because when selling one wants to sell above the expected value to make a profit. An amount of TRADING-UNIT assets is selected but if there are fewer collateralized assets left then the remaining amount of collateral is selected. See chapter 5 for the equation of the amount of collateralized assets.

Table 28: Ask-Offering parameters

Pre-Condition	$collateral > 0$
Asset-Price	random($limit_{collateral}, max_{collateral}$)
Asset-Amount	min(*collateralized assets*, *TRADING-UNIT*)

8.1.5 Match

Below the wealth-exchange table is given in case of a match between two agents on the new market. Note that the wealth is increased/decreased as given by the +/- signs.

Table 29: Wealth-Exchange on match

	Seller	Buyer
Loans Given	+ matching-amount	N/A
Loans Taken	N/A	- matching-amount
Assets holdings	- matching-amount	+ matching-amount
Cash holdings	+ matching-price	- matching-price

Note that regarding these formulae it is clear that the Collateral/Cash market can't work without the BP-mechanism because only with it is it allowed to increment the net-loans variable *loan* by incrementing *loans given*. Without the BP-mechanism incrementing *loans given* would simply be forbidden thus making this market unable to work. Because of this the thesis has the BP-mechanism hard-wired into the implementation and does not provide the user with an option to run the simulation without it.

8.2 Results

The plain results of the simulation with using the new market are given in this section where the interpretation of the results are given in the following one.

The same experiment-configuration is used as given in chapter 6 except that the new market is now also activated.

Table 30: Configuration for all experiments

Agent-Count	100
Bond-Type	0.5
Replication-Count	50
Matching-Round	max. 500 offering-rounds
Terminate after	1000 failed successive matching-rounds

8.2.1 Fully-Connected topology

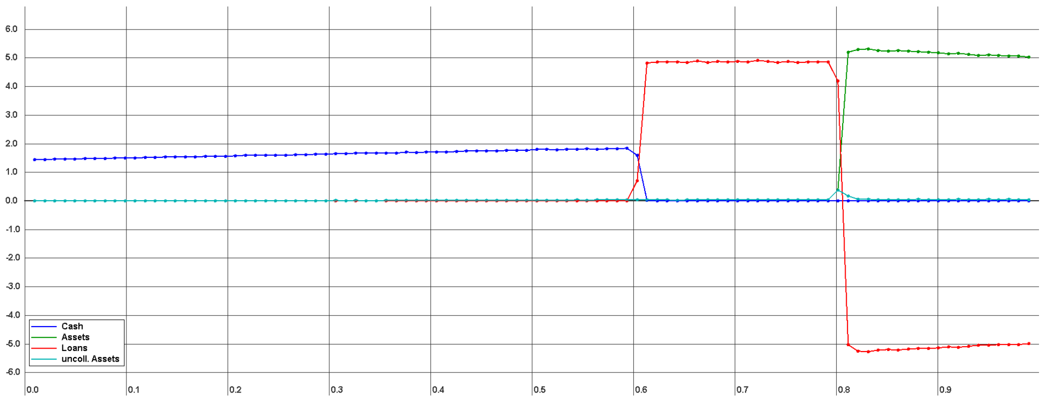

Figure 24: Wealth-Distribution of Fully-Connected topology with Collateral/Cash market enabled.

Table 31: Equilibrium of Fully-Connected topology with Collateral/Cash market enabled

Asset-Price p	0.718 (0.016)
Bond-Price q	0.383 (0.000)
Marginal Agent i1	0.603 (0.003)
Marginal Agent i2	0.812 (0.000)
Pessimist Wealth	1.640 (0.002)
Medianist Wealth	4.808 (0.074)
Optimist Wealth	5.171 (0.023)

Table 32: Performance of Fully-Connected topology with Collateral/Cash market enabled

Successful matching-rounds	1,803.86 (20.24)
Failed matching-rounds	1,365.88 (134.63)
Total matching-rounds	3,169.74 (137.60)
Ratio successful/total	0.57
Ratio failed/total	0.43

Table 33: Difference of Fully-Connected topology to theoretical equilibrium as given in Table 15

	Result	Reference	difference to Reference
Asset-Price p	0.718	0.717	+0.1%
Bond-Price q	0.383	0.375	+2.1%
Marginal Agent i1	0.603	0.584	+3.2%
Marginal Agent i2	0.812	0.803	+1.1%

Table 34: Difference of Fully-Connected topology to equilibrium without Collateral/Cash market as given in Table 16

	Result	Reference	difference to Reference
Asset-Price p	0.718 (0.016)	0.717 (0.015)	+0.1% (+6.6%)
Bond-Price q	0.383 (0.000)	0.382 (0.001)	+0.2%
Marginal Agent i1	0.603 (0.003)	0.596 (0.004)	+1.2% (-25%)
Marginal Agent i2	0.812 (0.000)	0.810 (0.004)	+0.24%
Pessimist Wealth	1.640 (0.002)	1.646 (0.004)	-0.3% (-50%)
Medianist Wealth	4.808 (0.074)	4.736 (0.089)	+1.5% (-16%)
Optimist Wealth	5.171 (0.023)	5.179 (0.082)	-0.1% (-71%)

8.2.2 Ascending-Connected topology

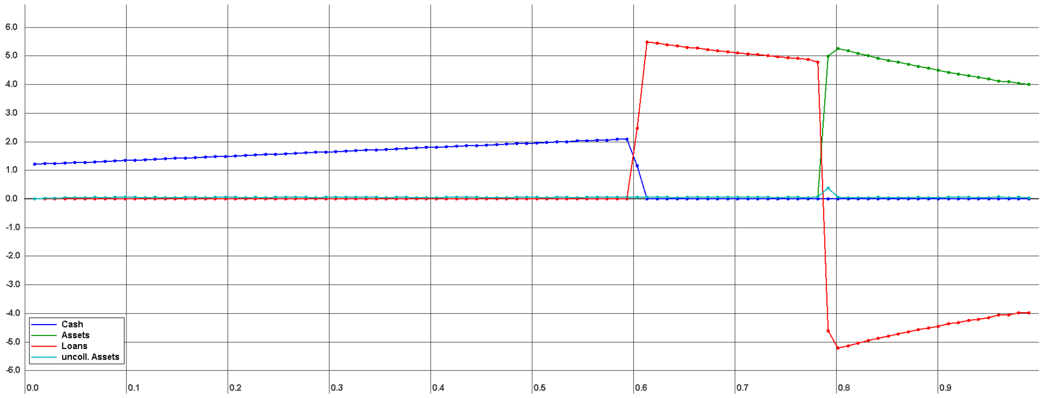

Figure 25: Wealth-Distribution of Ascending-Connected topology with Collateral/Cash market enabled.

Table 35: Equilibrium of Ascending-Connected topology

Asset-Price p	0.729 (0.004)
Bond-Price q	0.380 (0.001)
Marginal Agent i1	0.594 (0.001)
Marginal Agent i2	0.792 (0.000)
Pessimist Wealth	1.647 (0.003)
Medianist Wealth	4.999 (0.032)
Optimist Wealth	4.581 (0.013)

Table 36: Performance of Ascending-Connected topology

Successful matching-rounds	48,629.32 (197.20)
Failed matching-rounds	1,001.30 (1.31)
Total matching-rounds	49,630.62 (197.30)
Ratio successful/total	0.98
Ratio failed/total	0.02

Table 37: Difference of Ascending-Connected topology to theoretical equilibrium as given in Table 15

	Result	Reference	difference to Reference
Asset-Price p	0.729	0.717	+1.6%
Bond-Price q	0.380	0.375	+1.3%
Marginal Agent i1	0.594	0.584	+1.7%
Marginal Agent i2	0.792	0.802	-1.2%

Table 38: Difference of Ascending-Connected topology to equilibrium of Fully-Connected topology with Collateral/Cash as given in table 31

	Result	Reference	difference to Reference
Asset-Price p	0.729 (0.004)	0.718 (0.016)	+1.5% (-80%)
Bond-Price q	0.380 (0.001)	0.383 (0.000)	-0.8%
Marginal Agent i1	0.594 (0.001)	0.603 (0.003)	-1.4% (-60%)
Marginal Agent i2	0.792 (0.000)	0.812 (0.000)	-2.4%
Pessimist Wealth	1.647 (0.003)	1.640 (0.002)	+0.4% (+50%)
Medianist Wealth	4.999 (0.032)	4.808 (0.074)	+3.9% (-56%)
Optimist Wealth	4.581 (0.013)	5.171 (0.023)	-11.4% (-43%)

8.3 Interpretation of results

When interpreting the results the following questions must be answered:

- Does the Fully-Connected topology reach the theoretical equilibrium as well with Collateral/Cash market enabled?
- Does the new market repair the miss-allocation of wealth in the pessimists-range of the Ascending-Connected topology?
- If not why? If yes, does the Ascending-Connected topology approach theoretical equilibrium now?

Does the Fully-Connected topology reach the theoretical equilibrium as well with Collateral/Cash market enabled? Yes it does. Both visual and statistical results show that it reaches the theoretical equilibrium. The Medianist wealth is slightly higher with the new market but that difference, as well as the variations in the other variables are not statistically significant.

Does the new market repair the miss-allocation of wealth in the pessimists-range of the Ascending-Connected topology? Yes it does. The visual results are clear with no miss-allocations showing up within 50 replications. If there would have been miss-allocations within any replication they would have shown up in the final result.

If yes, does the Ascending-Connected topology approach theoretical equilibrium now? The miss-allocations are repaired but it does does not approach theoretical equilibrium. Both visual and statistical results show that it fails to reach the theoretical and Fully-Connected topology. Although p,q,i1 and i2 do not differ significantly, the wealth of the optimists and medianists are substantially different and show a different shape than in the Fully-Connected case and thus the theoretical equilibrium is not reached.

8.4 Simulation and Market dynamics

When implementing a new market the market-dynamics are of great importance and thus the following questions must be answered.

- Can the trading stages 1-4 be identified too as given in Breuer et al. (2015)?
- How does trading progress with this new market? Is it the same as without the new market?
- How does the new market resolve the miss-allocation (with and without deferred activation)?
- When and how much is each market active?
- How do the market-activities change when a new market is introduced?

To answer these questions one must look closely at the market-dynamics. There are trading stages to be identified but due to the new market and the different topology they are expected to be quite different from the ones found in Breuer et al. (2015). The method used to find these stages is through observation of a single run and refining and validating the derived facts over many additional runs. Note that replications provide no real value here as one needs to look very carefully into the dynamics of single runs instead of the mean of multiple runs.

8.4.1 Fully-Connected with new Market

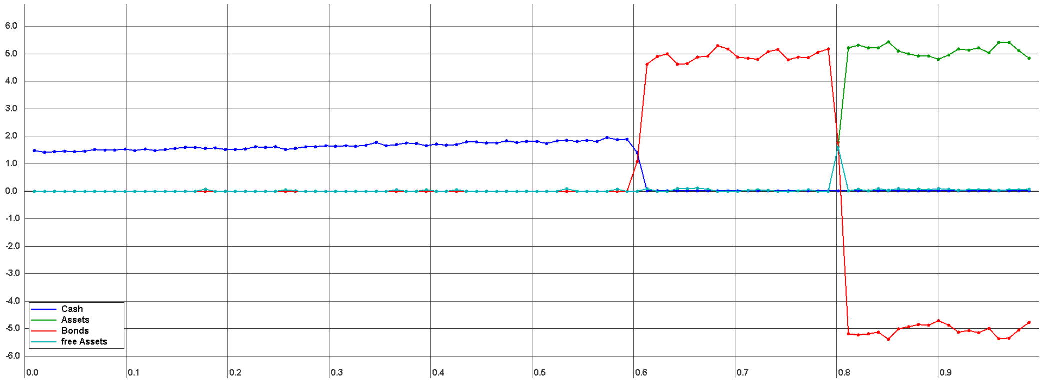

Figure 26: Final wealth-distribution of a single run of the Fully-Connected topology with Collateral/Cash market.

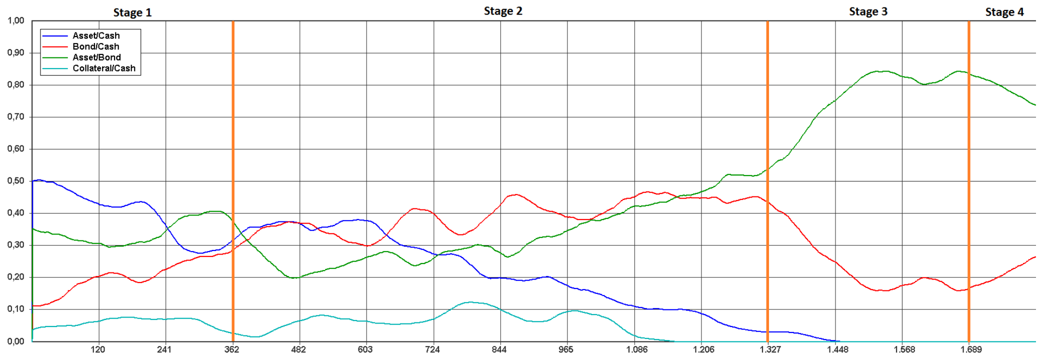

Figure 27: Market-activity stages of the single run of figure 26.

4 Stages were identified which resemble roughly the ones given in Breuer et al. (2015) as the new market doesn't make a big impact due to the fully-connectedness.

Stage 1 In this stage the pessimists become visible rapidly as they sell their assets and increase their cash wealth. One can also get a sense of the more optimistic range of agents as they gather assets both free and collateralized. The medianists are not visible yet. This picture is in stark contrast to the one given in Ascending-Connected topology where the wealth stabilizes from both the left and right ends towards the marginal agent i2. This is obviously due to the different topology as in the fully-connected one all agents can trade with each other thus the wealth-distribution approaches the theoretical equilibrium over all agents and not only on two frontiers from left and right.

The Asset/Cash market dominates but goes down slowly as fewer and fewer pessimists trade assets against cash compared to the very beginning. The Bond/Cash market is slightly increasing as more and more bonds are traded because they remain the alternative method of payment for assets and because the pessimists trade them up towards the optimists using the BP-mechanism. The Collateral/Cash market begins quite low and stays in the range of a 10% share as it is not heavily required for the pessimists due to the fully-connectedness. The Asset/Bond market starts quite strong and stays at this level as pessimists trade assets with optimists against bonds.

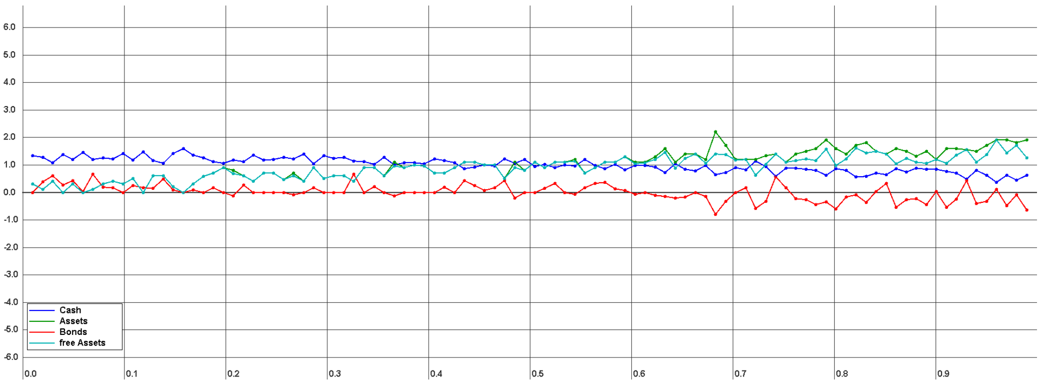

Figure 28: Wealth-Distribution during Stage 1 of the single run of figure 26.

Stage 2 The bonds, free- and collateralized assets are all traded from the pessimists towards the optimists and the optimists crystallize themselves even more but no medianists are visible yet.

The Asset/Cash market continues to go down as the cash holdings of the pessimists begin to decline. The Bond/Cash market raises even further as agents switch to this market because the pessimists have run out of cash. The Collateral/Cash market is still in the range of the 10% share because it is not really needed in the Fully-Connected topology. The Asset/Bond market raises fast towards the end of the stage as the optimists are then out of cash and need to distribute the collateralized assets between each other and the yet to come medianists.

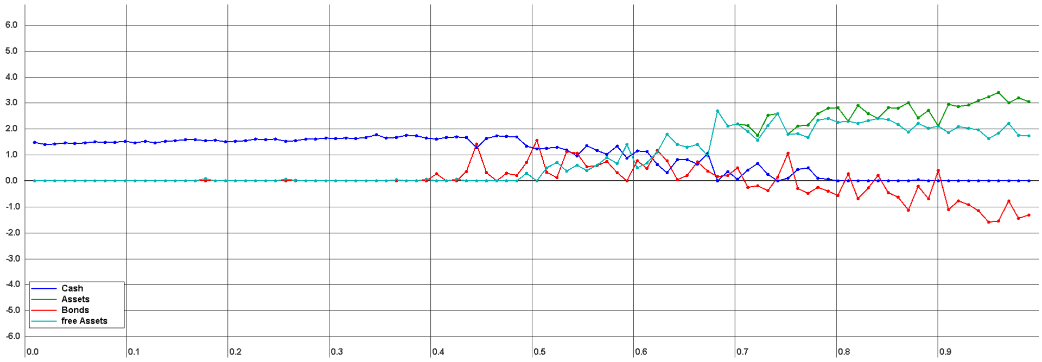

Figure 29: Wealth-Distribution during Stage 2 of the single run of figure 26.

Stage 3 The pessimists are now nearly inactive as only the more optimistic pessimists hold a few bonds which they trade towards the now emerging medianists. The i1-point is beginning to show up but is not yet refined.

Because the pessimists are nearly inactive now and hold no more assets and cash the Asset/Cash and Collateral/Cash markets go down and decline completely. The Bond/Cash market goes down but does not decline as bonds are still traded because of the emerging of medianists. The medianists and pure optimists that are emerging have no other possibility than to trade on the Asset/Bond market to further distribute their collateralized assets among each other which is the reason for the rise of the Asset/Bond market above all others and its heavy domination. Despite the heavy domination of the Asset/Bond market a few bonds are still traded against cash between the most optimistic pessimists and the medianists to refine the i1 point.

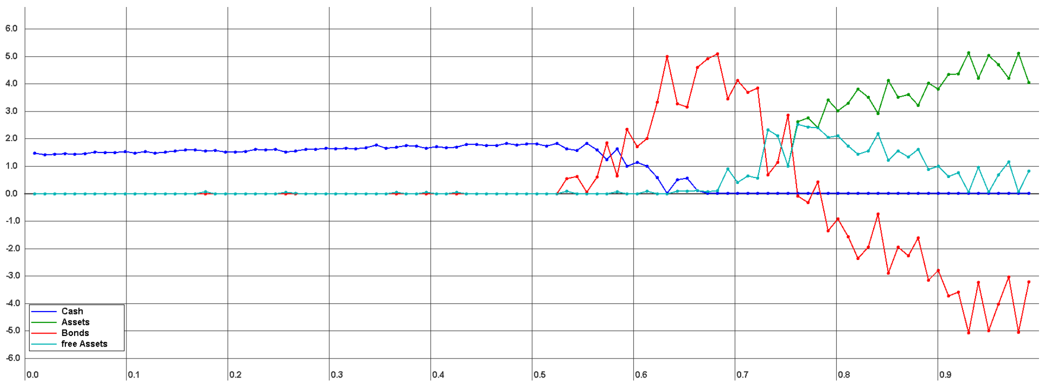

Figure 30: Wealth-Distribution during Stage 3 of the single run of figure 26.

Stage 4 The pessimists are finally inactive except a couple of most optimistic pessimists around the i1 point which still trade with the medianists on the Bond/Cash market to refine the i1 point to the final allocation as can be seen in figure 26. Also the i2 is now emerging and a couple of medianists and optimists are trading around it on the Asset/Bond market to achieve the final allocation.

Only the Bond/Cash and Asset/Bond markets are active any more. The Bond/Cash market is active due to the final transactions between the most optimistic pessimists that want to sell their final bonds to the medianists which is only possible through cash which the most pessimistic medianists still own. The Asset/Bond market is active due to the final transactions between the most optimistic medianists that want to sell their free assets to the optimists which is only possible through bonds because the most pessimistic optimists have no more cash left thus they have to trade assets against bonds.

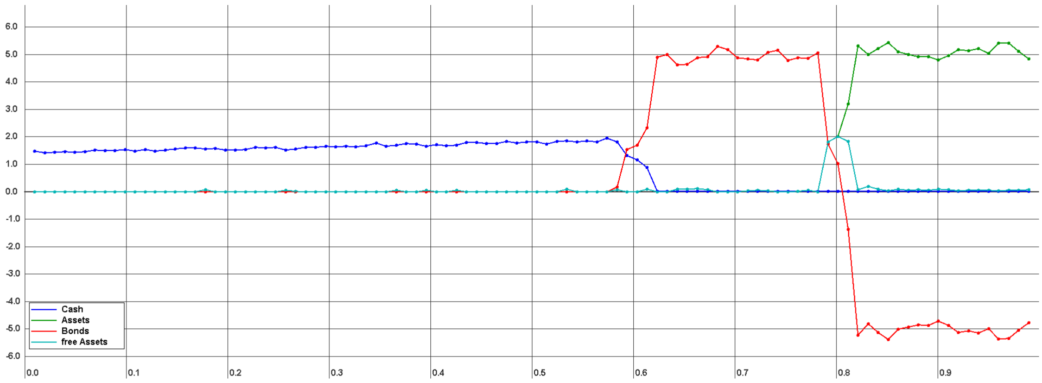

Figure 31: Wealth-Distribution during Stage 4 of the single run of figure 26.

8.4.2 Deferred new market enabling

Using the thesis-software it is possible to start a simulation-run with Ascending-Connected topology without the Collateral/Cash market and enabling it after 1,000 successive failed matching-rounds which gives interesting hints about how the spikes of collateralized assets in the pessimists-range are resolved and distributed over the already existing pure optimists.

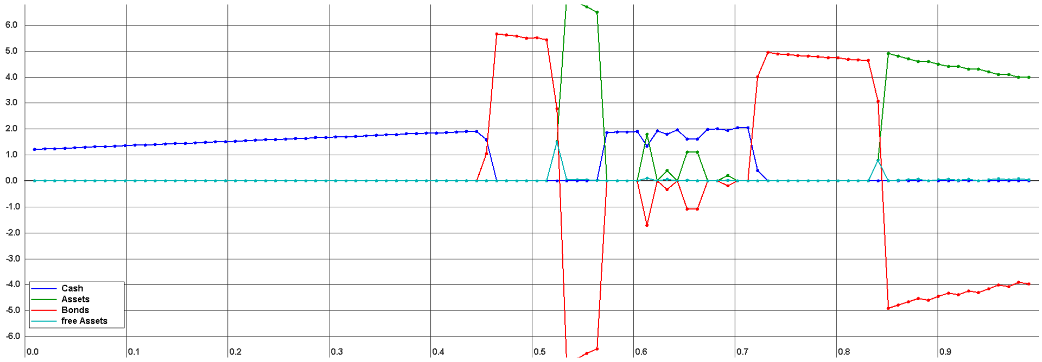

Figure 32: Wealth-distribution of a single run of Ascending-Connected topology before enabling the Collateral/Cash market.

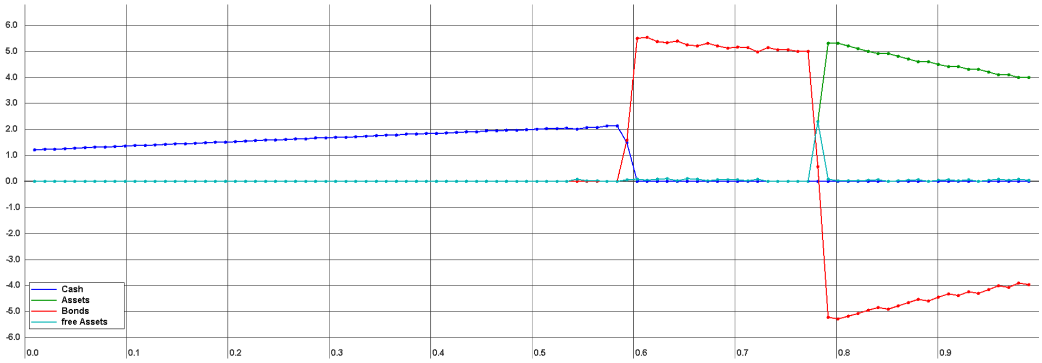

Figure 33: Final wealth-distribution after enabling of the Collateral/Cash market of the single run of figure 32.

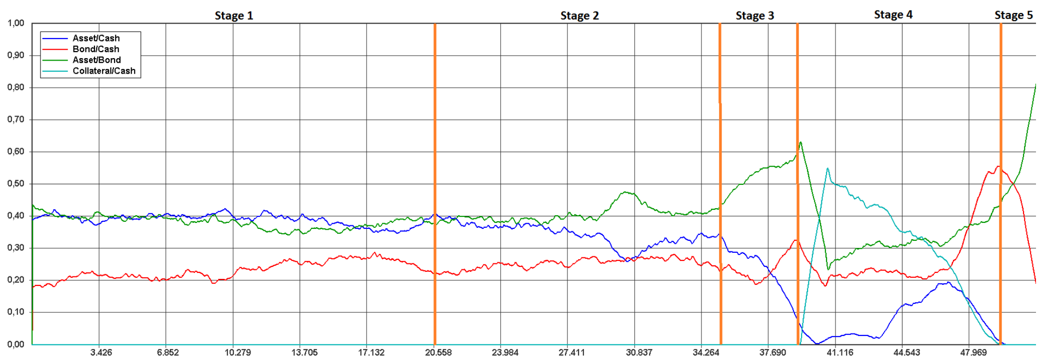

Figure 34: Market-activity stages of the single run of figure 33.

Of course there are the same 3 stages to be found as described already in section 7.2.1 whereas the deferred enabling of the Collateral/Cash market adds 2 new stages.

Stage 4 The free- and collateralized assets and bonds are traded by the pessimists towards the medianists and optimists and lead to a pattern similar to the end of stage 3.

The Collateral/Cash market kicks in and allows the blocked goods to be traded again thus changing the activity of the other markets too. The Asset/Bond market initially drops rapidly only to recover shortly after. The Asset/Cash market, previously dormant, becomes active again because the pessimists have freed their collateral first due to selling it through the Collateral/Cash market and then through the Asset/Bond market or Bond/Cash market. This results in them having free assets they don't want and thus trade them against cash towards the medianists.

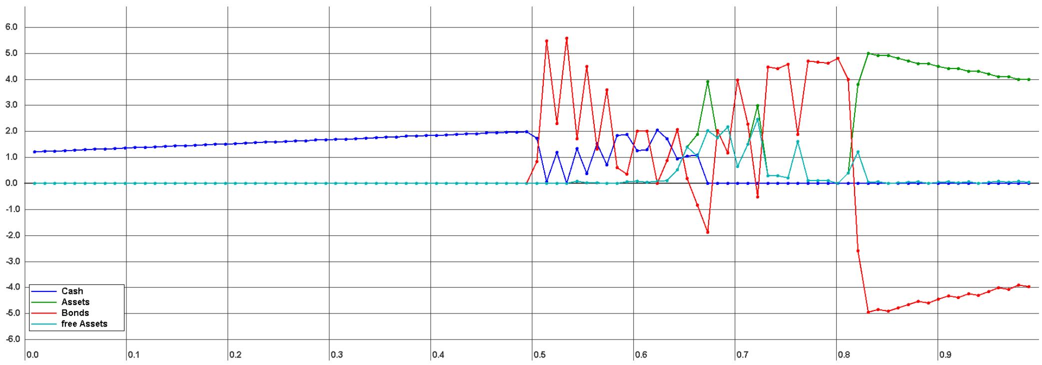

Figure 35: Wealth-Distribution during Stage 4 of the single run of figure 33.

Stage 5 The i1-point has finalized as no more pessimists are able to trade with their next medianists. Now only free assets in the range of medianists are traded towards the optimists which can only happen on the Asset/Bond market because both are out of cash. This will lead to the finalizing of the i2-point which can be seen in figure 33.

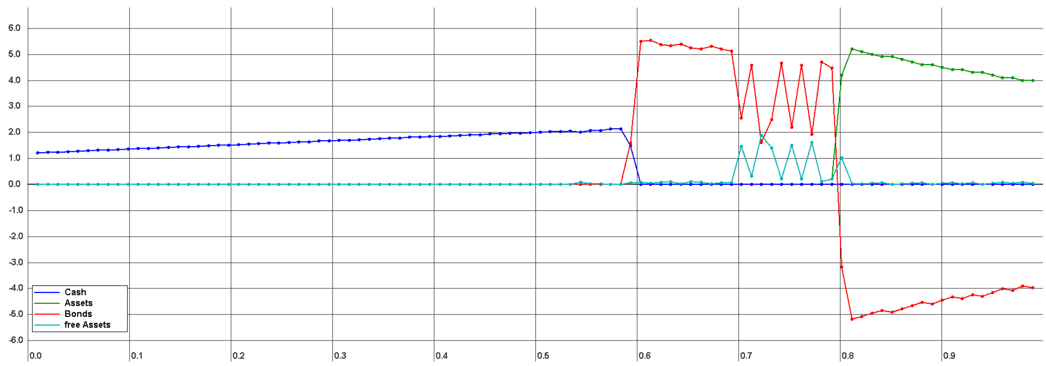

Figure 36: Wealth-Distribution during Stage 5 of the single run of figure 33.

8.4.3 Ascending-Connected with new Market

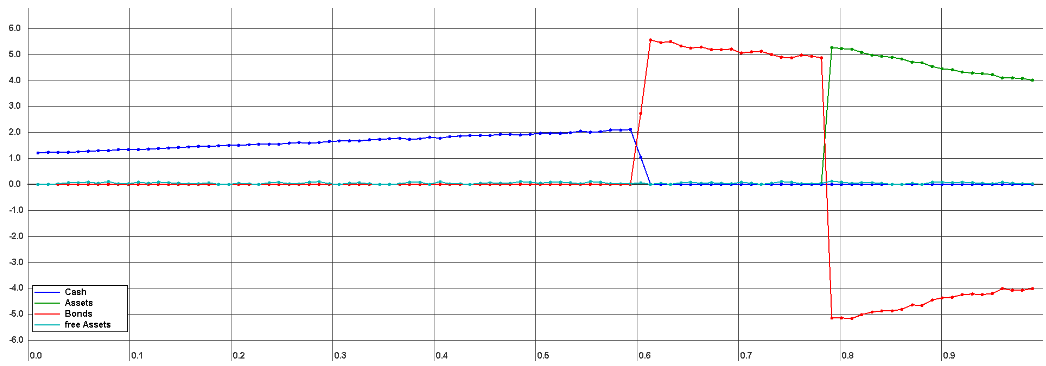

Figure 37: Final wealth-distribution of a single run of Ascending-Connected topology with Collateral/Cash market enabled.

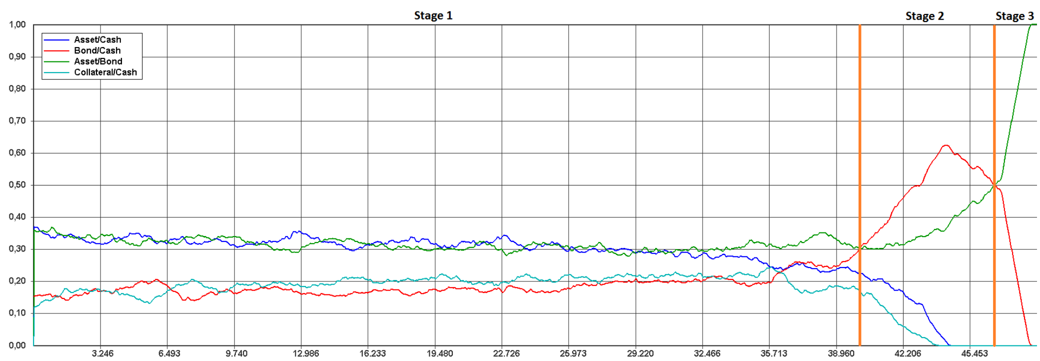

Figure 38: Market-activity stages of the single run of figure 37.

3 stages were identified as can be seen in the Market-Dynamics diagram in figure 38.

Stage 1 Pessimists and optimists emerge where the pessimists are gathering cash and selling all other goods. The optimists are buying assets against cash and against bonds. There are no medianists visible yet.

The Bond/Cash market is slightly increasing as pessimists trade their bonds towards the later emerging medianists and the optimists. The Asset/Bond market is already very active as optimists trade assets against bonds after they have ran out of cash. Collateral/Cash market is rising slightly until the end where it drops as it is used by the pessimists to trade their collateral gathered on the other markets towards the optimists. The Asset/Cash market starts quite high because optimists trade their assets first towards the pessimists against cash but is declining slowly as fewer and fewer pessimists trade on this market.

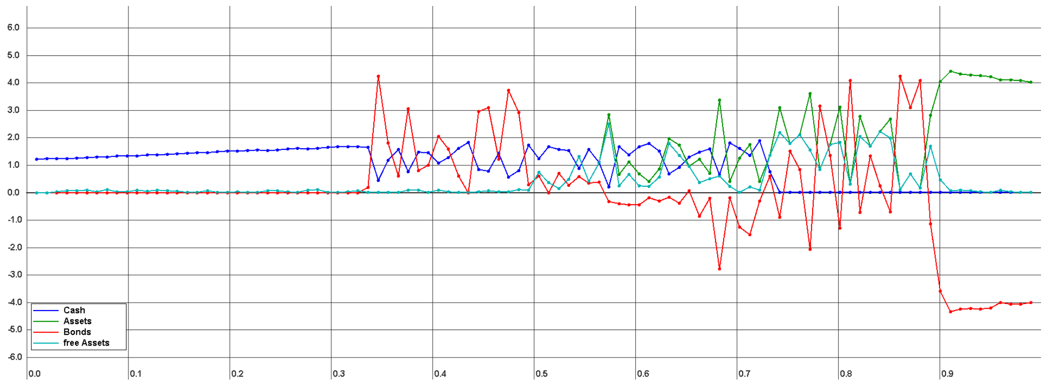

Figure 39: Wealth-Distribution during Stage 1 of the single run of figure 37.

Stage 2 The wealth of the pessimists has nearly reached final equilibrium and the pessimists hold only a few bonds which they trade towards the now emerging medianists. The medianists and optimists are approaching each other by trading towards the i2-point on the Asset/Bond market because both are out of cash and thus are only able to trade on this market with each other.

Due to the remaining bonds in the pessimists' range the Bond/Cash market's activity rises only to drop towards the end of this stage where all bonds have been traded up to the medianists and the pessimists have become dormant. The Asset/Bond market rises too due to increased activity between the medianists and optimists which can only trade between each other on this market because they are both out of cash making the other markets impossible choices. The Asset/Cash and Collateral/Cash market activities decline as the pessimists which are the only agents that hold cash finally become inactive.

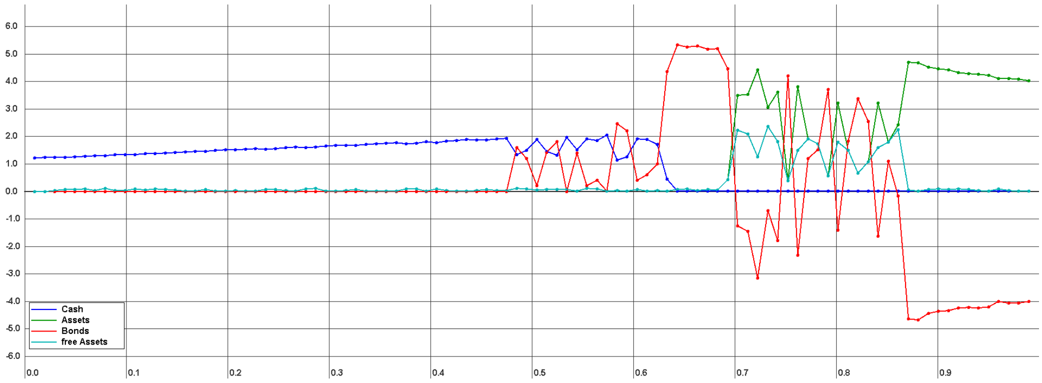

Figure 40: Wealth-Distribution during Stage 2 of the single run of figure 37.

Stage 3 Pessimists are now final and won't change any more and the i1-point has emerged and is final. Medianists and optimists trade on the Asset/Bond market to finalize i2-point which can be seen in the final wealth-distribution in figure 37

The only market which is active in this stage is the Asset/Bond market. The pessimists are already inactive and final thus generating no more activities on the Asset/Cash, Bond/Cash or Collateral/Cash markets. The medianists as well as the optimists have no more cash thus are only able to trade on the Asset/Bond market, to finally trade the remaining assets in the range of the medianists towards the optimists until the i2-point has finalized.

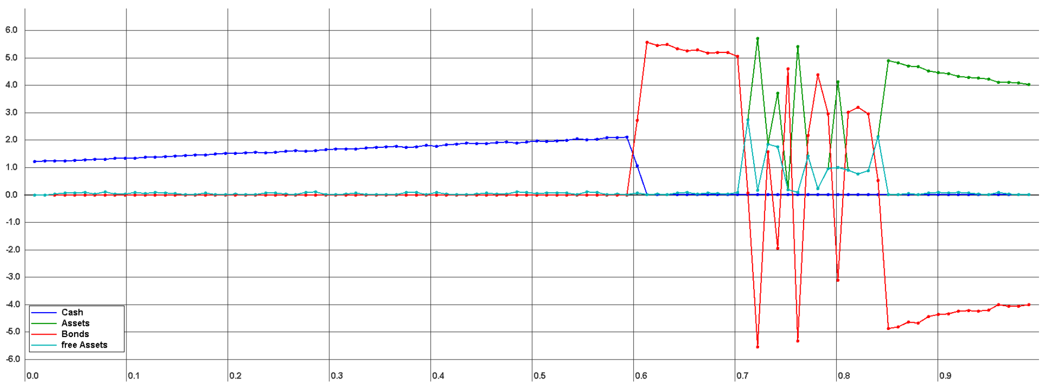

Figure 41: Wealth-Distribution during Stage 3 of the single run of figure 37.

Following these observations one can answer the questions.

Can the trading stages 1-4 be identified too as given in Breuer et al. (2015)? There are 4 stages in the case of both Fully-Connected and Ascending-Connected topology with the new market which are not the ones given in Breuer et al. (2015) but show up by pure chance and depend also a bit on the point-of-view on how to separate the stages from each other.

How does trading progress with this new market? Is it the same as without the new market? The progression of the trading is obviously very different with the new market as compared to the market-activities without as the usage of the new market changes the dynamics completely.

How does the new market resolve the miss-allocation (with and without deferred activation)? It becomes active during the formation of the pessimists agents as they gather wealth from collateralized assets which must be traded towards the optimists. The collateralized assets are traded from neighbour to neighbour until they reach the optimists-region.

When and how much is each market active? This can be seen clearly in the market-activity diagrams.

How do the market-activities change when a new market is introduced? They have less share on the overall activity and thus the new market is quite a heavy competitor in the overall share. The Asset/Bond market though is still the market on which the final trades occur.

8.5 Conclusions on new Market

The equilibrium of the Ascending-Connected topology with the new market is different than the Fully-Connected one which reaches the theoretical equilibrium. Thus the property of the hypothesis is still not sufficient because it predicted the Ascending-Connected topology to reach the theoretical equilibrium. This thesis can only speculate on the reason for this but it is most probably rooted in the fundamental different trading dynamics of Ascending-Connected topology compared to Fully-Connected as can be seen in the market-dynamics. This thesis leaves the question of market-dynamics open for further research.

Chapter 9

Conclusions

The motivation for this thesis was to investigate the influence of various types of network-topologies in continuous double-auctions on the convergence towards theoretical equilibrium. It builds upon findings and results of Breuer et al. (2015) and an equilibrium framework developed by Geanakoplos (2009). As a starting point a hypothesis was formulated which gives properties a network must exhibit to reach the theoretical equilibrium defined in the model. Interpreting the results of the thesis-software it turned out that the hypothesis is not feasible for the model of Breuer et al. (2015) as the Ascending-Connected topology - the most minimal topology which satisfies the properties postulated by the hypothesis - leads to serious miss-allocation of wealth in the range of the pessimist agents. The hope then was to be able to repair the miss-allocation by extending the original model by introducing a new market and that then theoretical equilibrium would be reached. This new market on which collateral can be traded against cash was able to remove the allocation inefficiencies but the equilibrium is still different from the theoretical one thus the hypothesis turns out to be invalid and infeasible for the extended model too.

The major conclusion on the findings of the thesis is that equilibrium seems only to be possible in a fully-connected trading-network. Thus as soon as a market-institution exhibits signs of restricting trades between agents to a reduced neighbourhood, it will lead to miss-allocation and fail reaching equilibrium and thus will be unfair. The introduction of a new market solves the miss-allocations in theory and to the best knowledge of the author of this thesis there already exist such markets in real-world trading thus it seems that it can be put into practice in the real world and that it and its products are already accepted by traders.

Further Research

Importance-Sampling

Importance sampling was used to increase the matching-probabilities for the special case of an Ascending-Connected network which led to a dramatic performance enhancement as much less matching-rounds were required to reach the point where no trading was possible any more. Further research could investigate a more general mechanism of increasing the matching-probabilities independent of the topology without introducing a bias. One way could be through sampling the price-ranges in different intervals or learning and adapting them on-the-fly during a simulation run.

In-depth analysis of market-activities

The market-activities as presented in chapter 7 and to a greater extent in chapter 8 are only covered by intuitive description - what is going on and why - no theory about market-activity is presented. Further research could dedicate its attention to develop a serious theory why the activities behave as they do, how they interact between each other and how they influence equilibrium. Furthermore it could establish connections to market-activity theory in economics to validate the simulation-results against theoretical frameworks.

Experiments with real subjects

Experimental economics has a long history and allows the verification of theoretical equilibrium frameworks or theories. The author of this thesis is part of a group which has conducted a pilot experiment with real subjects on the subject of trading collateralized assets based on the model of Breuer et al. (2015). To the best knowledge, this was the first time that an experiment in this field has ever been undertaken. Although it was only a pilot it gave already valuable insights on the models of Breuer et al. (2015) and Geanakoplos (2009), however more experiments need to be carried out to be able to provide serious results. Based upon the results and findings of this thesis the already existing trading software could be extended furthermore to restrict the trading between agents to a specific topology e.g. Ascending-Connected. It would be of much interest whether intelligent human traders are able to resolve the miss-allocations produced by the Ascending-Connected topology without the use of the new market and if they can approach theoretical equilibrium.

Equilibrium theory vs. equilibrium process

It was noted on multiple occasions in this thesis that there is a fundamental difference between the equilibrium theory upon which the model of the thesis is built upon and the equilibrium process implemented in the thesis-software. Further research could look for a formal definition of equilibrium in such a process and combine it with the given equilibrium theory of complex systems in general and economics in particular to find a definition for the equilibrium of such a trading process.

Appendix A

Topologies

In this chapter all simulated topologies are visualized and explained shortly. All topologies are demonstrated with N = 30 Agents instead of 100 for better visibility and clarity of the connections and nodes. All topologies have one connected component because otherwise equilibrium cannot be reached. Note that Erdos-Renyi could produce more than one connected component depending on the parameters to create it. The agents are always arranged in clockwise increasing optimism factor unless stated otherwise. All connections are undirected and there are no self-loops.

A.1 Metrics

For each network the following metrics are given:

Average degree Gives a measure of how many neighbours a vertex has on average.

Average path-length Gives a measure of how many edges a path between two vertices has on average.

Network diameter Is the longest shortest path between any two vertices.

Graph density Is the ratio of the number of edges to the number of possible edges.

Edge count Gives a formula to calculate the exact number of edges. Is omitted for the complex networks as there is no exact estimation possible.

Connected component Gives the number of components found in this network. All non-complex networks have always only 1 connected-component where the complex networks could result in more than 1.

A.2 Fully-Connected

Each agent is connected with each other agent. Included as major point-of-reference as Breuer et al. (2015) developed their model and equilibrium for fully-connected networks.

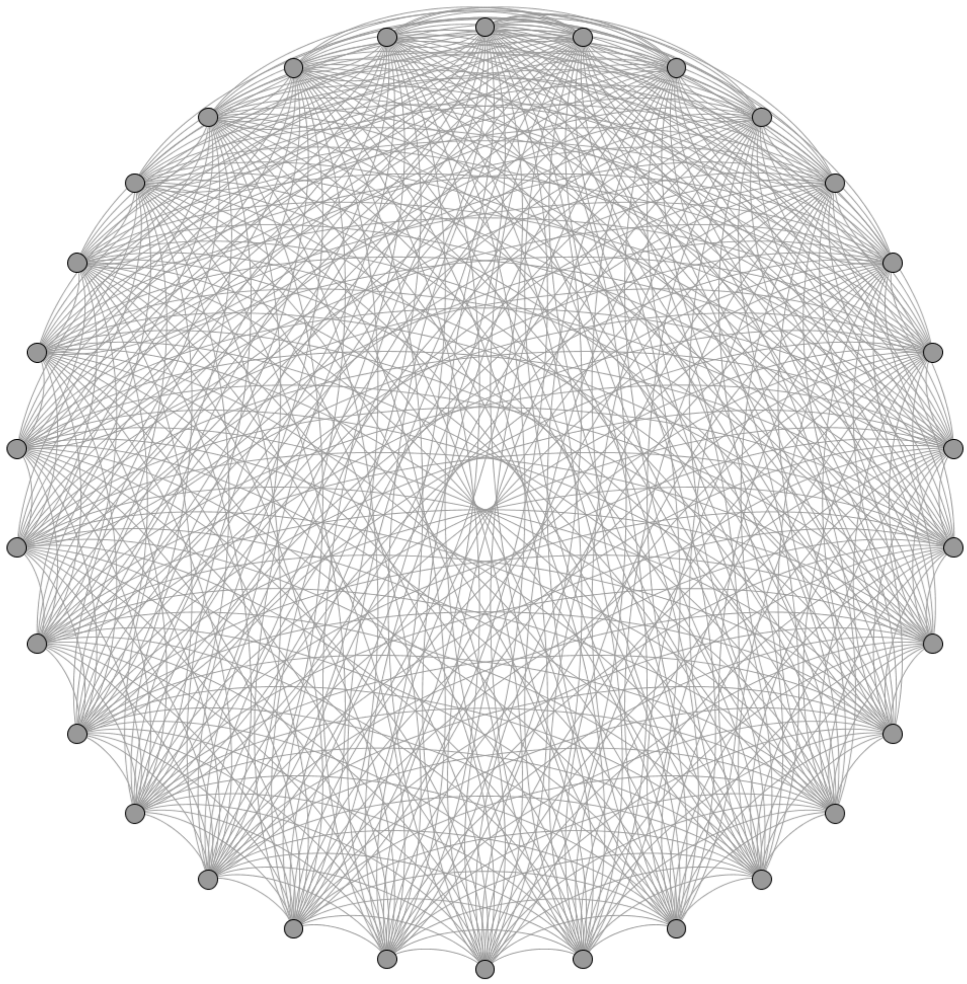

Figure 42: Fully-Connected topology

Table 39: Network metrics Fully-Connected topology

Avg. degree	29
Avg. path-length	1
Network diameter	1
Graph density	1
Edge count	$\frac{N(N-1)}{2}$

A.3 Half-Fully Connected

Agents with optimism-factor 0.5 to 1.0 are fully-connected and the others are connected to the agent with the next higher optimism-factor. The agents with highest and lowest optimism-factor are connected too, creating a closed circle. Included to investigate the influence of isolated agents.

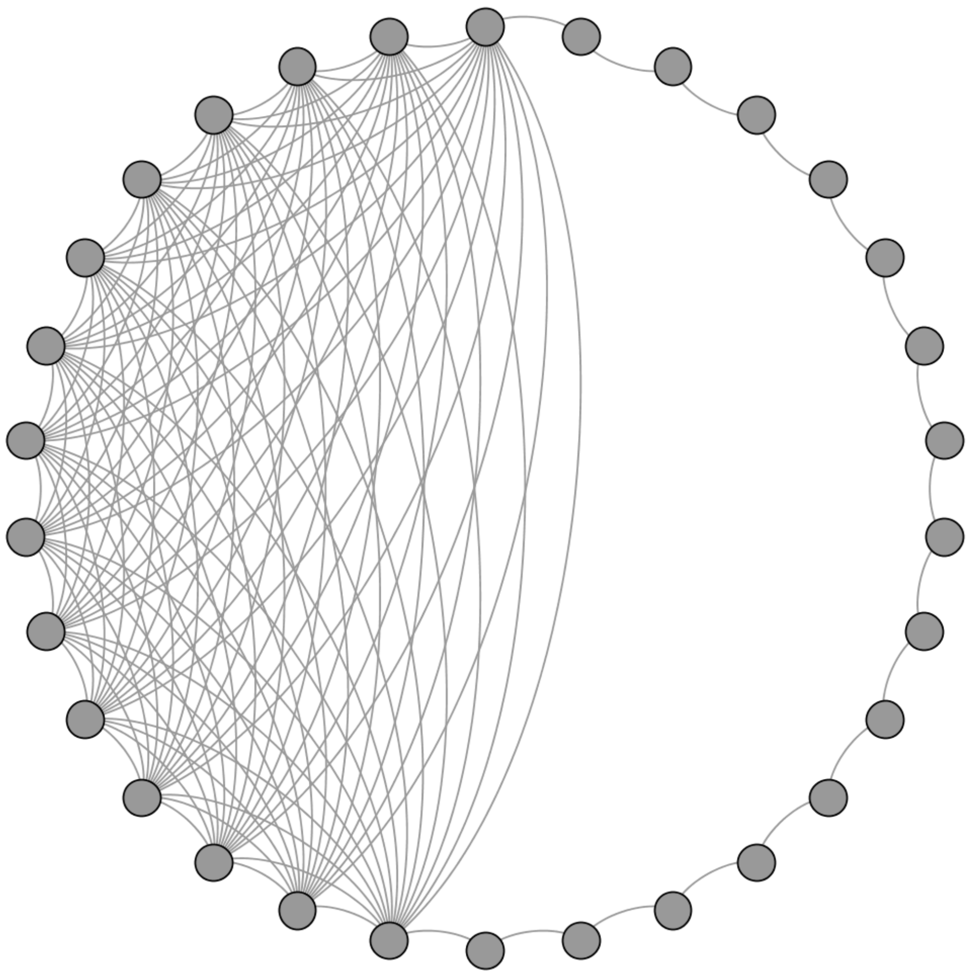

Figure 43: Half Fully-Connected topology

Table 40: Network metrics Half Fully-Connected topology

Avg. degree	8.067
Avg. path-length	4.007
Network diameter	9
Graph density	0.278
Edge count	$\frac{\frac{N}{2}(\frac{N}{2}-1)}{2}+\frac{N}{2}+1$

A.4 Ascending-Connected

Each agent is connected to the agent with the next higher optimism-factor. The agents with highest and lowest optimism-factor are not connected thus this network is not a closed circle. Included because it is the most minimal network which satisfies the property required by the hypothesis and is therefore the major network of interest throughout the thesis.

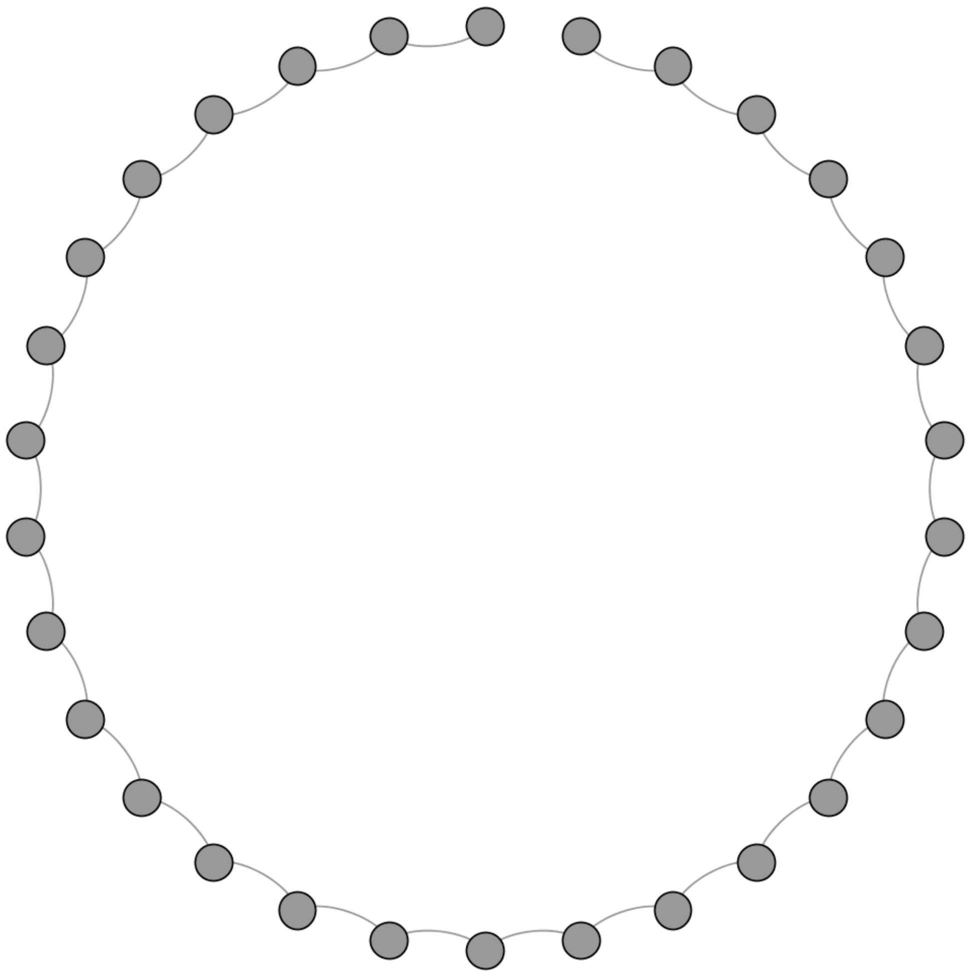

Figure 44: Ascending-Connected topology

Table 41: Network metrics Ascending-Connected topology

Avg. degree	1.933
Avg. path-length	10.33
Network diameter	29
Graph density	0.067
Edge count	$N-1$

A.5 Ascending-Connected with short-cuts

A.5.1 Full short-cuts

Each agent is connected to the K next neighbours in the clockwise arrangement. Thus agent N - K + 1 is connected to K - 1 higher optimism-agents and wraps around to the agent with lowest optimism-factor. Included to analyse the influence of increasing connectivity.

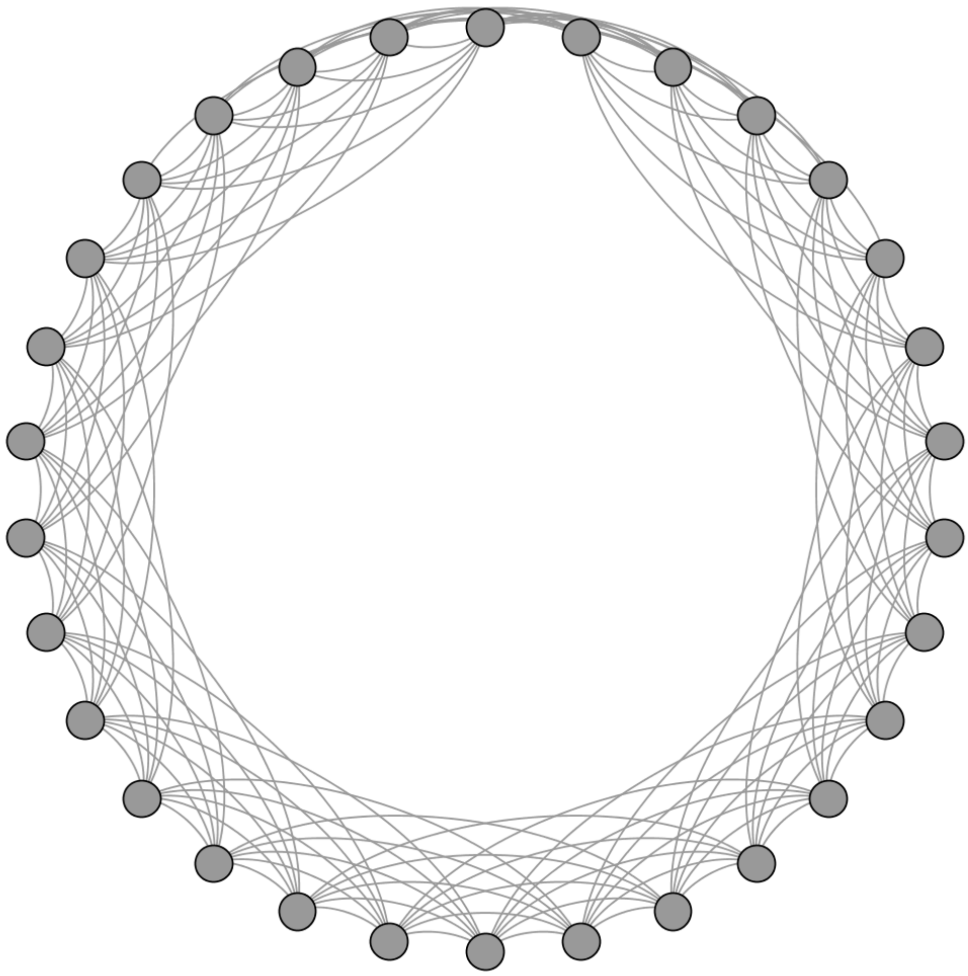

Figure 45: Ascending-Connected 5 full short-cuts topology

Table 42: Network metrics Ascending-Connected 5 full short-cuts topology

Avg. degree	10
Avg. path-length	1.966
Network diameter	3
Graph density	0.345
Edge count	NK

A.5.2 Regular short-cuts

The topology starts ascending-connected and each agent is additionally connected to one next neighbour in the clockwise arrangement where the distance to the next neighbour is K agents. Included to analyse the influence of trading-links to agents with much higher optimism-factor.

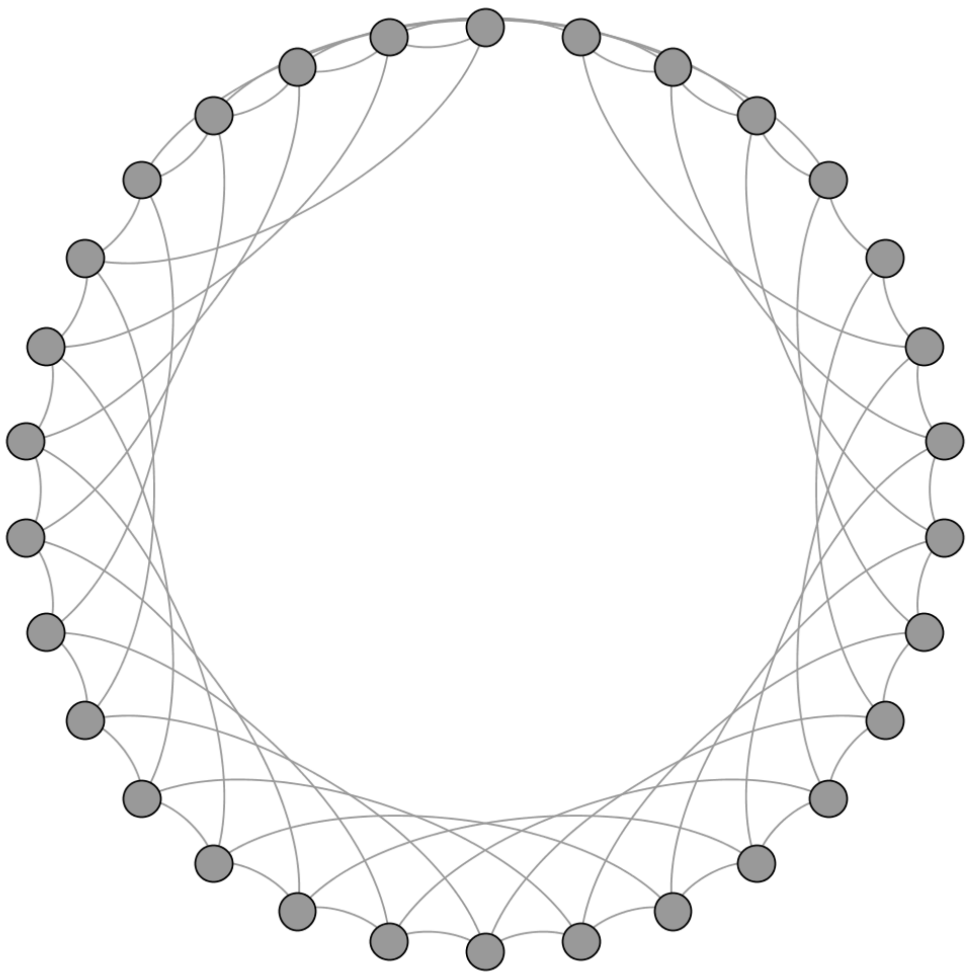

Figure 46: Ascending-Connected 5 regular short-cuts topology

Table 43: Network metrics Ascending-Connected 5 regular short-cuts topology

Avg. degree	3.867
Avg. path-length	2.839
Network diameter	6
Graph density	0.133
Edge count	$2N$

A.5.3 Random short-cuts

Starting with an ascending-connected network this topology adds one additional short-cut from each agent to another random agent with a given probability where a probability of 0.0 results in only the ascending-connectedness and a probability of 1.0 in each agent having an additional random short-cut. Self-loops and multi-edges are not allowed. Included to analyse the influence of randomness in ascending-connected topologies.

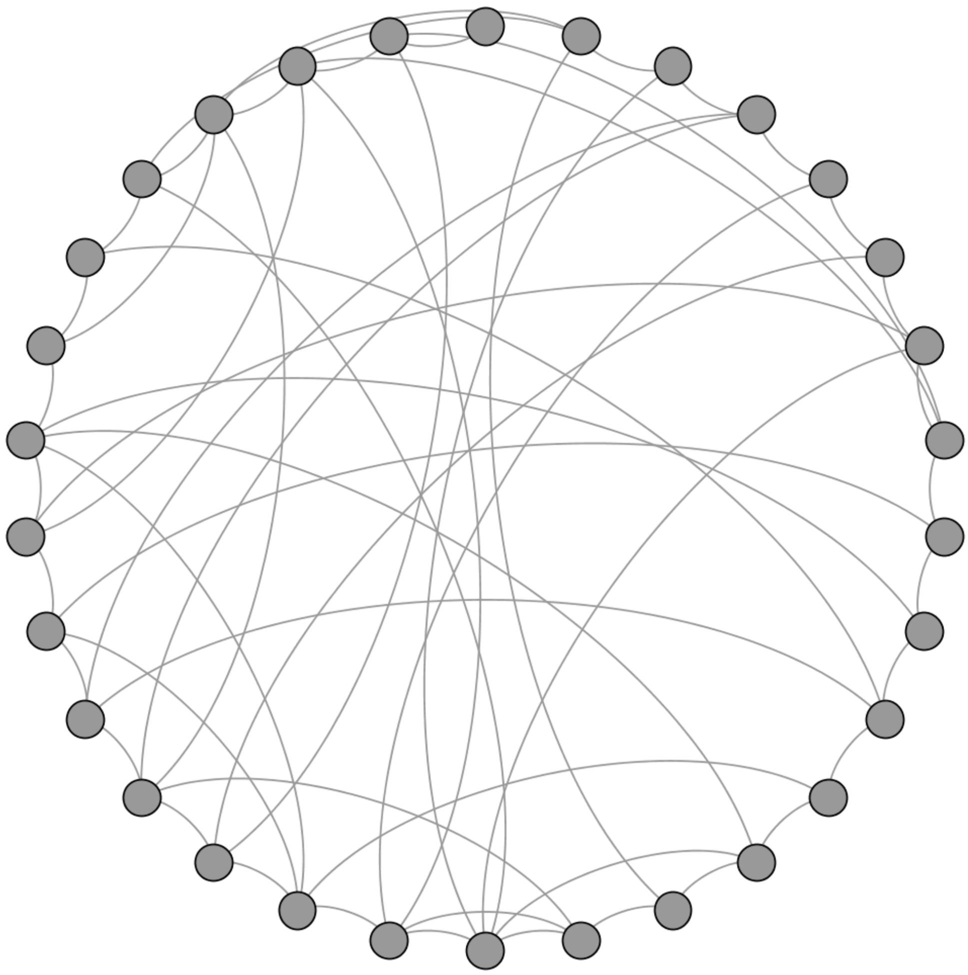

Figure 47: Ascending-Connected random short-cuts probability 1.0 topology

Table 44: Network metrics Ascending-Connected random short-cuts topology

Avg. degree	3.867
Avg. path-length	2.506
Network diameter	5
Graph density	0.133
Edge count	$\min N, \max 2N$

A.6 Hub-based topologies

A.6.1 3 Hubs

The agents are separated into 3 groups based upon their optimism-factor. Group 1 ranges from 0.0 to 0.33, group 2 from 0.33 to 0.66 and group 3 from 0.66 to 1.0. All agents within a group are fully-connected where the groups are interconnected between each other through a hub which is the agent with the highest optimism-factor of each group that is: 0.33, 0.66 and 1.0. Included as a point-of-reference as this topology was discussed too in Breuer et al. (2015).

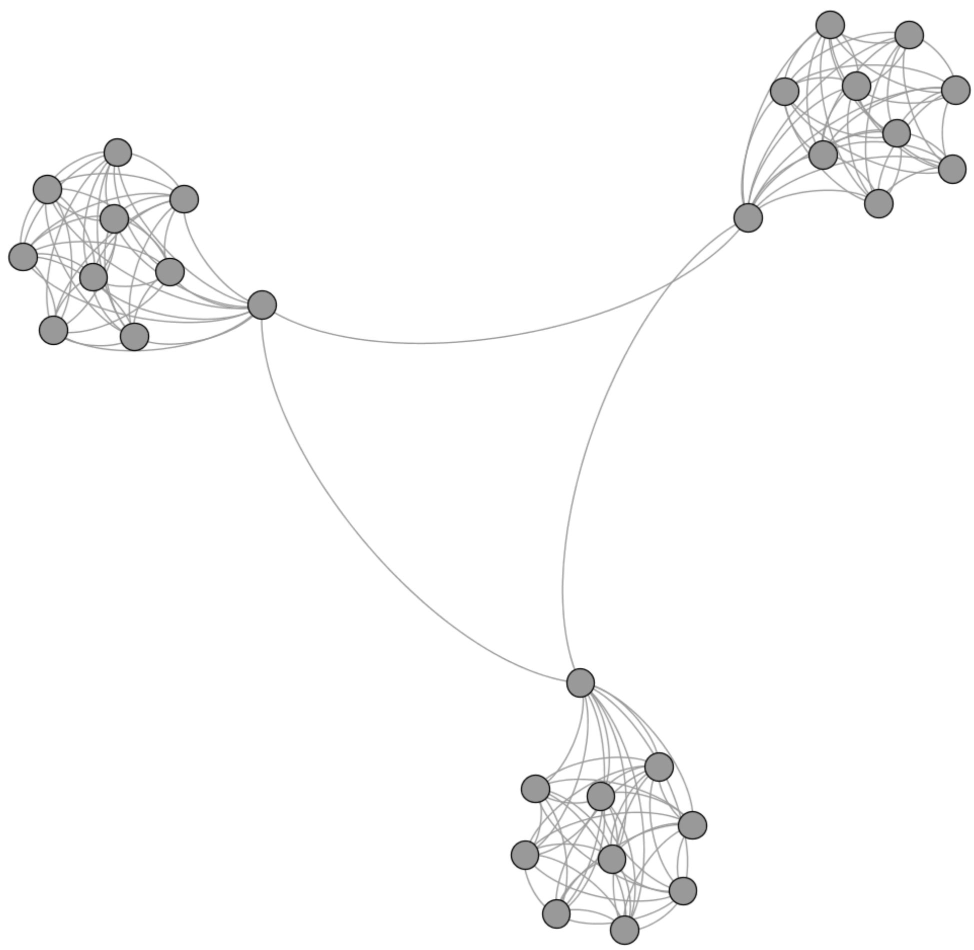

Figure 48: 3 Hubs topology

Table 45: Network metrics 3 Hubs topology

Avg. degree	9.2
Avg. path-length	2.241
Network diameter	3
Graph density	0.371
Edge count	$3(\frac{\frac{N}{3}(\frac{N}{3}-1)}{2})+3$

A.6.2 3 Median Hubs

There are 3 agents which act as median hubs which are the agent with the median optimism-factor and the next lower and higher ones. All three are connected to each other where the rest of the agents are randomly connected to one hub so that each hub has the same amount of agents. Included to see what happens if all agents can only trade through median agents which are the most active ones in the Fully-Connected topology.

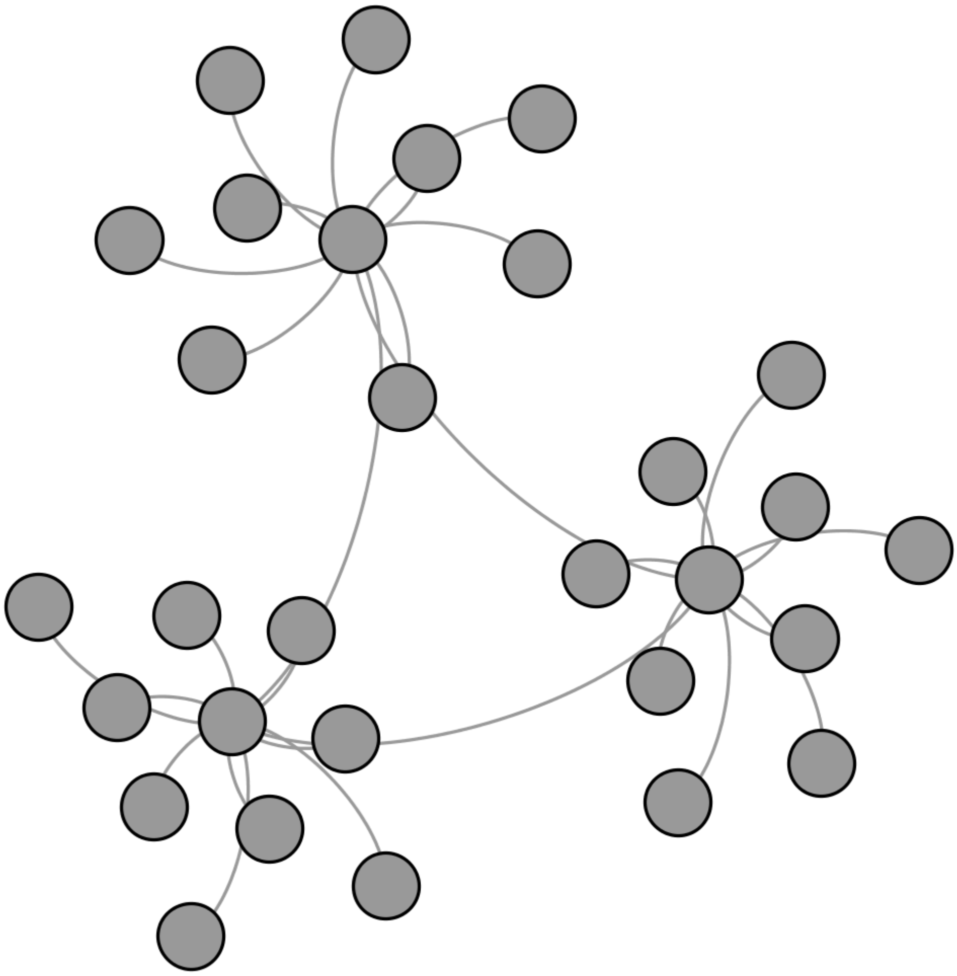

Figure 49: 3 Median Hub topology

Table 46: Network metrics 3 Median Hub topology

Avg. degree	2
Avg. path-length	2.49
Network diameter	3
Graph density	0.069
Edge count	N

A.6.3 Median Hub

All agents are connected to the agent with the median optimism-factor. Included for the same reason as in 3 median hubs but with just one median hub.

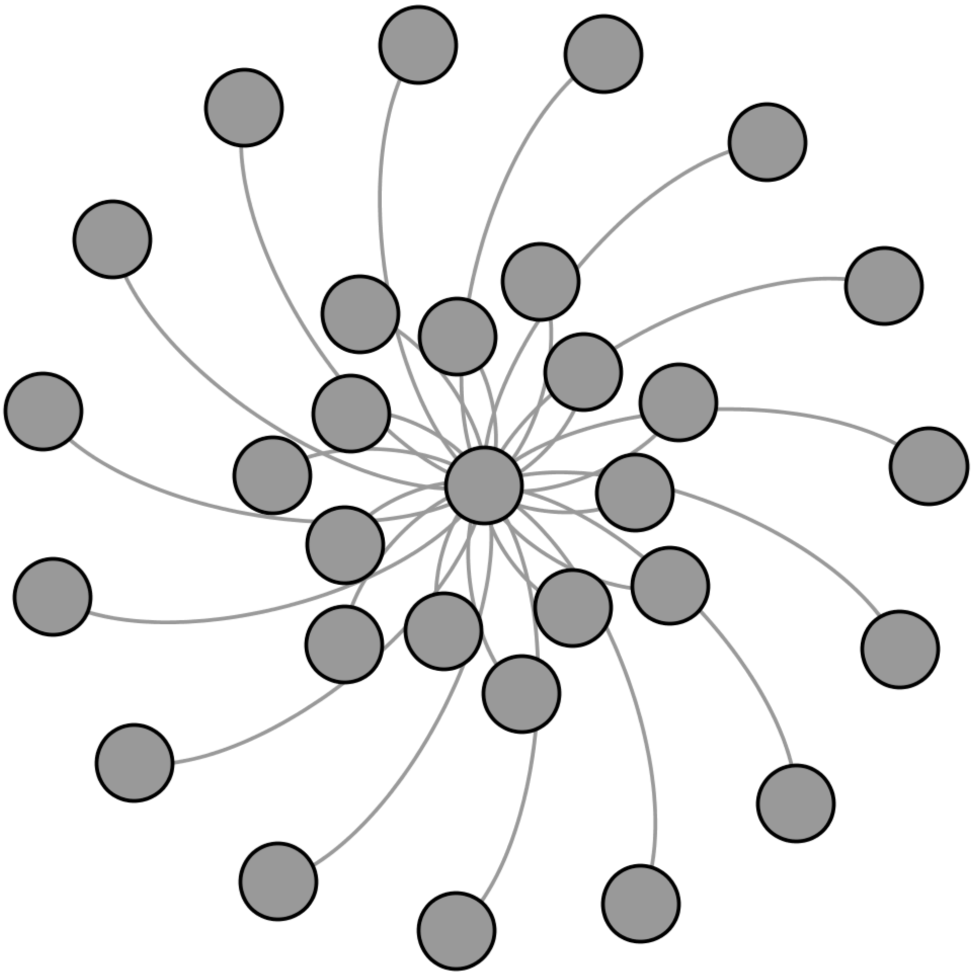

Figure 50: Median Hub topology

Table 47: Network metrics Median Hub topology

Avg. degree	1.933
Avg. path-length	1.933
Network diameter	2
Graph density	0.067
Edge count	$N-1$

A.6.4 Maximum Hub

Looks the same as 1 Median Hub but all edges are connected to the agent with the highest optimism-value. Has thus also the same metrics as the optimism-values have no functional influence on the metrics. Included just out of curiosity and has no real value as it is obviously clear that equilibrium is impossible to be reached in this case.

A.7 Complex network-topologies

A.7.1 Erods-Renyi

See section 2.4 for how this topology is created. Included to investigate the influence of randomness, small-world and scale-free effects upon equilibrium.

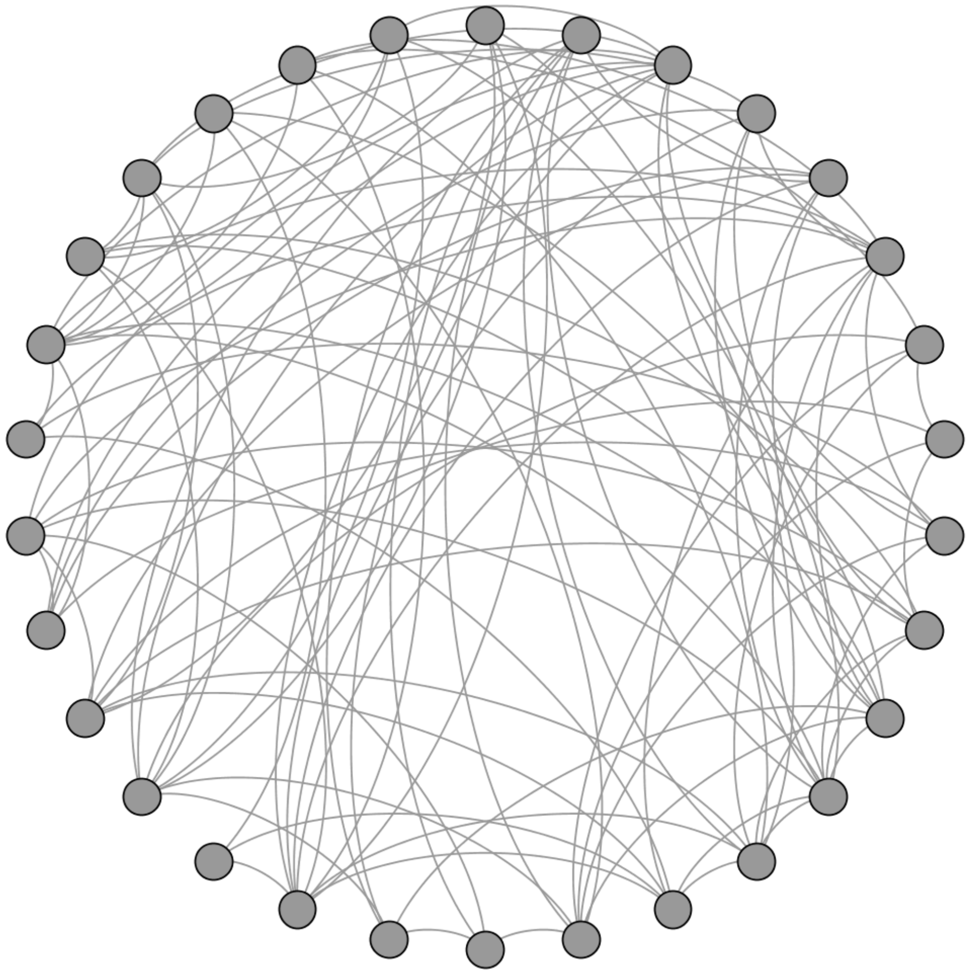

Figure 51: Erdos-Renyi topology with inclusion-probability of 0.2

Table 48: Network metrics Erdosy-Renyi 0.2

Avg. degree	6.8
Avg. path-length	1.913
Network diameter	3
Graph density	0.234
Connected component	1

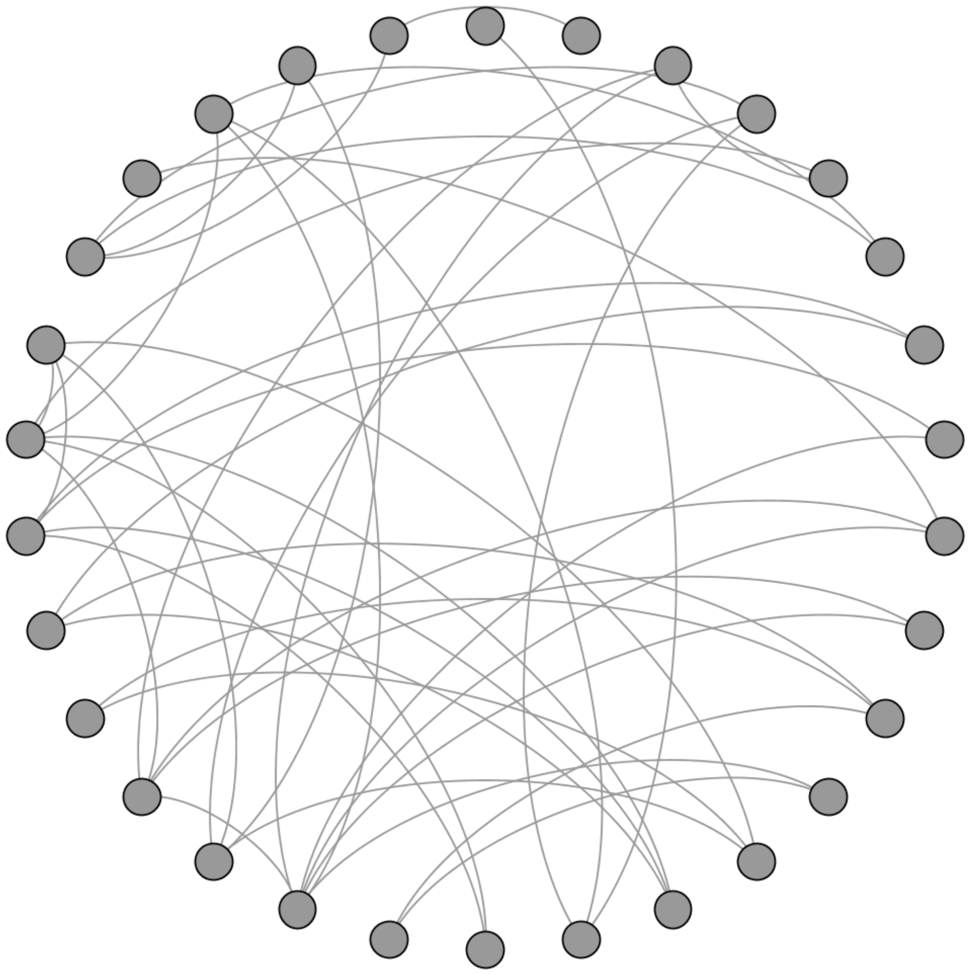

Figure 52: Erdos-Renyi topology with inclusion-probability of 0.1

Table 49: Network metrics Erdosy-Renyi 0.1

Avg. degree	2.933
Avg. path-length	3.262
Network diameter	7
Graph density	0.101
Connected component	1

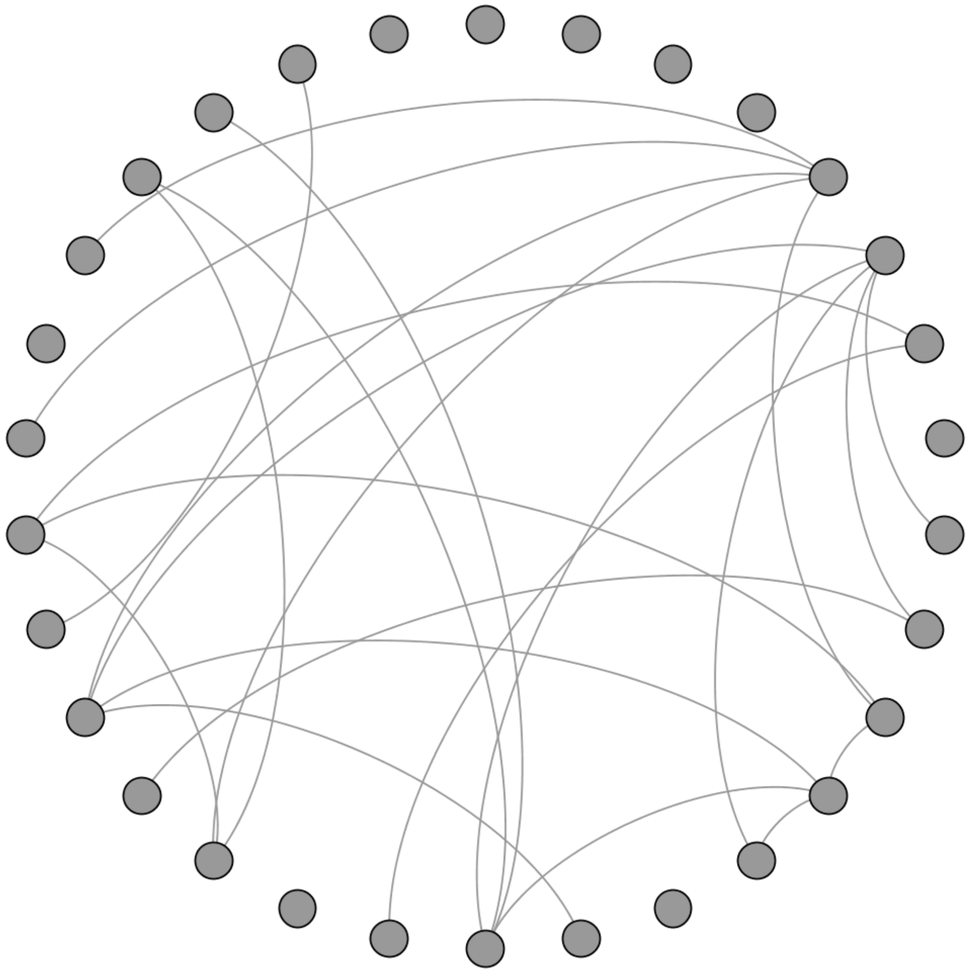

Figure 53: Erdos-Renyi topology with inclusion-probability of 0.05

Table 50: Network metrics Erdosy-Renyi 0.05

Avg. degree	1.6
Avg. path-length	3.052
Network diameter	8
Graph density	0.055
Connected component	11

A.7.2 Barbasi-Albert

See section 2.4 for how this topology is created. Included to investigate the influence of randomness, small-world and scale-free effects upon equilibrium.

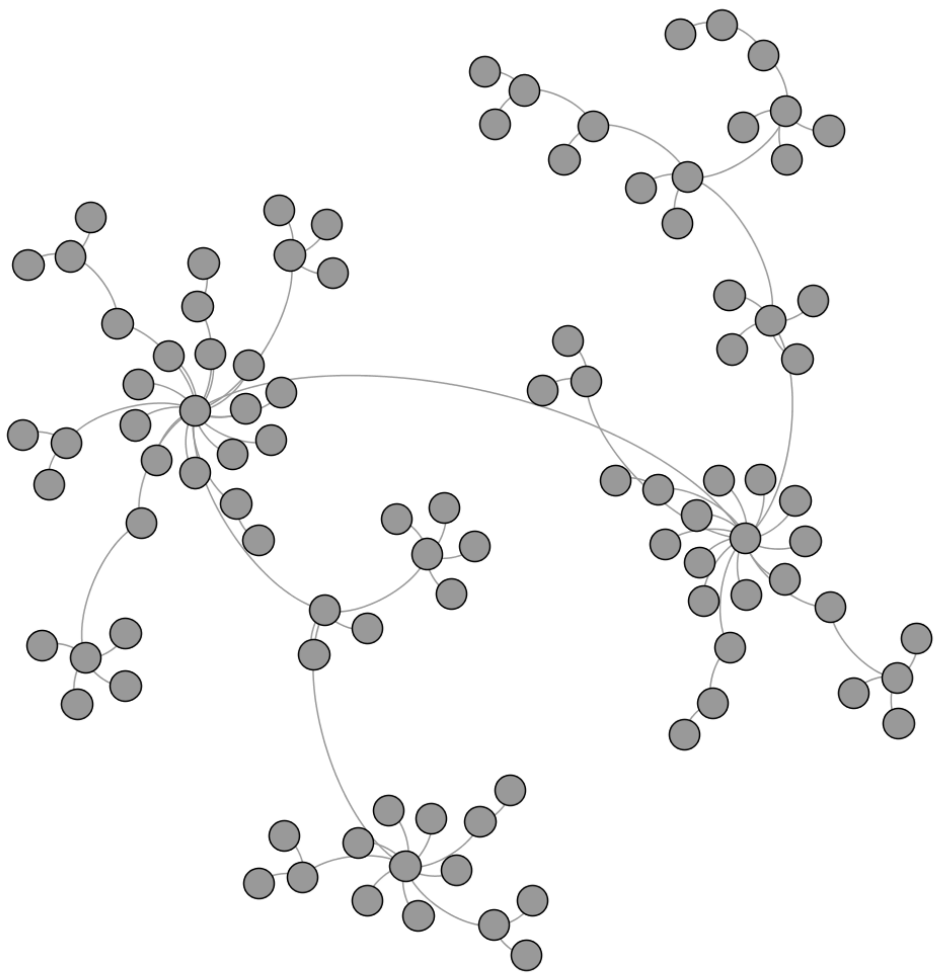

Figure 54: Barbasi-Albert topology with m0=3, m=1

Table 51: Network metrics Barbasi-Albert m0=3, m=1

Avg. degree	1.98
Avg. path-length	4.684
Network diameter	11
Graph density	0.02

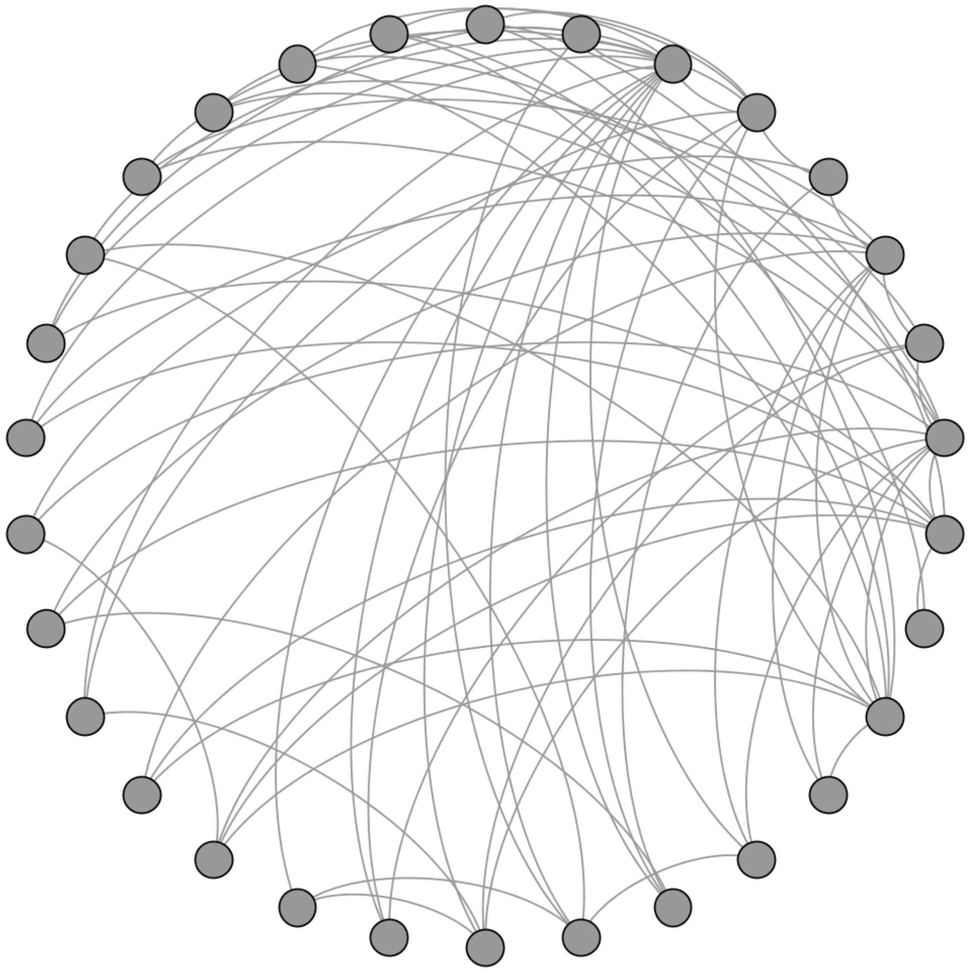

Figure 55: Barbasi-Albert topology with m0=9, m=3

Table 52: Network metrics Barbasi-Albert m0=9, m=3

Avg. degree	4.733
Avg. path-length	2.11
Network diameter	4
Graph density	0.163

A.7.3 Watts-Strogatz

See section 2.4 for how this topology is created. Included to investigate the influence of randomness, small-world and scale-free effects upon equilibrium.

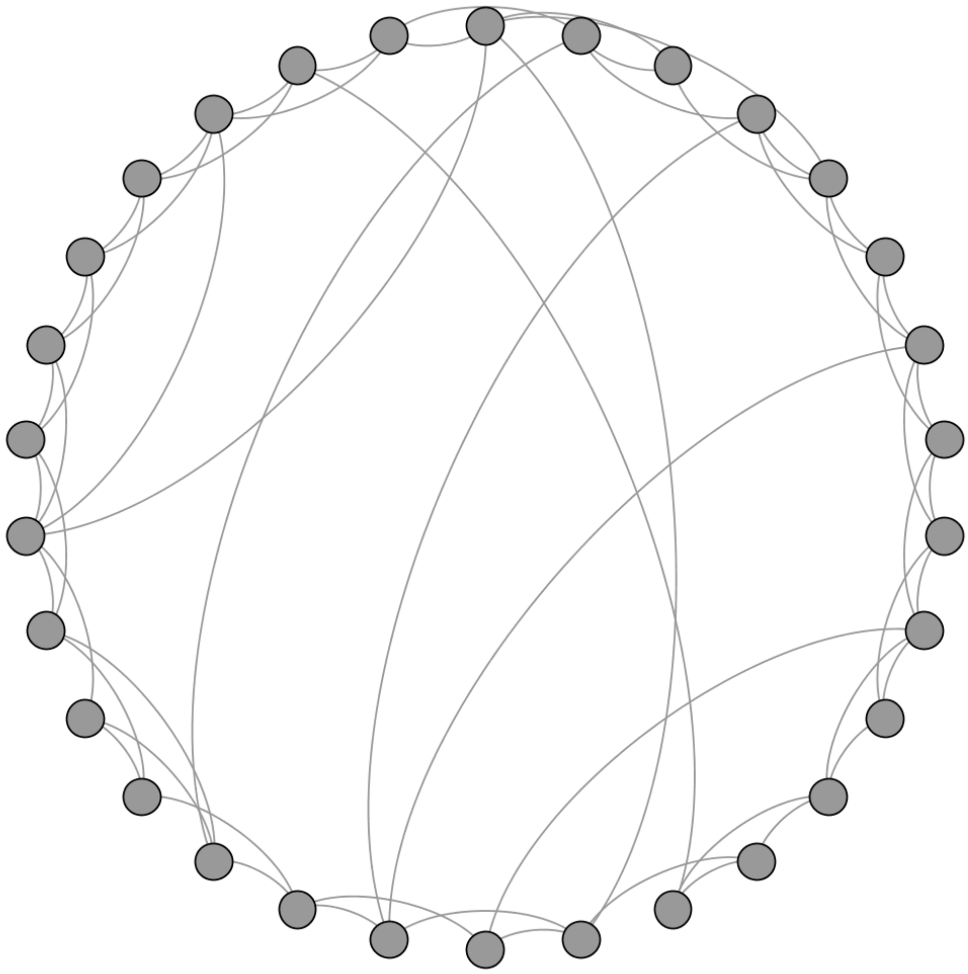

Figure 56: Watts-Strogatz topology with k=2, p=0.2

Table 53: Network metrics Watts-Strogatz k=2, p=0.2

Avg. degree	4
Avg. path-length	2.883
Network diameter	6
Graph density	0.138

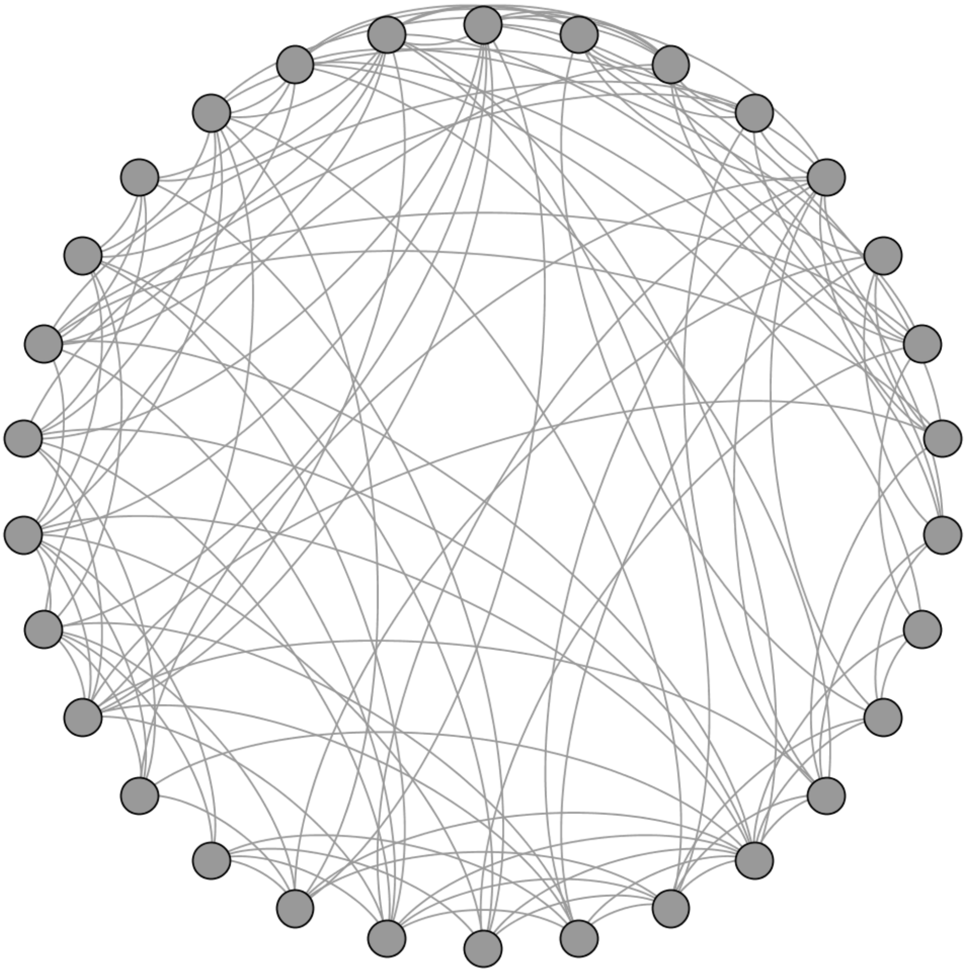

Figure 57: Watts-Strogatz topology with k=4, p=0.5

Table 54: Network metrics Watts-Strogatz k=4, p=0.5

Avg. degree	8
Avg. path-length	1.823
Network diameter	3
Graph density	0.276

Appendix B

Results of other networks

B.1 Half-Fully Connected

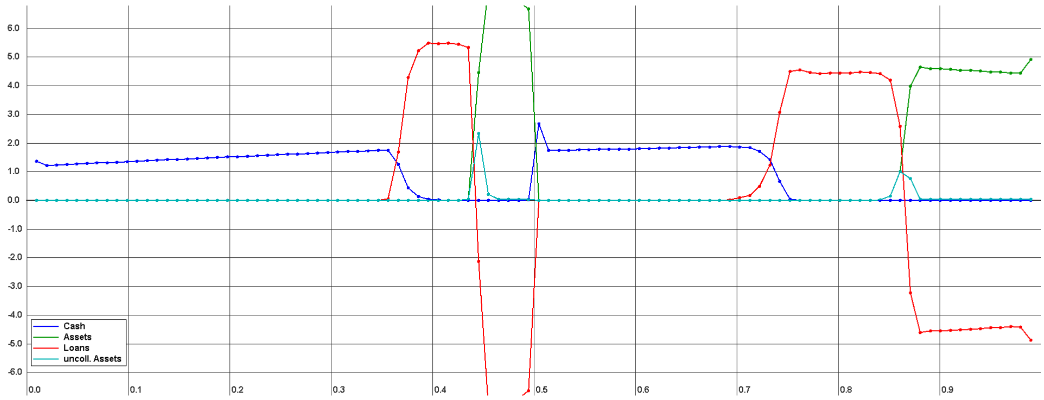

Figure 58: Wealth-Distribution of Half-Fully Connected topology

Table 55: Equilibrium of Half-Fully Connected topology

Asset-Price p	0.526 (0.011)
Bond-Price q	0.313 (0.003)
Marginal Agent i1	0.730 (0.011)
Marginal Agent i2	0.448 (0.005)
Pessimist Wealth	1.342 (0.022)
Medianist Wealth	4.542 (0.353)
Optimist Wealth	1.746 (0.074)

Table 56: Performance of Half-Fully Connected topology

Successful matching-rounds	16,373.48 (493.96)
Failed matching-rounds	1,000.00 (0.00)
Total matching-rounds	17,373.48 (493.96)
Ratio successful/total	0.94
Ratio failed/total	0.06

The equilibrium is clearly distinct from the theoretical and Fully-Connected one as miss-allocation can be found within the pessimists-range. Also the i1- and i2-points and the wealth-distributions differ both numerically and visually.

B.2 Ascending-Connected with short-cuts

B.2.1 Random short-cuts

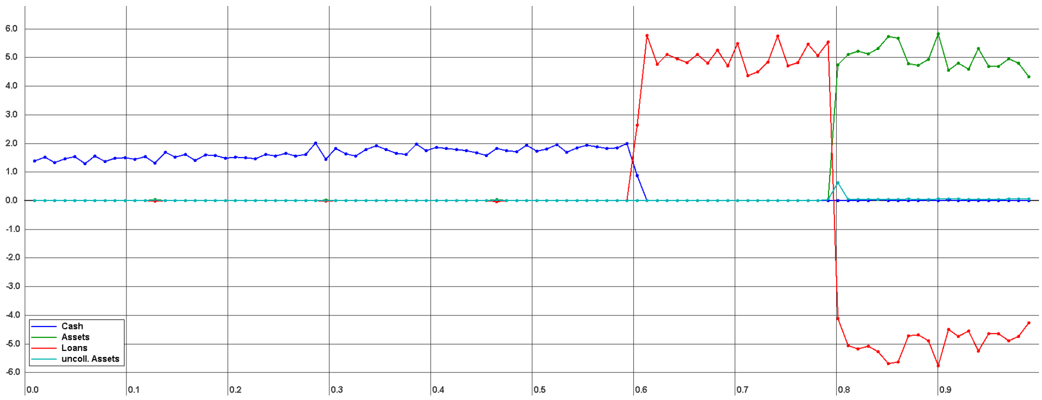

Figure 59: Wealth-Distribution of Ascending-Connected random short-cuts topology

Table 57: Equilibrium of Ascending-Connected random short-cuts topology

Asset-Price p	0.704 (0.008)
Bond-Price q	0.386 (0.003)
Marginal Agent i1	0.594 (0.000)
Marginal Agent i2	0.802 (0.000)
Pessimist Wealth	1.651 (0.003)
Medianist Wealth	4.517 (0.428)
Optimist Wealth	4.771 (0.249)

Table 58: Performance of Ascending-Connected random short-cuts topology

Successful matching-rounds	7,249.40 (148.18)
Failed matching-rounds	1,000.60 (0.86)
Total matching-rounds	8,250.00 (148.21)
Ratio successful/total	0.88
Ratio failed/total	0.12

Random short-cuts seem to reduce the miss-allocation of pessimists-wealth a bit but lead to a fundamental different equilibrium than the theoretical or fully-connected one as can clearly be seen both visually and numerically.

B.2.2 2 short-cuts

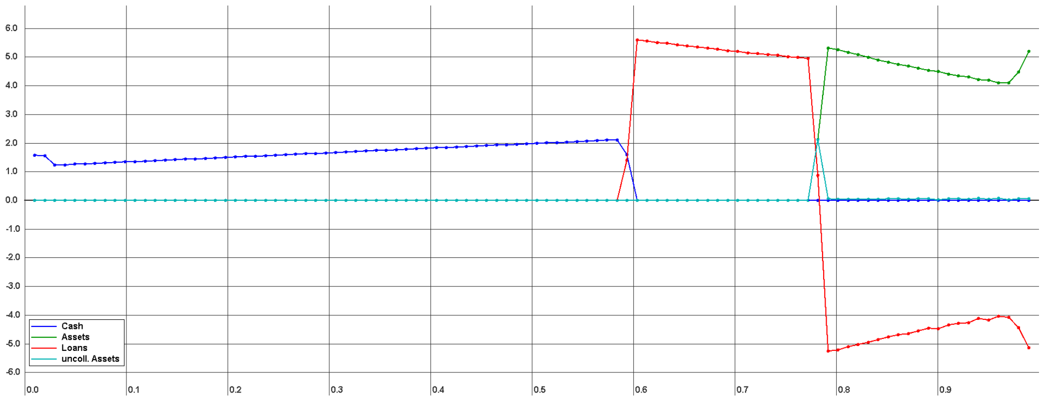

Figure 60: Wealth-Distribution of Ascending-Connected 2 short-cuts topology

Table 59: Equilibrium of Ascending-Connected 2 short-cuts topology

Asset-Price p	0.706 (0.010)
Bond-Price q	0.379 (0.001)
Marginal Agent i1	0.592 (0.004)
Marginal Agent i2	0.782 (0.001)
Pessimist Wealth	1.667 (0.001)
Medianist Wealth	5.207 (0.105)
Optimist Wealth	4.544 (0.039)

Table 60: Performance of Ascending-Connected random short-cuts topology

Successful matching-rounds	26,572.90 (91.85)
Failed matching-rounds	1,000.64 (1.03)
Total matching-rounds	27,573.54 (91.99)
Ratio successful/total	0.96
Ratio failed/total	0.04

This topology reduces the miss-allocation in the pessimists-range dramatically but doesn't solve it yet. Unfortunately it leads to a dramatically different wealth-distribution within the medianists and optimist.

B.2.3 5 full short-cuts

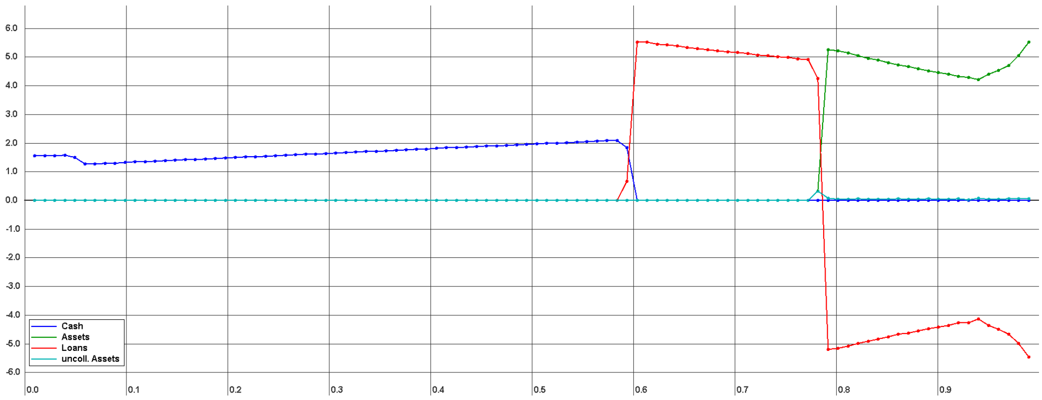

Figure 61: Wealth-Distribution of Ascending-Connected 5 full short-cuts topology

Table 61: Equilibrium of Ascending-Connected 5 full short-cuts

Asset-Price p	0.692 (0.010)
Bond-Price q	0.379 (0.001)
Marginal Agent i1	0.594 (0.000)
Marginal Agent i2	0.792 (0.000)
Pessimist Wealth	1.667 (0.000)
Medianist Wealth	5.160 (0.026)
Optimist Wealth	4.747 (0.013)

Table 62: Performance of Ascending-Connected 5 full short-cuts topology

Successful matching-rounds	11,157.56 (79.45)
Failed matching-rounds	1,000.92 (1.08)
Total matching-rounds	12,158.48 (79.36)
Ratio successful/total	0.92
Ratio failed/total	0.08

As can be clearly seen this topology seems to be able to solve miss-allocations in the pessimists-range seen in Ascending-Connected topology but is still different than the theoretical and Fully-Connected equilibrium.

B.2.4 15 full short-cuts

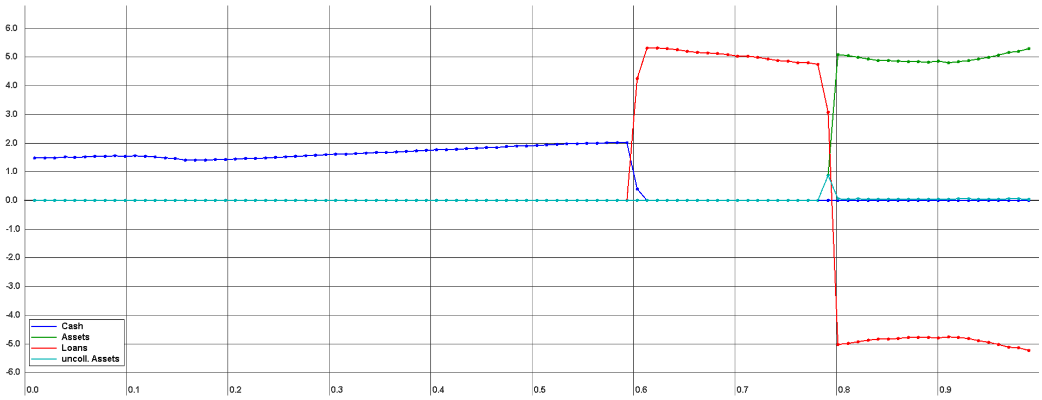

Figure 62: Wealth-Distribution of Ascending-Connected 15 full short-cuts topology

Table 63: Equilibrium of Ascending-Connected 15 full short-cuts topology

Asset-Price p	0.687 (0.012)
Bond-Price q	0.380 (0.001)
Marginal Agent i1	0.594 (0.000)
Marginal Agent i2	0.801 (0.002)
Pessimist Wealth	1.660 (0.003)
Medianist Wealth	4.922 (0.039)
Optimist Wealth	4.948 (0.052)

Table 64: Performance of Ascending-Connected 15 full short-cuts topology

Successful matching-rounds	3,716.52 (42.31)
Failed matching-rounds	1,000.66 (0.98)
Total matching-rounds	4,717.18 (42.18)
Ratio successful/total	0.79
Ratio failed/total	0.21

This topology comes very close to the theoretical equilibrium but is still a bit different as can be seen in the curved wealth-distributions of the pure optimists.

B.2.5 30 full short-cuts

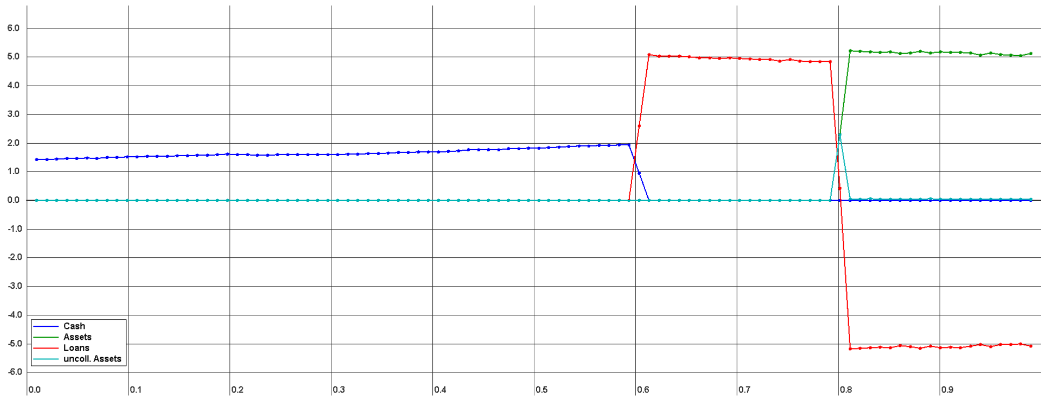

Figure 63: Wealth-Distribution of Ascending-Connected 30 full short-cuts topology

Table 65: Equilibrium of Ascending-Connected 30 full short-cuts topology

Asset-Price p	0.705 (0.015)
Bond-Price q	0.382 (0.001)
Marginal Agent i1	0.594 (0.001)
Marginal Agent i2	0.803 (0.003)
Pessimist Wealth	1.651 (0.004)
Medianist Wealth	4.812 (0.062)
Optimist Wealth	5.027 (0.068)

Table 66: Performance of Ascending-Connected 30 full short-cuts topology

Successful matching-rounds	2,062.92 (28.59)
Failed matching-rounds	1,000.42 (0.78)
Total matching-rounds	3,063.34 (28.49)
Ratio successful/total	0.67
Ratio failed/total	0.33

This topology is very close to the theoretical and Fully-Connected equilibrium although it differs in Asset-Price p and in the wealth-distributions. Of course with 30 fully short-cuts in a network of 100 agents one is already very close to fully connectedness.

B.2.6 5 regular short-cuts

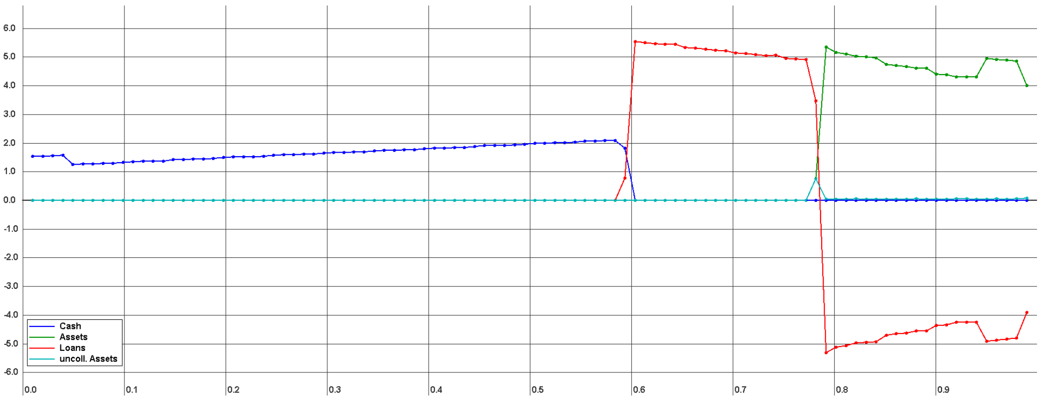

Figure 64: Wealth-Distribution of Ascending-Connected 5 regular short-cuts topology

Table 67: Equilibrium of Ascending-Connected 5 regular short-cuts topology

Asset-Price p	0.694 (0.013)
Bond-Price q	0.379 (0.002)
Marginal Agent i1	0.594 (0.000)
Marginal Agent i2	0.792 (0.000)
Pessimist Wealth	1.667 (0.000)
Medianist Wealth	5.130 (0.029)
Optimist Wealth	4.726 (0.014)

Table 68: Performance of Ascending-Connected 5 regular short-cuts topology

Successful matching-rounds	9,743.36 (52.60)
Failed matching-rounds	1,000.64 (0.69)
Total matching-rounds	10,744.00 (52.72)
Ratio successful/total	0.91
Ratio failed/total	0.09

As can be seen in the visual results this topology shows a different equilibrium than the theoretical and Fully-Connected one.

B.2.7 15 regular short-cuts

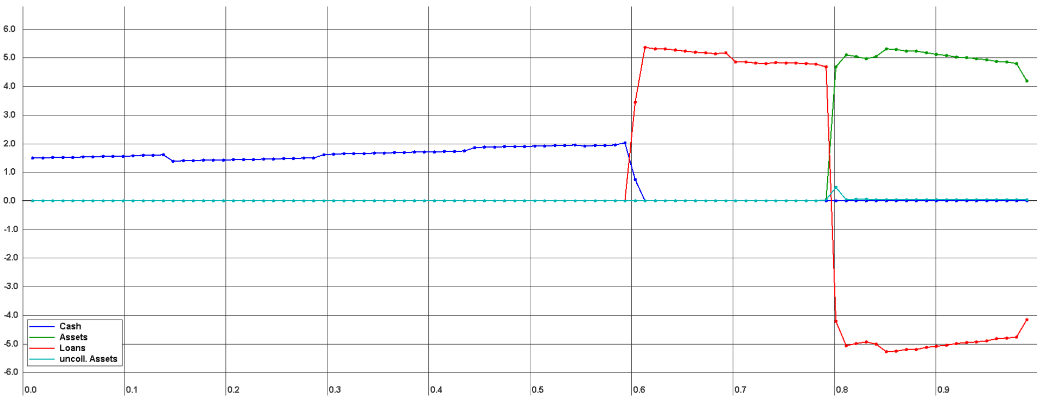

Figure 65: Wealth-Distribution of Ascending-Connected 15 regular short-cuts topology

Table 69: Equilibrium Ascending-Connected 15 regular short-cuts topology

Asset-Price p	0.678 (0.018)
Bond-Price q	0.382 (0.001)
Marginal Agent i1	0.594 (0.000)
Marginal Agent i2	0.802 (0.000)
Pessimist Wealth	1.654 (0.002)
Medianist Wealth	4.933 (0.014)
Optimist Wealth	4.998 (0.005)

Table 70: Performance of Ascending-Connected 15 regular short-cuts topology

Successful matching-rounds	4,090.94 (51.71)
Failed matching-rounds	1,007.40 (38.50)
Total matching-rounds	5,098.34 (73.30)
Ratio successful/total	0.80
Ratio failed/total	0.20

The equilibrium of this topology is falls very far from the theoretical and Fully-Connected one.

B.2.8 30 regular short-cuts

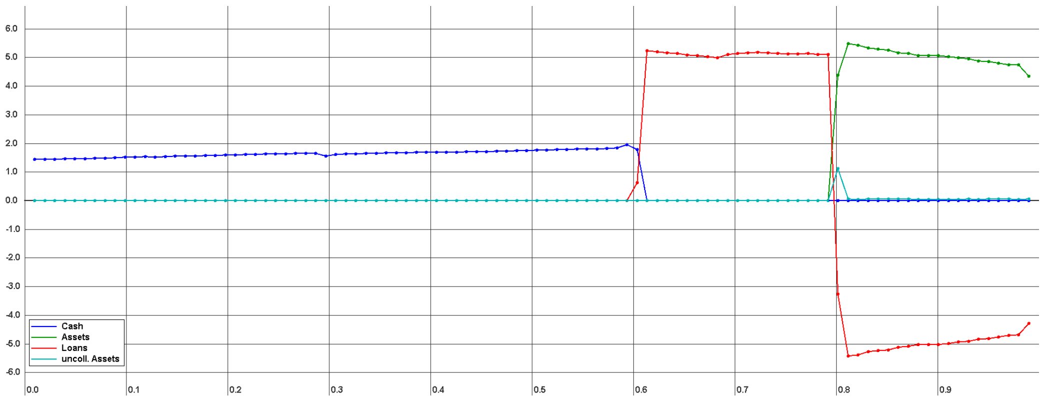

Figure 66: Wealth-Distribution of Ascending-Connected 30 regular short-cuts topology

Table 71: Equilibrium of Ascending-Connected 30 regular short-cuts topology

Asset-Price p	0.653 (0.045)
Bond-Price q	0.382 (0.001)
Marginal Agent i1	0.604 (0.001)
Marginal Agent i2	0.802 (0.000)
Pessimist Wealth	1.639 (0.001)
Medianist Wealth	5.120 (0.034)
Optimist Wealth	5.000 (0.000)

Table 72: Performance of Ascending-Connected 30 regular short-cuts topology

Successful matching-rounds	6,301.60 (162.67)
Failed matching-rounds	1,001.26 (1.38)
Total matching-rounds	7,302.86 (162.94)
Ratio successful/total	0.86
Ratio failed/total	0.14

The equilibrium of this topology is falls very far from the theoretical and Fully-Connected one.

B.3 Hub-Based topologies

The Hub-Based Topologies fail to come even close to equilibrium due to reasons given in chapter 4. This can be seen also very clearly in the visual results and thus no performance- and equilibrium-tables are listed as they would not make any sense.

B.3.1 3-Hubs

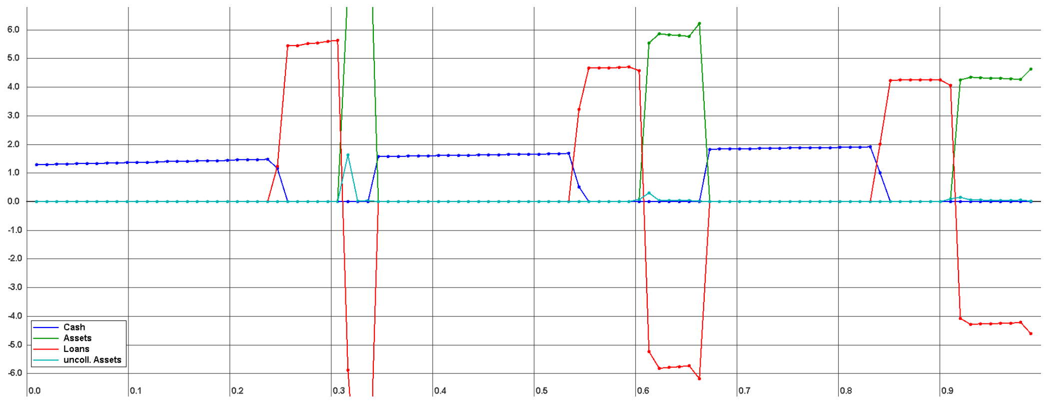

Figure 67: Wealth-Distribution of 3-Hubs topology

B.3.2 1-Median Hub

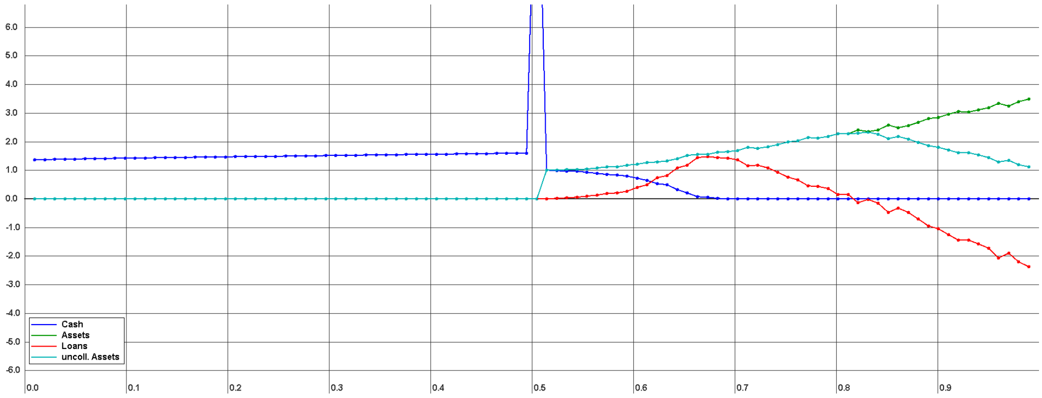

Figure 68: Wealth-Distribution of 1 Median-Hub topology

B.3.3 3-Median Hubs

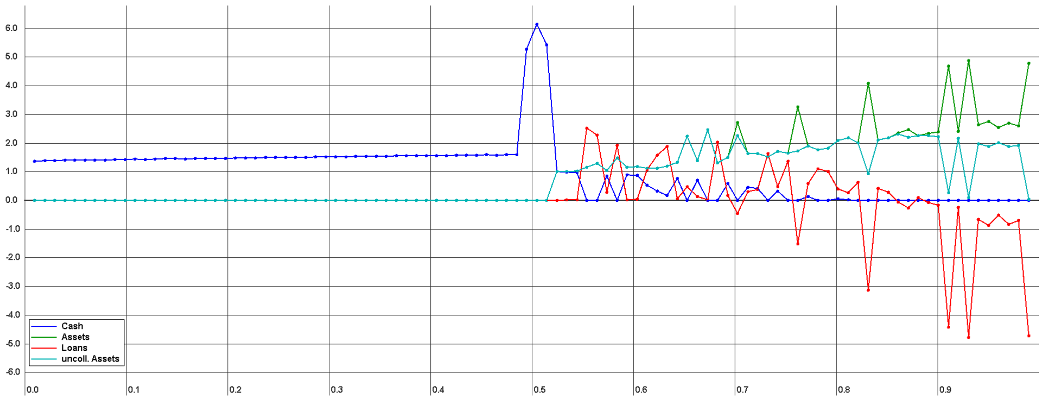

Figure 69: Wealth-Distribution of 3 Median-Hubs topology

B.3.4 Maximum Hub

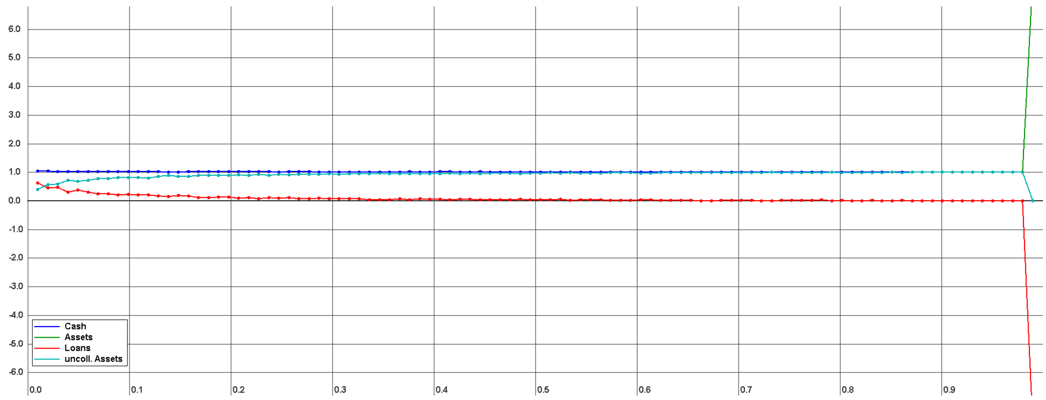

Figure 70: Wealth-Distribution of Maximum-Hub topology

B.4 Scale-Free and Small-World topologies

This topologies fail to come even close to equilibrium too due to reasons given in chapter 4. This can be seen also very clearly in the visual results and thus no performance- and equilibrium-tables are listed as they would not make any sense.

B.4.1 Erdos-Renyi

Note that with the correct parametrization this topology could satisfy the hypothesis by pure chance. The result would be a pure random network as an Ascending-Connected topology with random short-cuts but as already showed above this Ascending-Connected random short-cuts network fails from producing the theoretical and Fully-Connected equilibrium.

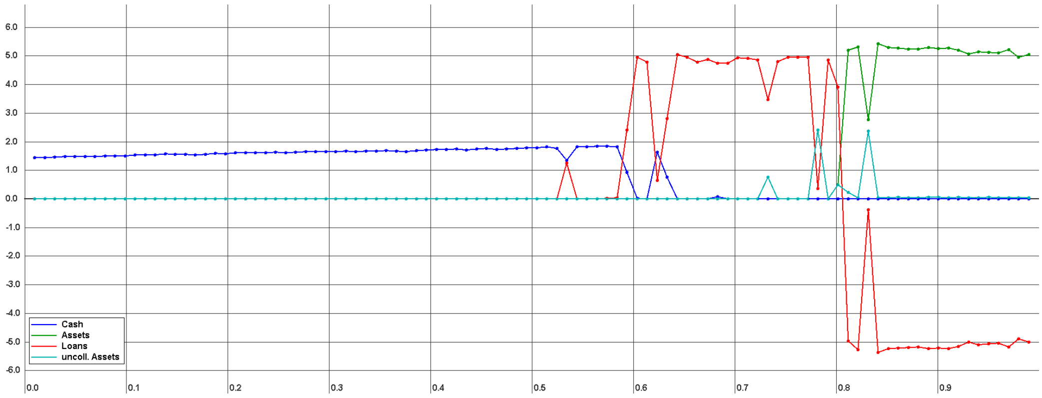

Figure 71: Wealth-Distribution of Erdos-Renyi 0.2 topology

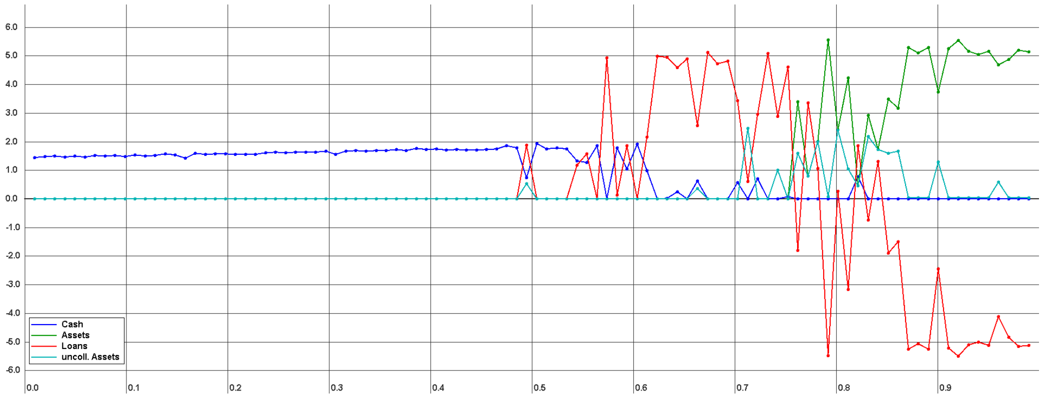

Figure 72: Wealth-Distribution of Erdos-Renyi 0.1 topology

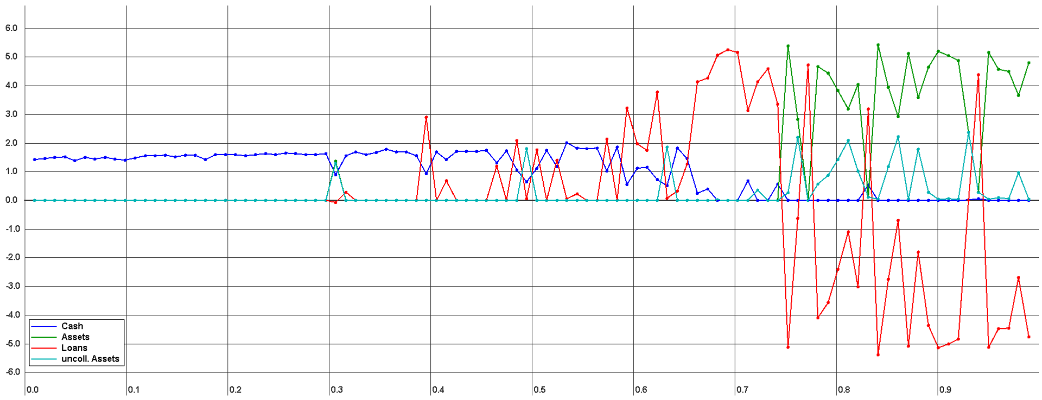

Figure 73: Wealth-Distribution of Erdos-Renyi 0.05 topology

B.4.2 Barbasi-Albert

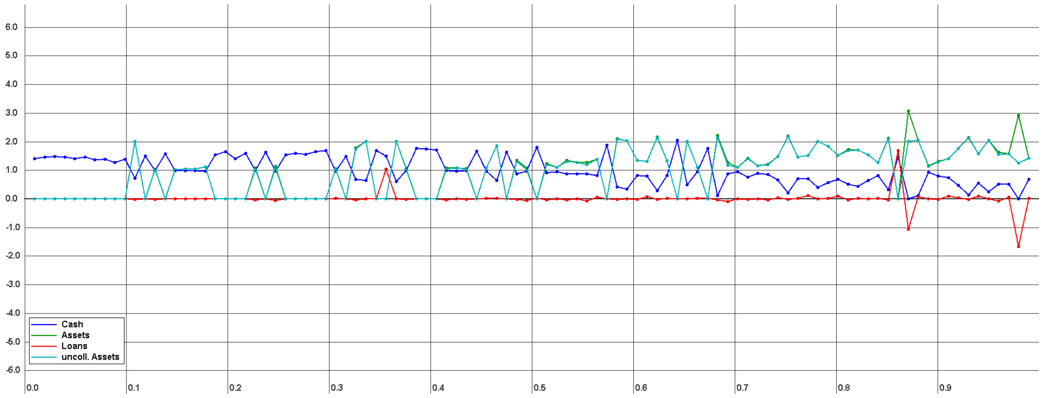

Figure 74: Wealth-Distribution of Barbasi-Albert m0=3, m=1 topology

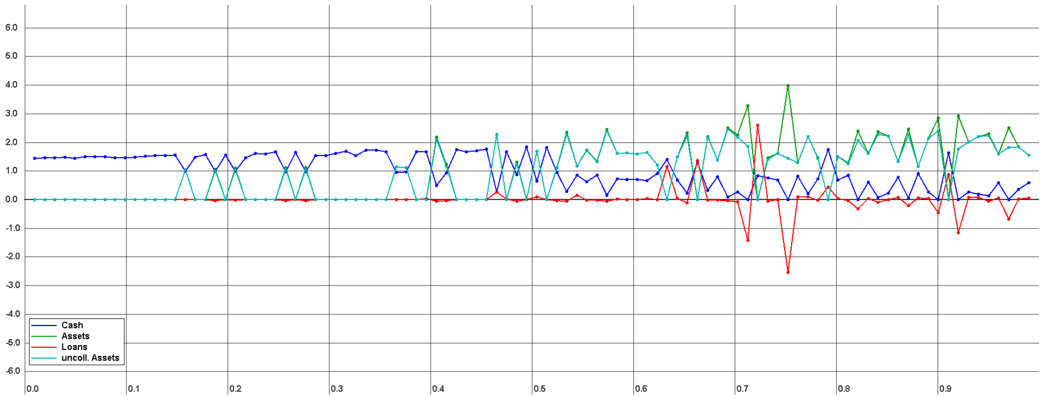

Figure 75: Wealth-Distribution of Barbasi-Albert m0=9, m=3 topology

B.4.3 Watts-Strogatz

Note that with the correct parametrization this topology could satisfy the hypothesis by pure chance too. The result would be a pure random network as an Ascending-Connected topology with random short-cuts but as already showed above this Ascending-Connected random short-cuts network fails from producing the theoretical and Fully-Connected equilibrium.

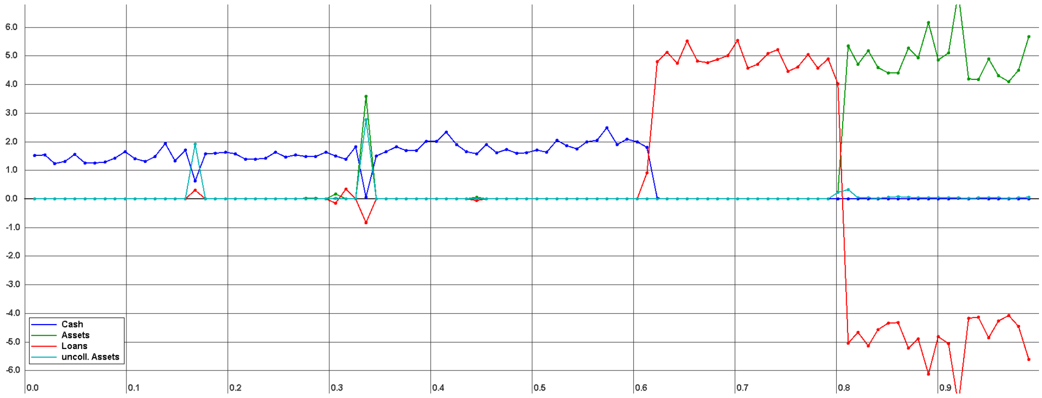

Figure 76: Wealth-Distribution of Watts-Strogatz k=2, b=0.2 topology

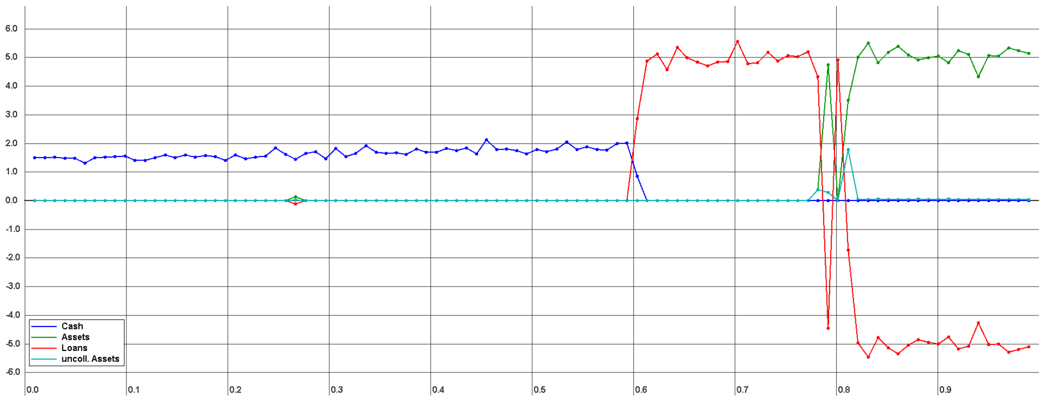

Figure 77: Wealth-Distribution of Watts-Strogatz k=4, b=0.5 topology

Appendix C

Increasing matching-probabilities

This appendix gives the formulas for increasing the matching-probabilities in Ascending-Connected topology. This technique was not developed by the author of the thesis but by the supervisor Mr. Hans-Joachim Vollbrecht and is included for completeness.

The idea is to adjust the price-ranges by a specific constant for each agent so that the shape of the matching-probabilities is kept but the absolute value of a match is dramatically increased where the edges as seen in 13 are at 1. So the process is to calculate the constants for each market for each agent and then in a next step adjust the price-ranges.

C.1 Calculating constants

The constants are named *ca* for the Asset/Cash market, *cl* for the Bond/Cash market and *cal* for the Asset/Bond market. The number of agents is denoted by R and the face-value of the used bond is V.

C.1.1 Asset/Cash market

$$E_i^{asset} = 0.8\frac{i}{R+1} + 0.2 \tag{C.1}$$

$$ca_{i+1} = \frac{(R-i)(i+2)(\frac{0.8}{R+1} + ca_i)}{(R+1-i)(i+1)} - \frac{0.8}{R+1} \tag{C.2}$$

C.1.2 Bond/Cash market

$$E_i^V = 0.2 + \frac{i}{R+i}(V - 0.2) \tag{C.3}$$

$$cl_{i+1} = \frac{(R-i)(i+2)(\frac{V-0.2}{R+1} + cl_i)}{(R+1-i)(i+1)} - \frac{V-0.2}{R+1} \tag{C.4}$$

C.1.3 Asset/Bond market

$$E_i^{asset,V} = \frac{E_i^{asset}}{E_i^V} \tag{C.5}$$

$$d_{i,V} = E_{i+1}^{asset,V} - E_i^{asset,V} \tag{C.6}$$

$$cal_{i+1} = \frac{d_{i,V}(5.0 - E_{i+1}^{asset,V})(E_{i+2}^{asset,V} - \frac{0.2}{V})(d_{i,V} + cal_i)}{d_{i+1,V}(5 - E_i^{asset,V}(E_{i+1}^{asset,V} - \frac{0.2}{V})} - d_{i+1,V} \tag{C.7}$$

C.1.4 Adjusting the ranges

Using the previously calculated constants *ca, cl, cal* for each agent on each market now the minimum and maximum values of the price-ranges of each agent are adjusted using the constants to increase the matching-probability. Note that only the min- and max-values are changed but the limit-price has obviously to be left untouched.

Ask-offering ranges

$$min\ asset\text{-}price\ agent_i = limit\text{-}price\ asset\ agent_i \tag{C.8}$$

$$max\ asset\text{-}price\ agent_i = limit\text{-}price\ asset\ agent_{i+1} + ca_{i+1} \tag{C.9}$$

$$min\ bond\text{-}price\ agent_i = limit\text{-}price\ bond\ agent_i \tag{C.10}$$

$$max\ bond\text{-}price\ agent_i = limit\text{-}price\ bond\ agent_{i+1} + cl_{i+1} \tag{C.11}$$

$$min\ asset/bond\text{-}price\ agent_i = \frac{limit\text{-}price\ asset\ agent_i}{limit\text{-}price\ bond\ agent_i} \tag{C.12}$$

$$max\ asset/bond\text{-}price\ agent_i = \frac{limit\text{-}price\ asset\ agent_{i+1}}{limit\text{-}price\ bond\ agent_{i+1}} + cal_{i+1} \tag{C.13}$$

Bid-offering ranges

$$min\ asset\text{-}price\ agent_i = limit\text{-}price\ asset\ agent_{i-1} \tag{C.14}$$

$$max\ asset\text{-}price\ agent_i = limit\text{-}price\ asset\ agent_i \tag{C.15}$$

$$min\ bond\text{-}price\ agent_i = limit\text{-}price\ bond\ agent_{i-1} \tag{C.16}$$

$$max\ bond\text{-}price\ agent_i = limit\text{-}price\ bond\ agent_i \tag{C.17}$$

$$min\ asset/bond\text{-}price\ agent_i = \frac{limit\text{-}price\ asset\ agent_{i-1}}{limit\text{-}price\ bond\ agent_{i-1}} \tag{C.18}$$

$$max\ asset/bond\text{-}price\ agent_i = \frac{limit - priceassetagent_i}{limit - pricebondagent_i} \tag{C.19}$$

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Glossary

allocative efficiency Is a state in which all products match the consumers preferences. In other words it means that goods are distributed optimally between all consumers reflecting the wants and tastes of the agents involved. If allocative efficiency is at 100% then it is not possible any more to increase the utilities of *both* traders in a case of a trade thus traders are no more willing to trade. In other words if two agents trade at an allocative efficiency of 100% then one agent will decrease its utility and will lose.. 16

auction Is a market institution in which messages from traders include some price information.. 13

buyer A trader who is willing to buy a given amount of good for a given price.. 13

clearing Is the process of finding a price in which all demands are matched to the given supplies thus clearing the market by leaving no unmatched demands or supplies.. 13

good A generic object which is traded between agents. Can be an asset, food, gold,.... 13

limit-price Is the private price a trader assigns to a good they want to exchange. This private price is different from the price in the offering and is higher in case of the buyer and lower in the case of the seller.. 13

market institution Defines how exchange between traders takes place by defining rules what traders can do.. 13

numeraire A generic form of money.. 13

offer A tuple of price and quantity on a given market which signals the willingness to trade by these given quantities.. 13

offer-book Keeps all offers made by the traders.. 13

round In each round all traders have the opportunity to place an offer. At the end of each round matching is applied and if a successful match is found the unmatched offers are deleted from the offer-book.. 13, 15

seller A trader who is willing to sell a given amount of good for a given price.. 13

transaction-price Is the price upon a buyer and a seller agree when trading with each other.. 15

zero-intelligence agents Place offers strictly in a range which increases their utility and do not learn. They are completely deterministic in a way that they never change their behaviour.. 14

Bibliography

Albert, R. and lászló Barabási, A. (2002). Emergence of scaling in random networks. *Science*, page 286(5439):509–512.

BBC (2007). The us sub-prime crisis in graphics. `http://news.bbc.co.uk/2/hi/business/7073131.stm`. [Online; accessed 23-July-2015].

Breuer, T., Summer, M., and Vollbrecht, H.-J. (2015). Endogenous leverage and asset pricing in double auctions.

Brigham, E. F. (2012). Fundamentals of financial management.

Drmota, M., Gittenberger, B., Karigl, G., and Panholzer, A. (2007). *Mathematik für Informatik*. Heldermann Verlag.

Easley, D. and Kleinberg, J. (2010). *Networks, Crowds and Markets*. Cambridge University Press.

Erdős, P. and Renyi, A. (1959). On random graphs. In *Publicationes Mathematicae*, pages 6:290—-297.

Erdős, P. and Renyi, A. (1960). On the evolution of random graphs. In *PUBLICATION OF THE MATHEMATICAL INSTITUTE OF THE HUNGARIAN ACADEMY OF SCIENCES*, pages 17–61.

Geanakoplos, J. (2009). The leverage cycle.

Geanakoplos, J. and Zame, W. (2014). Collateral equilibrium, i: a basic framework. *Economic Theory*, 56(3):443–492.

Gode, D. K. and Sunder, S. (1993). Allocative efficiency of markets with zero-intelligence traders: Market as a partial substitute for individual rationality. *Journal of Political Economy*, 101(1):119–37.

Jackson, M. O. (2008). *Social and Economic Networks*. Princeton University Press.

Kleinfeld, J. S. (2002). Could it be a big world after all ? *Society*, page 39(2):61–66.

Milgram, S. (1967). The small world problem. *Pyschology Today*, page 1:61–67.

Newman, M. E. J. (2003). The structure and function of complex networks. *SIAM REVIEW*, 45:167–256.

Parsons, S., Marcinkiewicz, M., Niu, J., and Phelps, S. (2006). Everything you wanted to know about double auctions, but were afraid to (bid or) ask.

StockMarketInvestors (2007). The subprime mortgage crisis explained. `http://www.stock-market-investors.com/stock-investment-risk/the-subprime-mortgage-crisis-explained.html`. [Online; accessed 23-July-2015].

Travers, J. and Milgram, S. (1969). An experimental study of the small world problem. *Sociometry*, page 32:425–443.

Watts, D. J. and Strogatz, S. H. (June 1998). Collective dynamics of 'small-world' networks. In *Nature*, page 393:440–442.