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Mid-Term Report

Option pricing models and their accuracy

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Contents

1 Introduction

The financial markets, characterized by their complexity and volatility, require robust mathematical models to price options and make informed investment decisions accurately. Among the plethora of models available, the Binomial Model and the Black-Scholes Model stand out for their widespread use and foundational contributions to financial theory.

This report compares the effectiveness of the Binomial and Black-Scholes models, illustrating their applications and performance through a detailed case study on the same assets.

1.1 Objectives

The primary objective of this study is to evaluate and compare the performance of the Binomial Model and the Black-Scholes Model in the context of option pricing. The key objectives include:

(a) Evaluating the efficacy of the Binomial and Black-Scholes Model in pricing European and American options.

(b) Conducting a comparative analysis using a case study on a specific asset to determine both models' accuracy and computational efficiency.

(c) Scrutinizing each model's relative strengths and limitations, considering both theoretical assumptions and practical application. This examination facilitates a practical comparison of their viability within real-world trading scenarios.

2 Options Pricing Models

2.1 The Binomial Options Pricing Model

2.1.1 Overview

The Binomial Model, introduced by Cox, Ross, and Rubinstein in 1979, is a discrete-time method for estimating the price of options. It constructs a binomial tree to represent possible future prices of the underlying asset. This model calculates the value of an option at time $t = 0$ and provides a payoff at a future date based on the value of non-dividend paying shares at that time.
The binomial option pricing model uses an iterative procedure, specifying nodes, or points in time, between the valuation date and the option's expiration date.

Assumptions:

- The key assumption is that there are only two possible outcomes for the stock price at each step: it can either move up or down.
- The model assumes no arbitrage opportunities exist in the market and that the stock price follows a random walk. The stock has a certain probability of moving up or down at each step.

By examining the binomial tree of values, a trader can determine when a decision on exercising the option may occur. If the option has a positive value, it might be exercised; if it has a value less than zero, it should be held longer.

2.1.2 How Does The Model Work?

Parameters:

- S_0: Price of the underlying asset at t_0
- X: Strike price of the option
- T: Time to maturity
- r: Risk-free interest rate
- σ: Volatility of the underlying asset
- N: Number of time steps in the binomial tree

Calculating the two possible outcomes:

- δt: Length of each time step, calculated as $\frac{T}{N}$
- u: Up factor, which is the factor by which the price increases in each step, calculated as $e^{\sigma\sqrt{\delta t}}$
- d: Down factor, which is the factor by which the price decreases in each step, calculated as $e^{-\sigma\sqrt{\delta t}}$
- p: Risk-neutral probability of an up move, calculated as $\frac{e^{r\delta t}-d}{u-d}$

Construct the Binomial Tree:

- **Stock Prices:** Build a binomial tree for stock prices from the initial price S_0 using the up and down factors. Each node in the tree represents a possible asset price at a specific time point.
- At node (i, j), the stock price is $S_0 \times u^j \times d^{i-j}$, where i is the step number and j is the number of up moves.

Calculate Option Payoff at Maturity:

- Calculate the option's payoff at each final node (N, j).
- For a call option: Payoff $= \max(S_N - X, 0)$
- For a put option: Payoff $= \max(X - S_N, 0)$

Backward Induction to Calculate Present Value:

- Starting from the final nodes, move backward through the tree to calculate the option value at each node.
- The option value at node (i, j) is the discounted expected value of the option values in the next step:
$$C_{i,j} = e^{-r\delta t}\left(p \times C_{i+1,j+1} + (1-p) \times C_{i+1,j}\right)$$
where $C_{i,j}$ is the option value at node (i, j).

Calculate the Option Value at the Root:

- Continue the backward induction until you reach the root of the tree $(0, 0)$, which gives the present value of the option.

2.1.3 Advantages

- The multi-period view allows users to visualize changes in asset prices over time and evaluate the option based on decisions made at different points.
- The binomial model helps determine when exercising the option may be advisable and when it should be held longer.
- One major advantage of the binomial option pricing model is its mathematical simplicity, even though it can become complex in a multi-period model.
- The binomial model offers flexibility as users can alter inputs at each step to account for differences in the ability to exercise options with non-standard features.

2.1.4 Disadvantages

- Binomial models are complex to construct and, depending on the number of steps, can become unwieldy regarding spreadsheet size and the computing power needed.
- They require predicting future prices, which can be challenging and uncertain.

2.2 The Black-Scholes Options Pricing Model

2.2.1 Overview

In contrast to the Binomial Model, the Black-Scholes Model, developed by Fischer Black, Myron Scholes, and Robert Merton in 1973, provides a continuous-time framework for pricing European-style options. This model is renowned for its analytical elegance and computational efficiency, offering a closed-form solution under the assumption of constant volatility and a risk-free rate.

The model uses current stock prices, expected dividends, the option's strike price, expected interest rates, time to expiration, and expected volatility to calculate the theoretical value of an option contract. It assumes that stock prices follow a lognormal distribution with a random walk, exhibiting constant drift and volatility.

Assumptions:

- The underlying asset pays no dividends.
- Markets are random and not influenced by external factors.
- The risk-free interest rate and volatility are constant over the option's life.
- Asset returns are normally distributed.

2.2.2 How Does The Model Work?

The Black-Scholes Model provides a closed-form solution to calculate the theoretical value of option prices.

Formula:

$$C = S_0 N(d_1) - X e^{-rT} N(d_2)$$

where:

$$d_1 = \frac{ln(S_0/x) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Parameters:

- S_0: Current price of the asset
- X: Strike price
- T: Time to maturity
- r: Risk-free interest rate
- σ: Volatility of the asset
- $N(\cdot)$: Cumulative distribution function of the standard normal distribution

2.2.3 Advantages

- **Closed-form solution:** The Black-Scholes Model provides an explicit formula to calculate option prices, making it straightforward to use and implement.
- **Widely accepted:** It is a standard model in financial markets, streamlining the process of option pricing and trading.
- **Market risk optimization:** By providing theoretical values, the model helps compare market prices and identify mispriced options, aiding in risk management.
- **Portfolio optimization:** The model also helps understand the risks associated with different options, allowing for better portfolio management.

2.2.4 Disadvantages

- **Many assumptions:** The model relies on several assumptions, such as constant volatility and risk-free rate, which may not hold true in real markets.
- **European options only:** It is designed for European options, which can only be exercised at expiration, limiting its applicability to American options.
- **Implied volatility:** The model does not account for changes in implied volatility, which can affect option prices.
- **No dividends:** It assumes the underlying asset does not pay dividends, which is a limitation for stocks that do.

3 Case Study

In this case study, we will evaluate the call and put options of Apple Inc. (AAPL) using two different models: the Black-Scholes Model (BSM) and the Cox-Ross-Rubinstein (CRR) Binomial Model. We will compare the predicted option prices with the actual market prices to assess the accuracy of these models.

3.1 Data Collection

1. Selection of Ticker and Option Dates
    - Ticker: We selected Apple Inc. (AAPL) due to its high trading volume and availability of options data.
    - Option Expiry Dates: We selected options expiring on September 20, 2024 (calls) and December 18, 2026 (puts).

2. Data Sources

    - Yahoo Finance retrieves historical and current stock prices, options chain data, and treasury yield data.

3.2 Methodology

We used Python and various libraries for data acquisition and model implementation. We used the `yfinance` library to fetch historical stock prices and treasury yield data to obtain the data required for our analysis, which is as follows:

- We used the `yfinance` library to fetch the latest closing price of the stock using its ticker symbol
- The risk-free rate was obtained using the 10-year Treasury yield, fetched using its ticker symbol `T̂NX`
- The time to expiration was calculated by finding the difference between the current date and the option's expiration date
- We calculated the stock's annualized volatility based on its historical daily returns over the past year

3.2.1 Model Implementation

We implemented two models to predict option prices: the Black-Scholes model and the Cox-Ross-Rubinstein (CRR) Binomial Model

Black-Scholes Model

- The Black-Scholes model is a closed-form solution used to estimate the price of European call and put options
- We defined the `d1` and `d2` functions and used them to calculate the call and put option prices

Cox-Ross-Rubinstein (CRR) Binomial Model

- The CRR Binomial Model is a discrete-time model used to price options by building a binomial tree of possible stock prices
- We defined the binomial model functions for both call and put options

3.3 Results and Comparison

We compared the valuations from both models with the actual market prices of the options. Below are the key findings:
Call Options (Expiring on 2024-09-20):

- The mean absolute error (MAE) for Black-Scholes: 14.36
- The mean absolute error (MAE) for Binomial: 11.33

The Black-Scholes model predicted negative values for some put options, indicating potential inaccuracies due to the long time to maturity and high strike prices. The Binomial model showed more realistic valuations but still had significant deviations from the market prices.

3.4 Visualizing Results

The scatter plots and histograms below illustrate the differences between the actual market prices and the predicted prices for call options from both models.

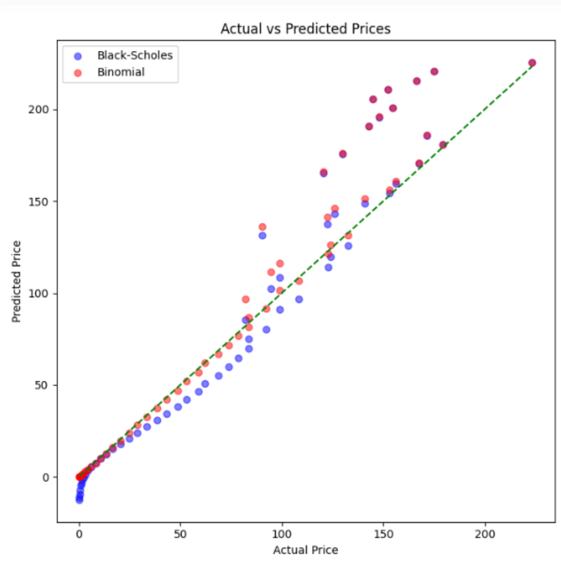

Figure 1: Actual vs. Predicted Prices

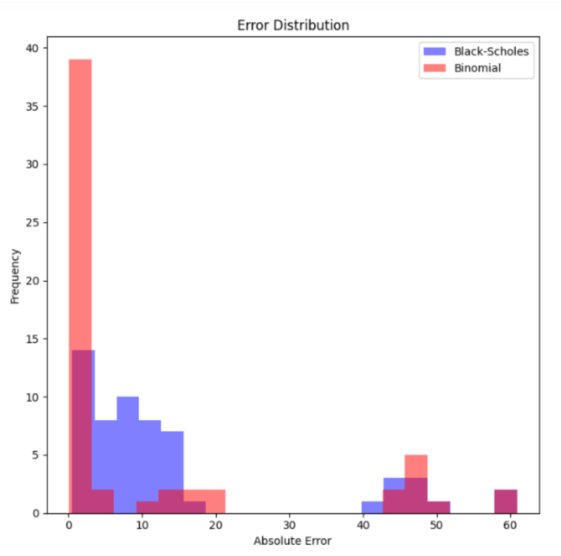

Figure 2: Error Distribution

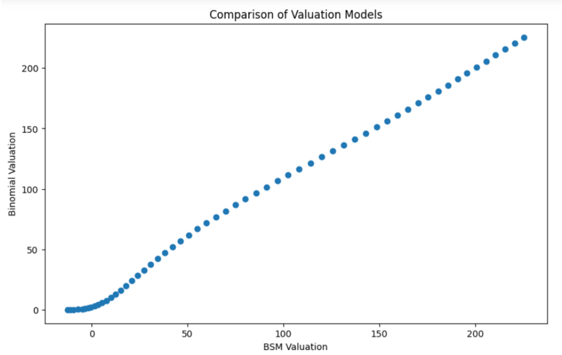

Figure 3: Comparison of Valuation Models

3.5 Discussion

In-The-Money Calls Options

rike	lastPrice	bid	ask	change	percentChange	impliedVolatility	inTheMoney	bsmValuation	binValuation
5.0	223.40	226.65	227.10	0.0	0.0	3.539064	True	226.859503	226.859503
10.0	175.16	180.50	184.15	0.0	0.0	0.000010	True	221.898999	221.898999
15.0	166.23	157.55	158.95	0.0	0.0	0.000010	True	216.938485	216.938495
20.0	152.13	144.00	146.25	0.0	0.0	0.000010	True	211.977824	211.977990
25.0	144.84	145.05	147.25	0.0	0.0	0.000010	True	207.016209	207.017486
30.0	154.79	160.70	164.35	0.0	0.0	0.000010	True	202.051103	202.056982
35.0	147.99	162.05	162.65	0.0	0.0	0.000010	True	197.077113	197.096478
40.0	142.89	157.15	157.70	0.0	0.0	0.000010	True	192.085444	192.135973
45.0	171.47	186.70	187.35	0.0	0.0	1.608400	True	187.064251	187.175469
50.0	179.19	181.75	182.40	0.0	0.0	1.543948	True	181.999767	182.214965

Figure 4: Call Options

The call options we examined were primarily in-the-money (ITM). For a growing company like Apple Inc., these ITM call options are expected to have high prices due to the positive outlook and anticipated returns. The Black-Scholes and Binomial models reflect this expectation, showing relatively high predicted prices for these options.

Out-of-the-Money Put Options

strike	lastPrice	bid	ask	change	percentChange	impliedVolatility	inTheMoney	bsmValuation	binValuation
50.0	0.18	0.15	0.27	0.00	0.000000	0.430181	False	0.000000e+00	0.000006
60.0	0.45	0.21	0.46	0.00	0.000000	0.409918	False	0.000000e+00	0.000107
70.0	0.45	0.36	0.58	0.00	0.000000	0.377936	False	0.000000e+00	0.000840
80.0	0.60	0.51	0.58	0.00	0.000000	0.338141	False	0.000000e+00	0.004474
85.0	0.67	0.60	0.87	0.00	0.000000	0.342292	False	2.557954e-13	0.009172
90.0	0.75	0.65	0.85	0.00	0.000000	0.322883	False	5.599077e-12	0.017428
95.0	0.80	0.67	1.00	0.00	0.000000	0.314948	False	9.191581e-11	0.030633
100.0	1.06	0.84	1.28	-0.06	-5.357148	0.312629	False	1.146304e-09	0.049902
105.0	1.27	1.03	1.49	0.00	0.000000	0.305427	False	1.122640e-08	0.083779
110.0	1.52	1.40	1.55	0.00	0.000000	0.291755	False	8.878604e-08	0.131019

Figure 5: Put Options

Conversely, the put options analyzed were mostly out-of-the-money (OTM). This results in significantly lower prices, as the market does not expect the stock price of Apple Inc. to decrease substantially in the near future. The Black-Scholes model, in particular, predicts very low prices for these OTM put options, sometimes even resulting in negative values, which indicates the model's limitation in handling certain market conditions. With its stepwise approach, the Binomial model provides a more realistic floor to these predictions but still reflects the low valuation in line with market expectations.

These observations underline the influence of market sentiment and the current stock trajectory on option pricing. The models' predictions align with the logical financial market behavior for a growth stock, where ITM calls are valued higher, and OTM puts hold little value.

3.6 Reasons for Inaccuracies and Variances

Model Assumptions
The Black-Scholes Model assumes constant volatility, interest rates, log-normally distributed returns, and continuous trading. Real market conditions often deviate from these assumptions. The Binomial Model provides more flexibility by allowing discrete time intervals and adjusting for early exercise in American options, leading to more accurate pricing in some cases.

Volatility Estimation
The volatility input is critical for both models. Historical volatility may not always predict future volatility well, leading to inaccuracies in model predictions.

Interest Rates
The risk-free rate was derived from the 10-year Treasury yield, which might not perfectly align with the specific time frames of the options analyzed.

3.7 Conclusion

The Binomial Model generally provided more accurate valuations than the Black-Scholes Model, especially for the put options analyzed. The discrepancies highlight the importance of understanding the limitations and assumptions of each model and the need for continuous model calibration to reflect current market conditions.
The inaccuracies observed stem from the inherent limitations of both models in capturing real-world complexities. The Black-Scholes Model, while elegant, often falls short in volatile or less liquid markets. Though more flexible, the Binomial Model still relies on accurate inputs and assumptions.

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