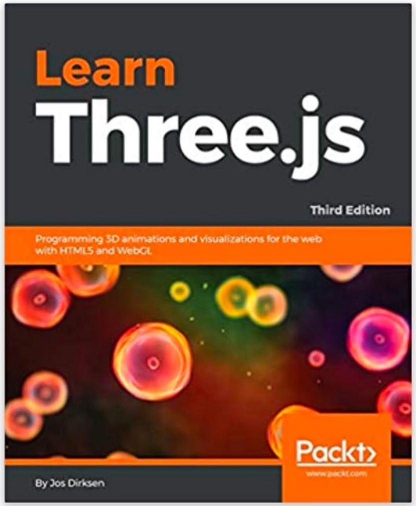
# COMPUTER GRAPHICS




* Adapted from CISC 3326 lecture by Michael Mandel

# PARAMETRIC LINES AND COLOR INTERPOLATION

Based on this CS 307 reading and this CS 307 lecture*

# THREE.JS SUPPORT FOR COLOR INTERPOLATION

# Reminder: Color mixing

- Problem: Suppose that vertex A is red (1,0,0) and vertex B is magenta (1,1,0).
  - What is the color of the point that is 2/3 of the way from A to B?

# Solution

- Solution: We can use the same mixture equation that we just used to find coordinates:
  - $P(2/3) = A(1/3) + B(2/3)$
  - $P(2/3) = (1,0,0)(1/3) + (1,1,0)(2/3)$
  - $P(2/3) = (1,2/3,0)$
- **Note:**
  - if a problem calls for more than one line, each line gets its own parameter, such as r, s, or u.
  - Each parameter has a meaning:
    - t=0 means the initial point of the line
      - so s=0 would be the initial point of the other line.

# Interpolation

- "a method of constructing new *data points* within the range of a discrete set of known data points"
- Linear interpolation:
  - Finding the points between points $p_s$ and $p_f$
  - Parametric equations enable doing this
    - $\mathrm{B}(t) = p_s + (p_f - p_s)t$

# Three.js Support for Interpolation

- Three.js has some useful functions for doing interpolation on [Vector3](Vector3) objects.
- Suppose v1, v2, and v3 are all Vector3 objects:
  - v1.add(v2) adds vector v2 to v1
  - v1.addVectors(v2,v3) sets v1 to the sum v2+v3
  - v1.multiplyScalar(s) multiplies v1 by a scalar s
  - v1.sub(v2) subtracts vector v2 from v1
  - v1.subVectors(v2,v3) sets v1 to the difference v2-v3
  - v1.lerp(v2,theta) moves v1 towards v2, by a fraction $\theta$, using linear interpolation

# Three.js Support for Interpolation

- Warning, these methods all modify the object
  - so if you want to compute a new vertex, it’s best to .clone() the vertex first.

# Computing with Three.js interpolation

- Here's an example of:
  - computing a new point $B$, given a point $A$ and a vector $V$
  - then computing the midpoint of the segment from $A$ to $B$
    - var A = new THREE.Vector3(1,3,5);
    - var V = new THREE.Vector3(10,20,30);
    - var B = A.clone();
    - B.add(V);
    - alert("B is " + JSON.stringify(B));
    - var Mid1 = A.clone();
    - Mid1.lerp(B,0.5); alert("midpoint is " + JSON.stringify(Mid1));

# Computing with Three.js interpolation

- Here’s another way to do the same thing
  - var A = new THREE.Vector3(1,3,5);
  - var V = new THREE.Vector3(10,20,30);
  - var B = A.clone();
  - var Mid2 = A.clone();
  - var Vhalf = V.clone();
  - Vhalf.multiplyScalar(0.5);
  - Mid2.add(Vhalf);
  - alert("midpoint is also " + JSON.stringify(Mid2));

# Interpolating the steeple coordinate using Three.js

- The code below carries out our earlier computation of the coordinates of the point B for the steeple, given the vertices R and S.
    - var R = new THREE.Vector3(15,55,0);
    - var S = new THREE.Vector3(0,30,0);
    - var Ans1 = S.clone();
    - Ans1.lerp(R,0.8);
    - alert("Ans1 is " + JSON.stringify(Ans1));

# Interpolating the steeple coordinate using Three.js

- An alternative method:
  - var R = new THREE.Vector3(15,55,0);
  - var S = new THREE.Vector3(0,30,0);
  - var V = new THREE.Vector3();
  - V.subVectors(S,R); // vector down the roof
  - V.multiplyScalar(0.2);
  - var Ans2 = R.clone();
  - Ans2.add(V);
  - alert("Ans2 is " + JSON.stringify(Ans2));

# Colors, Interpolation and RGB

- RGB color is a three-dimensional system just like our 3D spatial coordinates.
- Color interpolation works pretty much the same way as spatial interpolation.
- Assuming the vertex S is cyan and R is red, what color is B?
    - var cyan = new THREE.Vector3(0,1,1);
    - var red = new THREE.Vector3(1,0,0);
    - var mix = cyan.clone();
    - mix.lerp(red,0.8);
    - alert("mix is " + JSON.stringify(mix));

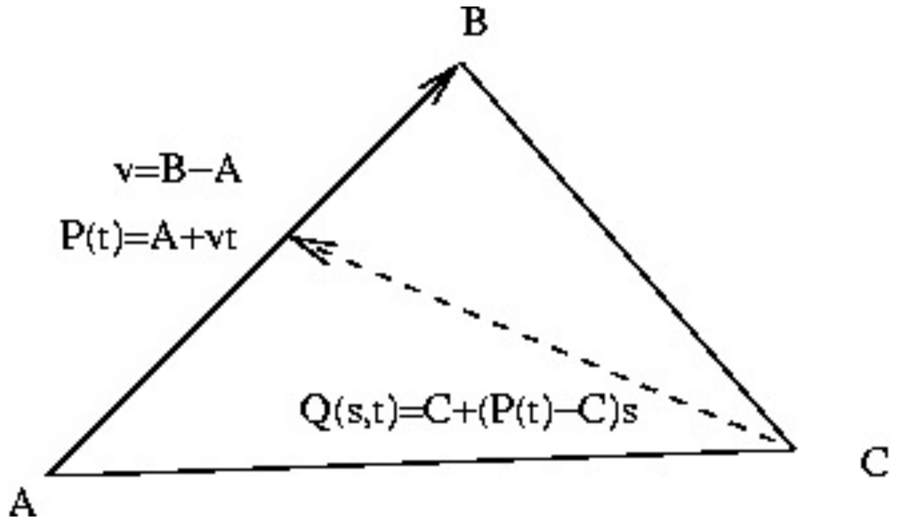
# Parametric Equation for a Triangle

- Since a triangle is a 2D thing, the parametric equation for a triangle will have *two* parameters
  - the first parameter, say t, moves you along one side of the triangle, from vertex A to vertex B.
    - Let P(t) be that point along the AB edge of the triangle.
  - The second parameter, say s, is the parameter of a line from vertex C to P(t).
    - That is, the endpoint of the second line is a moving target.
  - The point Q(s,t) is a point in the triangle, on a line between C and P(t).

# Parametric equation for a triangle

- $Q(s,t) = C + (P(t) - C)s$
- $Q(s,t) = C + (P(t)s - Cs)$
- $Q(s,t) = [A(1-t) + B(t)]s + C(1-s)$
- $Q(s,t) = A(1-t)s + Bts + C(1-s)$

• Lines to set up parametric equation of a triangle:



# Choices

- Notice that we have several choices:
  - The line from A to B could instead go from B to A.
  - Similarly, the line from C to P(t) could go from P(t) to C.
- These yield equivalent equations
  - just as the equation of a line from A to B is equivalent to the equation of a line from B to A.

# Parametric triangle as a weighted sum

$$Q(s,t) = A(1-t)s + Bts + C(1-s)$$

- This is a three-way mixture of the vertices.
- Meaning a triangle is all points in the *convex sum* of the vertices.
- A *convex sum* is a weighted sum of N things, where the weights all add up to 1.0:
- $S = w1 \cdot A + w2 \cdot B + w3 \cdot C$
- $1 = w1 + w2 + w3$

# Parametric triangle as weighted sum

- Do the weights sum to 1 for

$$Q(s,t) = A(1-t)s + Bts + C(1-s)$$

- Let's see

$$(1-t)s + ts + (1-s) = 1$$

- Incidentally, the center of the triangle is where all the weights are equal: one-third.

# Example: Equation of a Triangle from Three Points

- Suppose we have a triangle ABC whose vertices are:
  - $A = (1,2,3)$
  - $B = (2,4,1)$
  - $C = (3,1,5)$
- We could write down the following equation for the triangle:
  - $Q(s,t) = A(1-t)s + B(t)s + C(1-s)$
  - $Q(s,t) = (1,2,3)(1-t)s + (2,4,1)ts + (3,1,5)(1-s)$

# Example: Equation of a Triangle from Three Points

- We could write down the following equation for the triangle:
  - $Q(s,t) = A(1-t)s + B(t)s + C(1-s)$
  - $Q(s,t) = (1,2,3)(1-t)s + (2,4,1)ts + (3,1,5)(1-s)$
- Each coordinate separately is:
  - $x(s,t) = (1-t)s + 2ts + 3(1-s)$
  - $y(s,t) = 2(1-t)s + 4ts + (1-s)$
  - $z(s,t) = 3(1-t)s + ts + 5(1-s)$

# Example: Equation of a Triangle from Three Points

- We can simplify this algebraically to:
  - $x(s,t) = (1-t)s + 2ts + 3(1-s) =$
  - $= s - ts + 2ts + 3 - 3s =$
  - $= ts - 2s + 3$

  - $y(s,t) = 2(1-t)s + 4ts + (1-s) =$
  - $= 2s - 2ts + 4ts + 1 - s =$
  - $= 2ts + s + 1$

  - $z(s,t) = 3(1-t)s + ts + 5(1-s) =$
  - $= 3s - 3ts + ts + 5 - 5s =$
  - $= -2ts - 2s + 5$

# Example: Equation of a Triangle from Three Points

- Suppose a point's parameters with respect to that triangle are (0.5,0.5).
  - What does that mean?
  - It means that the point is halfway between C and the midpoint of AB.
- The coordinates are:
    - $x(0.5,0.5) = (0.5)(0.5) - 2(0.5) + 3 = 2.25$
    - $y(0.5,0.5) = 2(1 - 0.5)0.5 + 4(0.5)(0.5) + (1 - 0.5) = 2$
    - $z(0.5,0.5) = 3(1 - 0.5)0.5 + (0.5)(0.5) + 5(1 - 0.5) = 3.25$
  - => the coordinates of $Q(0.5,0.5)$ are $(2.25,2,3.25)$

# Equivalent solution

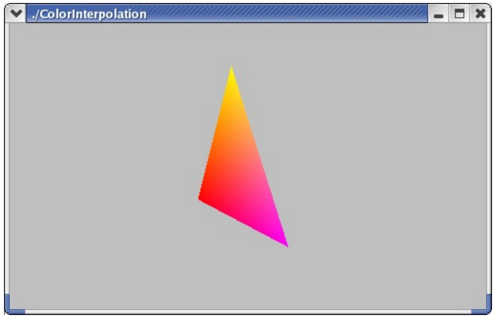
- Q(s,t) can be computed as a weighted sum of the triangles' vertices:
  - $Q(s,t) = A(1-t)s + B(t)s + C(1-s)$
  - $Q(0.5,0.5) = A(1-0.5)(0.5) + B(0.5)(0.5) + C(1-0.5)$
  - $Q(0.5,0.5) = A(0.25) + B(0.25) + C(0.5)$
- To compute the coordinates of Q(0.5,0.5):
  - substitute the coordinates of ABC
  - calculate the weighted sum

# Color Interpolation in a Triangle

- If the colors of the vertices are different, OpenGL interpolates them
  - using the same equations that we used for calculating coordinates.

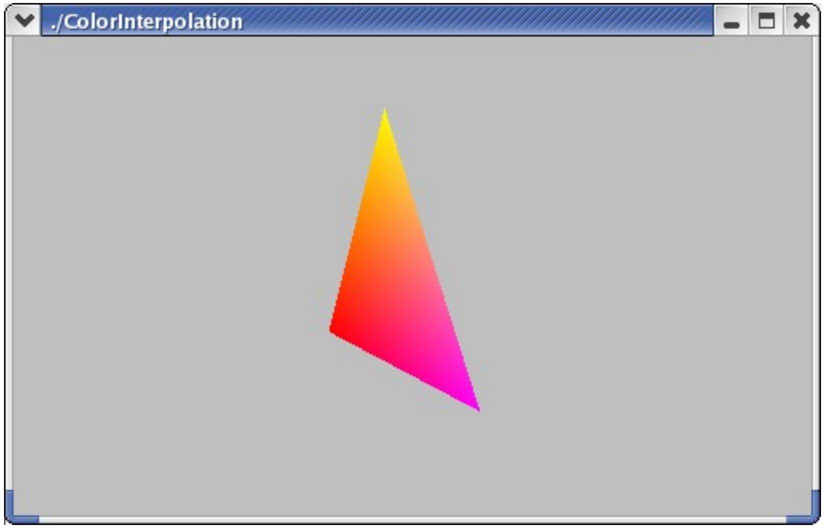
# Color Interpolation in a Triangle

- Suppose A is red (1,0,0), B is magenta (1,0,1), and C is yellow (1,1,0)
- We can compute the color of the middle point, $Q(0.5,0.5)$ , as:
  - Q(0.5,0.5) = A(0.25) + B(0.25) + C(0.5)
  - Q(0.5,0.5) = (1,0,0)(0.25) + (1,0,1) (0.25) + (1,1,0) (0.5)
  - Q(0.5,0.5) = (1, 0.5, 0.75)
- The triangle as a whole looks like this:



# Color Interpolation in a Triangle

- The triangle as a whole looks like this:




- A triangle with smooth interpolation

# Color Interpolation in Three.js

- To achieve interpolation in Three.js, you need to do the following:
  - Create a colors array, with as many entries as vertices in your mesh.
  - Set the colors array as the **vertexColors** property of the geometry
  - Using **THREE.MeshBasicMaterial**, set the **vertexColors** property to **THREE.VertexColors**

# Color Interpolation in Three.js

- The THREE.Geometry() object has a:
  - vertexColors property that is an *array* of colors
  - an array of THREE.Face3() objects
- Each THREE.Face3() object has a three-element array of colors
  - each is the color of the corresponding face vertex
- Using THREE.MeshBasicMaterial, we set the vertexColors property to THREE.VertexColors
  - The value of this property alerts Three.js that the vertices of a face could have different colors
    - The face is a triangle

# Color interpolation RGB triangle

- Triangle interpolation

# Color interpolation RGB triangle

- Triangle interpolation on a square

# Inconsistent color interpolation

- Inconsistent triangle interpolation on a square

# Inconsistent color interpolation

- Notice that at the lower right we have:
  - vertex B, coordinates (1,0,0), color THREE.ColorKeywords.lime
  - vertex B2, coordinates (1,0,0), color THREE.ColorKeywords.blue

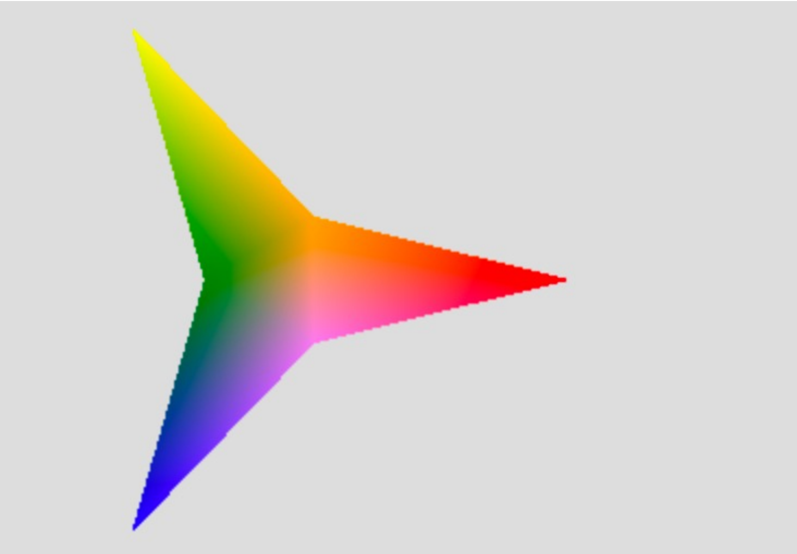
# THREE.JS EXERCISES

# Exercise: Colorful Stars

- This stars-start pen contains a function starGeometry()
  - that creates and returns a Three.Geometry object for a three-pointed star.
- Let’s take a minute to understand that geometry.

# Exercise: Colorful Stars

- Modify this code to create a star that uses *color interpolation* of the triangular faces
- and adds it to the scene.
- Your result might look like this:



# Exercise: Colorful Stars

- Suggestions:
  - The starting code includes an array of THREE.Color objects named colors
    - You can change the colors to whatever you want!
  - When creating the material for the star using THREE.MeshBasicMaterial:
    - add a second property to the input object
      - in addition to the vertexColors property
    - Property should tell Three.js to render both sides of the triangular faces: side: THREE.DoubleSide

# Exercise: Add stars to the scene

- Add six additional stars to the scene that each have a uniform color
- and which are placed around the central star
- Something like this:

# Exercise: Add stars to the scene

- Suggestions:
  - Think about how this can be done with a loop
  - Use the same array of colors that you used for the central star
  - Recall that position.set() can be used to place a mesh at a desired location
  - Remember to adjust the bounding box supplied to TW.cameraSetup() to see the additional stars

Questions?