Engineering Optimization Homework

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January 2024

1 Big M AND The Two-Phase Method

Max $z = 2x_1 + 3x_2$
s.t.

- $x_1 + 2x_2 \leq 4$
- $x_1 + x_2 = 3$
- $x_1, x_2 \geq 0$

Initialization

$$\begin{array}{lrrrr} z & -2x_1 - 3x_2 & & + M\bar{x}_4 & = 0 \\ & x_1 + 2x_2 & +x_3 & & = 4 \\ & x_1 + x_2 & & + \bar{x}_4 & = 3 \end{array} \tag{1}$$

Iteration 0

$$\begin{array}{rl} & Z - 2x_1 - 3x_2 + M\bar{x}_4 = 0 \\ & -M(x_1 + x_2 + \bar{x}_4 = 3) \\ \textbf{new} & Z - (M+2)x_1 - (M+3)x_2 = -3M \end{array} \tag{2}$$

Iteration 1

$$\begin{array}{l} x_2 = 0 \\ x_3 = 4 - x_1 \geq 0 \Rightarrow x_1 \leq 4 \\ \bar{x}_4 = 3 - x_1 \geq 0 \Rightarrow x_1 \leq 3 \quad \textit{minimum} \end{array} \tag{3}$$

$x_1 + \bar{x}_4 = 3$

$$\begin{array}{lrrrr} z & -(M+2)x_1 & & +(M+2)\bar{x}_4 & = 6 \\ & 2x_2 & +x_3 - \bar{x}_4 & & = 1 \\ & x_1 & & + \bar{x}_4 & = 3 \end{array} \tag{4}$$

Iteration 2

$$\begin{array}{l} x_1 = 0 \\ x_3 = 4 - 2x_2 \geq 0 \Rightarrow x_2 \leq \dfrac{4}{2} = 2 \quad \textit{minimum} \\ \bar{x}_4 = 3 - x_2 \geq 0 \Rightarrow x_2 \leq 3 \end{array} \tag{5}$$

$x_2 + \frac{1}{2}x_3 = 2$

$$\begin{array}{lrrrr} z & -(M+2)x_1 & +\dfrac{M+3}{2}x_3 & & = 6 - m \\ & \dfrac{1}{2}x_1 + x_2 & +\dfrac{1}{2}x_3 & & = 2 \\ & x_1 & -\dfrac{1}{2}x_3 + \bar{4}_4 & & = 1 \end{array} \tag{6}$$

Iteration	Basis Variable	Eq.	Coefficient of:					Right Side
			Z	x_1	x_2	x_3	$\bar{x}_4$	
0	Z	(0)	1	$-(M+2)$	$-(M+3)$	0	0	$-3M$
	x_3	(1)	0	0	2	1	-1	1
	$\bar{x}_4$	(2)	0	1	1	0	1	3
1	Z	(0)	1	0	$-(M+3)$	0	$M+2$	6
	x_1	(1)	0	1	0	1	0	4
	$\bar{x}_4$	(2)	0	1	0	0	1	3
2	Z	(0)	1	$-(M+2)$	0	$\frac{M+3}{2}$	0	$6-M$
	x_2	(1)	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	2
	$\bar{x}_4$	(2)	0	1	0	$-\frac{1}{2}$	1	1