Engineering Optimization Homework

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1 Is the intersection of two convex sets convex?

- lf the intersection is empty, or consists of a single point, it is true bydefinition
- Otherwise, take any two points A, B in the intersection. The line ABjoining these points must also lie wholly within their intersection
- Therefore, the intersection is a convex set
- The intersection of any number of convex sets is also convex

Proof. Let A and B be two convex sets. Show by $A \cap B$ is also convex.

Consider any two points x and y, $\forall x, y \in A \cap B$.

$x \in A, x \in B, y \in A, y \in B$

To prove that $A \cap B$ is convex, we need to show that for any $\lambda \in [0, 1]$, the point $\lambda x + (1 - \lambda)y$ is also in $A \cap B$.

Since $x, y \in A$, and A is convex, we have $\lambda x + (1 - \lambda)y \in A(\lambda \in [0, 1])$.

Similarly, since $x, y \in B$, and B is convex, we have $\lambda x + (1 - \lambda)y \in B(\lambda \in [0, 1])$.

Therefore, $\lambda x+(1-\lambda)y$ is in both A and B, which implies that $\lambda x+(1-\lambda)y \in A \cap B$.

Since this holds for any two points x and y in $A \cap B$, it follows that $A \cap B$ is convex.

Thus, we have proven that the intersection of two convex sets is convex.

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