

Engineering Optimization Homework

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1 Max

$$\begin{aligned} \text{Max } z &= 4x_1 + 3x_2 + 6x_3 \\ \text{s.t.} \end{aligned}$$

- $3x_1 + x_2 + 3x_3 \leq 30$
- $2x_1 + 2x_2 + 3x_3 \leq 40$
- $x_1, x_2, x_3 \geq 0$

Iteration 0

$$\begin{array}{rcccccl} z & -4x_1 & -3x_2 & -6x_3 & & = 0 \\ & 3x_1 & +x_2 & +3x_3 & +x_4 & = 30 \\ & 2x_1 & +2x_2 & +3x_3 & & +x_5 = 40 \end{array} \quad (1)$$

Iteration 1

$$\begin{aligned} x_1 &= 0, x_3 = 0 \\ x_4 &= 30 - x_2 \geq 0 \Rightarrow x_2 \leq 30 \\ x_5 &= 40 - 2x_2 \geq 0 \Rightarrow x_2 \leq \frac{40}{2} = 20 \quad \text{minimum} \end{aligned} \quad (2)$$

$$x_2 + \frac{1}{2}x_5 = 20$$

$$\begin{array}{rcccccl} z & -4x_1 & & -6x_3 & +\frac{3x_5}{2} & = 60 \\ & 3x_1 & & +3x_3 & +x_4 & +\frac{x_5}{2} = 10 \\ & x_1 & +x_2 & +\frac{3x_3}{2} & & +\frac{x_5}{2} = 20 \end{array} \quad (3)$$

Iteration 2

$$x_2 = 0, x_3 = 0$$

$$x_4 = 30 - 3x_1 \geq 0 \Rightarrow x_1 \leq \frac{30}{3} = 10 \quad \text{minimum} \quad (4)$$

$$x_5 = 40 - 2x_1 \geq 0 \Rightarrow x_1 \leq \frac{40}{2} = 20$$

$$x_1 + \frac{1}{3}x_4 = 10$$

$$\begin{array}{rcccccl} z & & -3x_2 & -6x_3 & +\frac{16x_4}{3} & = 40 \\ & x_1 & +\frac{x_2}{3} & +x_3 & +\frac{x_4}{3} & = 10 \\ & & 2x_2 & +3x_3 & -\frac{2x_4}{3} & +x_5 = 20 \end{array} \quad (5)$$

Iteration 3

$$x_1 = 0, x_2 = 0$$

$$x_4 = 30 - 3x_3 \geq 0 \Rightarrow x_3 \leq \frac{30}{3} = 10 \quad \text{minimum} \quad (6)$$

$$x_5 = 40 - 3x_3 \geq 0 \Rightarrow x_3 \leq \frac{40}{3}$$

$$x_3 + \frac{1}{3}x_4 = 10$$

$$\begin{array}{rcccccl} z & -4x_1 & -3x_2 & & +2x_4 & = 60 \\ & x_1 & +\frac{x_2}{3} & +x_3 & +\frac{x_4}{3} & = 10 \\ & 2x_1 & +2x_2 & +3x_3 & -x_4 & +x_5 = 10 \end{array} \quad (7)$$

Iteration	Basis Variable	Eq.	Coefficient of:						Right Side
			Z	x_1	x_2	x_3	x_4	x_5	
0	Z	(0)	1	-4	-3	-6	0	0	0
	x_4	(1)	0	3	1	3	1	0	30
	x_5	(2)	0	2	2	3	0	1	40
1	Z	(0)	1	-4	0	-6	0	$\frac{3}{2}$	60
	x_4	(1)	0	3	0	3	1	$\frac{1}{2}$	10
	x_2	(2)	0	1	1	$\frac{3}{2}$	0	$\frac{1}{2}$	20
2	Z	(0)	1	0	-3	-6	$\frac{4}{3}$	0	40
	x_2	(1)	0	1	$\frac{1}{3}$	1	$\frac{1}{3}$	0	10
	x_5	(2)	0	0	2	3	$\frac{2}{3}$	1	20
3	Z	(0)	1	-4	-3	0	2	0	60
	x_3	(1)	0	1	$\frac{2}{3}$	1	$\frac{4}{3}$	0	10
	x_5	(2)	0	2	2	0	-1	1	10