FH Vorarlberg
University of Applied Sciences

Masterthesis in the study program
Informatik – Software and Information Engineering

Influence of network-topologies on equilibrium in continuous double-auctions.

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Statuatory Declaration

I declare that I have developed and written the enclosed work completely by myself, and have not used sources or means without declaration in the text. Any thoughts from others or literal quotations are clearly marked. This Master Thesis was not used in the same or in a similar version to achieve an academic degree nor has it been published elsewhere.

Widmung

Ich widme diese Arbeit meinen beiden liebevollen Eltern, die den verlorenen Sohn nach 11 Jahren in Wien wie selbstverständlich wieder mit offenen Armen zu Hause in Vorarlberg aufgenommen haben und ihm so ein entspanntes Masterstudium ermöglichten und ihm dadurch halfen ein völlig neues Kapitel in seinem Leben aufzuschlagen.

Contents

Abstract

In the paper of [BSV13] a model for endogenous leverage in a continuous double-auction is introduced and it is shown under which circumstances holdings and trading prices approach an equilibrium. One main criteria is the trading network the agents use where Breuer et al. examine only two topologies and report that the prices come to an equilibrium only in the case of a fully connected network. They leave the question open on how the model behaves with different kind of networks and which network topology exactly allows an equilibrium to be reached for further research. This thesis builds upon this model and gives a hypothesis for a necessary condition a network must satisfy to allow the model to approach an equilibrium. Then a few network-topologies are examined in regard of their ability to allow equilibria to be reached or not through computer-driven simulation. As will be shown in this thesis through validation by computer-driven simulation the hypothesis turns out to be correct only after extending the simulation-model by an additional market. This result raises questions this thesis tries to answer about market-mechanisms and market-types when agents don't trade in a fully informed network.

Chapter 1

Introduction

TODO: überarbeiten, passt so noch nicht In 2008 the so called "Subprime Mortgage Crisis" struck the world. It was caused by declining house prices which rose during the US Housing Market Bubble in 2006 to an all-time high. Borrowers used their asset as collateral for the mortgage which constantly increased in value which guaranteed them a low payment-rate because the rate was coupled to the value of the asset. Banks granted "subprime" mortgages to more and more highly risky borrowers. In 2007 borrowers started to default which led to falling prices as the banks reclaimed the collateral and wanted to sell it again on the market to compensate for the loss. This led to a flood of assets which led to a decline of housing prices overall. As the prices fell dramatically the payment-rates rose dramatically to compensate for the cheaper asset. This in turn resulted in even more borrowers going default which resulted in a dramatic downward spiral. Even worse the banks were selling these collateralized products between each other and even insured themself against defaults of borrowers which led to an even more dramatic kick-back.

This mechanism of borrowing money to buy goods which in turn act as a security for the borrowed money is called leverage which was determined as the primary driving force behind systemic risk in the aftermath of the "Subprime Mortgage Crisis". See Chapter 2.1 "Leverage and Systemic Risk" for a more in-depth discussion.

Up until 2010 leverage was always exogenous in the literature on collateralized credit but recently Geanakoplos and Zame (TODO: cite) proposed theories which endogenized leverage within a general equilibrium framework.

[BSV13] developed a simulation on top of the model of Geanakoplos in which zero-intelligence agents trade assets and loans in a continuous double auction. They wanted to better understand the dynamic of such a theoretical process and how prices develop instead of being predicted through an equilibrium theory. They TODO: zitierne "ask whether the competitive theory of trade in leveraged assets has decriptive and predictive power in a double auction environment."

4 contributions: 1. double auctions for leveraged assets is new 2. details of institutional specification matter a lot 3. limits of the endogenous leverage model 4.

They could show that in their simulation trading prices and wealth-distribution approach the theoretical equilibrium of Geanakoplos. In their simulation only a fully connected network and a hub-network of agents was investigated where the equilibrium was only reached in the case of the fully connected network. See Chapter 3. "The Leverage Cycle" for a thorough description of the simulation-model of [BSV13].

This thesis investigates more topologies of networks and their states of equilibrium. Furthermore it presents a hypothesis about the necessary property a topology of a network must satisfy to reach the theoretical equilibrium predicted in the theory of Geanakoplos. Interestingly it is shown experimentally that the hypothesis alone does not guarantee the reach of the theoretical equilibrium but further mechanisms needs to be implemented. See Chapter 4 "Hypothesis" and Chapter 6 "Results" for an in-depth explanation of both the hypothesis and why it does not hold and needs to be extended by means of an additional market-mechanism.

For experimental investigation a software was built for this thesis which implemented the exact simulation model of [BSV13] but extended it further to be applicable to arbitrary topologies. See Chapter 5 "Implementation" on details of the software.

In Chapter **??** "Theory" the theoretical background involved with this thesis is presented. First Leverage and systemic risk and its implications are discussed. Then an introduction into the mechanics of Continuous Double Auction as market-mechanisms and equilibrium theory in economics is given. Finally an overview of abstract networks, network-generating algorithms and and their properties is given.

In Chapter **??** "The Leverage Cycle" the theoretical model [BSV13] built their simulation upon is discussed in-depth.

In Chapter 4 "Hypothesis" all topologies which are investigated are introduced and the conjecture about the type of topology necessary to reach the theoretical equilibirum is presented and discussed whether the given topologies could ever approach it or not.

Chapter 5 ”Implementation” gives an in-depth explanation of the implementation of the computer-driven simulation presented in [BSV13] including a description of the architecture, implementation of the markets and trading mechanisms.

Chapter 6 ”Results” shows the results of simulations of all implemented topologies.

Chapter 7 ”Interpretation and Discussions” connects the content of the previous chapters to show that the initial hypothesis of Chapter 4 does not satisfy the equilibrium and shows how it can be reached by introducing an additional market. Then results of simulations with this market are given and discussed where will be shown that using the additional market an equilibrium will be reached but that it is different from the theoretical predictions.

In Chapter 8 ”Conclusions” a short sum-up of the thesis and questions left for further research are presented.

Chapter 2

Theory

TODO: der theorie-teil. Soll in die verwendete Theorie des Hauptteils einführen und darauf hinweisen, aber nicht völlig trocken und losgelöst vom hauptteil sein. Soll immer den kontext des hauptteils berücksichtigen und schon gewisse anwendungsfälle vorwegnehmen.

2.1 Systemic risk and Leverage

Both are tightly coupled in a way that leverage increases systemic risk dramatically as was the case in the "Subprime Mortgage Crisis".

Systemic Risk

WIKI: It refers to the risks imposed by interlinkages and interdependencies in a system or market, where the failure of a single entity or cluster of entities can cause a cascading failure, which could potentially bankrupt or bring down the entire system or market.

[Bor10]

Leverage

WIKI: In finance, leverage (sometimes referred to as gearing in the United Kingdom and Australia) is any technique to multiply gains and losses.

Accounting Leverage Notational Leverage Economic Leverage

2.2 Equilibrium Theory

theoretisches: utiliy-funktionen und clearing preis in der simulation: ungeklärt, immer individuell, "steckenbleiben" vs. gleichgewicht, am ende an theoretischem gleichgewicht orientiert

TODO: What is equilibrium in a process? It is NOT when no agents are able to trade anymore. TODO: Can we give an equilibrium-definition for the dynamic-process? HV: hier ein paar schlaue gedanken dazu?

2.3 Continuous Double Auction

Paper: gode and sunders auszüge aus dem Breuer et al. Paper und Everything you wanted to know about Continous Double-Auctions

2.4 Complex Networks

small-world power-law distribution generation algorithms dient hauptsächlich zur kategorisierung

TODO: In "State of the art" an overview of abstract networks and their properties is given. Also network-generating algorithms are presented and discussed. Because continuous double-auctions are the type of market which is used for matchings a short introduction is given on this topic too.

TODO: ziel hier eine theoretische übersicht über netzwerk-theorie zu geben wobei hauptaugenmerk auf die entwicklungen der letzten jahre (scale-free, small-world, ...)

Regular Graphs: [AlB99, vgl.] [New03, vgl.]

Random Graphs: but since then, most large scale networks with no apparent design principle were described as random graphs introduced by two Hungarian mathematicians Paul Erdos and Alfred Renyi [ER59, vgl.] [ER60, vgl.] Have small-world properties.

Small World Graphs or Average Path Length: Stanley Milgram [TM69] [Mil67] [Kle00]

Clustering Coefficient or Transitivity [WS98]

Degree Distribution [AlB02] Generally, it was believed that the degree distribution in most networks follows a Poisson distribution but in reality, real world networks have a highly skewed degree distribution following power-laws. Power-laws are expressions of the form y / x, where is a constant, x and y are the measures of interest [152].

Small World and Scale Free Network: A small world network as deined by Watts and Strogatz [WS98], is a network with high clustering coeffcient and small average path length. A scale free network as deined by Barabasi and

Albert [AlB02], is a network where the degree distribution follows a power law.

Complex Networks: are Small-World and/or Scale-Free [BW00] [ASBS00] [Kle02] http://www.cs.princeton.edu/ chazelle/courses/BIB/big-world.htm

introduce Metrics: - Average degree - average path-length - average clustering coefficient - network diameter - graph density

Mathematical stuff [New06] [ACL01] [EMB02] [GP04]

2.4.1 Network-Generating Algorithms

- fully connected - ascending connected - ascending connected with shortcuts - hubs - erdos-renyi - barbasi-albert - watts-strogatz

TODO: reference to appendix a for concrete pictures of topologies

Chapter 3

The Leverage Cycle

TODO: optimism factor which denotes the probability an agents assigns the event of up-state tomorrow

Definition des Modells Märkte, Marktmechanismen, clearing, utiliy funktionen,.... alles theoretisch, um des dann in implementierung praktisch zu zeigen Bestehende Resultate mit Bezug auf paper Fully-Connected: prozess und endverteilung, erreicht theoretisches Gleichgewicht approximativ

Chapter 4

Hypothesis

Eigentliche Fragestellung: Wie wichtig ist die Vollvernetzung? Allgemeine Netzwerkstrukturen untersuchen aber mit hauptaugenmerk auf Ascending-Connected d.h. reicht ascending-connected aus?

If there exists a path between each pair of agents in which each visited agent has a monotonous increasing optimism factor than the previously visited one then theoretical equilibrium will be reached.

Chapter 5

Implementation

For this thesis a software was written to be able to investigate the behaviour and results of the different types of networks and get visual and numerical results to be embedded in this written text. In this chapter the implementation-details of the software are discussed.

5.1 Requirements

The author of this thesis had access to the software of [BSV13] which was written in C++ therefore the question is why a new software had to be written and why not the original could be used. The reason for it was that the original software supported only a very narrow feature-set focusing only on the numerical results of a fully-connected network. Thus a complete redevelopment in Java was the option used. The following requirements were identified:

- Java based.
- Comprehensive GUI functionality.
- Emulate the functionality of [BSV13] and its results.
- Represent arbitrary networks in the simulation.
- Step-through forward and backwards in a simulation-run.
- Run replications of simulations.
- Store results of replications to be opened again for later usage.
- Command-line mode to run previously defined number of replications.

Java based The original software was written in C++ which offers very much power and highest speed if used correctly but comes with a very high responsibility regarding memory-management. Java offers a much more relaxed programming model regarding memory-management as it is garbage collected. This does not mean that the programmer can waste memory without giving thought to it but that not as many aufwand into it. debugging is easier. The most compelling argument for Java are the vast libraries which are included in the JDK which are missing in standard C++. As complete GUI-functionality for the whole software is a requirement Java is the way to go. Although multi-purpose libraries like Boost and GUI-frameworks like Qt are available for C++ too, it takes quite some time to set them up correctly for the target platform one develops for which implies that in C++ the development would always have been for just one platform whereas Java runs on every platform - if not platform-dependent stuff was used - without recompilation. As one will see later in the "Command-line mode"-Feature this is a major requirement to make this feature practical. Thus the reasons for using Java were:

- Platform-independence which applies to 3rd party libraries too.
- GUI-framework provided by JDK.
- Relaxed memory-management which emphasises fast iterations and easy debugging.
- Support for smooth XML-Serialization.
- 3rd party libraries for network-modelling and -visualization.

GUI functionality All functionality should be accessible through a GUI where some features e.g. "Inspection" are only possible to use through a GUI.

Emulate [BSV13] functionality Of course the whole software should be a super-set of the functionality of the one found in [BSV13] so this was the point to start from.

Arbitrary networks It should be possible to restrict trading between the agents to arbitrary networks. As network-modelling and -visualization library JUNG is used.

Step-through simulation The software should support to go through a simulation-run step-by-step and storing all steps of the simulation to jump back and forth between steps.

Real-time visualisation and information The original C++ software didn't provide real-time information about the current wealth-distribution and market-dynamics and provided the user just with the numerical results in the end through the means of a command-line output. For better understanding of dynamics of both wealth and markets a real-time visualisation of both are necessary together with extensive information on the current state like the offering-book, agent-information, network-activity, history of matches and the current equilibrium. Also the real-time visualisation is necessary to provide this written part of the thesis with diagrams of various results and processes.

Replications Because the whole trading-process includes randomness the results are subject to noise thus replications are an absolute must-have feature to be able to give reliable results. Each replications is independent from all others thus it is a candidate for parallel programming to speed up the already very time-consuming process of running replications. According to the number of CPU-cores the software should spawn threads up to the number of cores and run replications in parallel thus speeding up by a considerable amount of time.

Store results When running a bunch of replications for a given set-up the results of it should be automatically stored as XML to be accessible for later inspection thus conserving state and eliminating the necessity to re-run time-consuming simulation-runs with a high number of replications.

Command-line mode As already described replications are required to be implemented for parallel processing where up to the number of CPU-cores replications can run in parallel. When running a vast number of replications one does not need to GUI-functionality and most probably the machine is so occupied by the heavy work-load that a fließender usage of GUI would be not possible any-more. Thus replications should be runnable through a separate command-line mode of the thesis-software which reads information from a XML-File in which one can specify multiple simulation set-ups for which replications should be run. The command-line mode iterates through all configurations and runs the required replications and writes the result out for a later inspection. Obviously the more CPU-cores the faster a simulation-run with e.g. 50 replications finishes. For this reason most of the final replication-runs were done on a 40-core machine of the FH Vorarlberg which runs on Linux on which the thesis-software could be run easily because of Java's platform-independence.

5.2 Functionality

In this section the functionality of the thesis-software is explained to get an understanding of the implemented features.

5.2.1 Emulate [BSV13] functionality

TODO markets, bp, abm, equilibrium. not: abbruchbedingung is different, more strict

5.2.2 Inspection

TODO step-by-step and matching-history

5.2.3 Replications

TODO

5.2.4 Experiments

TODO results öffnen experimente per xml definieren über command-line laufen lassen

5.3 Architecture

In this section the basic architecture of the thesis-software is discussed. It is important to note that this thesis is a fat-client and has its major emphasis not on the software-development aspect but on the visual- and numerical results where the accompanying software is just a tool for the means to calculate theses required results. Thus the architecture is guided by a simple division of layers into front-end, controller and back-end. The front-end is responsible for input and output of the user through GUI or command-line. The controller-layer provides encapsulated chunks of functionality of the back-end to the front-end. It is necessary to abstract, encapsulate and combine the stateful nature of communication with the back-end into a separate layer instead of polluting the front-end with it and creating unnecessary dependencies. The back-end layer provides the functionality where the real work is done e.g. simulation is executed. Theoretically the dependencies are top-down where the front-end includes only controller-functionality, the controller includes only back-end functionality and the back-end has no dependencies to the preceding layers. In this thesis-software a more pragmatic approach was chosen so this dogma was not followed where over-engineering and over-complication would have resulted when sticking strictly to the separation of dependencies. Thus in very few cases the controller- and back-end layer include front-end layer functionality to create different types of network topologies in a more convenient way. Also the front-end accesses instances of pure back-end classes for graphical visualisation and information purposes. Despite the seemingly flawed architecture the development-process has proven to be very smooth and expansions and refactorings went quite smoothly and always resulted in a better and clearer structure with reduced code-smell which is always a sign for a good architecture.

TODO: frontend-controller-backend package diagram

5.3.1 Frontend

TODO GUI functionality and visualisation emphasis on massive re-use of gui-components thus massive use of sub-classing and pulling into separate class

5.3.2 Controller

TODO steering of the backend through the frontend: holds state and knows what to do next. inspection-controler, replications-controler and experimenter-controler, network-exporter

5.3.3 Backend

TODO where the real simulation happens: auction, markets, network, matching, agents, offerings, transactions

5.4 Agents

Although agents are used in this software it is not an agent-based simulation in the classical way as these agents are zero-intelligence ones and have no states of behaviour. That is each agent makes bid- and ask-offers on all markets if the constraints allow it where the prices are selected from random ranges which improve the utility of the agent - that means it makes always offers which would result in a profit.

Each agent is characterized by its state where the main variables are:

- Id - the unique id of the agent in the range of the natural numbers in the range of [1..number of agents].
- Optimism-factor h - defines how optimistic the agent is in the range of [0..1] where 0 is most pessimistic and 1 is most optimistic. The distribution of the optimism-factor among the agents is linearly ascending with the id and is defined through the following equation: $\textit{optimism-factor} = \frac{id}{\textit{number of agents}+1}$
- Cash holdings - the current cash holdings of the agent.
- Assets holdings - the current asset holdings of the agent not including the assets granted as securities for giving loans.
- Loan given - the amount of loans bought from / granted to other agents. For a given amount of loans the equal amount of assets are granted as security to the buyer of the loan. Thus this variable increases the amount of assets the agent can trade with.
- Loan taken - the amount of loans sold to other agents. For a given amount of loans the equal amount of assets the equal amount of assets are granted as security to the buyer of the loan. Thus this variable decreases the amount of assets the agent can trade with as this amount of loans needs to be kept as securities.

There are three important derived variables which are calculated from the previous ones:

- Collateralized assets - is the amount of assets which are bound through collateral obligations because of taken bonds. $\textit{collateralized assets} = max(0, \textit{loans taken} - \textit{loans given})$
- Free assets - is the amount of assets which are unbound and act not as security and are owned completely by the agent. $\textit{free assets} = \textit{Assets holding} - \textit{Collateralized assets}$

- Loans - is the net number of loans and calculated through $loans = loans\ given - loans\ taken$. Thus this value is positive if the agent has granted more loans to other agents than received and it is negative if the agent has received more loans from other agents than granted.

Note that all variables cannot go negative except loans.

5.5 Markets

In this section each market and its implementation is described.

5.5.1 Asset/Cash

Products

Free assets are traded against cash. The buyer gets a specific amount of free assets for a given amount of cash where the seller gives away the specific amount of free assets and gets the given amount of cash.

Price-Range

For an agent to be able to place profit-making offers on the market the according price-ranges need to be defined. Note that all prices must be obviously in the unit of cash according to the previously defined products.

minimum The minimum value of one asset is the down-value pD tomorrow as defined in chapter 3 "The Leverage Cycle". This value is obviously a constant for all agents.

$$min\ asset\text{-}price = pD$$

maximum The maximum value of one asset is the up-value pU tomorrow as defined in chapter 3 "The Leverage Cycle". This value is obviously a constant for all agents.

$$max\ asset\text{-}price = pU$$

limit The limit-price of an asset depends on the optimism-factor h of the agent where the most optimistic agent assigns pU and the most pessimistic pD as the price. Thus applying linear interpolation one receives the following equation.

$$limit\text{-}price\ asset = h * pU + (1.0 - h) * pD$$

Bid-Offering

The price is drawn randomly between the minimum price and the limit-price because when buying one wants to buy below the expected value to make a profit. As amount one TRADING-UNIT of assets is selected - in the thesis-implementation 0.1 - but if there is not enough cash left to buy one TRADING-UNIT of assets then no bid-offer is made.

Table 1: Bid-Offering parameters

Pre-Condition	*cash holdings* > *price of TRADING-UNIT assets*
Asset-Price	random(*min asset-price*, *limit-price asset*)
Asset-Amount	*TRADING-UNIT*

Ask-Offering

Ask offers are generated only when the agent has at least one TRADING-UNIT of free assets left. The price is drawn randomly between the limit-price and maximum price because when selling one wants to sell above the expected value to make a profit. As amount one TRADING-UNIT of assets is selected - in the thesis-implementation 0.1 - but if there are fewer free assets left then no offer is made.

Table 2: Ask-Offering parameters

Pre-Condition	*free assets* > *TRADING-UNIT*
Asset-Price	random(*limit-price of asset*, *max asset-price*)
Asset-Amount	*TRADING-UNIT*

Match

Below the wealth-exchange table is given in case of a match between two agents on the Asset/Cash market. Note that the wealth is increased/decreased as given by the +/- signs.

Table 3: Wealth-Exchange during a match on Asset/Cash market

	Seller	Buyer
Assets holdings	- matching-amount	+ matching-amount
Cash holdings	+ matching-price	- matching-price

5.5.2 Bond/Cash

Products

Bonds are traded against cash. The buyer grants the seller a loan in buying a bond from the seller thus the buyer gets a given bond-amount from the seller and gives a given cash-amount to the seller. For a given amount of sold bonds the equal amount of assets need to be hold as securities.

Price-Range

For an agent to be able to place profit-making offers on the market the according price-ranges need to be defined. Note that all prices must be obviously in the unit of cash according to the previously defined products.

minimum The minimum value of one bond is the down-value pD tomorrow as defined in chapter 3 "The Leverage Cycle". This value is obviously a constant for all agents.

$$\textit{min bond-price} = pD$$

maximum The maximum value of one bond is the face-value V tomorrow as defined in chapter 3 "The Leverage Cycle". This value is obviously a constant for all agents.

$$\textit{max bond-price} = \textit{facevalue}\ V$$

limit The limit-price of a bond depends on the optimism-factor h of the agent where the most optimistic agent assigns *facevalue* V and the most pessimistic pD as the price. Thus applying linear interpolation one receives the following equation.

$$\textit{limit-price bond} = h * V + (1.0 - h) * pD$$

Bid-Offering

Bid offers are generated only when the agent has any cash holdings. The price is drawn randomly between the minimum price and the limit-price because when buying one wants to buy below the expected value to make a profit. As amount one TRADING-UNIT of bonds is selected - in the thesis-implementation 0.2 - but if there is not enough cash left to buy one TRADING-UNIT of bonds then the amount of bonds is selected which can be bought with the remaining cash holdings.

Table 4: Bid-Offering parameters

Pre-Condition	$\textit{cash holdings} > 0$
Bond-Price	random(*min bond-price*, *limit-price of bonds*)
Bond-Amount	$\min(\frac{\textit{cash holdings}}{\textit{Bond-Price}}, \textit{TRADING-UNIT})$

Ask-Offering

Ask offers are generated only when the agent has any free assets because when selling the agent needs to reserve as many free assets as security as bonds sold. The price is drawn randomly between the limit-price and maximum price because when selling one wants to sell above the expected value to make a profit. As amount one TRADING-UNIT of bonds is selected - in the thesis-implementation 0.2 - but if there are fewer free assets left then the remaining amount of free assets is selected.

Table 5: Ask-Offering parameters

Pre-Condition	$\textit{free assets} > 0$
Bond-Price	random(*limit-price of bonds*, *max bond-price*)
Bond-Amount	min(*free assets*, *TRADING-UNIT*)

Match

Below the wealth-exchange table is given in case of a match between two agents on the Bond/Cash market. Note that the wealth is increased/decreased as given by the +/- signs.

Table 6: Wealth-Exchange during a match on Bond/Cash market

	Seller	Buyer
Loan Given	N/A	+ matching-amount
Loans Taken	+ matching-amount	N/A
Cash holdings	+ matching-price	- matching-price

5.5.3 Asset/Bond

Products

Assets are traded against bonds. The buyer gets a specific amount of free assets for a given amount of bonds where the seller gives away the specific amount of free assets and gets the given amount of bonds.

Price-Range

For an agent to be able to place profit-making offers on the market the according price-ranges need to be defined. Note that all prices must be obviously in the unit of bonds according to the previously defined products.

minimum The minimum value of one asset in bonds is defined as the ratio of the minimum asset-price in cash to the maximum bond-price in cash. This value is obviously a constant for all agents.

$$\textit{min asset-price in bonds} = \frac{\textit{minimum asset-price}}{\textit{maximum bond-price}} = \frac{pD}{V}$$

maximum The maximum value of one asset in bonds is defined in the ratio of the maximum asset-price in cash to the minimum bond-price in cash.

$$\textit{max asset-price in bonds} = \frac{\textit{maximum asset-price}}{\textit{minimum bond-price}} = \frac{pU}{pD}$$

limit The limit-price in bonds of an asset depends on the optimism-factor h of the agent where the most optimistic agent assigns the previously defined maximum and the most pessimistic the previously defined minimum as the price.

$$\textit{limit-price asset/bond} = \frac{limit-priceasset}{limit-priceloans}$$

Bid-Offering

Bid offers are generated only when the agent is not negative on free assets after the trade. The price is drawn randomly between the minimum price and the limit-price because when buying one wants to buy below the expected value to make a profit. As amount one TRADING-UNIT of a assets is selected - in the thesis-implementation 0.1. Thus to calculate the free assets after the trade one calculates

1. Start with current free assets holdings.
2. Subtract current collateral obligations.
3. Subtract taken loans after trade.
4. Add assets bought through trade.

Table 7: Bid-Offering parameters

Pre-Condition	*free assets* $>= 0$*after trade*
Asset/Bond-Price	random(*min asset/bond price*, *limit-price of asset/bond*)
Asset/Bond-Amount	*TRADING-UNIT*

Ask-Offering

The price is drawn randomly between the limit-price and maximum price because when selling one wants to sell above the expected value to make a profit. As amount one TRADING-UNIT of bonds is selected - in the thesis-implementation 0.2 - but if there are fewer free assets left then the remaining amount of free assets is selected.

1. Start with current free assets holdings.
2. Subtract current collateral obligations.
3. Add given loans after trade.
4. Subtract assets sold through trade.

Table 8: Bid-Offering parameters

Pre-Condition	*free assets* $>= 0$*after trade*
Asset/Bond-Price	random(*limit-price of asset/bond*, *max asset/bond price*)
Asset/Bond-Amount	*TRADING-UNIT*

Match

Below the wealth-exchange table is given in case of a match between two agents on the Asset/Loan market. Note that the wealth is increased/decreased as given by the +/- signs.

Table 9: Wealth-Exchange during a match on Asset/Loan market

	Seller	Buyer
Loan Given	+ matching-amount	N/A
Loans Taken	N/A	+ matching-amount
Asset holdings	- matching-price	+ matching-price

5.6 Simulation

TODO: discuss the fact that not 1.0 units but small subchunks are traded. always talking about 1.0 units of assets/loans/cash but actually its a continuous double-auction. TODO: conserving the wealth TODO: explain why with 0.1 assets are traded down to 0.0

5.6.1 Sweeping and Matching

TODO: matching-amount

5.7 Performance improvement

Matching Wahrscheinlichkeiten Importance Sampling Lokales vs Globales Offerbook

5.8 Calculating theoretical Equilibrium

TODO analyse matlab-script of martin jandacka

Chapter 6

Results

In this chapter the results of the experiments are given. Each topology-type introduced in appendix A "Topologies" was simulated where in this chapter only Fully-Connected and Ascending-Connected topologies are handled as the Ascending-Connected topology - both with and without importance sampling - is the most minimal network which satisfies the requirements for the hypothesis. The results for the other topologies can be found in appendix B "Results for Hub-Based, Scale-Free and Small-World Topologies".

Note: The numbers in tables resemble always a median-value with the standard-deviation given in parentheses.

6.1 Validating simulation results

As a point-of-reference and as an experimental proof for the correctness of the implementation of the thesis-software the results of a validation against both the theoretical equilibrium and the equilibrium found in [BSV13] are given. Because equilibrium differs across the number of agents and the type of bond traded to be comparable the same amount of agents and the same bond-type has to be used in the experiments. Thus 1000 agents and a bond with face-value of 0.5 were chosen because [BSV13] report their equilibria for this number of agents and bonds with face-value between 0.1 to 0.5.

6.1.1 References

Table 10: Theoretical equilibrium for 1,000 agents and 0.5 bond

Asset-Price p	0.715
Bond-Price q	0.374
Marginal agent i1	0.583
Marginal agent i2	0.802

Table 11: Equilibrium in [BSV13] for 1,000 agents and 0.5 bond

Asset-Price p	0.716
Bond-Price q	0.375
Marginal agent i1	0.583
Marginal agent i2	0.801
Pessimist Wealth	1.716
Medianist Wealth	4.578
Optimist Wealth	5.032

6.1.2 Thesis results

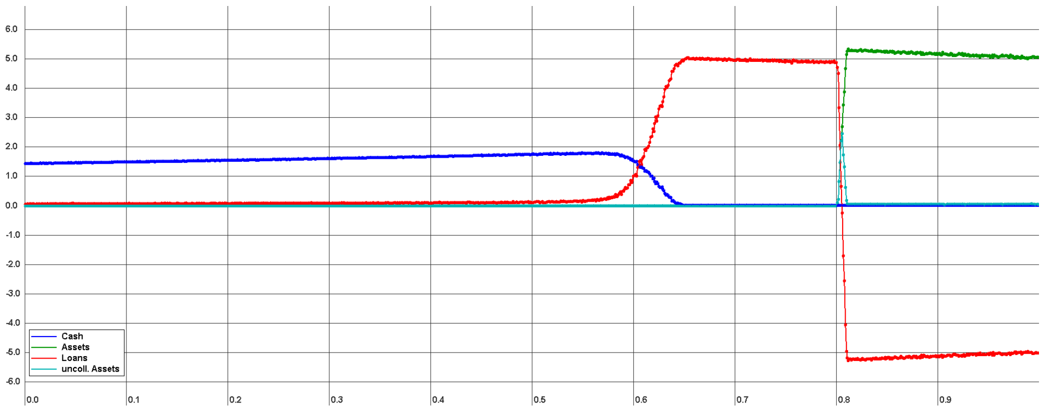

Figure 1: Wealth-Distribution of thesis-implementation of Fully-Connected topology for 1,000 agents and 0.5 bond

Table 12: Equilibrium of thesis-implementation for 1,000 agents and 0.5 bond

Asset-Price p	0.700 (0.005)
Bond-Price q	0.389 (0.002)
Marginal agent i1	0.616 (0.004)
Marginal agent i2	0.805 (0.001)
Pessimist Wealth	1.582 (0.01)
Medianist Wealth	4.578 (0.031)
Optimist Wealth	5.105 (0.025)

Table 13: Difference of Fully-Connected topology price-equilibrium as given in table 12 to theoretical equilibrium as given in table 10

	Result	Reference	difference to Reference
Asset-Price p	0.700	0.715	-2.1%
Bond-Price q	0.389	0.374	+4.0%
Marginal agent i1	0.616	0.583	+5.6%
Marginal agent i2	0.805	0.802	+0.4%

Table 14: Difference of Fully-Connected topology wealth-equilibrium as given in table 12 to wealth-equilibrium as given in [BSV13] from table 11

	Result	Reference	difference to Reference
Pessimist Wealth	1.582	1.716	-7.8%
Medianist Wealth	4.578	4.578	0.0%
Optimist Wealth	5.105	5.032	+1.5%

Although marginal agent i1 and bond-price q are quite different than from theoretical equilibrium and the pessimists wealth is 7.8% less than given in [BSV13] these results are nonetheless accepted as reaching the equilibrium. The differences emerge from the reasons that the thesis-simulation runs were terminated earlier than in [BSV13] which results in the i1 and i2 edges to be not as sharp as reported in [BSV13]. It would be necessary to run the simulation an order of magnitude longer as the matching probabilities are reduced rapidly when only direct neighbours are able to trade any more within a network of 1000 agents. See section 5.7 "Performance improvement" for details on matching-probabilities.

6.1.3 Performance and termination measurements

As noted in section 5.6.1 "Sweeping and Matching" a matching-round performs up to 500 offering-rounds where during one round all agents make an offer to find a match. If a match occurs during one offering-round the current matching-round is terminated and marked as successful. If no match occurs during all 500 offering-rounds the current matching-round is terminated too but marked as failed. Thus the following terminology is defined:

Successful matching-round a match occurred within maximal 500 offering-rounds where in each offering-round all agents make an offer.

Failed matching-round no match occurred within 500 offering-rounds where in each offering-round all agents make an offer.

Termination criteria after 1,000 successive failed matching-rounds it is expected that no more trading will occur thus the simulation is terminated.

Table 15: Performance of thesis-implementation with 1000 agents and 0.5 bond

Successful matching-rounds	19,300.04 (101.68)
Failed matching-rounds	10,306.78 (2914.11)
Total matching-rounds	29,606.82 (2938.82)
Ratio successful/total	0.65
Ratio failed/total	0.35

6.2 Experiments configuration

In the following experiments 100 agents were used, all markets (Asset/Cash, Bond/Cash, Asset/Bond) were enabled, a bond with face-value of 0.5 was selected and in each experiment 50 replications were run. A replication was terminated after 1000 failed matching-rounds in a row. Note that if trading is not possible any more before this criteria is met the simulation is terminated and thus it is possible that it halts earlier as can be seen for the Ascending-Connected Importance Sampling topology.

[BSV13] showed that equilibrium can be reached already with 30 agents so this was the minimum number of agents to start with but for a smoother visual result 100 were chosen. Also one simulation-run takes not very much time with 100 as compared to the 1,000 agents thus it is a very good match between visual accurateness and processing-power requirements.

The 0.5 bond was selected because it is a risky one which is important as with risk-less loans which have a face-value less than or equal 0.2 the results are indifferent and not unique and won't show the characteristic distribution of equilibrium.

As already described in section 5.6.1 "Sweeping and Matching" the whole simulation-process is a random-process with an equilibrium different for each topology as the fixed-point solution thus one needs replications to reduce noise. The number of 50 replications was chosen because it is a good match between processing-power requirements and overall reduction of noise. Thus increasing the number e.g. to 100 or 200 would not result in much better results - both visual and numerical - but would need much longer to run. All facts can already be seen and derived when using 50 replications thus for all figures 50 replications were used unless stated otherwise e.g. a single run.

Table 16: Configuration for all experiments

Agent-Count	100
Bond-Type	0.5
Replication-Count	50
Matching-Round	max. 500 offering-rounds
Terminate after	1,000 failed successive matching-rounds

Table 17: Theoretical Equilibrium for 100 agents and 0.5 bond

Asset-Price p	0.717
Bond-Price q	0.375
Marginal agent i1	0.584
Marginal agent i2	0.802

6.3 Fully-Connected

This topology serves as the major point-of-reference for the other experiments as it reaches the theoretical equilibrium for 1,000 agents as demonstrated and explained.

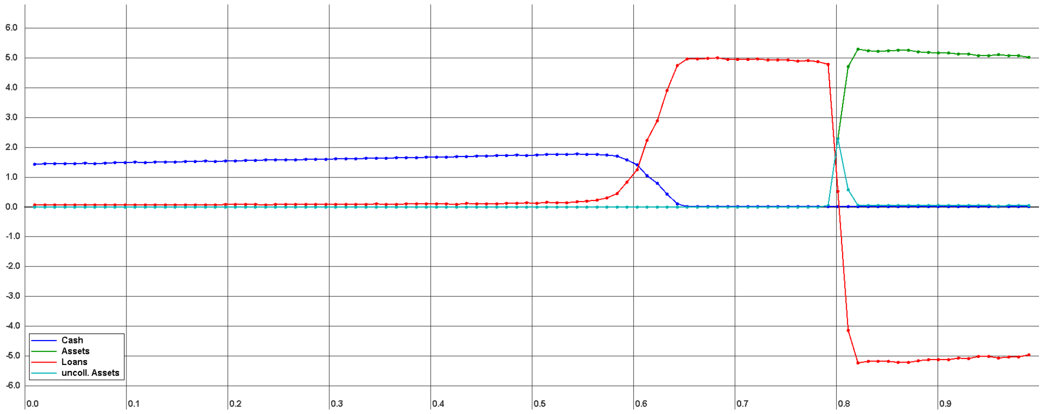

Figure 2: Wealth-distribution of Fully-Connected topology

Table 18: Equilibrium of Fully-Connected topology

Asset-Price p	0.689 (0.01)
Bond-Price q	0.384 (0.004)
Marginal agent i1	0.603 (0.007)
Marginal agent i2	0.803 (0.003)
Pessimist Wealth	1.597 (0.015)
Medianist Wealth	4.565 (0.113)
Optimist Wealth	5.021 (0.064)

Table 19: Performance of Fully-Connected topology

Successful matching-rounds	1916.14 (31.42)
Failed matching-rounds	4448.66 (1668.93)
Total matching-rounds	6364.8 (1679.21)
Ratio successful/total	0.3
Ratio failed/total	0.7

Table 20: Difference to theoretical equilibrium

	Result	Reference	difference to Reference
Asset-Price p	0.689	0.717	-3.9%
Bond-Price q	0.384	0.375	+2.4%
Marginal agent i1	0.603	0.584	+3.2%
Marginal agent i2	0.803	0.802	+0.1%

6.4 Ascending-Connected topology

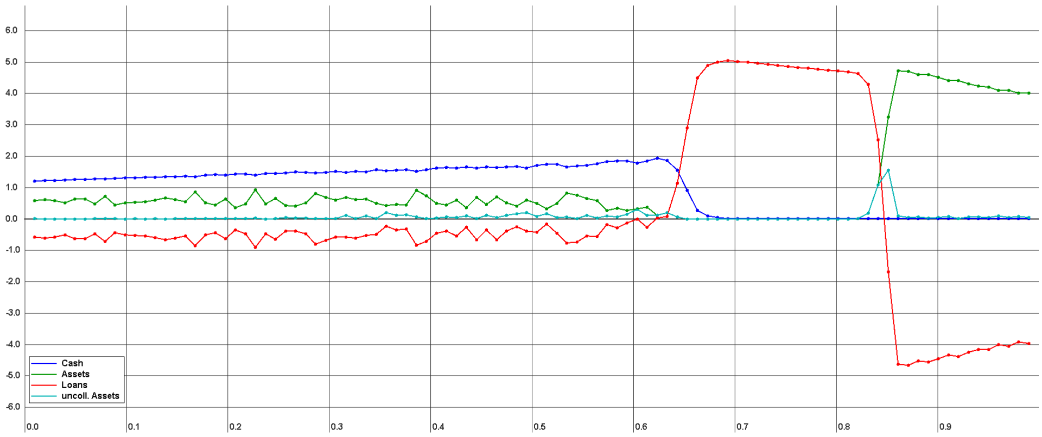

Figure 3: Wealth-distribution of Ascending-Connected topology

Table 21: Equilibrium of Ascending-Connected topology

Asset-Price p	0.711 (0.016)
Bond-Price q	0.391 (0.005)
Marginal agent i1	0.646 (0.012)
Marginal agent i2	0.850 (0.008)
Pessimist Wealth	1.166 (0.072)
Medianist Wealth	1.869 (0.243)
Optimist Wealth	4.307 (0.07)

Table 22: Performance of Ascending-Connected topology

Successful matching-rounds	36,940.96 (1948.69)
Failed matching-rounds	1176.08 (98.01)
Total matching-rounds	38,117.04 (1934.06)
Ratio successful/total	0.97
Ratio failed/total	0.03

Table 23: Difference to theoretical equilibrium

	Result	Reference	difference to Reference
Asset-Price p	0.711	0.717	-0.8%
Bond-Price q	0.391	0.375	+4.2%
Marginal agent i1	0.646	0.584	+10.6%
Marginal agent i2	0.850	0.802	+6.0%

Table 24: Difference to Fully-Connected topology equilibrium

	Result	Reference	difference to Reference
Asset-Price p	0.711 (0.016)	0.689 (0.01)	+3.2% (+60%)
Bond-Price q	0.391 (0.005)	0.384 (0.004)	+1.8% (+25%)
Marginal agent i1	0.646 (0.012)	0.603 (0.007)	+6.9% (+71%)
Marginal agent i2	0.850 (0.008)	0.803 (0.003)	+6.0% (+166%)
Pessimist Wealth	1.166 (0.072)	1.597 (0.015)	-27.0% (+380%)
Medianist Wealth	1.869 (0.243)	4.565 (0.113)	-59% (+115%)
Optimist Wealth	4.307 (0.070)	5.021 (0.064)	-14.2% (+9.3%)

6.4.1 Ascending-Connected Importance Sampling

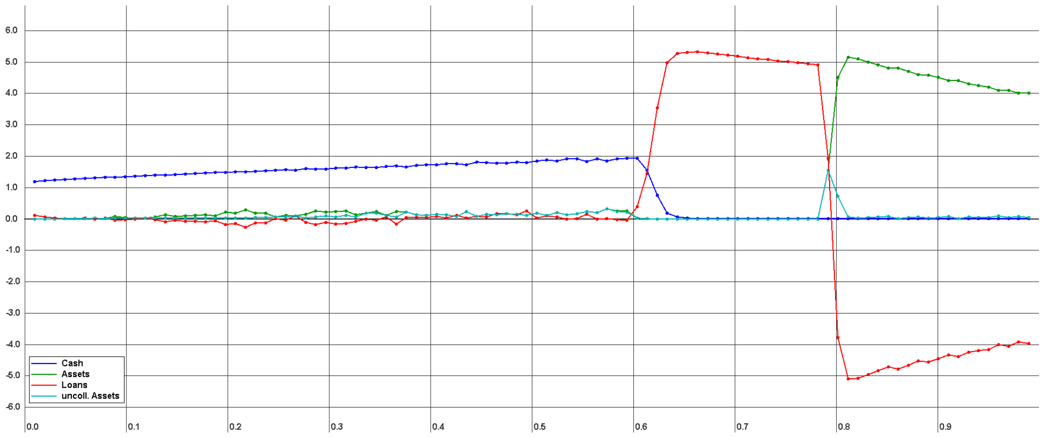

Figure 4: Wealth-distribution of Ascending-Connected Importance Sampling topology

Table 25: Equilibrium of Ascending-Connected Importance Sampling topology

Asset-Price p	0.691 (0.009)
Bond-Price q	0.383 (0.004)
Marginal agent i1	0.614 (0.009)
Marginal agent i2	0.799 (0.006)
Pessimist Wealth	1.497 (0.072)
Medianist Wealth	3.934 (0.505)
Optimist Wealth	4.519 (0.051)

Table 26: Performance of Ascending-Connected Importance Sampling topology

Successful matching-rounds	49,881.6 (1733.33)
Failed matching-rounds	1.0 (0.00)
Total matching-rounds	49,882.6 (1733.33)
Ratio successful/total	0.9999
Ratio failed/total	0.0001

Note that in this case the matching-probabilities are such that upon the first failed matching-round the equilibrium is reached as no agent can trade with each other any more which results in just on single failed matching-round.

Table 27: Difference to theoretical equilibrium

	Result	Reference	difference to Reference
Asset-Price p	0.691	0.717	-3.6%
Bond-Price q	0.383	0.375	+2.1%
Marginal agent i1	0.614	0.584	5.1%
Marginal agent i2	0.799	0.802	-0.4%

Table 28: Difference to Fully-Connected topology equilibrium

	Result	Reference	difference to Reference
Asset-Price p	0.691 (0.009)	0.689 (0.01)	+0.3% (-10%)
Bond-Price q	0.383 (0.004)	0.384 (0.004)	-0.3% (0.0%)
Marginal agent i1	0.614 (0.009)	0.603 (0.007)	+1.8% (28.6%)
Marginal agent i2	0.799 (0.006)	0.803 (0.003)	-0.5% (+100%)
Pessimist Wealth	1.497 (0.072)	1.597 (0.015)	-6.2% (+380%)
Medianist Wealth	3.934 (0.505)	4.565 (0.113)	-13.8% (+346%)
Optimist Wealth	4.519 (0.051)	5.021 (0.064)	-10% (-20.3%)

Chapter 7

Interpretation

In this chapter the interpretation of the results of chapter 6 ”Results” are given and discussed where the central question is whether the Ascending-Connected topology satisfies the hypothesis or not. Thus only this topology is handled - both with and without importance sampling - because it is the most minimal network which satisfies the requirements for the hypothesis. The interpretations for the results of Hub-, Scale-Free and Small-World Topologies are handled in appendix B ”Results for Hub-Based, Scale-Free and Small-World” but only to a minimal extent as they turn out to fall far from satisfying the hypothesis and the equilibrium because almost all of them do not meet the requirements but show interesting behaviour.

7.1 Validating the Hypothesis

When comparing the results of Ascending-Connected topology with and without importance sampling from Chapter 6 ”Results” of figure 4 and 3 with the results of the Fully-Connected topology of figure 2 it becomes immediately clear that the equilibrium is different from the one of the Fully-Connected network and thus theoretical equilibrium is not reached in the case of Ascending-Connected topology neither with or without importance sampling. Although the visual results come quite close to the Fully-Connected one - there is a clear distinction between pessimists, medianists and optimists and the wealth-distribution looks about the same as in fully-connected - there remain serious artefacts in the range of the pessimists. Thus the hypothesis is proven wrong by experiment.

7.2 Analysing artefacts

Obviously the artefacts in the range of the pessimists indicate a miss-allocation of wealth, which are in fact collateralized assets. Pessimists, as noted in Chapter 3 "The Leverage Cycle", are maximally short on assets and bonds and hold only cash, thus it is clearly a miss-allocation. As will be shown it comes from the fact that the pessimists want to sell but no neighbour is able to buy any more - a scenario which is not possible in Fully-Connected topology and is thus unique to Ascending-Connected networks with and without importance sampling.

7.2.1 Dynamics of a single run

To better understand how such artefacts arise one needs to investigate the dynamics of a single run of the Ascending-Connected topology. The tools used are both the market-activity and wealth-distribution diagrams where the former one shows during which points in time - which are the successful matching-rounds - of the simulation each market is active. Being active means a successful match on a given market which implies that in a successful matching-round only one market can be active as only one match on a specific market happens during a successful matching round. Because of this a moving window of size 100 is used to create a moving-average filter over all active markets where the result is normalized and all market-activity sums to 1.0 at each point in time of the diagram. This allows for a very good visual analysis of distinct trading-stages because noise is reduced but the overall trend of a market can be still seen clearly.

3 trading-stages can be identified in the market-activity diagram of Ascending-Connected topology.

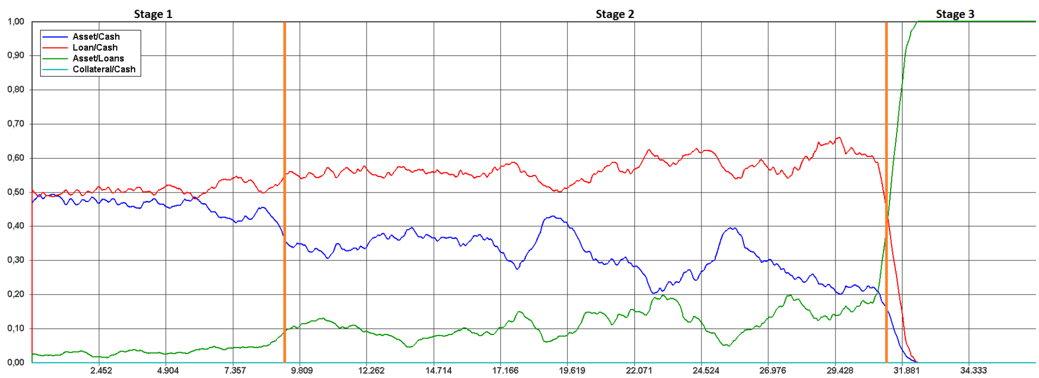

Figure 5: Market-activity stages of Ascending-Connected topology

Stage 1 The allocations are very chaotic overall but pessimists can be identified already as they sell their free assets against cash thus holding primarily cash but lots of collateralized assets are in the pessimists-range as well. Real distinction of optimists is not yet visible and medianists are far from showing up.

The Asset/Cash and Loan/Cash markets are very dominant in this stage as the pessimists try to get cash for their free assets where the Asset/Loan market is hardly active but contributes enough to create the miss-allocation of the collateralized assets in the pessimists-range already.

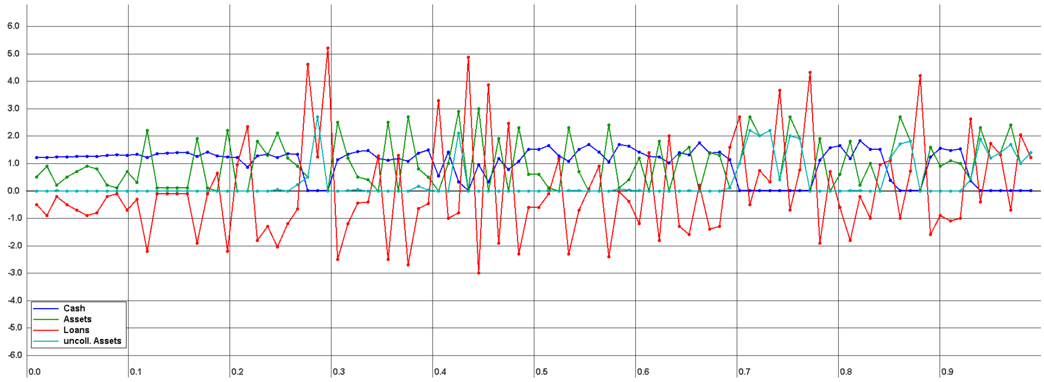

Figure 6: Wealth-Distribution of Ascending-Connected topology during Stage 1

Stage 2 The pessimists which hold collateralized assets try to trade them up to the optimists which looks like waves when visualisizing it in the thesis-software. The optimists are now about to emerge as most of them are maximally short on cash and hold either free or collateralized assets. The medianists are still not visible yet.

The Asset/Cash market seems to go down in the long term while the Loan/Cash and Asset/Loan markets seems to increase. This is because fewer and fewer assets can be traded against cash because the optimists are already very low on cash thus the Asset/Loan market is naturally increasing as they can trade only on this market any more.

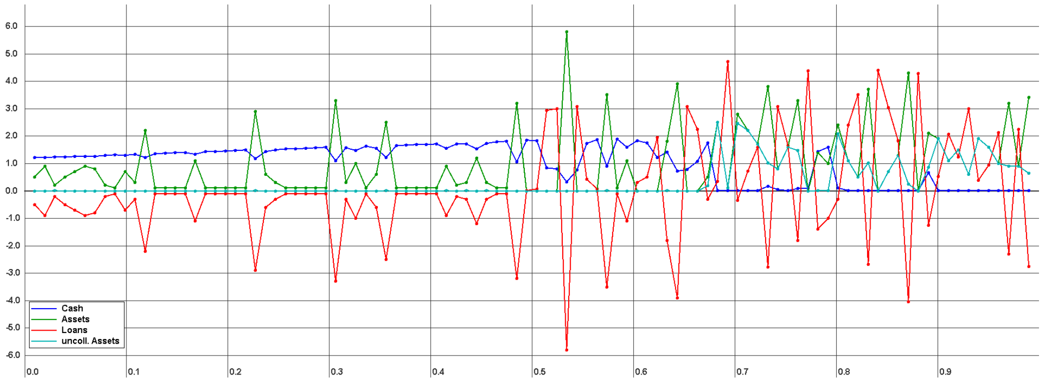

Figure 7: Wealth-Distribution of Ascending-Connected topology during Stage 2

Stage 3 The pessimists lie dormant and are completely inactive. The medianists begin to show up holding only bonds and the real optimists begin to crystallize holding only collateralized assets. These two frontiers move towards each other as only collateralized assets can be traded any more as both medianists and optimists hold only collateralized assets and bonds.

The Asset/Cash market lies dormant because the pessimists are no more able to trade and the optimists are maximally low on cash. The Loan/-Cash market is inactive too whereas the Asset/Loan market takes over and dominates 100% as only collateralized assets are traded any more as stated above.

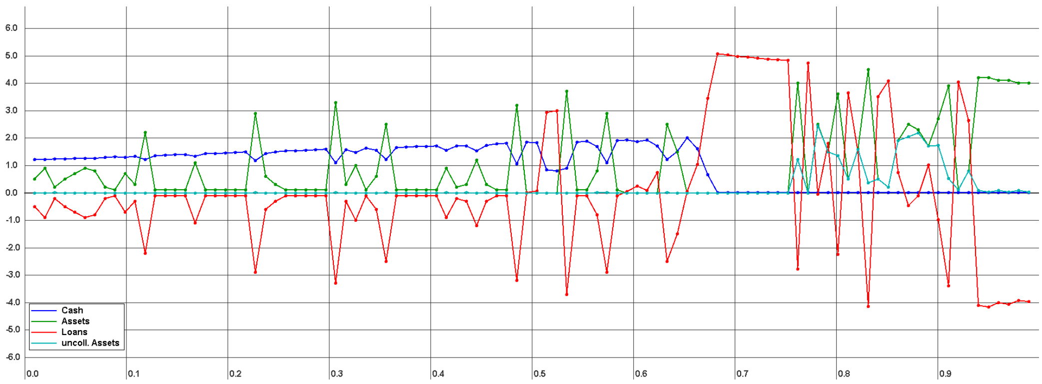

Figure 8: Wealth-Distribution of Ascending-Connected topology during Stage 3

Deriving the emerging of the artefacts The wealth stabilizes from both the left and the right end of the optimism-scale towards the i2-point where medianists become optimists - around this point the last trades will happen.

Pessimists try to sell all their assets against cash to the neighbour with higher optimism-factor.

Optimists try to buy as much assets as they can get from the neighbour with lower optimism-factor. In the beginning they use cash and after they've run out of cash they buy assets against bonds.

The medianists serve as connection between the pessimists and optimists transferring the assets to the optimists by buying from agents with lower optimism and selling to higher ones either through asset against cash or asset against bond.

Thus the assets move from the pessimists through the ascending chain of optimism to the optimists as no direct connection between these two groups exists with the medianists in between. Thus waves of uncollateralized assets can be seen moving from pessimists to optimists.

It is important to understand that all agents despite their optimism factor make offers on all markets if they are able to and their cash, collateral or bond constraints are satisfied. This implies that pessimists trade bonds as well as assets against bonds although they turn out to be pessimists. Note that the agents are not defined exogenous as pessimists/medianist/optimists but this property emerges during the simulation.

Thus pessimists gain wealth in collateralized assets too which can be seen by the green spikes with the same amount of negative bonds as those assets are bought against bond. Of course they try to sell it to neighbours with higher optimism factor but this is only possible if these neighbours are able to buy which they can only if they hold enough positive bonds to buy the offered asset for the offered amount of bonds.

Whether an agent has enough wealth to buy from a seller is more or less random and depends on its trading history. Matching happens randomly and thus it is possible that the neighbourhood of a seller "dries up" as the potential buyers sold all their goods to the next agent with higher optimism factor and become thus unable to buy from the potential seller because they have no more positive bonds to buy assets against bonds. In such a case a potential pessimist seller of collateralized assets is then cut from its environment and becomes unable to trade any more resulting in a miss-allocation in collateralized assets.

It is also possible for a group of agents to get cut from its environment through this random trading-process. In this case the agents within this "island" still trade between each other resulting in the uncollateralizing of assets which immediately are traded towards optimists but as soon as a point is reached where no buyer is available with enough positive bonds to buy collateralized assets this island is also incapable of trading any more resulting in an island of miss-allocated wealth.

An important fact to notice is that the artefacts must not necessarily show up. It is possible for a single run to finish without these artefacts showing up. This is due to the random-process of sweeping and matching and thus the artefacts are subject to this random process too as noted in section 5.7 "Sweeping and Matching". Importance sampling elevates this problem a bit as it allows for more trades as the matching probabilities are very much increased but fails in the end for the same reason as the simulation without it - the artefacts are just "smaller" but show up almost always.

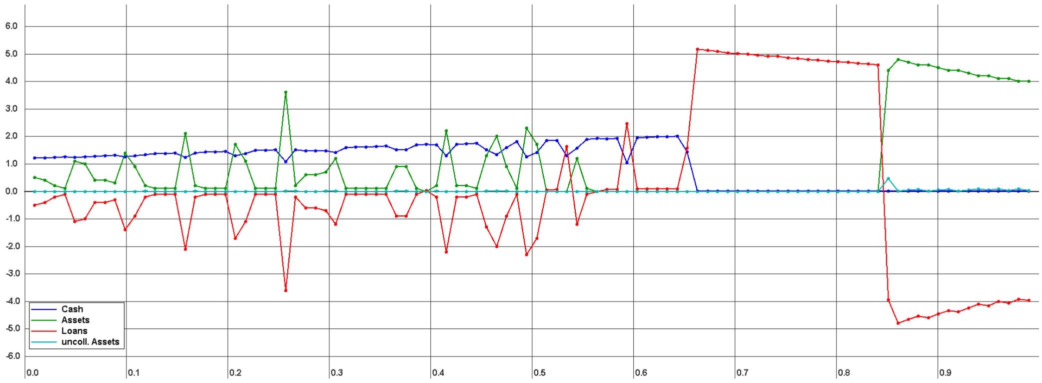

Figure 9: Final wealth-distribution of Ascending-Connected topology after a single run

7.3 Extending the Hypothesis

After it has become clear that the hypothesis is wrong the question arises what needs to be done to correct it. It is clear that a mechanism needs to be found which prevents or resolves the arising of the artefacts within the pessimist wealth-range. Obviously two solutions are available.

7.3.1 Approaching fully connectedness

Increasing the connectedness of the topology increases the probability of global-optimal trades and allows more agents to trade between each other and thus the probability of resolving islands or artefacts of wealth miss-allocation

is increased with the density of connectedness. The experiments of ascending-connected topology with short-cuts were desigend to develop an understanding how the simulation behaves with increasing connectedness and also how the two types of fully- and regular-connectedness influence the results. It seems that full short-cuts seem to help dramatically in reducing the miss-allocations where the number of full short-cuts seems to be dependent on the number of agents which this thesis leaves for further research. See section B.2 "Ascending-Connected with short-cuts" in appendix B for a short overview of the results and interpretation of short-cut based Ascending-Connected topologies.
Of course in real environments approaching fully connectedness is not always possible and thus only the other mechanism is left as an option to resolve the artefacts.

7.3.2 Re-Enabling trading

Another way to look at the arising of the artefacts is in identifying them as suboptimal trades. [BSV13] were confronted with this circumstance when they introduced the Asset/Bond Market where they found that the equilibrium was fundamentally different from the theoretical one because agents were trapped in suboptimal trades and couldn't reverse their decisions made earlier. The trades where suboptimal because each agent assigns depending on its optimism factor a different bond-value to each asset. As a solution they introduced the "Bonds-Pledgeability" (BP) mechanism which allows to trade bonds in both ways instead of only gathering them and not being able to sell them - see chapter 3 "The Leverage Cycle" for a more in-depth discussion of the BP-Mechanism.

Thus if those artifacts are treated as suboptimal trades one needs to introduce a mechanism similar to BP to allow the reversibility of suboptimal trades in the context of collateralized assets. The only possibility without altering the network-topology is to re-enable the pessimists to trade their collateralized assets against cash as all pessimists hold cash and are thus able to buy and sell collateralized assets against cash. This new mechanism is expected to repair the miss-allocated wealth and to restore the validity of the previously disproved hypothesis.

See Chapter 8 "A new Market" for the implementation and results of this new mechanism.

Chapter 8

A new Market

As already introduced in section 7.3.2 "Re-Enabling trading" a new market is necessary to repair the miss-allocation of collateralized assets in the range of the pessimist agents by enabling the agents to trade collateralized assets against cash.

8.1 Definition

8.1.1 Products

Collateralized assets are traded against cash. The buyer gets a specific amount of collateralized assets for a given amount of cash where the seller gives away the specific amount of collateralized assets and gets the given amount of cash.

8.1.2 Price-Range

As within all other three markets the price-ranges of the offers must be defined. Note that all prices must be obviously in the unit of cash according to the previously defined products.

minimum When calculating the minimum price of a collateralized asset - that is how much is the collateralized asset minimally worth - it is important to include the collateral-aspect of the asset. Thus one starts with the minimum asset-price in cash which is the down-price pD and subtracts the maximum possible amount of cash which is bound through a bond as collateral which is the face-value V. This value is a constant for all agents.

$$\textit{min collateralized asset-price} = \min(0, pD - V)$$

maximum To calculate the maximum price of a collateralized asset - that is how much is the collateralized asset maximally worth - one needs to include the collateral-aspect of the asset too. Equal to calculating the minimum one starts now with the maximum asset-price in cash which is the up-price pU and subtracts the minimum possible amount of cash which is bound through a bond as collateral which is the face-value pD. This value is a constant for all agents.

$$\textit{max collateralized asset-price} = pU - pD$$

limit Applying the same rules as in minimum and maximum to the limit price calculation one needs to subract the limit-price of loans from the limit-price of asset to receive the limit-price of a collateralized asset. This value is individual for each agent as the limit-prices differ across the agents both for assets and loans.

$$\textit{limit-price of collateralized asset} = \textit{limit-price of asset} - \textit{limit-price of loan}$$

8.1.3 Bid-Offering

The way bid-offers are generated is very similar to the Bond/Cash market. Bid offers are generated only when the agent has any cash holdings. The price is drawn randomly between the minimum price and the limit-price because when buying one wants to buy below the expected value to make a profit. As amount one TRADING-UNIT of an asset is selected - in the thesis-implementation 0.1 - but if there is not enough cash left to buy one TRADING-UNIT of assets then the amount of assets is selected which can be bought with the remaining cash holdings.

Table 29: Bid-Offering parameters

Pre-Condition	$\textit{cash holdings} > 0$
Asset-Price	$\text{random}(\textit{min coll. asset-price}, \textit{limit-price of coll. asset})$
Asset-Amount	$\min(\frac{\textit{cash holdings}}{\textit{Asset-Price}}, \textit{TRADING-UNIT})$

8.1.4 Ask-Offering

The way ask-offers are generated is very similar to the Bond/Cash market. Ask offers are generated only when the agent has any collateralized assets. The price is drawn randomly between the limit-price and maximum price because when selling one wants to sell above the expected value to make a profit. As amount one TRADING-UNIT of an asset is selected - in the thesis-implementation 0.1 - but if there are fewer collateralized assets left then the remaining amount of collateral is selected. See Chapter 5 "Implementation" for the equation of collateral holdings.

Table 30: Ask-Offering parameters

Pre-Condition	*collateralized assets* > 0
Asset-Price	random(*limit-price of coll. asset*, *max coll. asset-price*)
Asset-Amount	min(*collateralized assets*, *TRADING-UNIT*)

8.1.5 Match

Below the wealth-exchange table is given in case of a match between two agents on the new market. Note that the wealth is increased/decreased as given by the +/- signs.

Table 31: Wealth-Exchange on match

	Seller	Buyer
Loan Given	+ matching-amount	N/A
Loans Taken	N/A	- matching-amount
Assets holdings	- matching-amount	+ matching-amount
Cash holdings	+ matching-price	- matching-price

8.2 Results

Of most importance are the results of the simulation when using the new market. The plain results are given in this section where the interpretation of the results are given in the following section.

As experiment-configuration the same is used as given in Chapter 6 "Results" except that the new market is now activated too.

Table 32: Configuration for all experiments

Agent-Count	100
Bond-Type	0.5
Replication-Count	50
Matching-Round	max. 500 offering-rounds
Terminate after	1000 failed successive matching-rounds

8.2.1 Fully-Connected topology

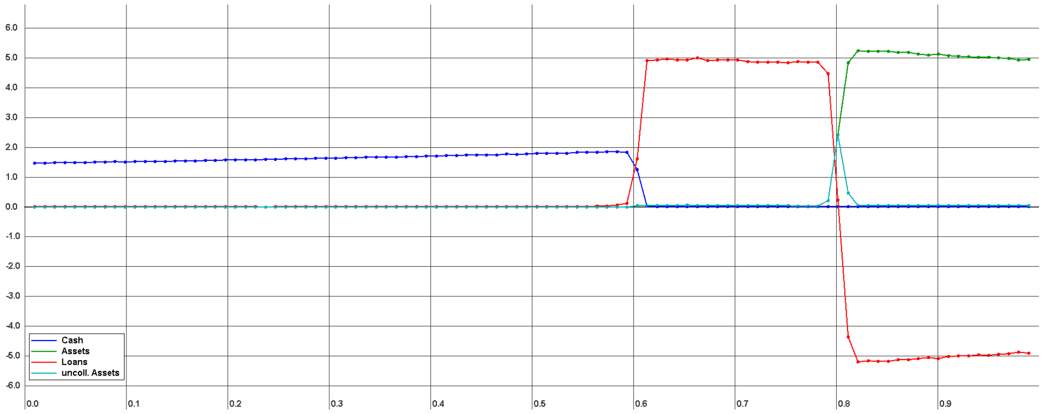

Figure 10: Wealth-Distribution of Fully-Connected topology with Collateral/Cash market

Table 33: Equilibrium of Fully-Connected topology with Collateral/Cash market

Asset-Price p	0.688 (0.008)
Bond-Price q	0.381 (0.002)
Marginal Agent i1	0.597 (0.005)
Marginal Agent i2	0.803 (0.003)
Pessimist Wealth	1.597 (0.009)
Medianist Wealth	4.76 (0.1)
Optimist Wealth	4.963 (0.052)

Table 34: Performance of Fully-Connected topology with Collateral/Cash market

Successful matching-rounds	1916.14 (31.42)
Failed matching-rounds	4448.66 (1668.93)
Total matching-rounds	6364.8 (1679.21)
Ratio successful/total	0.3
Ratio failed/total	0.7

Table 35: Difference of Fully-Connected topology to theoretical equilibrium as given in Table 17 of Chapter 6 ”Results”

	Result	Reference	difference to Reference
Asset-Price p	0.688	0.717	-4.0%
Bond-Price q	0.381	0.375	+1.6%
Marginal Agent i1	0.597	0.584	+2.2%
Marginal Agent i2	0.802	0.803	+0.1%

Table 36: Difference of Fully-Connected topology to equilibrium without Collateral/Cash market as given in Table 18 of Chapter 6 ”Results”

	Result	Reference	difference to Reference
Asset-Price p	0.688 (0.008)	0.689 (0.01)	-0.1% (-20%)
Bond-Price q	0.381 (0.002)	0.384 (0.004)	-0.7% (-50%)
Marginal Agent i1	0.597 (0.005)	0.603 (0.007)	-1.0% (-28%)
Marginal Agent i2	0.803 (0.003)	0.803 (0.003)	0.0% (0%)
Pessimist Wealth	1.597 (0.009)	1.597 (0.015)	0.0% (-40%)
Medianist Wealth	4.76 (0.1)	4.565 (0.113)	+4.2% (-11%)
Optimist Wealth	4.963 (0.052)	5.021 (0.064)	-1.1% (-19%)

8.2.2 Ascending-Connected topology

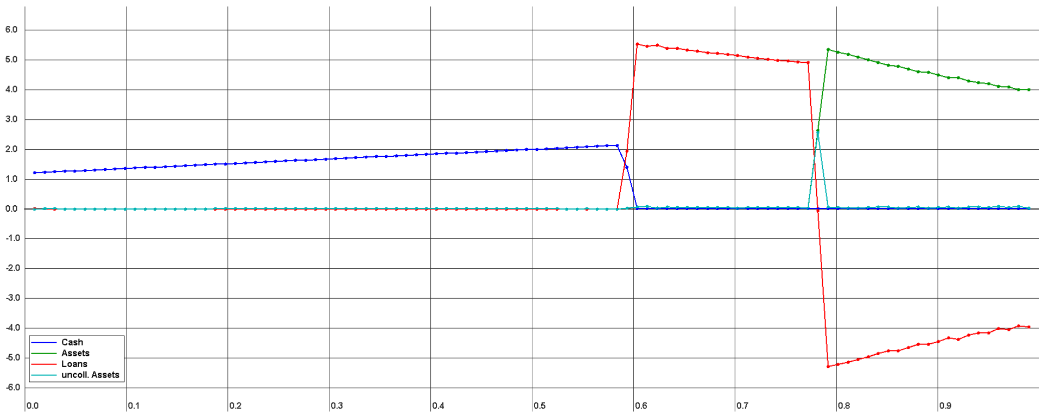

Figure 11: Wealth-Distribution of Ascending-Connected topology with Collateral/Cash market

Table 37: Equilibrium of Ascending-Connected topology

Asset-Price p	0.713 (0.013)
Bond-Price q	0.383 (0.005)
Marginal Agent i1	0.584 (0.0)
Marginal Agent i2	0.782 (0.0)
Pessimist Wealth	1.671 (0.0)
Medianist Wealth	5.032 (0.013)
Optimist Wealth	4.508 (0.006)

Table 38: Performance of Ascending-Connected topology

Successful matching-rounds	51,838.74 (1613.36)
Failed matching-rounds	1124.76 (28.31)
Total matching-rounds	52,963.5 (1612.31)
Ratio successful/total	0.98
Ratio failed/total	0.02

Table 39: Difference of Ascending-Connected topology to theoretical equilibrium as given in Table 17 of Chapter 6 "Results"

	Result	Reference	difference to Reference
Asset-Price p	0.713	0.717	-0.5%
Bond-Price q	0.383	0.375	+2.1%
Marginal Agent i1	0.584	0.584	0.0%
Marginal Agent i2	0.782	0.802	-2.5%

Table 40: Difference of Ascending-Connected topology to equilibrium without Collateral/Cash market as given in Table 21 of Chapter 6 "Results"

	Result	Reference	difference to Reference
Asset-Price p	0.713 (0.013)	0.711 (0.016)	+0.3% (19%)
Bond-Price q	0.383 (0.005)	0.391 (0.005)	-2.0% (0%)
Marginal Agent i1	0.584 (0.0)	0.646 (0.012)	-9.6% (-100%)
Marginal Agent i2	0.782 (0.0)	0.85 (0.008)	-8.0% (-100%)
Pessimist Wealth	1.671 (0.0)	1.166 (0.072)	+43.3% (-100%)
Medianist Wealth	5.032 (0.013)	1.869 (0.243)	+169.2% (95%)
Optimist Wealth	4.508 (0.006)	4.307 (0.07)	+4.6% (-91%)

Table 41: Difference of Ascending-Connected to equilibrium of fully-connected topology with Collateral/Cash market as given above

	Result	Reference	difference to Reference
Asset-Price p	0.713 (0.013)	0.688 (0.008)	+3.6% (+62%)
Bond-Price q	0.383 (0.005)	0.381 (0.002)	+0.5% (+150%)
Marginal Agent i1	0.584 (0.0)	0.597 (0.005)	-2.2% (-100%)
Marginal Agent i2	0.782 (0.0)	0.803 (0.003)	-2.6% (-100%)
Pessimist Wealth	1.671 (0.0)	1.597 (0.009)	+4.6% (-100%)
Medianist Wealth	5.032 (0.013)	4.76 (0.1)	+5.7% (-87%)
Optimist Wealth	4.508 (0.006)	4.963 (0.052)	-9.2% (88%)

Table 42: Difference of Ascending-Connected to equilibrium of fully-connected without Collateral/Cash market as given in Table 18 of Chapter 6 "Results"

	Result	Reference	difference to Reference
Asset-Price p	0.713 (0.013)	0.689 (0.01)	+3.5% (+30%)
Bond-Price q	0.383 (0.005)	0.384 (0.004)	-0.3% (+25%)
Marginal Agent i1	0.584 (0.0)	0.603 (0.007)	-3.2% (-100%)
Marginal Agent i2	0.782 (0.0)	0.803 (0.003)	-2.6% (-100%)
Pessimist Wealth	1.671 (0.0)	1.597 (0.015)	+4.6% (-100%)
Medianist Wealth	5.032 (0.013)	4.565 (0.113)	+10.23% (88%)
Optimist Wealth	4.508 (0.006)	5.021 (0.064)	-10.22% (91%)

8.3 Interpretation of results

When interpreting the results the following questions must be answered:

- Does the Fully-Connected topology reach the theoretical equilibrium as well?
- Does the new market repair the miss-allocation of wealth in the pessimists-range?
- If not why? If yes, does the Ascending-Connected topology approach theoretical equilibrium now?

Does the Fully-Connected topology reach the theoretical equilibrium as well? Yes it does. Both visual and statistical results show that it reaches the theoretical equilibrium. The medianist wealth is slightly higher with the new market but that difference, as well as the variations in the other variables are not statistically significant.

Does the new market repair the miss-allocation of wealth in the pessimists-range? Yes it does. The visual results are clear with no miss-allocations showing up within 50 replications. If there would have been miss-allocations within any replication they would have shown up in the final result.

If yes, does the Ascending-Connected topology approach theoretical equilibrium now? The miss-allocations are repaired but it does does not approach theoretical equilibrium. Both visual and statistical results show that it misses to reach the theoretical and Fully-Connected topology with or without new market equilibrium. Because of the way the new market works the wealth-distributions in medianists and pure optimists show a different shape than in Fully-Connected and thus the theoretical equilibrium is not reached. The reason for the different shape of the wealth-distributions is rooted in the way the market-dynamics work which is discussed in the following section.

8.4 Simulation and Market dynamics

When implementing a new market the market-dynamics are of very importance and thus the following questions must be answered.

- Can the trading stages 1-4 be identified too as given in [BSV13]?
- How does trading progresses with this new market? Is it the same as without the new market?
- How does the new market resolve the miss-allocation (with and without deferred activation)?
- When and how much is each market active?
- How do the market-activities change when a new market is introduced?

To answer these questions one must look closely at the market-dynamics. There are trading stages to be identified but due to the new market and the different topology they are expected to be quite different from the ones found in [BSV13]. The method used to find these stages is through observation of a single run and refining and validating the derived facts over many additional runs. Note that replications provide no real value here as one needs to look very carefully into the dynamics of single runs instead of the mean of multiple runs.

8.4.1 Fully-Connected with new Market

4 Stages were identified which are quite different from the ones given in [BSV13] as the new market makes quite a big impact.

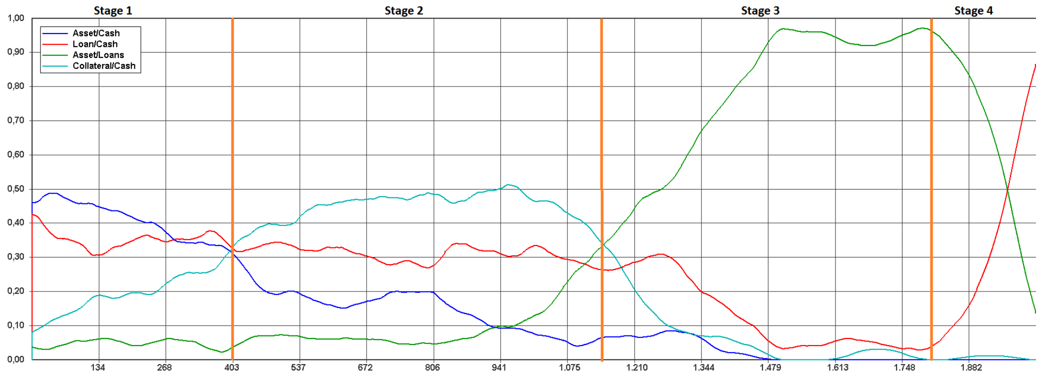

Figure 12: Market-activity stages of Fully-Connected topology with Collateral/Cash market

Stage 1 In this stage the pessimists become visible rapidly as they sell their assets and increase their cash wealth. One can get also a sense of the more optimistic range of agents as they gather assets both free and collateralized. The medianists are not visible yet.

The Asset/Cash market dominates but goes down slowly as fewer and fewer pessimists trade assets against cash compared to the very beginning. The Bond/Cash market fluctuates around the same point as loans are traded more or less equally the same. The Collateral/Cash market begins quite low and raises as more and more collateralized assets are created by pessimists and need to be sold again using the new market. The Asset/Bond market is hardly active as there is no strong need for its features yet.

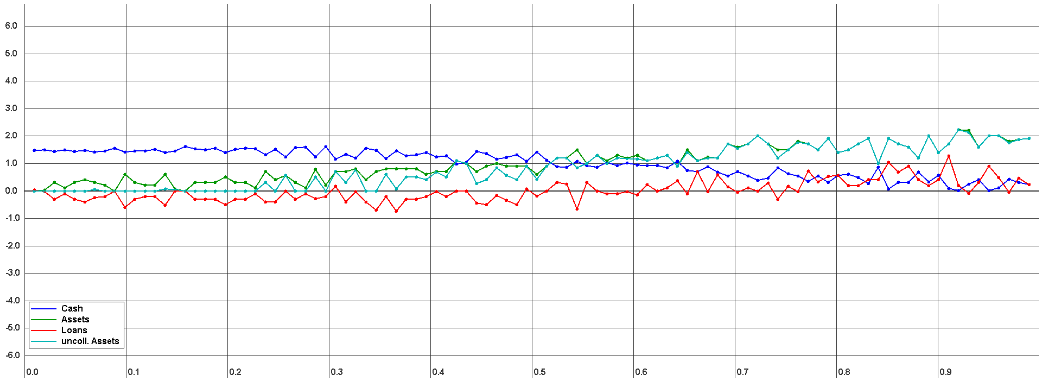

Figure 13: Wealth-Distribution of Fully-Connected topology with Collateral/Cash market during Stage 1

Stage 2 The collateralized assets are traded from the pessimists towards the optimists and the optimists crystallize themselves even more but no medianists are visible yet.

The Asset/Cash market continues to go down as the cash holdings of the pessimists begin to decline. The Bond/Cash market fluctuates around the same point as before. Now the Collateral/Cash market becomes very active as more and more collateralized assets need to be traded towards the optimists. At the beginning of this stage the Asset/Bond market is hardly active but raises fast towards the end of it as the optimists are then out of cash and need to distribute the collateralized assets between each other and the yet to come medianists.

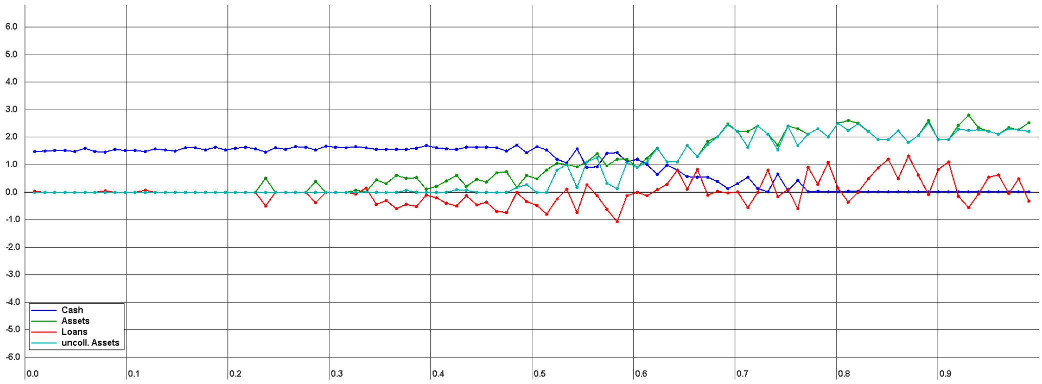

Figure 14: Wealth-Distribution of Fully-Connected topology with Collateral/Cash market during Stage 2

Stage 3 The pessimists have gone inactive as they hold no more wealth they can trade. The i1-point is emerging and the medianists and pure optimists begin to show up.

Because the pessimists are inactive now and hold no more assets the Asset/Cash and Collateral/Cash markets go down and decline completely. The Bond/Cash market goes down but does not decline as bonds are still traded because of the emerging of medianists. The medianists and pure optimists which are emerging have no other possibility than to trade on the Asset/Bond market to further distribute their collateralized assets among each other which is the reason for the raise of the Asset/Bond market above all others and its heavy domination. Despite the heavy domination of the Asset/Bond market still a few bonds are traded against cash in the range of the medianists.

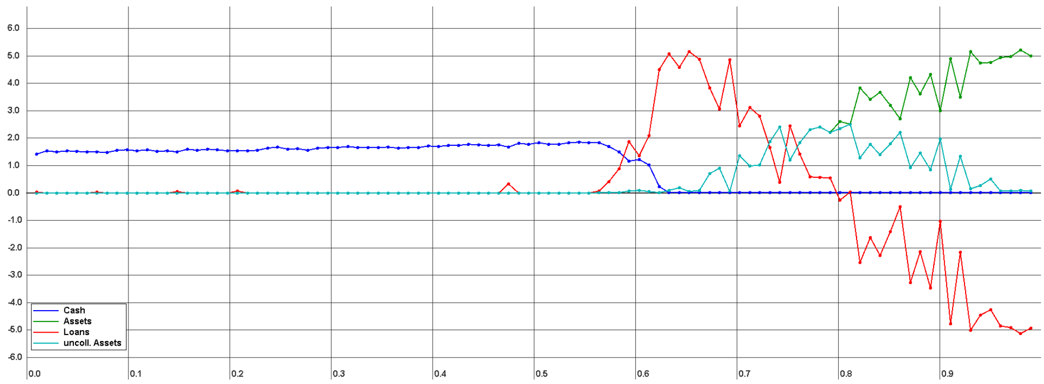

Figure 15: Wealth-Distribution of Fully-Connected topology with Collateral/Cash market during Stage 3

Stage 4 Finally the i2-point has emerged and both i1 and i2 are finalizing. The only active agents remaining are around these two points where the pure optimists trade with the medianists to finalize i2 and the medianists with the next closest pessimists to finalize i1 where the very last transactions occur around i1.

The Asset/Bond market goes down until i2 has finalized as the collateralized asset allocation has nearly reached its equilibrium. The Bond/Cash market goes up as agents around i1 are still refining the point as the equilibrium of the medianists is not reached yet. Thus bonds are traded against cash as i1 is the connecting point between pessimists with cash and medianists with bonds which enables the Bond/Cash market to trade again.

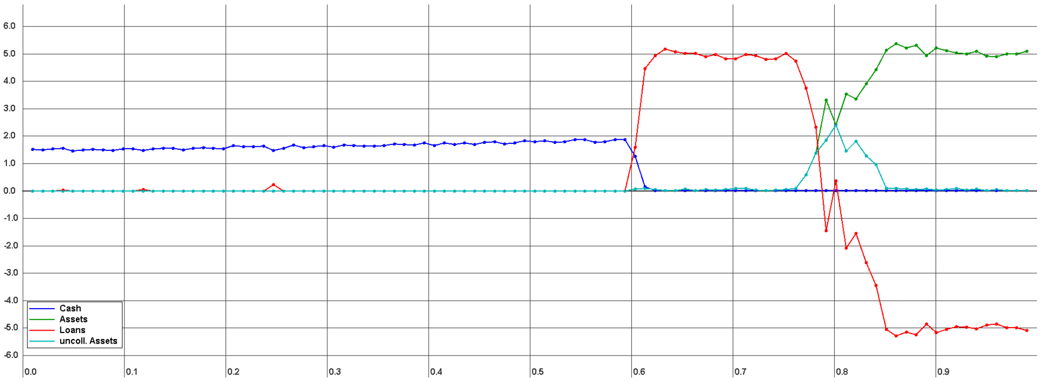

Figure 16: Wealth-Distribution of Fully-Connected topology with Collateral/Cash market during Stage 4

8.4.2 Ascending-Connected with new market enabled after 1,000 successive failed matching-rounds

Using the thesis-software it is possible to start a simulation-run on Ascending-Connected topology without the Collateral/Cash market and enabling it after 1,000 successive failed matching-rounds which gives interesting hints about how the spikes of collateralized assets in the pessimists-range are resolved and distributed over the already existing pure optimists.

Of course there are the same 3 stages to be found as described already in section 7.2.1 "Dynamics of a single run" whereas the deferred enabling of the Collateral/Cash market adds 2 new stages.

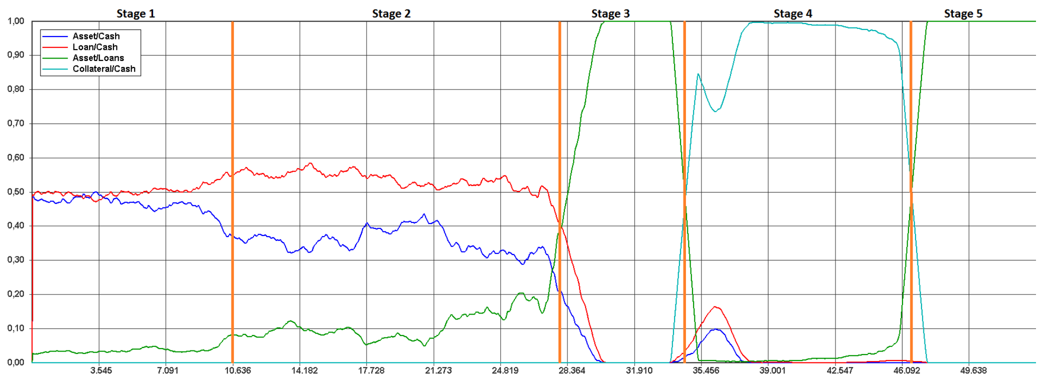

Figure 17: Market-activity stages of Ascending-Connected topology with deferred activated Collateral/Cash market

Stage 4 The collateralized assets are traded against cash and sum up at the i1-point where the first agent has no more cash.

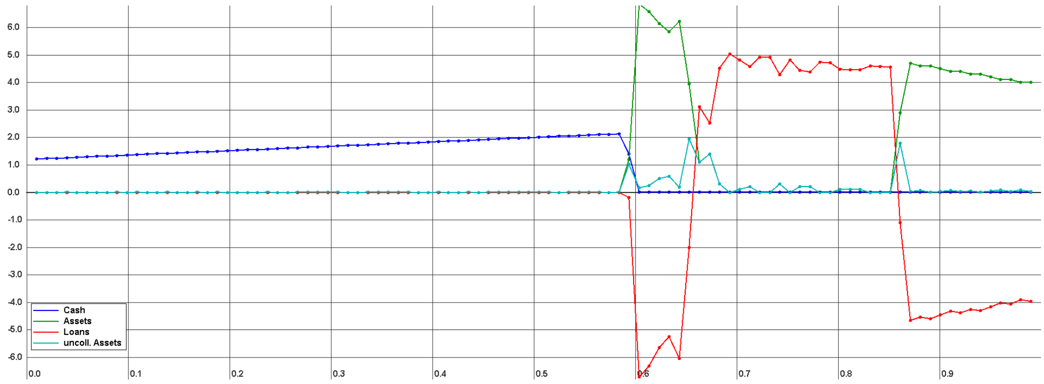

Figure 18: Wealth-Distribution of Ascending-Connected topology with deferred activated Collateral/Cash market during Stage 4

Stage 5 Now the Asset/Bond market becomes 100% dominant again and the collateralized assets are traded through the medianists towards the pure optimists as they have no more cash and can only trade anymore on this remaining active market.

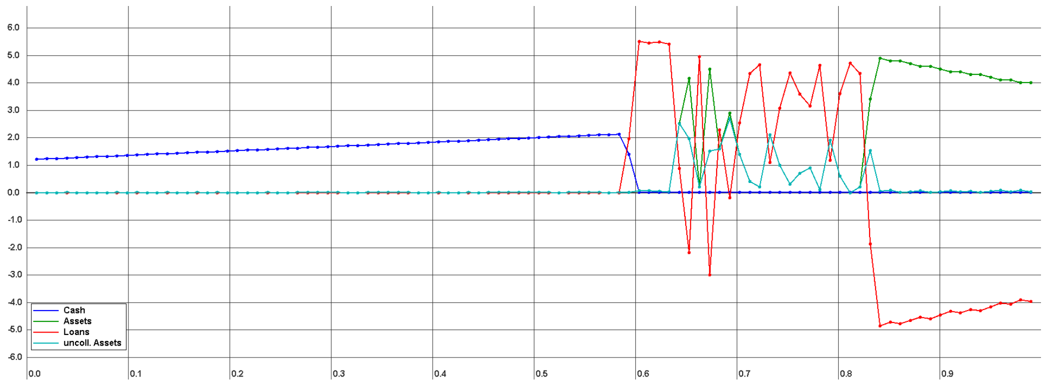

Figure 19: Wealth-Distribution of Ascending-Connected topology with deferred activated Collateral/Cash market during Stage 5

8.4.3 Ascending-Connected with new Market

4 stages were identified where only 3 of them can be seen in the Market-Dynamics diagram as stage 3 and 4 have indifferent market-activities.

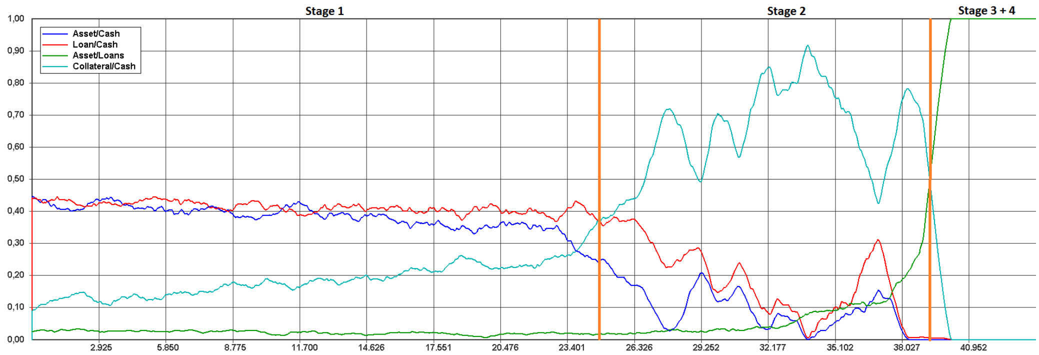

Figure 20: Market-activity stages of Ascending-Connected topology with Collateral/Cash market

Stage 1 Pessimists and optimists begin to emerge where the pessimists are gathering cash and collateralized assets and the optimists are gathering free assets against cash and a few collateralized assets. There are no medianists visible yet.

The Asset/Cash and Bond/Cash markets start around 40% slightly decreasing because the pessimists are slowly going low on assets. The Collateral/Cash market starts around 10% with increasing tendency because the amount of collateralized assets which move towards the optimists increases. The Asset/Bond market is hardly active as its features are not yet very necessary.

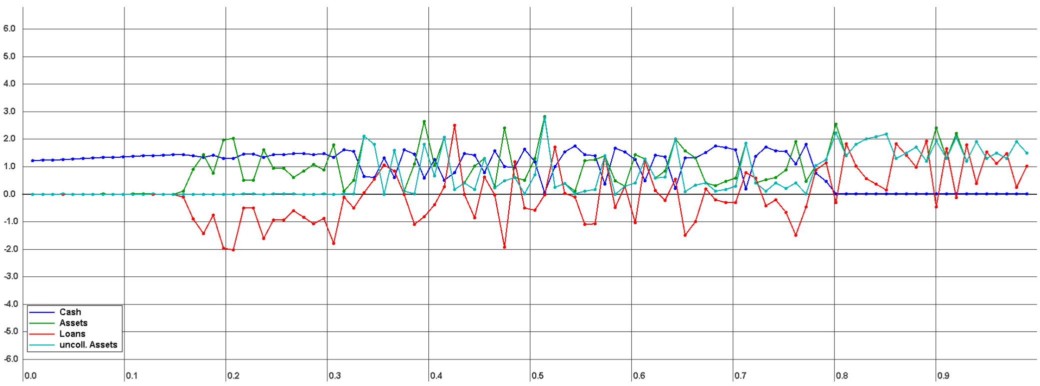

Figure 21: Wealth-Distribution of Ascending-Connected topology with Collateral/Cash market during Stage 1

Stage 2 In the pessimist-range large amount of collateralized assets have gathered which are traded now towards the optimists as the pessimists try to get maximally short on any assets and maximally plus on cash. Those collateralized assets are traded towards the i1-point - that is the first agent who holds no more cash - which can be seen by a spike in the wealth-distribution of figure 22 around 0.65. There are no medianists visible yet.

The Collateral/Cash market raises above the Asset/Cash and Bond/-Cash markets and either fluctuates or stays quite constant. In figure 20 the fluctuating variant is shown. If the market fluctuates the Asset/Cash and Bond/Cash fluctuate inverse in that if Collateral/Cash market raises they go down and vice versa. If the Collateral/Cash market stays quite constant in this stage it raises above 90% and Asset/Cash and Bond/Cash markets decline to 0%. Why the Collateral/Cash market either fluctuates or stays constant is not clear but is most probably dependent on the allocation of collateralized assets in the pessimists-range. The Asset/Bond market becomes a bit more active as more collateralized assets are traded.

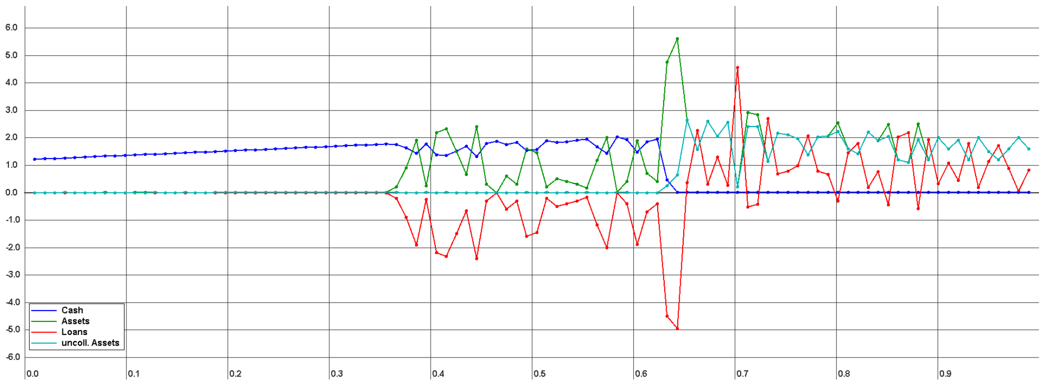

Figure 22: Wealth-Distribution of Ascending-Connected topology with Collateral/Cash market during Stage 2

Stage 3 Pessimists are now final and won't change any more. The optimists-range is now clearly visible and holds a large amount of collateralized assets from the pessimists which needs now to be traded and distributed to the remaining optimists. Medianists are still not visible yet.

Because the pessimists are no more able to trade and the optimists hold no more cash the activity of the Collateral/Cash market drops rapidly and Asset/Bond market raises to 100% activity as only collateralized-assets are tradeable any more.

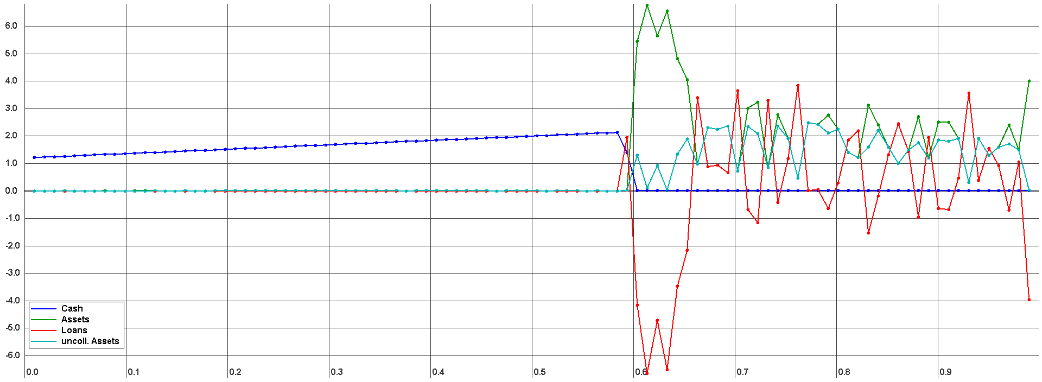

Figure 23: Wealth-Distribution of Ascending-Connected topology with Collateral/Cash market during Stage 3

Stage 4 Medianists begin to show up and to distinguish themselves from the pure optimists. This is no more visible on the market-dynamics as only Asset/Bonds are traded any more and thus only this market is active any more.

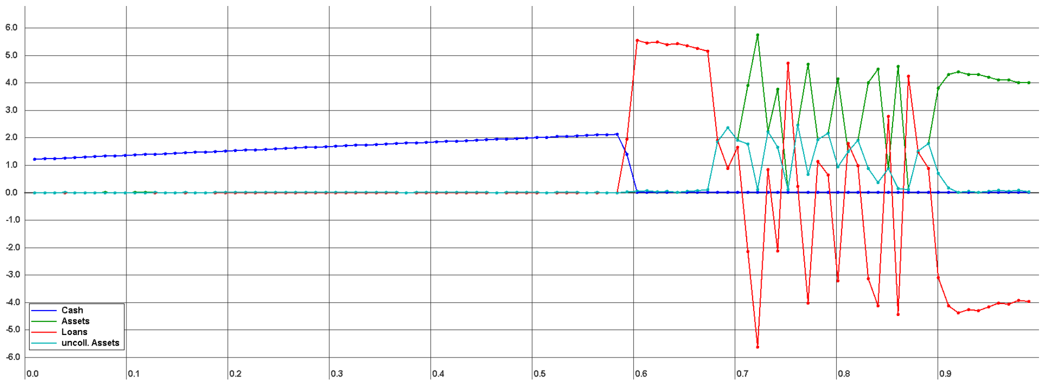

Figure 24: Wealth-Distribution of Ascending-Connected topology with Collateral/Cash market during Stage 4

After these observations made one can answer the questions.

Can the trading stages 1-4 be identified too as given in [BSV13]? There are 4 stages in the case of both Fully-Connected and Ascending-Connected topology with the new market which are not the ones given in [BSV13] but show up by pure chance and depend also a bit on the point-of-view on how to separate the stages from each other.

How does trading progress with this new market? Is it the same as without the new market? The progression of the trading is obviously very different with the new market as compared to the market-activities without as the usage of the new market changes the dynamics completely.

How does the new market resolve the miss-allocation (with and without deferred activation)? It becomes active during the formation of the pessimists agents as they gather collateralized assets wealth which must be traded towards the optimists. The collateralized assets are traded from neighbour to neighbour until they reach the optimists-region.

When and how much is each market active? This can be seen clearly in the market-activity diagrams.

How do the market-activities change when a new market is introduced? They have less share on the overall activity and thus the new market is quite a heavy competitor in the overall share. The Asset/Bond market though is still the market on which the final trades occur.

8.5 Conclusions on new Market

The equilibrium of the Ascending-Connected topology with the new market is different than the Fully-Connected one which reaches the theoretical equilibrium. Thus the hypothesis is still wrong because it predicted the Ascending-Connected topology to reach the theoretical equilibrium. This thesis can only speculate on the reason for this it is most probably rooted in the fundamental different trading dynamics of Ascending-Connected topology compared to Fully-Connected as can be seen in the market-dynamics. This thesis leaves the question of market-dynamics open for further research.

Chapter 9

Conclusion, Summary and further Research

9.1 Conclusion

9.2 Summary

9.3 Further Research

9.3.1 In-depth analysis of market-activities

je nach markt-aktivität kommt sicher ein anderes gleichgewicht heraus bzw. ist das so? hier bedarf es sicher weiterer foschungen und ist sicher auch ein ergiebiges und interessantes thema.

9.3.2 Imporance-Sampling

importance-sampling allgemein

9.3.3 Experiments with real subjects

experimentelle simulationen mit echten menschen: einschränken der handelsbeziehungen wie lokal bzw. global muss die vernetzung sein (ascending-connected full shortcuts)

9.3.4 Mathematical proof of hypothesis

beweisbarkeit der ascending-connected (MIT/OHNE neuem Markt)

9.3.5 Equilibrium definition for continous double-auction process

Appendix A

Topologies

In this chapter all simulated topologies are visualized and explained shortly. All topologies are demonstrated with N = 30 Agents instead of 100 for better visibility and clarity of the connections and nodes. All topologies have one connected component because otherwise equilibrium cannot be reached. Note that Erdos-Renyi could produce more than one connected component depending on the parameters to create it. The agents are always arranged in clockwise increasing optimism factor unless stated otherwise. All connections are undirected and there are no self-loops.

A.1 Fully-Connected

Each agent is connected with each other agent. Included as major point-of-reference as [BSV13] developed their model and equilibrium for fully-connected networks.

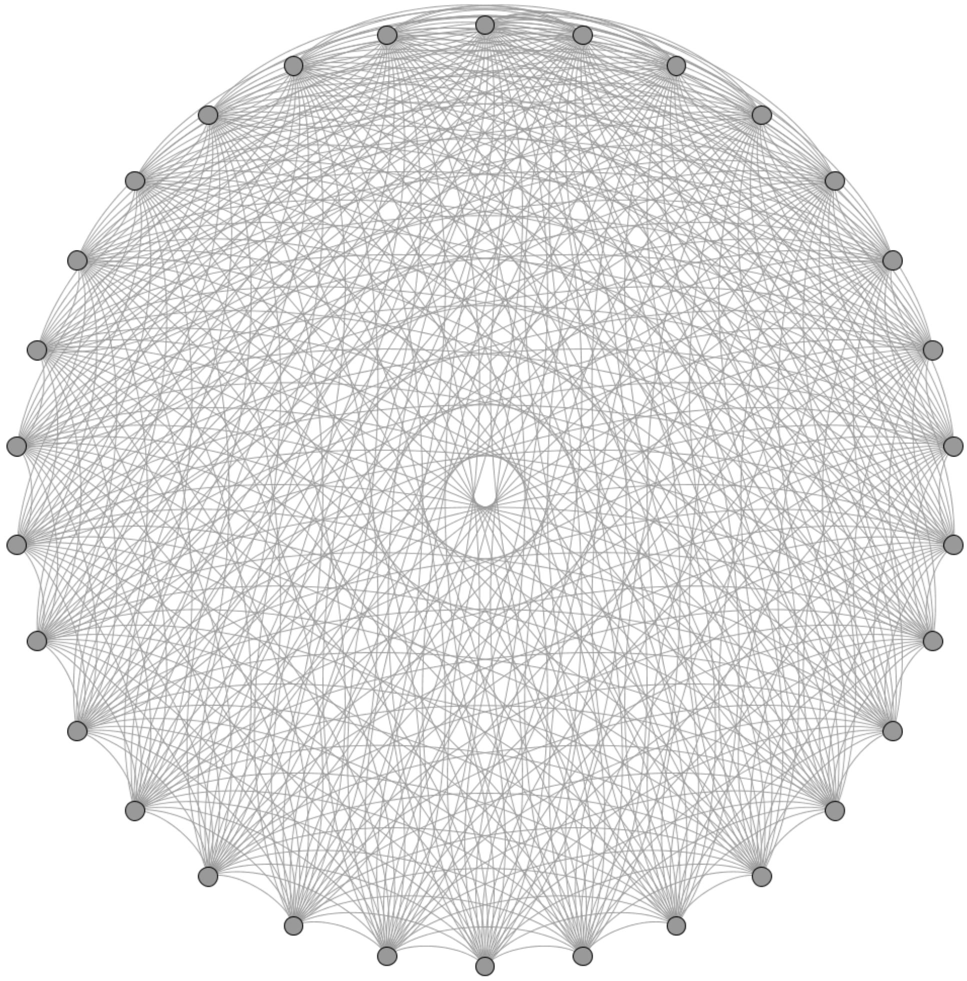

Figure 25: Fully-Connected topology

Table 43: Network metrics Fully-Connected topology

Avg. degree	29
Avg. path-length	1
Avg. clustering coefficient	1
Network diameter	1
Graph density	1
Edge count	$\frac{N(N-1)}{2}$

A.2 Half-Fully Connected

Agents with optimism-factor 0.5 to 1.0 are fully-connected and the others are connected to the agent with the next higher optimism-factor. The agents with highest and lowest optimism-factor are connected too, creating a closed circle. Included to investigate the influence of isolated agents.

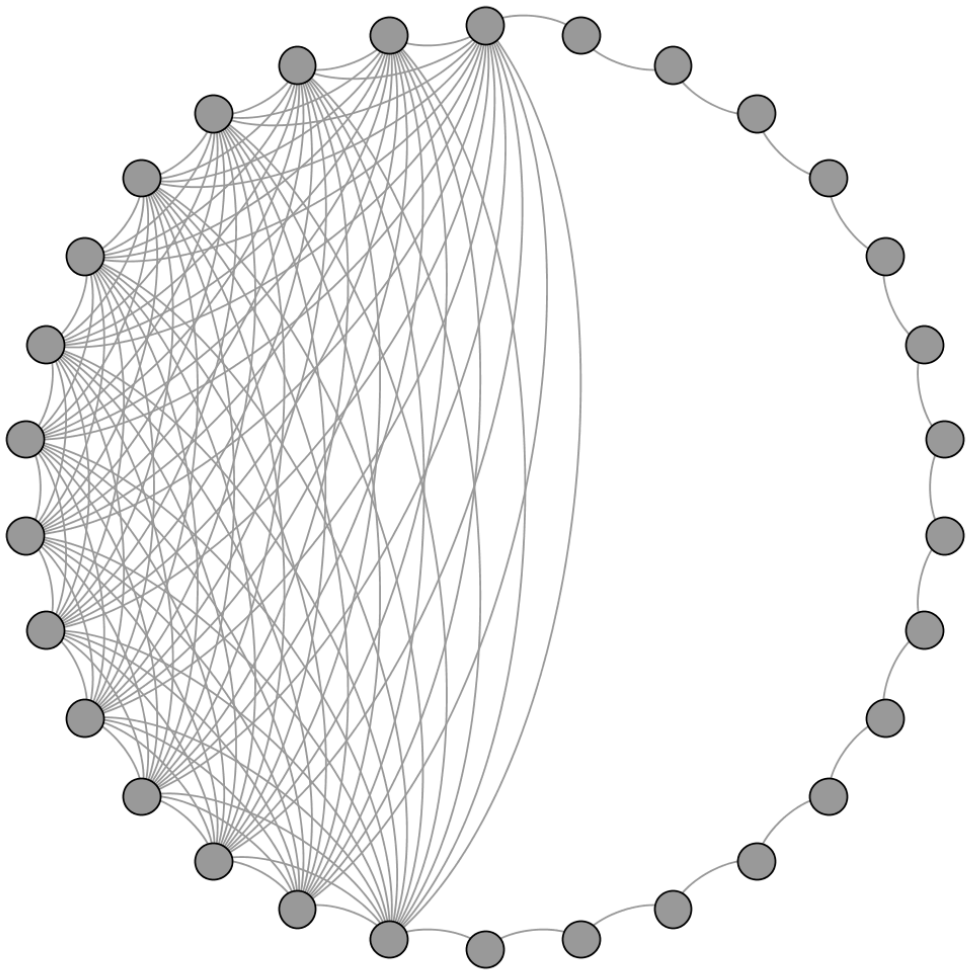

Figure 26: Half Fully-Connected topology

Table 44: Network metrics Half Fully-Connected topology

Avg. degree	8.067
Avg. path-length	4.007
Avg. clustering coefficient	0.491
Network diameter	9
Graph density	0.278
Edge count	$\frac{\frac{N}{2}(\frac{N}{2}-1)}{2}+\frac{N}{2}+1$

A.3 Ascending-Connected

Each agent is connected to the agent with the next higher optimism-factor. The agents with highest and lowest optimism-factor are not connected thus this network is not a closed circle. Included because it is the most minimal network which satisfies the theory and is therefore the major network of interest throughout the thesis.

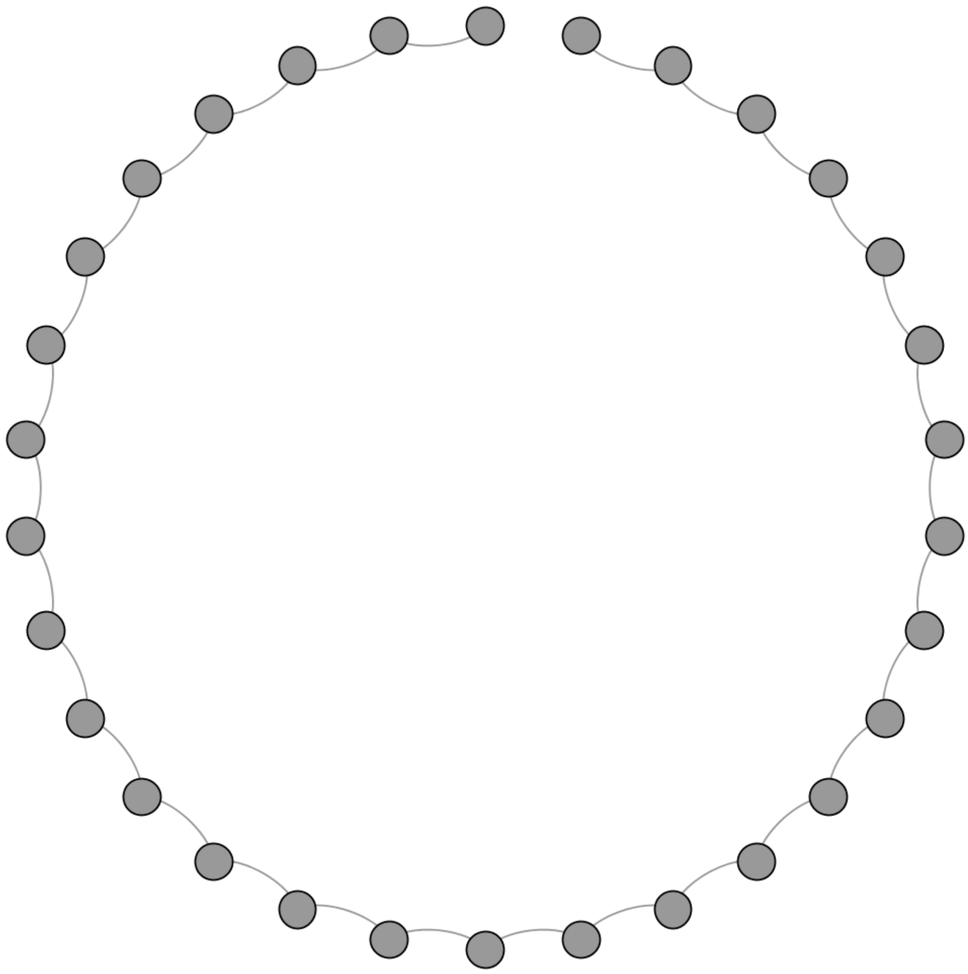

Figure 27: Ascending-Connected topology

Table 45: Network metrics Ascending-Connected topology

Avg. degree	1.933
Avg. path-length	10.33
Avg. clustering coefficient	0
Network diameter	29
Graph density	0.067
Edge count	$N-1$

A.4 Ascending-Connected with short-cuts

A.4.1 Full short-cuts

Each agent is connected to the K next neighbours in the clockwise arrangement. Thus agent N - K + 1 is connected to K - 1 higher optimism-agents and wraps around to the agent with lowest optimism-factor. Included to analyse the influence of increasing connectivity.

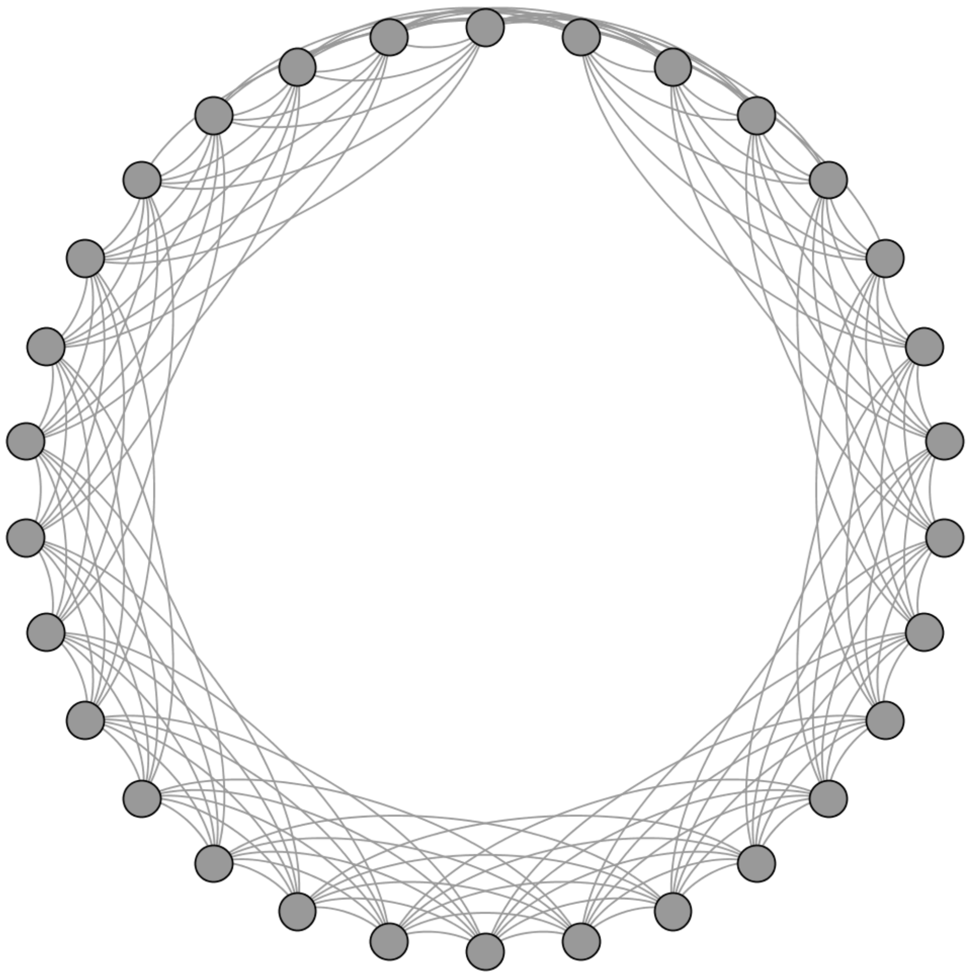

Figure 28: Ascending-Connected 5 full short-cuts topology

Table 46: Network metrics Ascending-Connected 5 full short-cuts topology

Avg. degree	10
Avg. path-length	1.966
Avg. clustering coefficient	0.667
Network diameter	3
Graph density	0.345
Edge count	NK

A.4.2 Regular short-cuts

The topology starts ascending-connected and each agent is additionally connected to one next neighbour in the clockwise arrangement where the distance to the next neighbour is K agents. Included to analyse the influence of trading-links to agents with much higher optimism-factor.

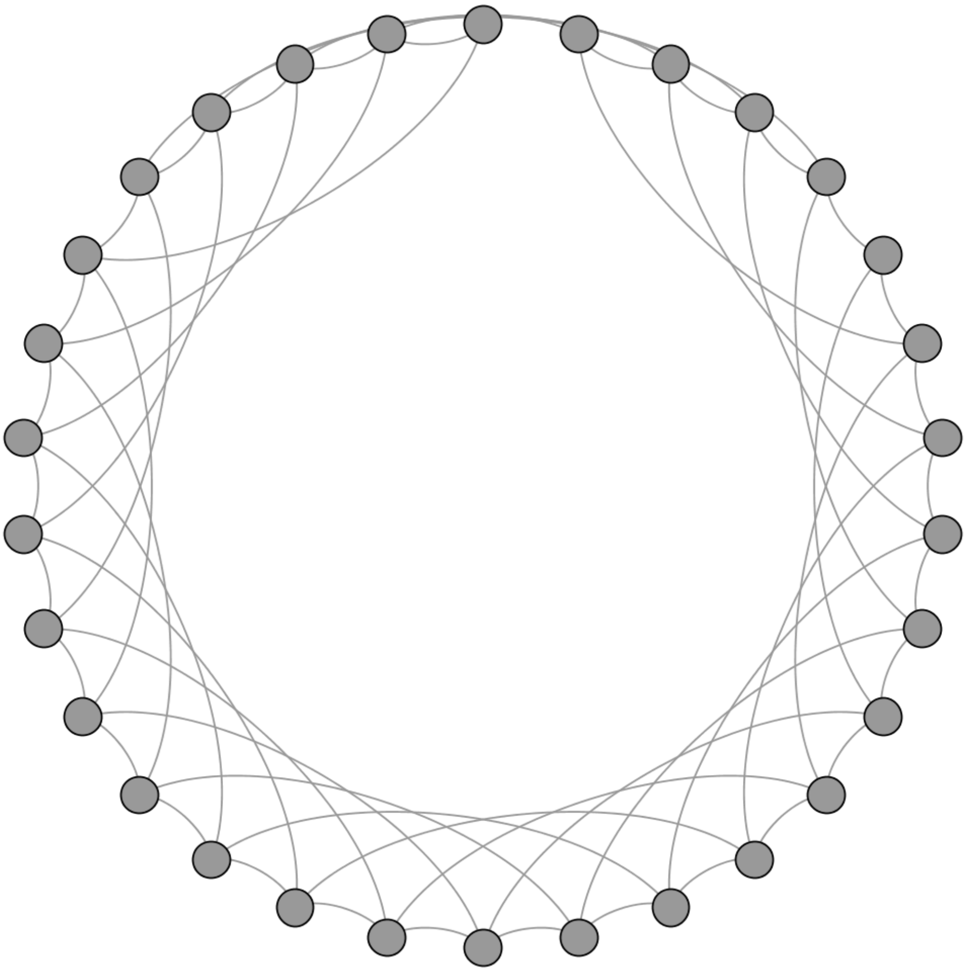

Figure 29: Ascending-Connected 5 regular short-cuts topology

Table 47: Network metrics Ascending-Connected 5 regular short-cuts topology

Avg. degree	3.867
Avg. path-length	2.839
Avg. clustering coefficient	0
Network diameter	6
Graph density	0.133
Edge count	$2N$

A.4.3 Random short-cuts

Starting with an ascending-connected network this topology adds one additional short-cut from each agent to another random agent with a given probability where a probability of 0.0 results in only the ascending-connectedness and a probability of 1.0 in each agent having an additional random short-cut. Self-loops and multi-edges are not allowed. Included to analyse the influence of randomness in ascending-connected topologies.

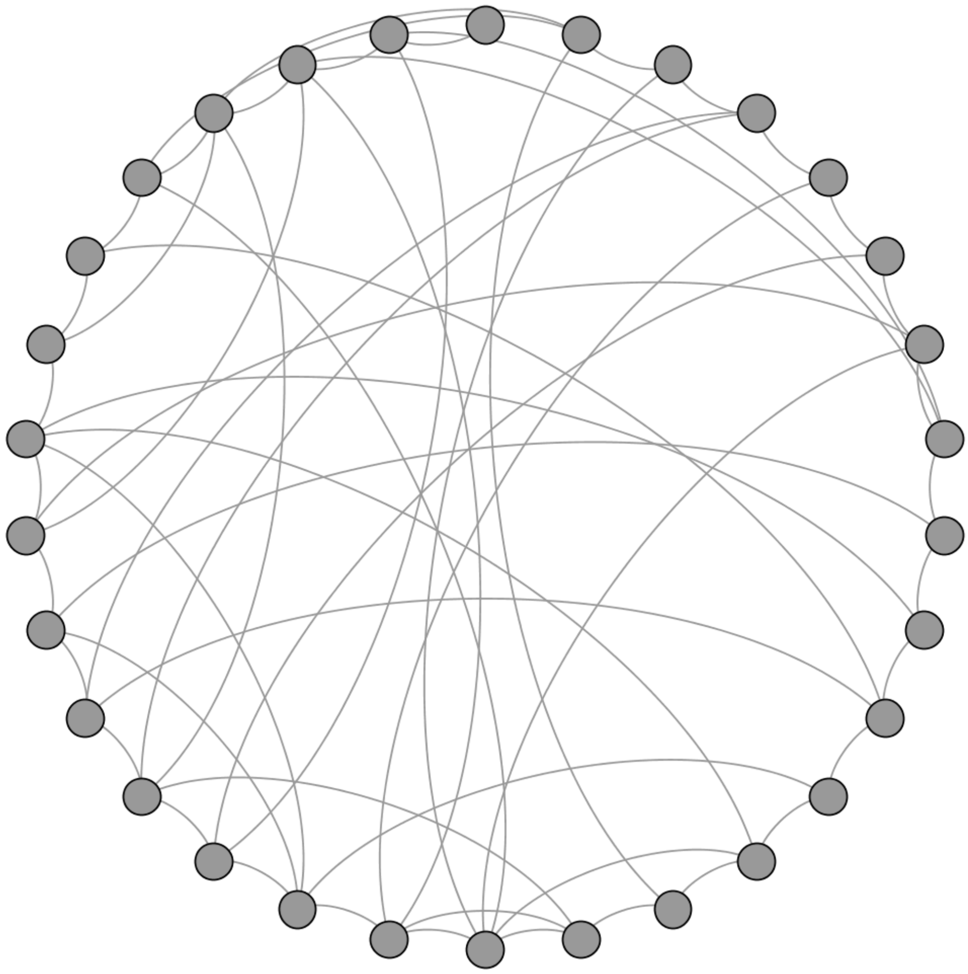

Figure 30: Ascending-Connected random short-cuts probability 1.0 topology

Table 48: Network metrics Ascending-Connected random short-cuts topology

Avg. degree	3.867
Avg. path-length	2.506
Avg. clustering coefficient	0.056
Network diameter	5
Graph density	0.133
Edge count	$\min N, \max 2N$

A.5 Hub-based topologies

A.5.1 3 Hubs

The agents are separated into 3 groups based upon their optimism-factor. Group 1 ranges from 0.0 to 0.33, group 2 from 0.33 to 0.66 and group 3 from 0.66 to 1.0. All agents within a group are fully-connected where the groups are interconnected between each other through a hub which is the agent with the highest optimism-factor of each group that is: 0.33, 0.66 and 1.0. Included as a point-of-reference as this topology was discussed too in [BSV13].

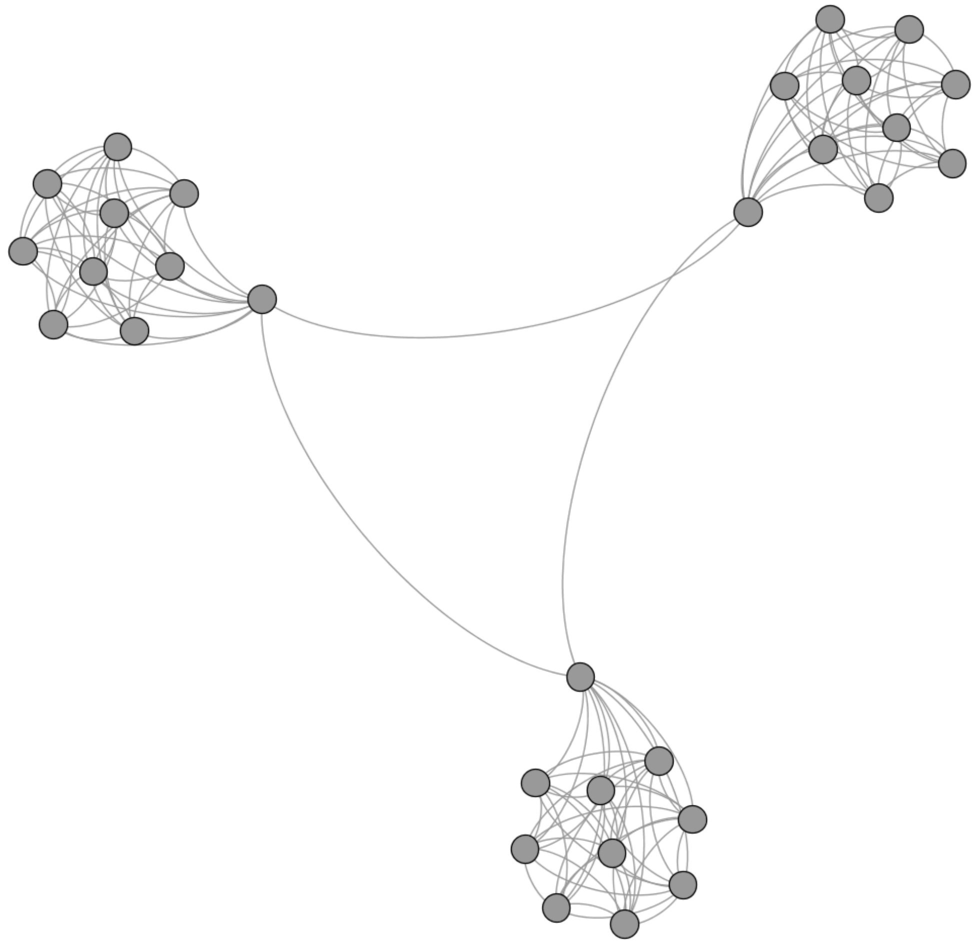

Figure 31: 3 Hubs topology

Table 49: Network metrics 3 Hubs topology

Avg. degree	9.2
Avg. path-length	2.241
Avg. clustering coefficient	0.976
Network diameter	3
Graph density	0.371
Edge count	$3(\frac{\frac{N}{3}(\frac{N}{3}-1)}{2})+3$

A.5.2 3 Median Hubs

There are 3 agents which act as median hubs which are the agent with the median optimism-factor and the next lower and higher ones. All three are connected to each other where the rest of the agents are randomly connected to one hub so that each hub has the same amount of agents. Included to see what happens if all agents can only trade through median agents which are the most active ones in the Fully-Connected topology.

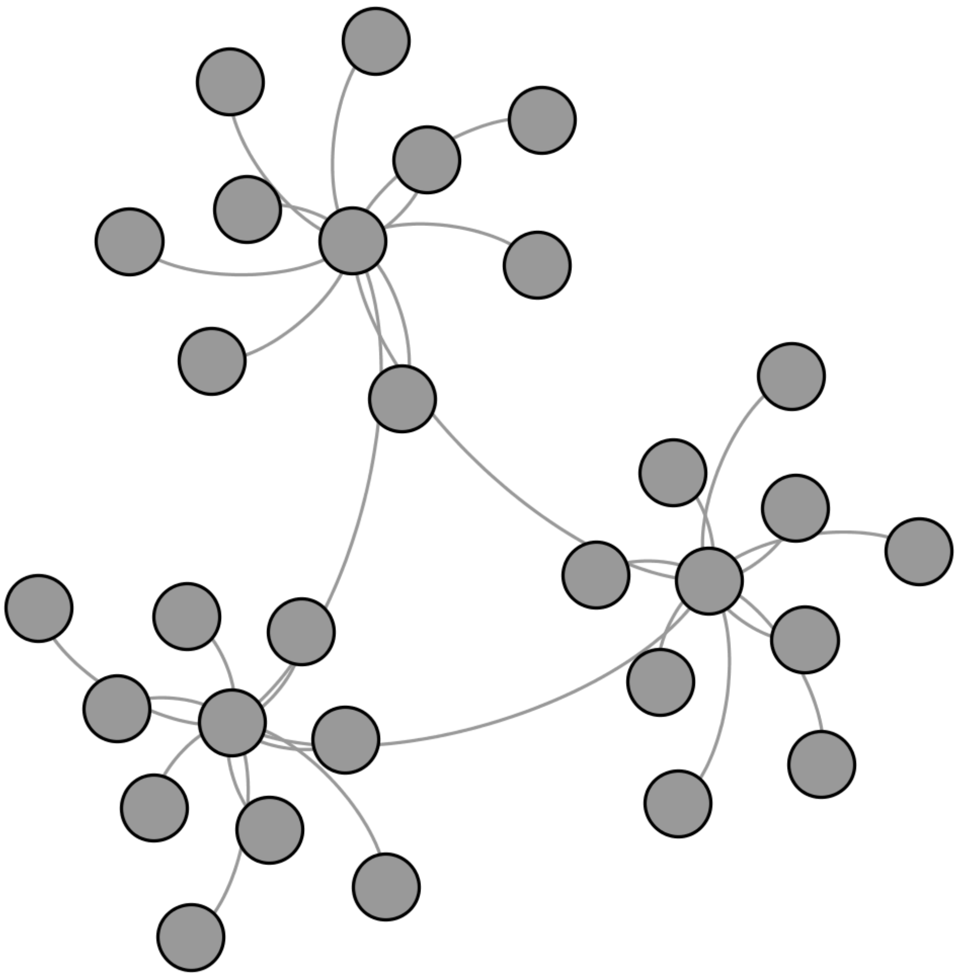

Figure 32: 3 Median Hub topology

Table 50: Network metrics 3 Median Hub topology

Avg. degree	2
Avg. path-length	2.49
Avg. clustering coefficient	0.018
Network diameter	3
Graph density	0.069
Edge count	N

A.5.3 Median Hub

All agents are connected to the agent with the median optimism-factor. Included for the same reason as in 3 median hubs but with just one median hub.

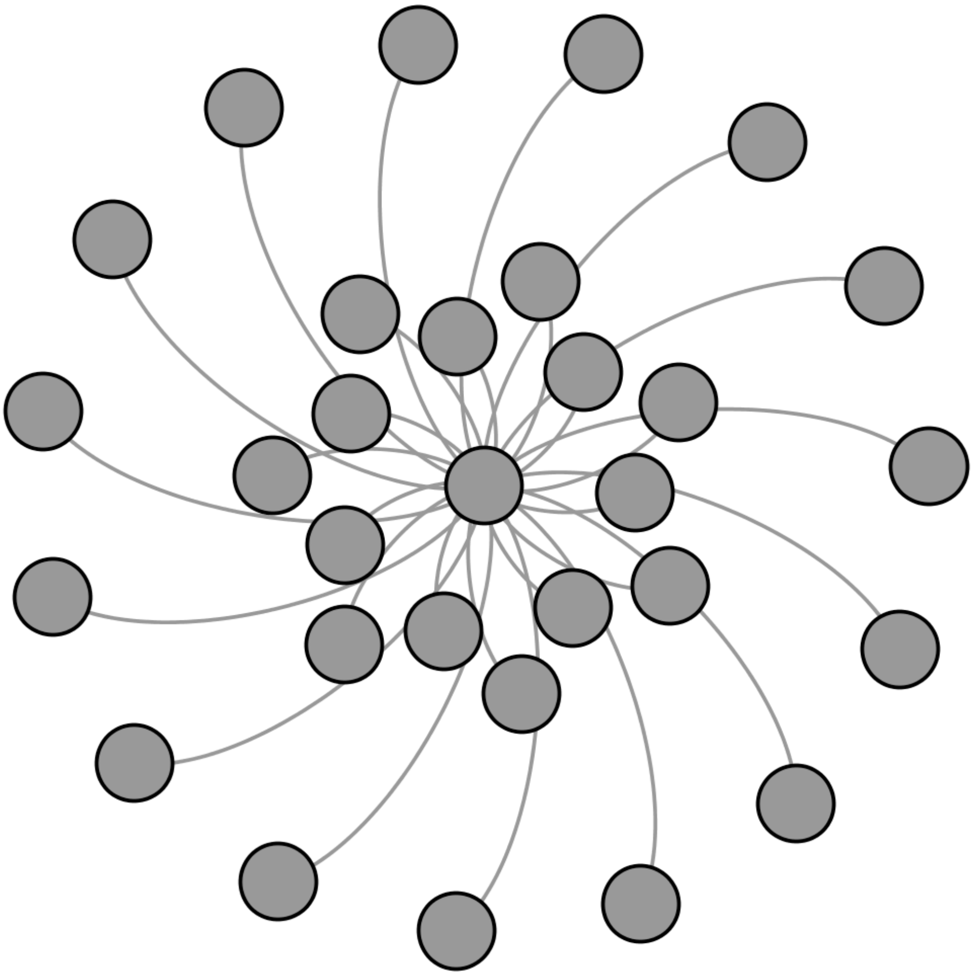

Figure 33: Median Hub topology

Table 51: Network metrics Median Hub topology

Avg. degree	1.933
Avg. path-length	1.933
Avg. clustering coefficient	0
Network diameter	2
Graph density	0.067
Edge count	$N-1$

A.5.4 Maximum Hub

Looks the same as 1 Median Hub but all edges are connected to the agent with the highest optimism-value. Has thus also the same metrics as the optimism-values have no functional influence on the metrics. Included just out of curiosity and has no real value as it is obviously clear that equilibrium is impossible to be reached in this case.

A.6 Small-World and Scale-Free topologies

A.6.1 Erods-Renyi

Each possible edge is included with a given probability where self-loops and multi-edges are omitted. See section 2.4 for a discussion of this topology. Included to investigate the influence of small-world an scale-free effects upon equilibrium and because of the randomness of the network.

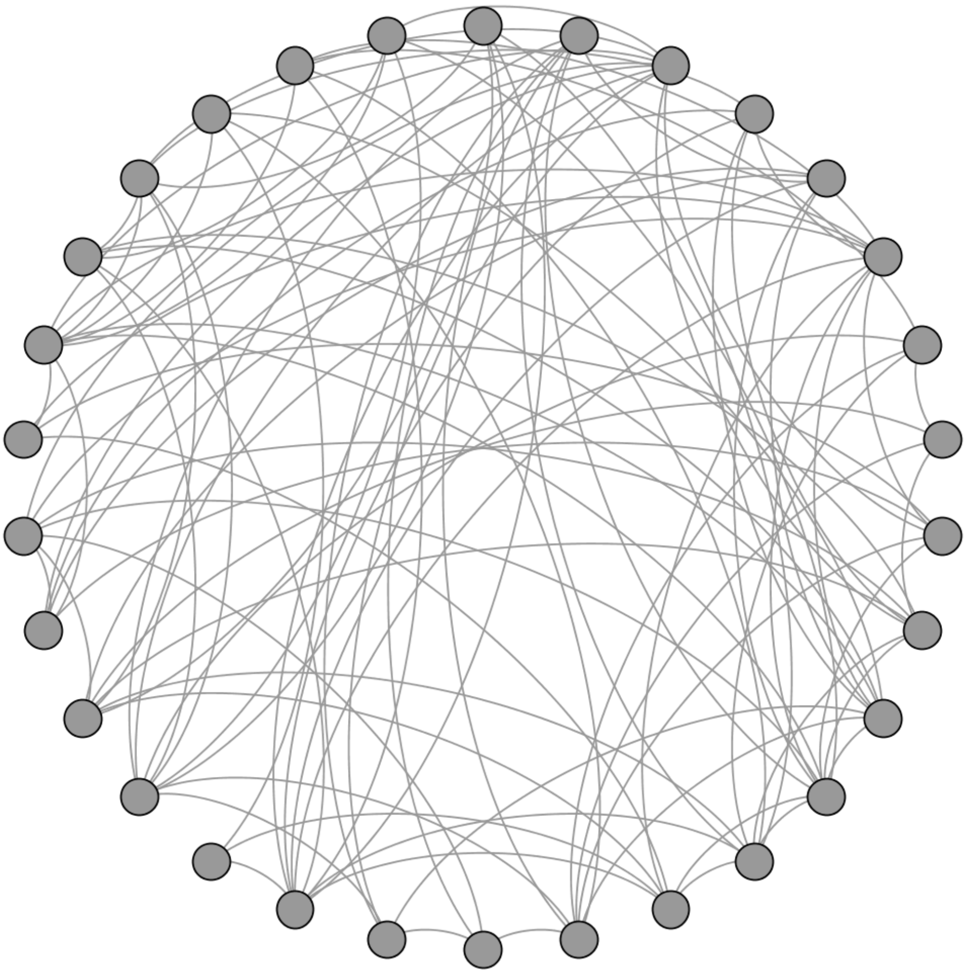

Figure 34: Erdos-Renyi topology with inclusion-probability of 0.2

Table 52: Network metrics Erdosy-Renyi 0.2

Avg. degree	6.8
Avg. path-length	1.913
Avg. clustering coefficient	0.266
Network diameter	3
Graph density	0.234
Connected component	1

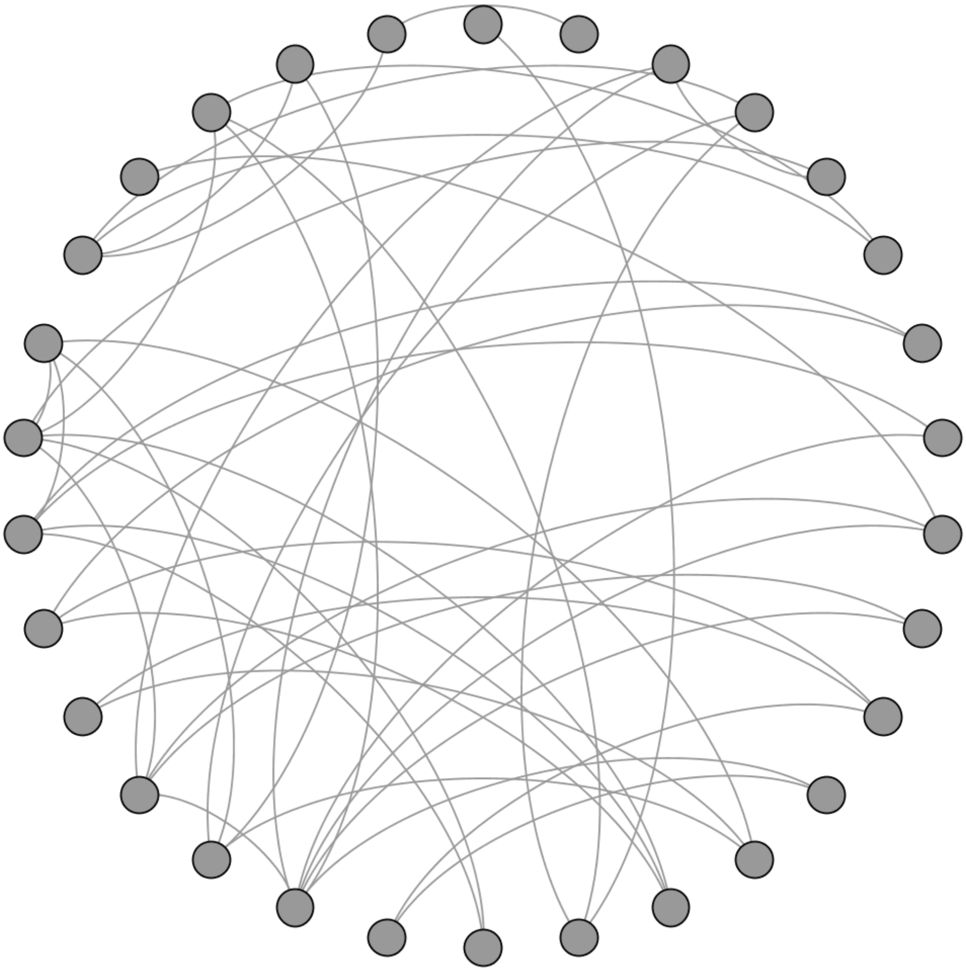

Figure 35: Erdos-Renyi topology with inclusion-probability of 0.1

Table 53: Network metrics Erdosy-Renyi 0.1

Avg. degree	2.933
Avg. path-length	3.262
Avg. clustering coefficient	0.103
Network diameter	7
Graph density	0.101
Connected component	1

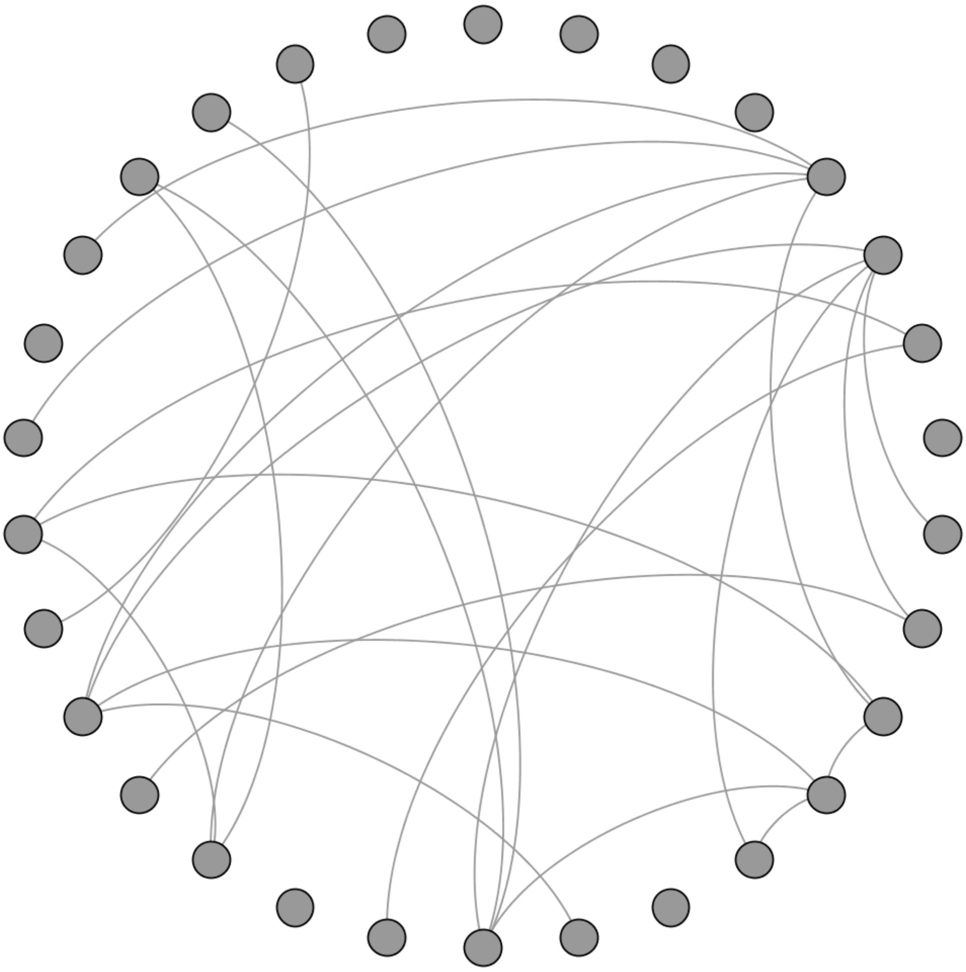

Figure 36: Erdos-Renyi topology with inclusion-probability of 0.05

Table 54: Network metrics Erdosy-Renyi 0.05

Avg. degree	1.6
Avg. path-length	3.052
Avg. clustering coefficient	0
Network diameter	8
Graph density	0.055
Connected component	11

A.6.2 Barbasi-Albert

One starts with m0 nodes and attaches each new node randomly to m existing nodes where the selection-probability of a node depends on its degree. See section 2.4 for a discussion of this topology. Included to investigate the influence of small-world an scale-free effects upon equilibrium and because of the randomness of the network.

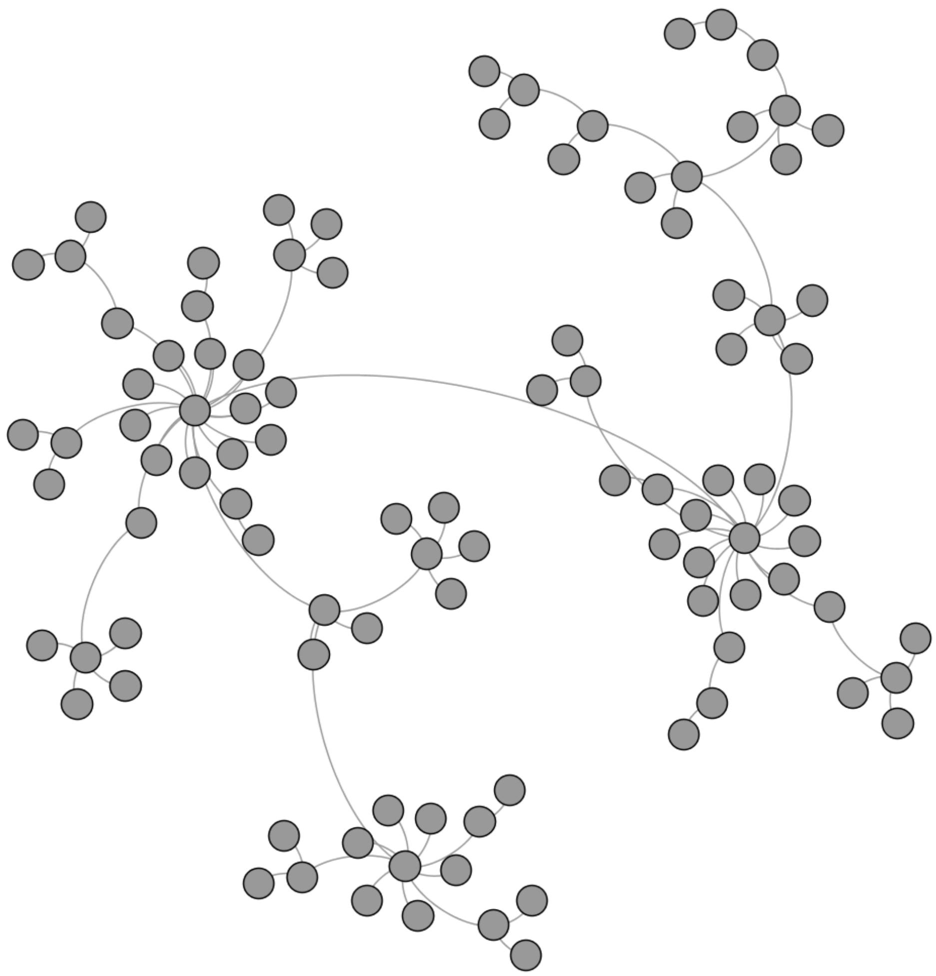

Figure 37: Barbasi-Albert topology with m0=3, m=1

Table 55: Network metrics Barbasi-Albert m0=3, m=1

Avg. degree	1.98
Avg. path-length	4.684
Avg. clustering coefficient	0
Network diameter	11
Graph density	0.02

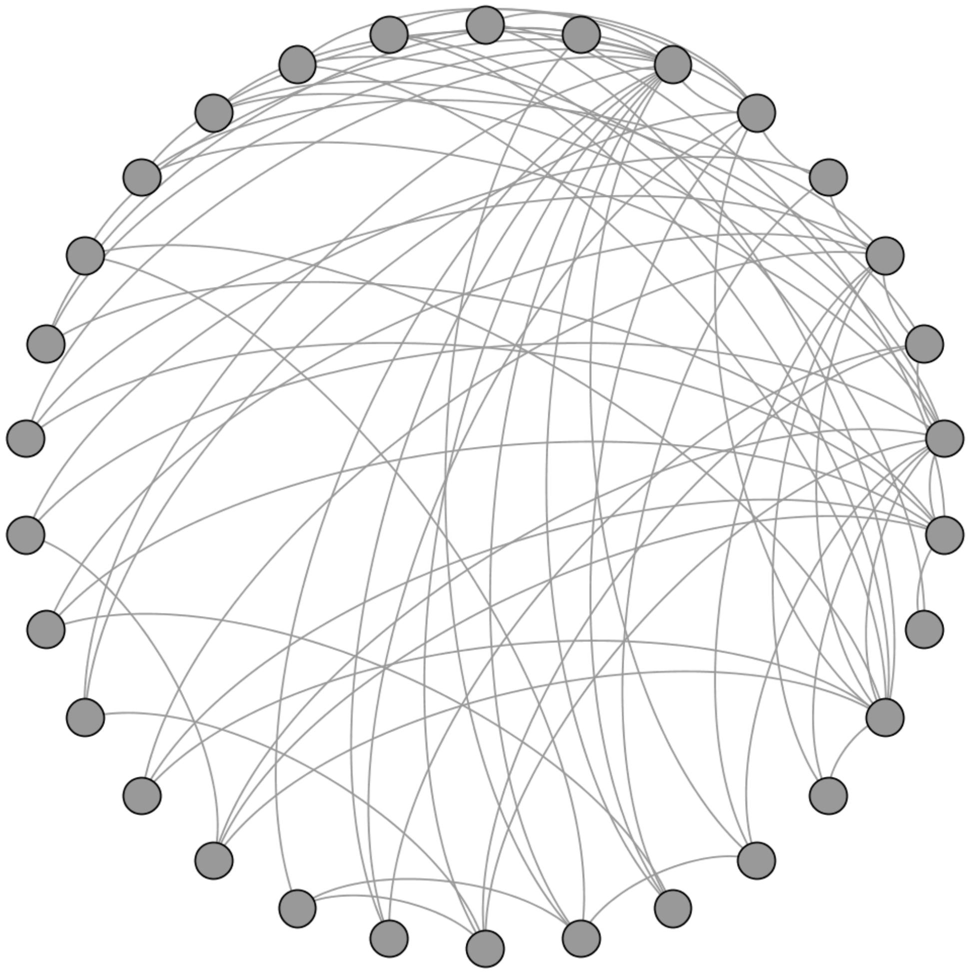

Figure 38: Barbasi-Albert topology with m0=9, m=3

Table 56: Network metrics Barbasi-Albert m0=9, m=3

Avg. degree	4.733
Avg. path-length	2.11
Avg. clustering coefficient	0.279
Network diameter	4
Graph density	0.163

A.6.3 Watts-Strogatz

Creates N nodes and connects each to K neighbours and rewires each then existing edge with a probability of 0.2 to a younger node. See section 2.4 for a discussion of this topology. Included to investigate the influence of small-world an scale-free effects upon equilibrium and because of the randomness of the network.

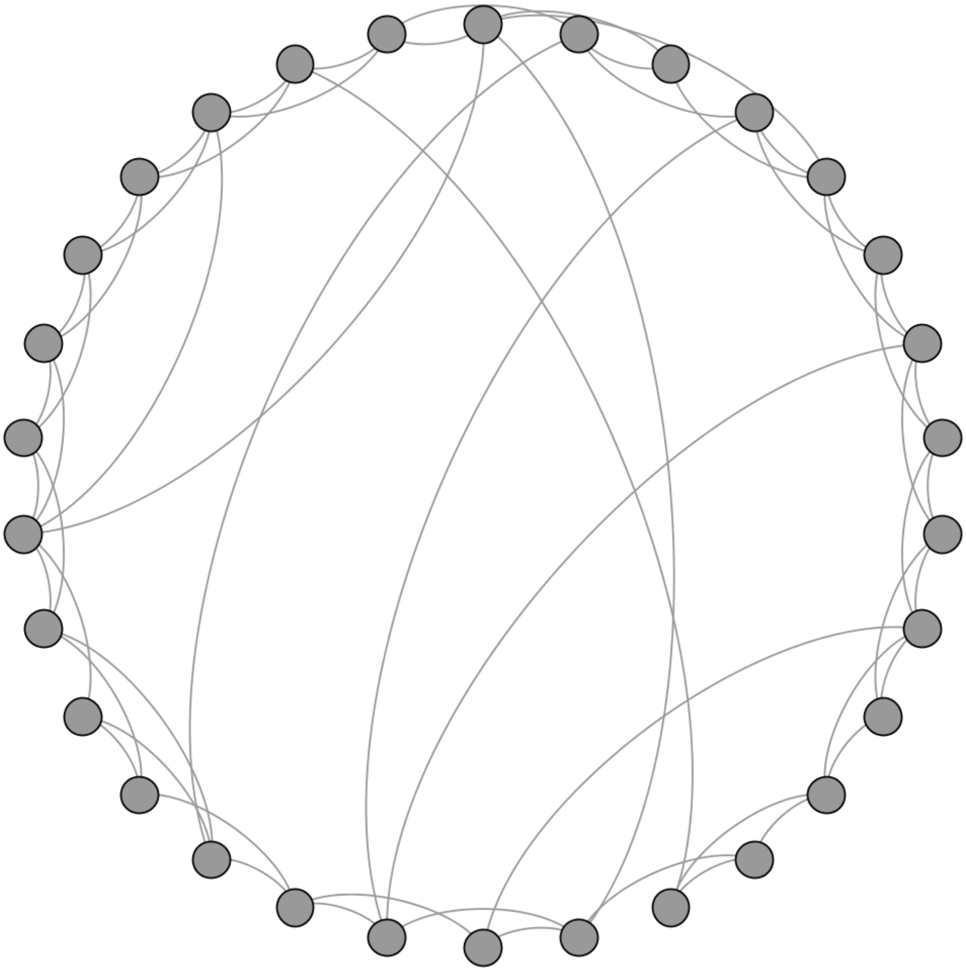

Figure 39: Watts-Strogatz topology with k=2, p=0.2

Table 57: Network metrics Watts-Strogatz k=2, p=0.2

Avg. degree	4
Avg. path-length	2.883
Avg. clustering coefficient	0.259
Network diameter	6
Graph density	0.138

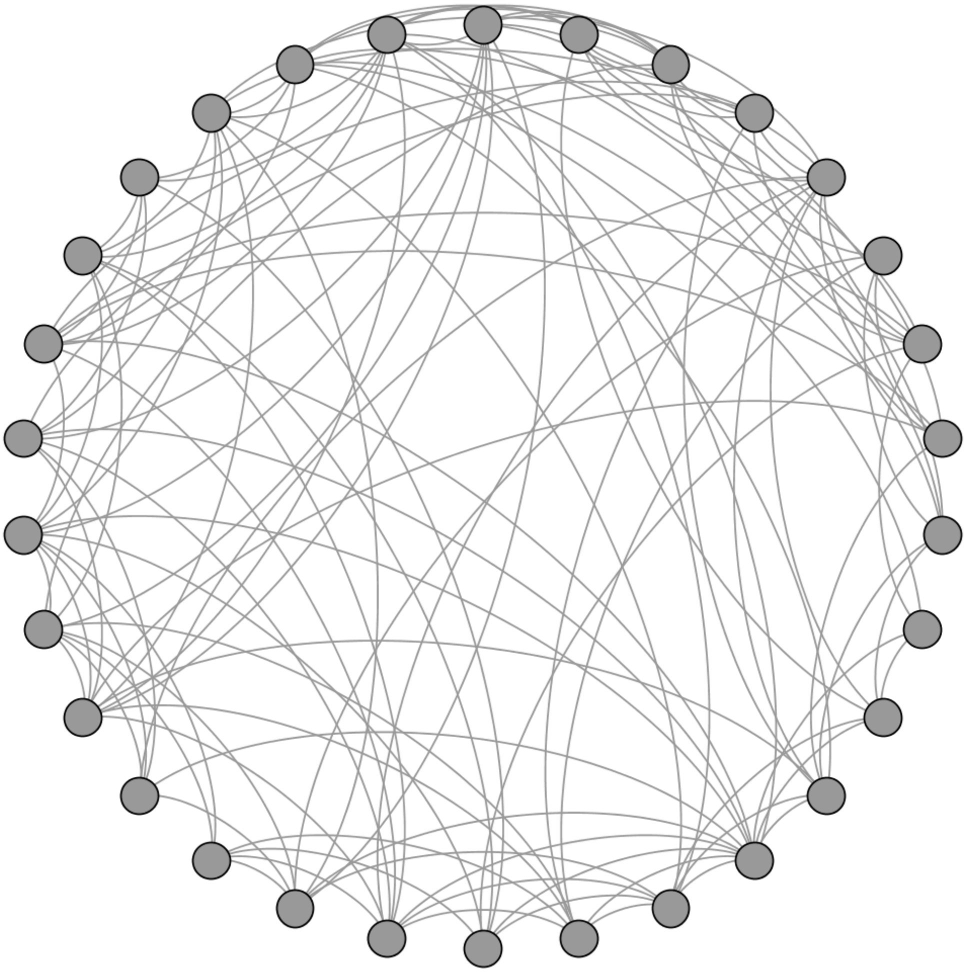

Figure 40: Watts-Strogatz topology with k=4, p=0.5

Table 58: Network metrics Watts-Strogatz k=4, p=0.5

Avg. degree	8
Avg. path-length	1.823
Avg. clustering coefficient	0.241
Network diameter	3
Graph density	0.276

Appendix B

Results for Hub-Based, Scale-Free and Small-World

B.1 Half-Fully Connected

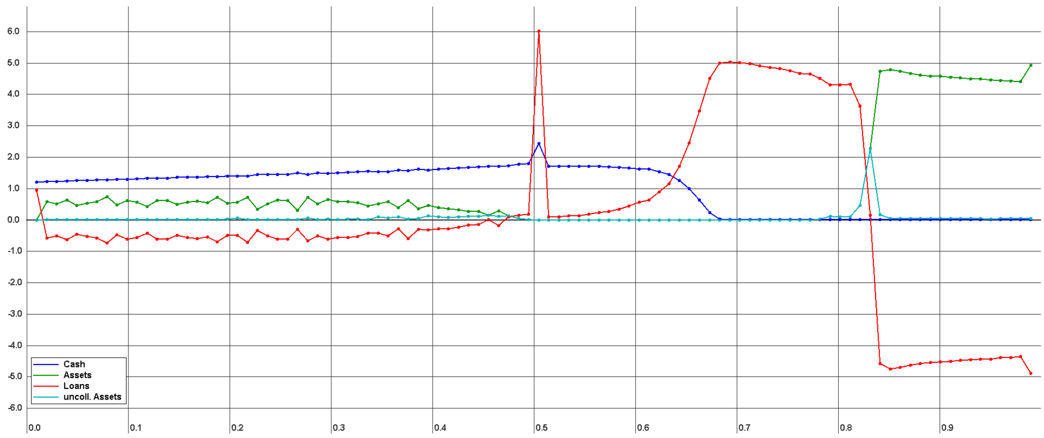

Figure 41: Wealth-Distribution of Half-Fully Connected topology

Table 59: Equilibrium of Half-Fully Connected topology

Asset-Price p	0.651 (0.027)
Bond-Price q	0.362 (0.013)
Marginal agent i1	0.640 (0.015)
Marginal agent i2	0.833 (0.09)
Pessimist Wealth	1.22 (0.096)
Medianist Wealth	2.258 (0.409)
Optimist Wealth	4.526 (0.071)

Table 60: Performance of Half-Fully Connected topology

Successful matching-rounds	14,218.9 (4621.74)
Failed matching-rounds	1034.12 (22.99)
Total matching-rounds	15,253.02 (4633.44)
Ratio successful/total	0.93
Ratio failed/total	0.07

The equilibrium is clearly distinct from the theoretical and Fully-Connected one as miss-allocation can be found within the pessimists-range. Also the i1- and i2-points and the wealth-distributions differ both numerically and visually.

B.2 Ascending-Connected with short-cuts

B.2.1 Random short-cuts

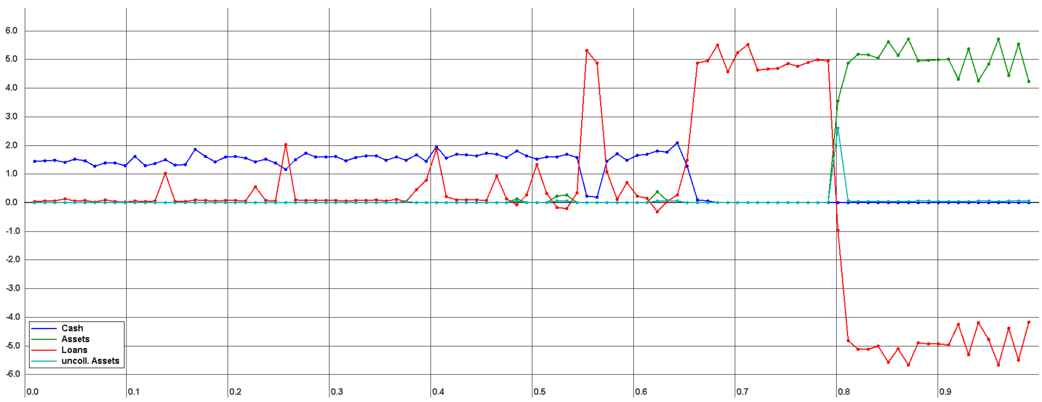

Figure 42: Wealth-Distribution of Ascending-Connected random short-cuts topology

Table 61: Equilibrium of Ascending-Connected random short-cuts topology

Asset-Price p	0.731 (0.019)
Bond-Price q	0.393 (0.009)
Marginal agent i1	0.649 (0.005)
Marginal agent i2	0.804 (0.004)
Pessimist Wealth	1.441 (0.03)
Medianist Wealth	4.282 (0.278)
Optimist Wealth	4.974 (0.038)

Table 62: Performance of Ascending-Connected random short-cuts topology

Successful matching-rounds	8314.78 (229.85)
Failed matching-rounds	1182.06 (29.23)
Total matching-rounds	9496.84 (228.23)
Ratio successful/total	0.87
Ratio failed/total	0.13

Random short-cuts seem to reduce the miss-allocation of pessimists-wealth a bit but lead to a fundamental different equilibrium than the theoretical or fully-connected one as can clearly be seen both visually and numerically.

B.2.2 2 short-cuts

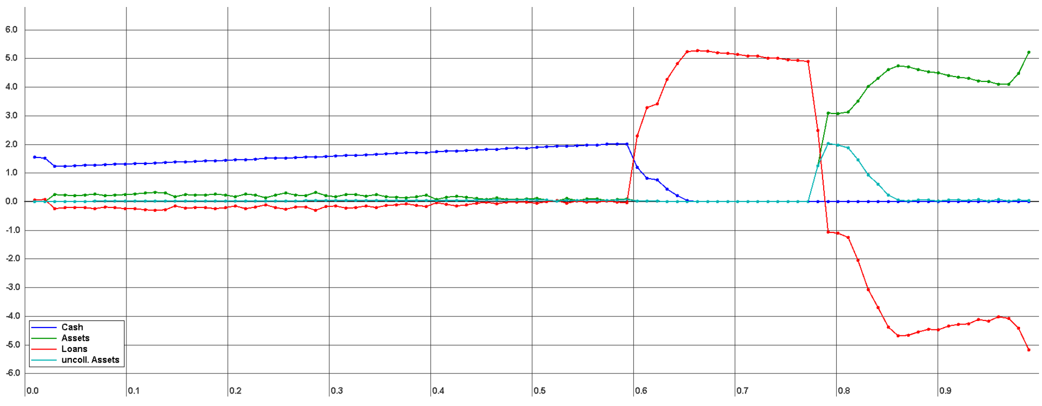

Figure 43: Wealth-Distribution of Ascending-Connected 2 short-cuts topology

Table 63: Equilibrium of Ascending-Connected 2 short-cuts topology

Asset-Price p	0.662 (0.024)
Bond-Price q	0.376 (0.006)
Marginal agent i1	0.608 (0.018)
Marginal agent i2	0.805 (0.028)
Pessimist Wealth	1.441 (0.21)
Medianist Wealth	3.978 (1.442)
Optimist Wealth	4.514 (0.063)

Table 64: Performance of Ascending-Connected random short-cuts topology

Successful matching-rounds	37,093.64 (12,864.4)
Failed matching-rounds	1021. (18.85)
Total matching-rounds	38,115.54 (12,851.53)
Ratio successful/total	0.97
Ratio failed/total	0.03

This topology reduces the miss-allocation in the pessimists-range dramatically but doesn't solve it yet. Unfortunately it leads to a dramatically different wealth-distribution within the medianists and optimist.

B.2.3 5 full short-cuts

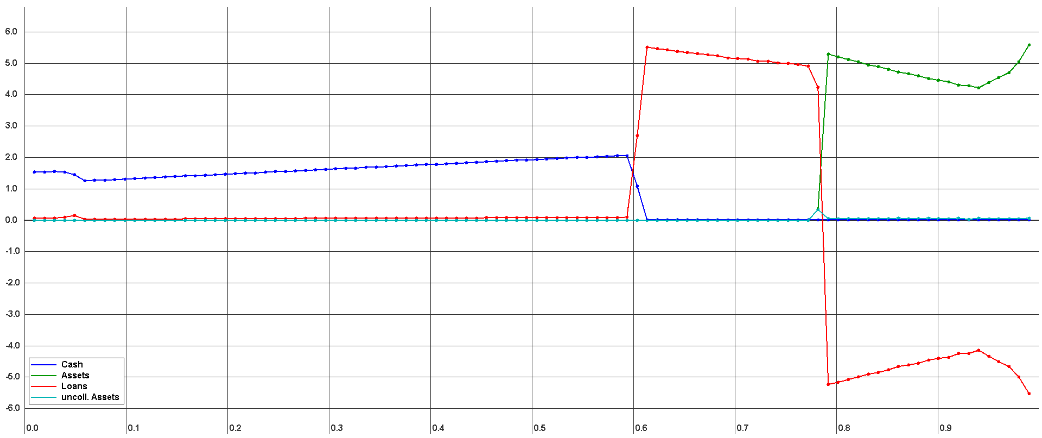

Figure 44: Wealth-Distribution of Ascending-Connected 5 full short-cuts topology

Table 65: Equilibrium of Ascending-Connected 5 full short-cuts

Asset-Price p	0.656 (0.019)
Bond-Price q	0.371 (0.003)
Marginal agent i1	0.594 (0.0)
Marginal agent i2	0.792 (0.0)
Pessimist Wealth	1.649 (0.002)
Medianist Wealth	5.013 (0.018)
Optimist Wealth	4.746 (0.011)

Table 66: Performance of Ascending-Connected 5 full short-cuts topology

Successful matching-rounds	16,971.34 (228.0)
Failed matching-rounds	1026.92 (22.68)
Total matching-rounds	17,998.26 (225.23)
Ratio successful/total	0.94
Ratio failed/total	0.06

As can be clearly seen this topology seems to be able to solve miss-allocations in the pessimists-range seen in Ascending-Connected topology but is still different than the theoretical and Fully-Connected equilibrium.

B.2.4 15 full short-cuts

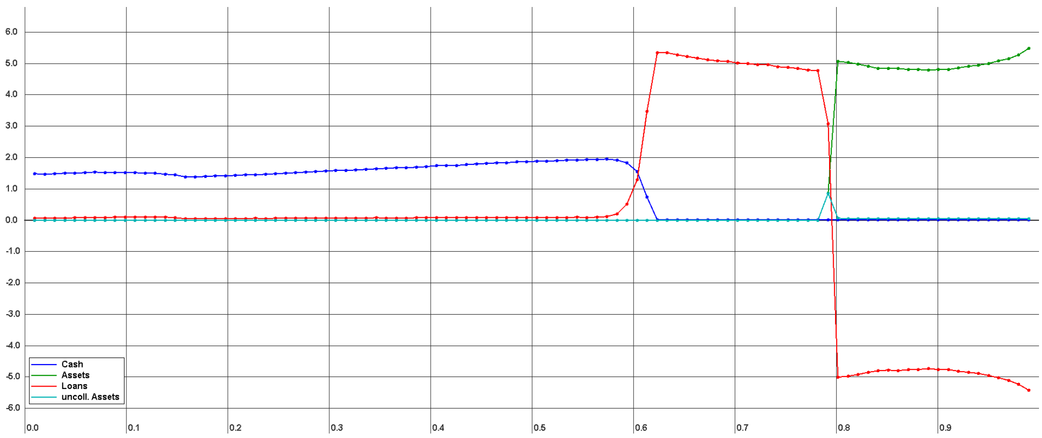

Figure 45: Wealth-Distribution of Ascending-Connected 15 full short-cuts topology

Table 67: Equilibrium of Ascending-Connected 15 full short-cuts topology

Asset-Price p	0.658 (0.024)
Bond-Price q	0.366 (0.009)
Marginal agent i1	0.601 (0.004)
Marginal agent i2	0.802 (0.0)
Pessimist Wealth	1.649 (0.004)
Medianist Wealth	4.811 (0.092)
Optimist Wealth	4.957 (0.021)

Table 68: Performance of Ascending-Connected 15 full short-cuts topology

Successful matching-rounds	4498.08 (58.67)
Failed matching-rounds	1024.78 (17.3)
Total matching-rounds	5522.860 (64.72)
Ratio successful/total	0.81
Ratio failed/total	0.19

This topology comes very close to the theoretical equilibrium but is still a bit different as can be seen in the curved wealth-distributions of the pure optimists.

B.2.5 30 full short-cuts

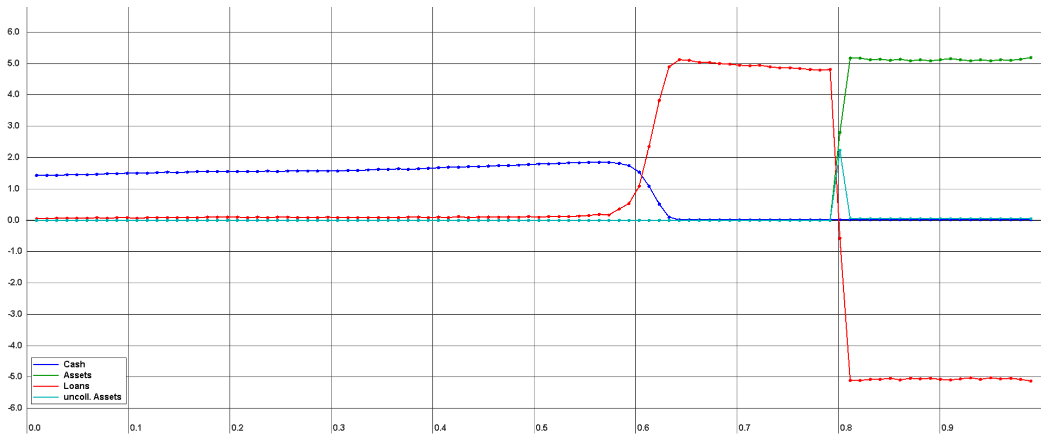

Figure 46: Wealth-Distribution of Ascending-Connected 30 full short-cuts topology

Table 69: Equilibrium of Ascending-Connected 30 full short-cuts topology

Asset-Price p	0.681 (0.012)
Bond-Price q	0.378 (0.006)
Marginal agent i1	0.603 (0.006)
Marginal agent i2	0.802 (0.1)
Pessimist Wealth	1.649 (0.009)
Medianist Wealth	4.702 (0.112)
Optimist Wealth	5.004 (0.025)

Table 70: Performance of Ascending-Connected 30 full short-cuts topology

Successful matching-rounds	2211.08 (35.88)
Failed matching-rounds	1014.68 (10.55)
Total matching-rounds	3225.76 (40.18)
Ratio successful/total	0.68
Ratio failed/total	0.32

This topology is very close to the theoretical and Fully-Connected equilibrium although it differs in asset-price p and in the wealth-distributions. Of course with 30 fully short-cuts in a network of 100 agents one is already very close to fully connectedness.

B.2.6 5 regular short-cuts

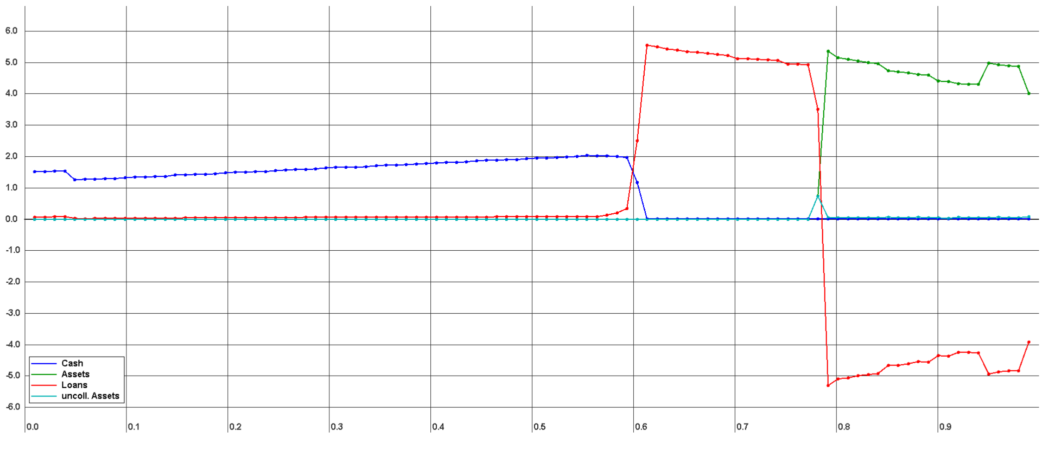

Figure 47: Wealth-Distribution of Ascending-Connected 5 regular short-cuts topology

Table 71: Equilibrium of Ascending-Connected 5 regular short-cuts topology

Asset-Price p	0.665 (0.016)
Bond-Price q	0.364 (0.007)
Marginal agent i1	0.595 (0.003)
Marginal agent i2	0.792 (0.0)
Pessimist Wealth	1.649 (0.003)
Medianist Wealth	4.991 (0.045)
Optimist Wealth	4.727 (0.011)

Table 72: Performance of Ascending-Connected 5 regular short-cuts topology

Successful matching-rounds	14,570.44 (157.61)
Failed matching-rounds	1064.24 (29.88)
Total matching-rounds	15,634.68 (166.21)
Ratio successful/total	0.93
Ratio failed/total	0.07

As can be seen in the visual results this topology shows a different equilibrium than the theoretical and Fully-Connected one.

B.2.7 15 regular short-cuts

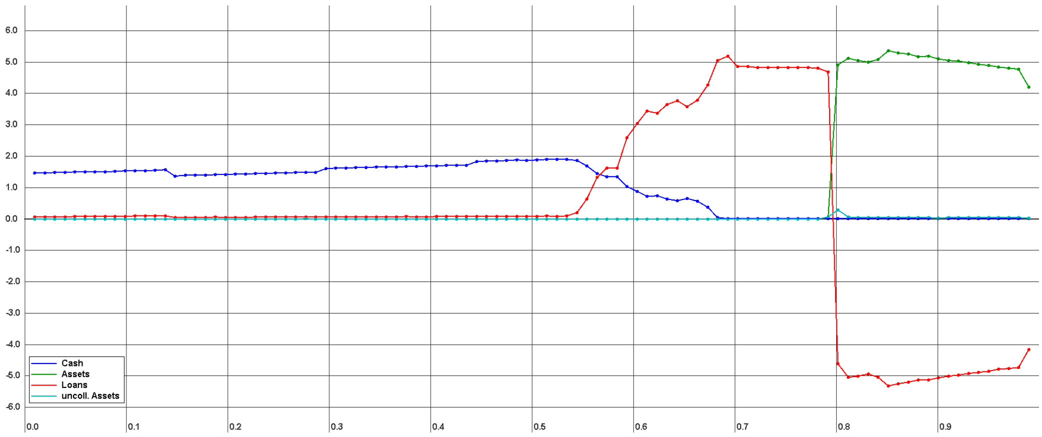

Figure 48: Wealth-Distribution of Ascending-Connected 15 regular short-cuts topology

Table 73: Equilibrium Ascending-Connected 15 regular short-cuts topology

Asset-Price p	0.705 (0.020)
Bond-Price q	0.357 (0.018)
Marginal agent i1	0.586 (0.023)
Marginal agent i2	0.802 (0.0)
Pessimist Wealth	1.649 (0.051)
Medianist Wealth	4.146 (0.101)
Optimist Wealth	4.997 (0.007)

Table 74: Performance of Ascending-Connected 15 regular short-cuts topology

Successful matching-rounds	4373.28 (50.13)
Failed matching-rounds	1129.24 (19.2)
Total matching-rounds	5502.52 (52.11)
Ratio successful/total	0.79
Ratio failed/total	0.21

The equilibrium of this topology is falls very far from the theoretical and Fully-Connected one.

B.2.8 30 regular short-cuts

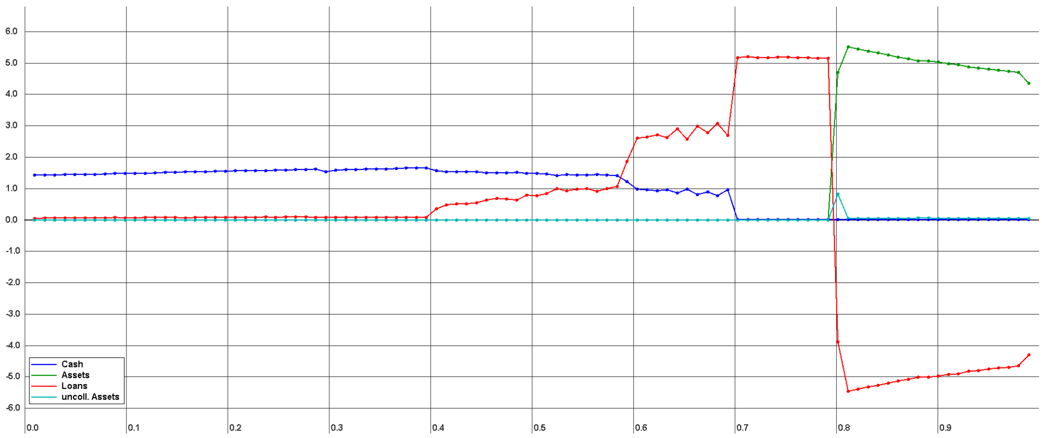

Figure 49: Wealth-Distribution of Ascending-Connected 30 regular short-cuts topology

Table 75: Equilibrium of Ascending-Connected 30 regular short-cuts topology

Asset-Price p	0.710 (0.021)
Bond-Price q	0.398 (0.008)
Marginal agent i1	0.589 (0.021)
Marginal agent i2	0.802 (0.0)
Pessimist Wealth	1.479 (0.049)
Medianist Wealth	3.713 (0.125)
Optimist Wealth	5.0 (0.0)

Table 76: Performance of Ascending-Connected 30 regular short-cuts topology

Successful matching-rounds	5427.02 (90.82)
Failed matching-rounds	1139.04 (27.74)
Total matching-rounds	6566.06 (96.04)
Ratio successful/total	0.82
Ratio failed/total	0.18

The equilibrium of this topology is falls very far from the theoretical and Fully-Connected one.

B.3 Hub-Based topologies

The Hub-Based Topologies fail to come even close to equilibrium due to reasons given in chapter 4 "Hypothesis". This can be seen also very clearly in the visual results and thus no performance- and equilibrium-tables are listed as they would not make any sense.

B.3.1 3-Hubs

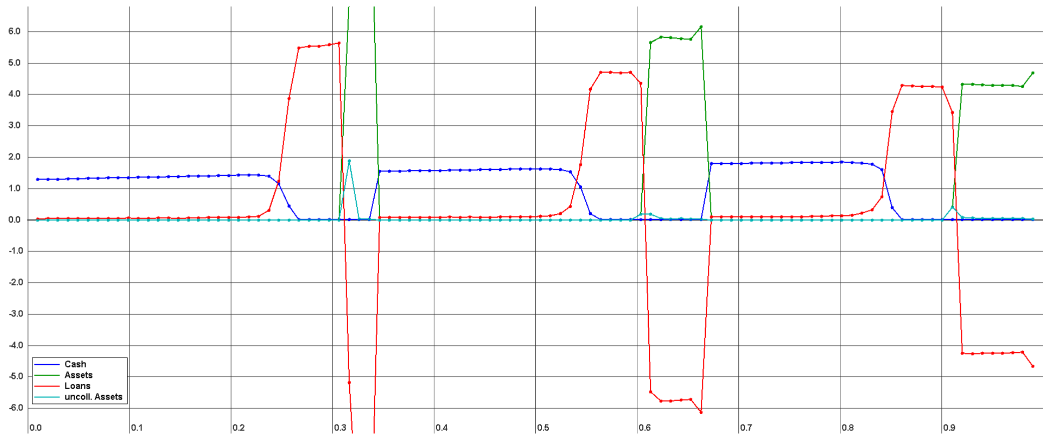

Figure 50: Wealth-Distribution of 3-Hubs topology

B.3.2 1-Median Hub

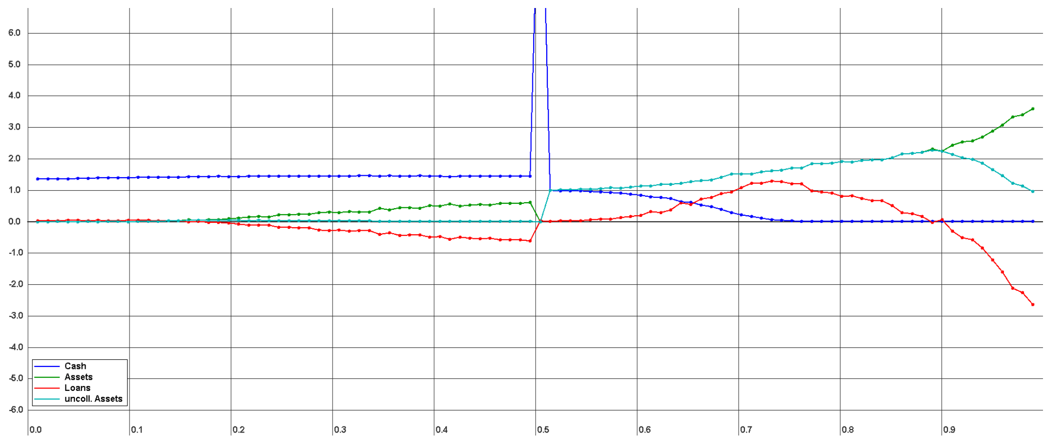

Figure 51: Wealth-Distribution of 1 Median-Hub topology

B.3.3 3-Median Hubs

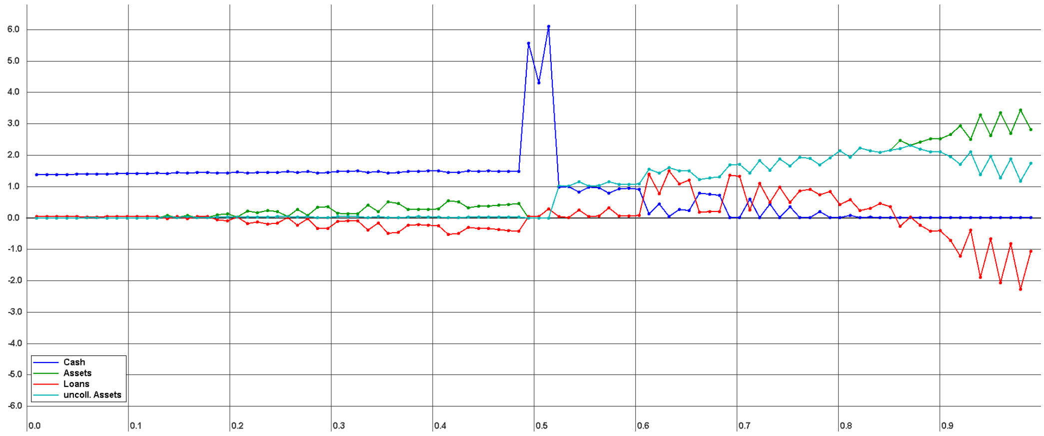

Figure 52: Wealth-Distribution of 3 Median-Hubs topology

B.3.4 Maximum Hub

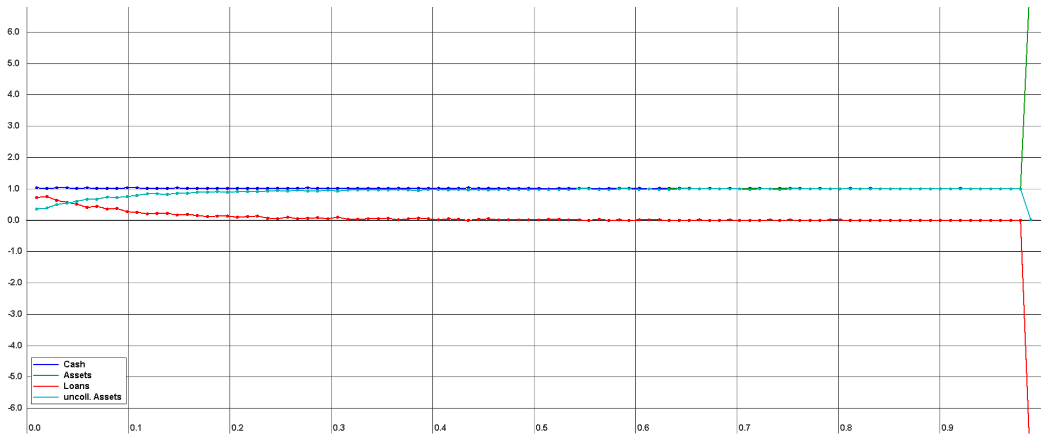

Figure 53: Wealth-Distribution of Maximum-Hub topology

B.4 Scale-Free and Small-World topologies

This topologies fail to come even close to equilibrium too due to reasons given in chapter 4 ”Hypothesis”. This can be seen also very clearly in the visual results and thus no performance- and equilibrium-tables are listed as they would not make any sense.

B.4.1 Erdos-Renyi

Note that with the correct parametrization this topology could satisfy the hypothesis by pure chance. The result would be a pure random network as an Ascending-Connected topology with random short-cuts but as already showed above this Ascending-Connected random short-cuts network fails from producing the theoretical and Fully-Connected equilibrium.

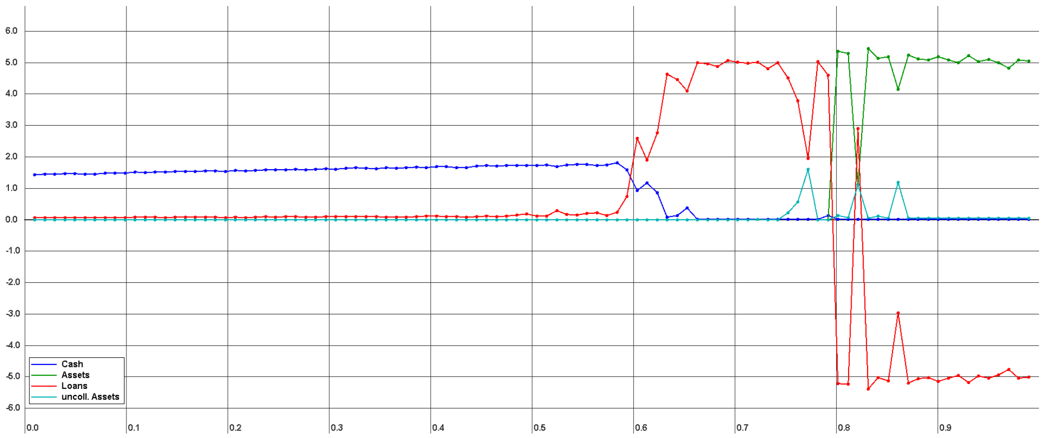

Figure 54: Wealth-Distribution of Erdos-Renyi 0.2 topology

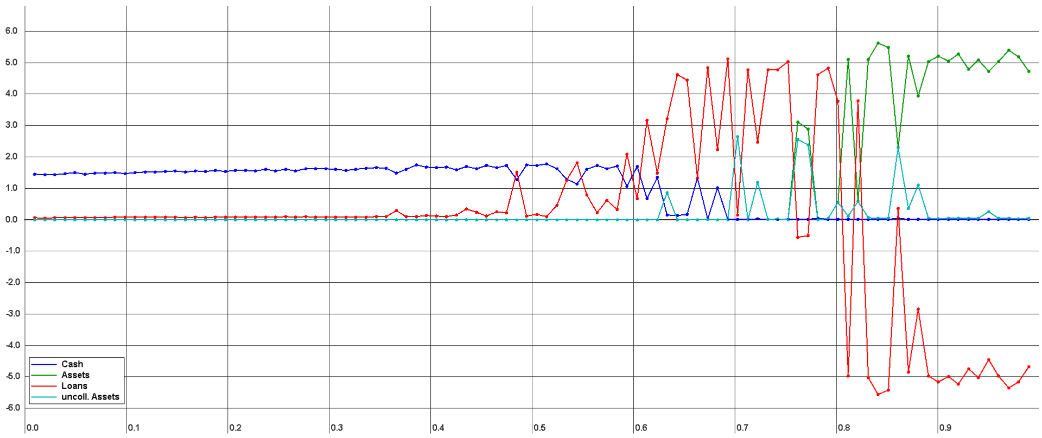

Figure 55: Wealth-Distribution of Erdos-Renyi 0.1 topology

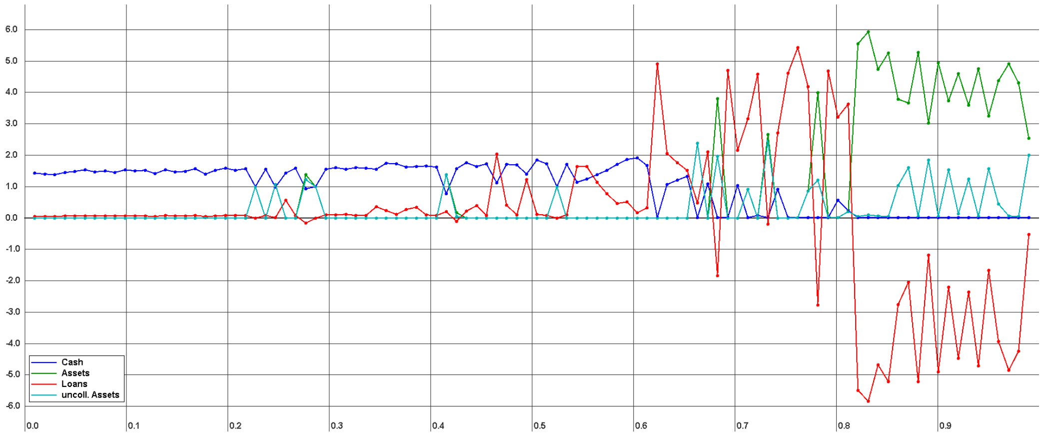

Figure 56: Wealth-Distribution of Erdos-Renyi 0.05 topology

B.4.2 Barbasi-Albert

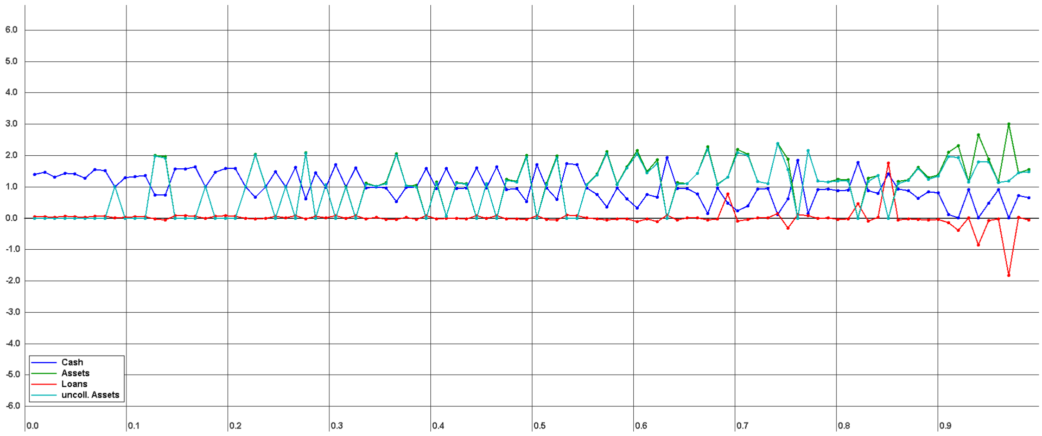

Figure 57: Wealth-Distribution of Barbasi-Albert m0=3, m=1 topology

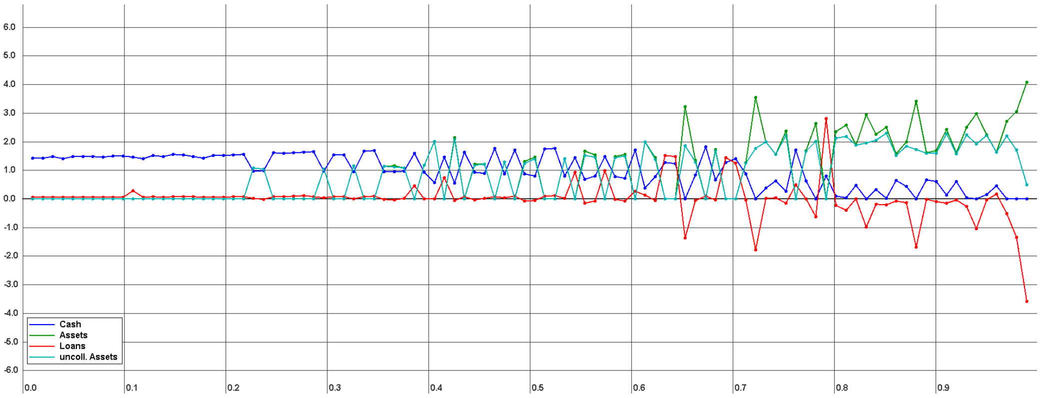

Figure 58: Wealth-Distribution of Barbasi-Albert m0=9, m=3 topology

B.4.3 Watts-Strogatz

Note that with the correct parametrization this topology could satisfy the hypothesis by pure chance too. The result would be a pure random network as an Ascending-Connected topology with random short-cuts but as already showed above this Ascending-Connected random short-cuts network fails from producing the theoretical and Fully-Connected equilibrium.

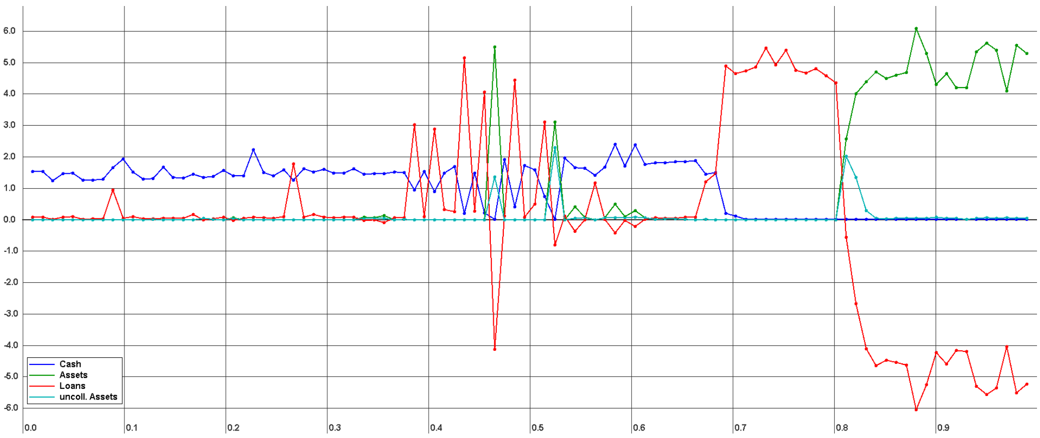

Figure 59: Wealth-Distribution of Watts-Strogatz k=2, b=0.2 topology

Figures

List of Tables

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