Linear Regression & Logistic Regression

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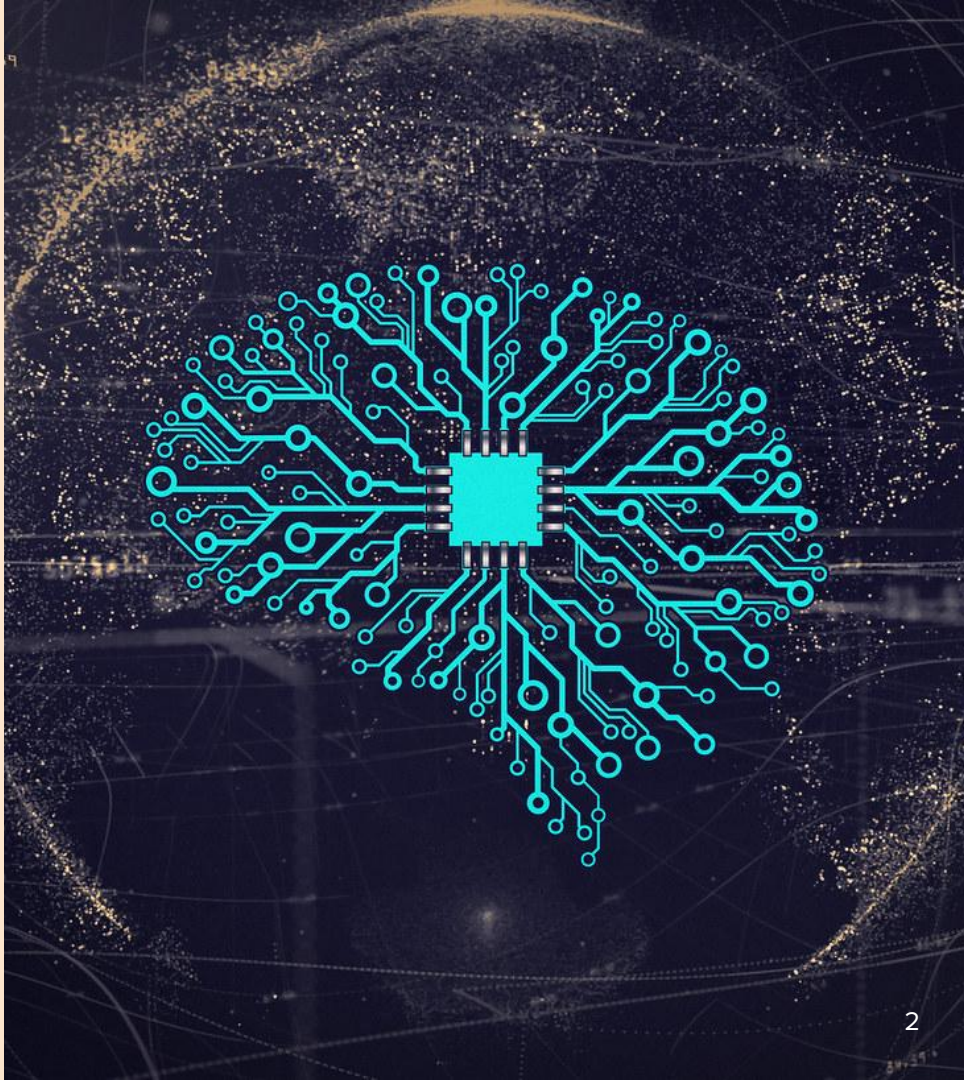
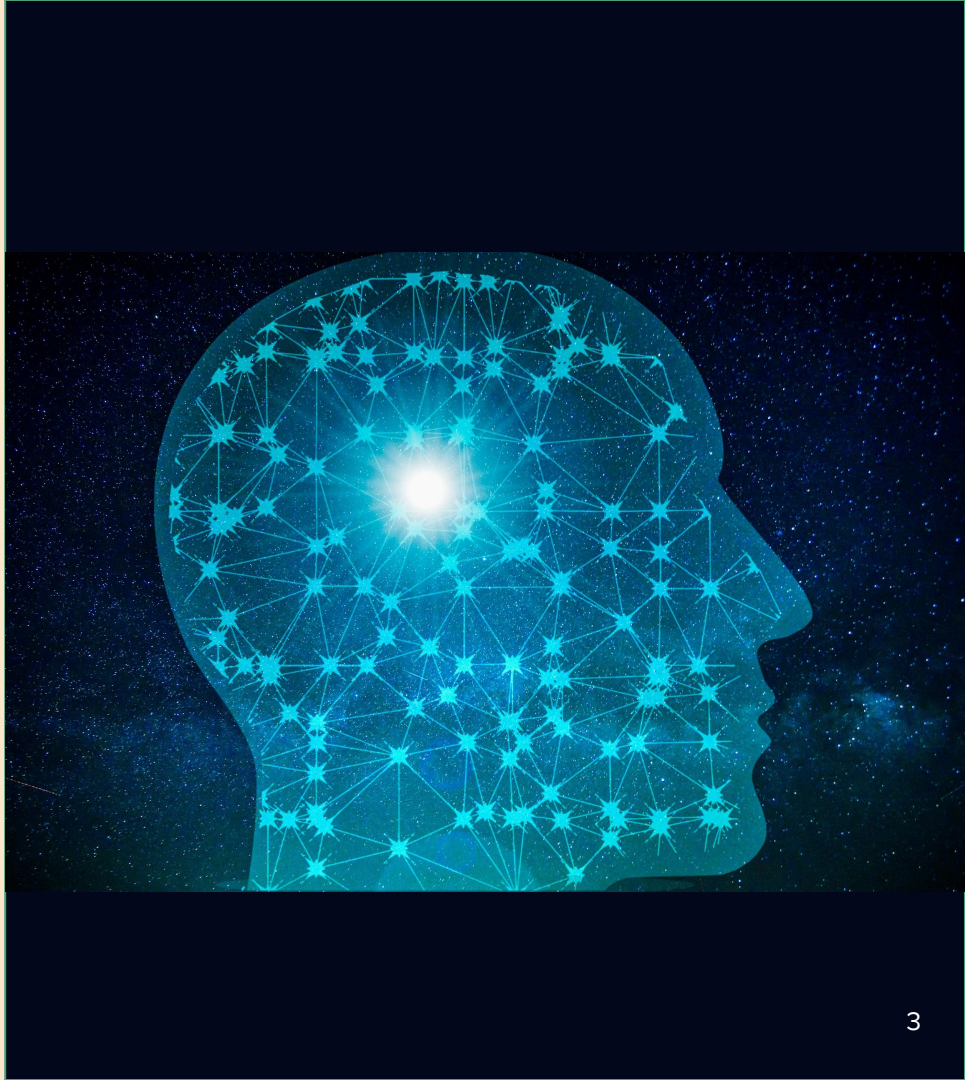
Fibo, Kmutt

Outline

- Machine learning basics
- Cost function
- Over fitting
- Linear regression
- Regression performance
- Logistic regression
- Classification performance



Machine Learning Basics



The basic example

An agent has been selling 300 houses in the last years and want to be able to predict of a house by just knowing the size of the house.

i	Size(feet^2)	Price (dollar)
1	8450	208500
2	9600	181500
3	11250	223500
4	9550	140000
...	...	...

The basic example

In general,

x_i: one feature (input) at i index

y_i: one target (output) at i index

i: sample index

i	x Size(feet^2)	y Price (Dollar)
1	8450 = x_1	208500 = y_1
2	9600 = x_2	181500 = y_2
3	11250 = x_3	223500 = y_3
4	9550 = x_4	140000 = y_4
...	...	...

The basic example

In general,

x^j_i: multi feature (input) at i index

y^k_i: multi target (output) at i index

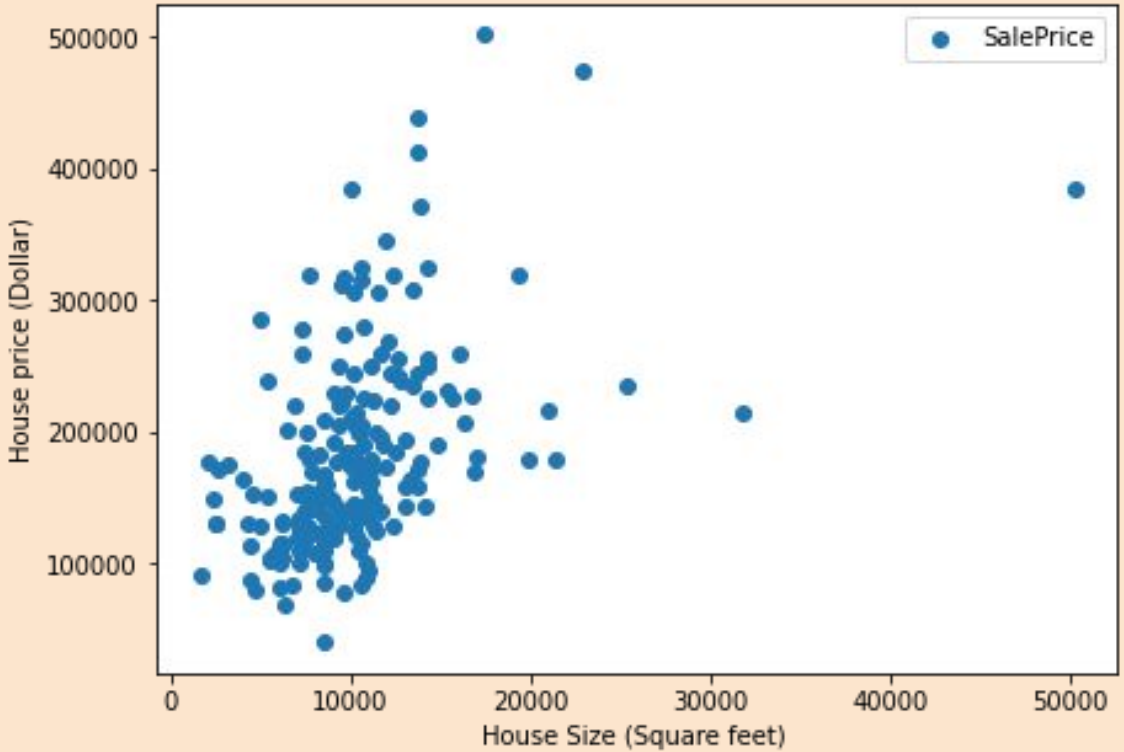
j: feature index

k: target index

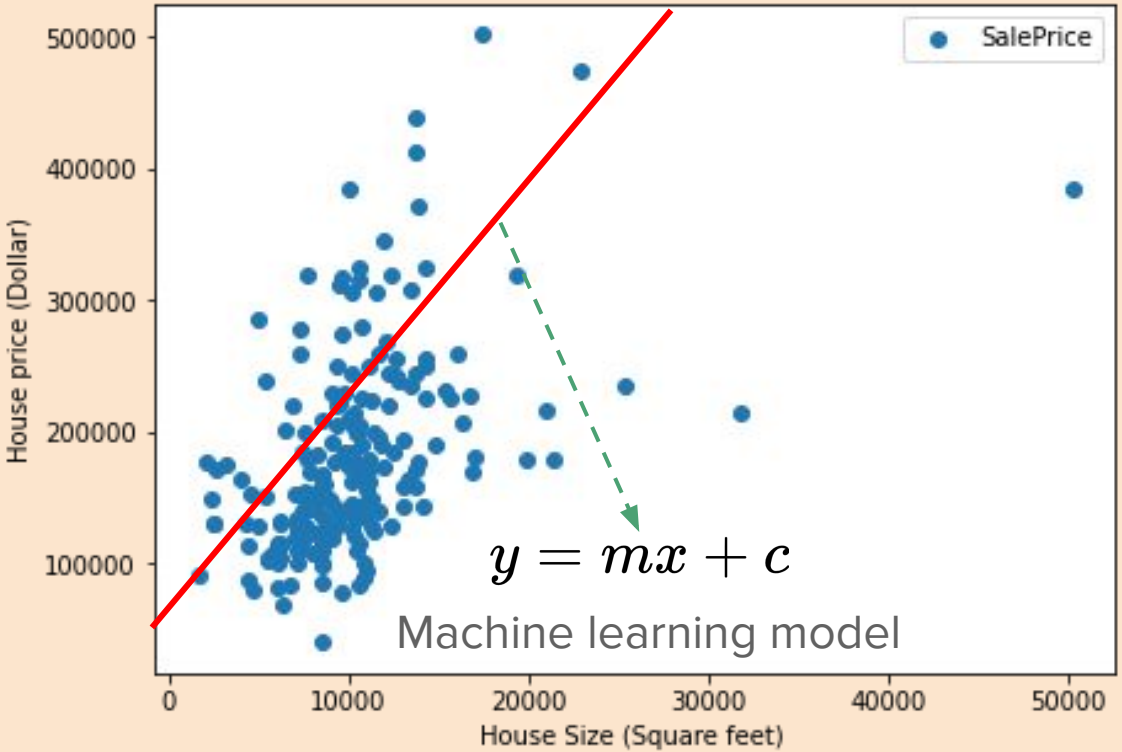
i: sample index

i	$x^{(1)}$ # Bath	$x^{(2)}$ #Bed	$x^{(3)}$ Size(feet^2)	$y^{(1)}$ Sell price (Dollar)	$y^{(2)}$ Buy price (Dollar)
1	$1= x^{(1)}_1$	$1= x^{(2)}_1$	$8450 = x^{(3)}_1$	$208500 = y^{(1)}_1$	$207000 = y^{(2)}_1$
2	$1= x^{(1)}_2$	$2 = x^{(2)}_2$	$9600 = x^{(3)}_2$	$181500 = y^{(1)}_2$	$180000 = y^{(2)}_2$
3	$1= x^{(1)}_3$	$1= x^{(2)}_3$	$11250 = x^{(3)}_3$	$223500 = y^{(1)}_3$	$221500 = y^{(2)}_3$
4	$2 = x^{(2)}_4$	$3= x^{(2)}_4$	$9550 = x^{(3)}_4$	$140000 = y^{(1)}_4$	$120500 = y^{(2)}_4$
...			...	...	

The basic example



The basic example



The basic example

Output

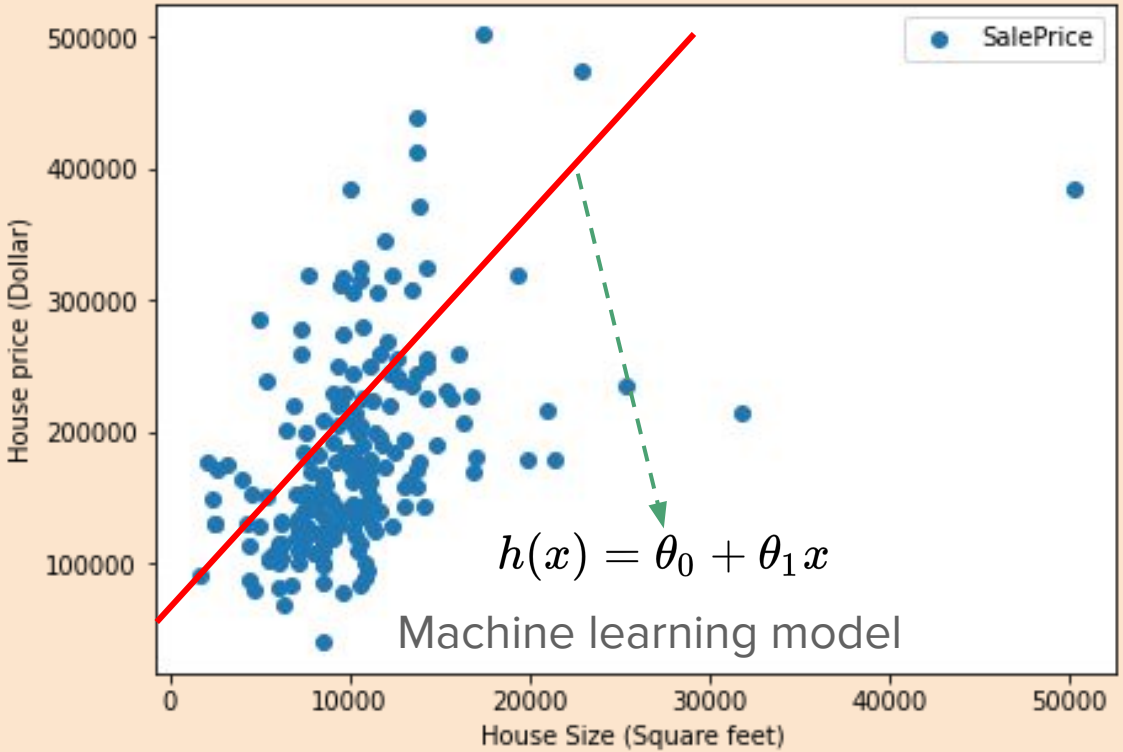
Input

$$h(x) = \theta_0 + \theta_1 x$$

Parameters of the model

$\theta_0 = 50000$

$\theta_1 = 15$

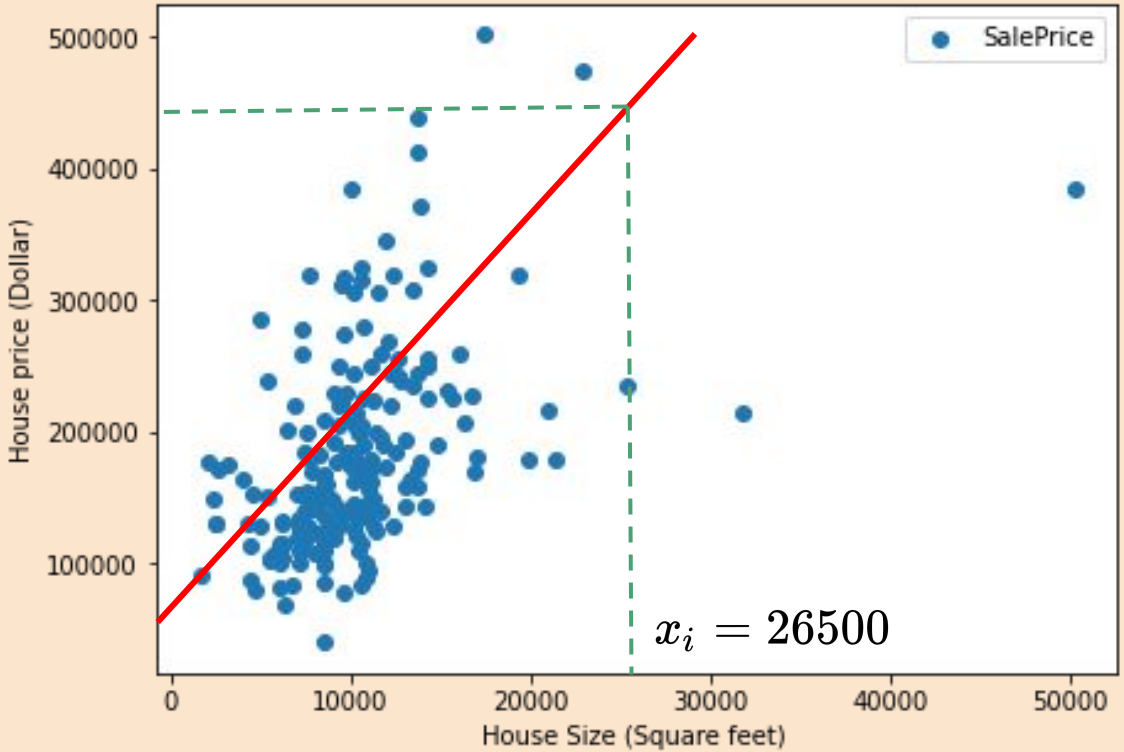


The basic example

$$Model : h(x_i) = 50000 + 15x_i$$

$$x_i = 26500$$

$$h(x_i = 26500) = 50000 + 15 \times 26500$$
$$= 447500$$



The basic example

$$Model: h(x_i) = 50000 + 15x_i$$

$$x_i = 26500$$

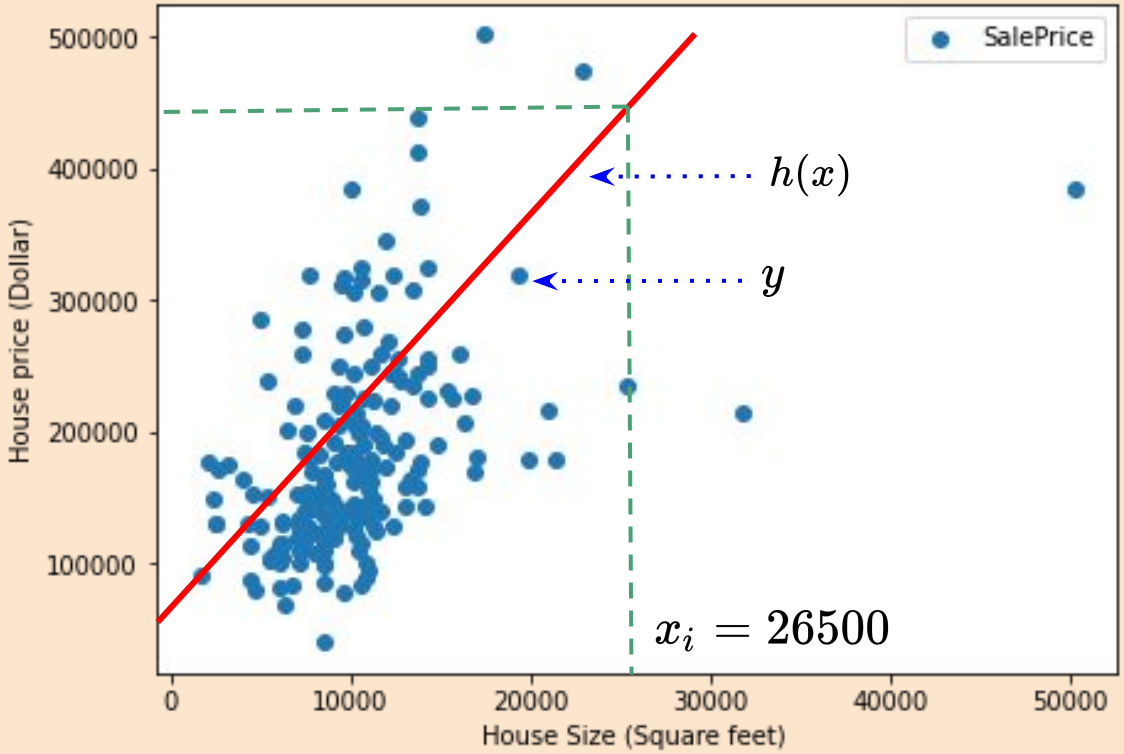
$$h(x_i = 26500) = 50000 + 15 \times 26500$$
$$= 447500$$

Notice that

$h(x)$ and y are not the same

$h(x)$: predicted values

y : actual values



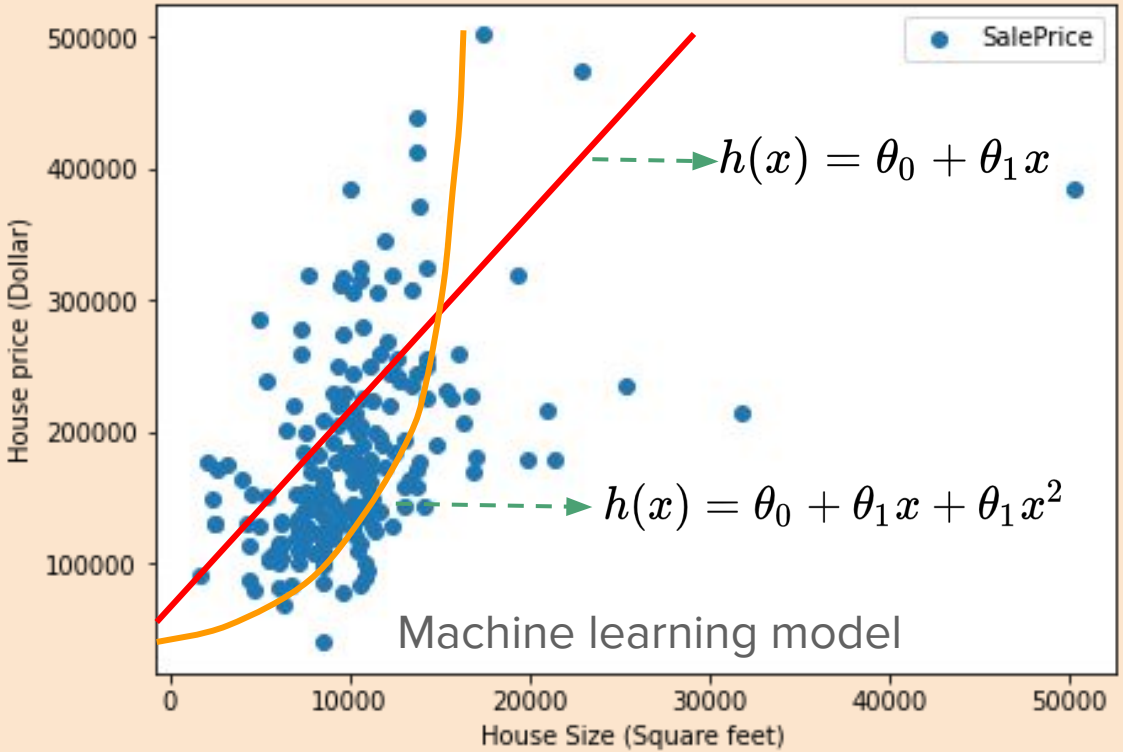
The basic example

$$h(x) = \theta_0 + \theta_1 x$$

$$h(x) = \theta_0 + \theta_1 x + \theta_1 x^2$$

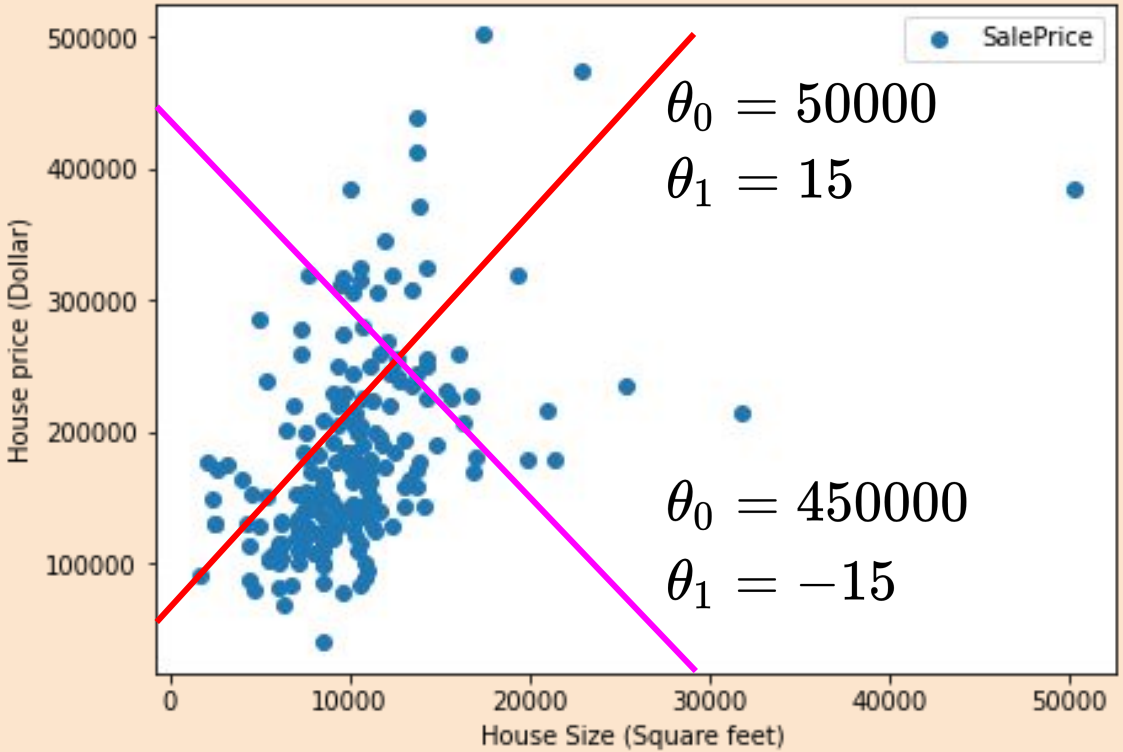
Model is a function of both features(x) and parameters (theta).

$$h(x; \theta)$$

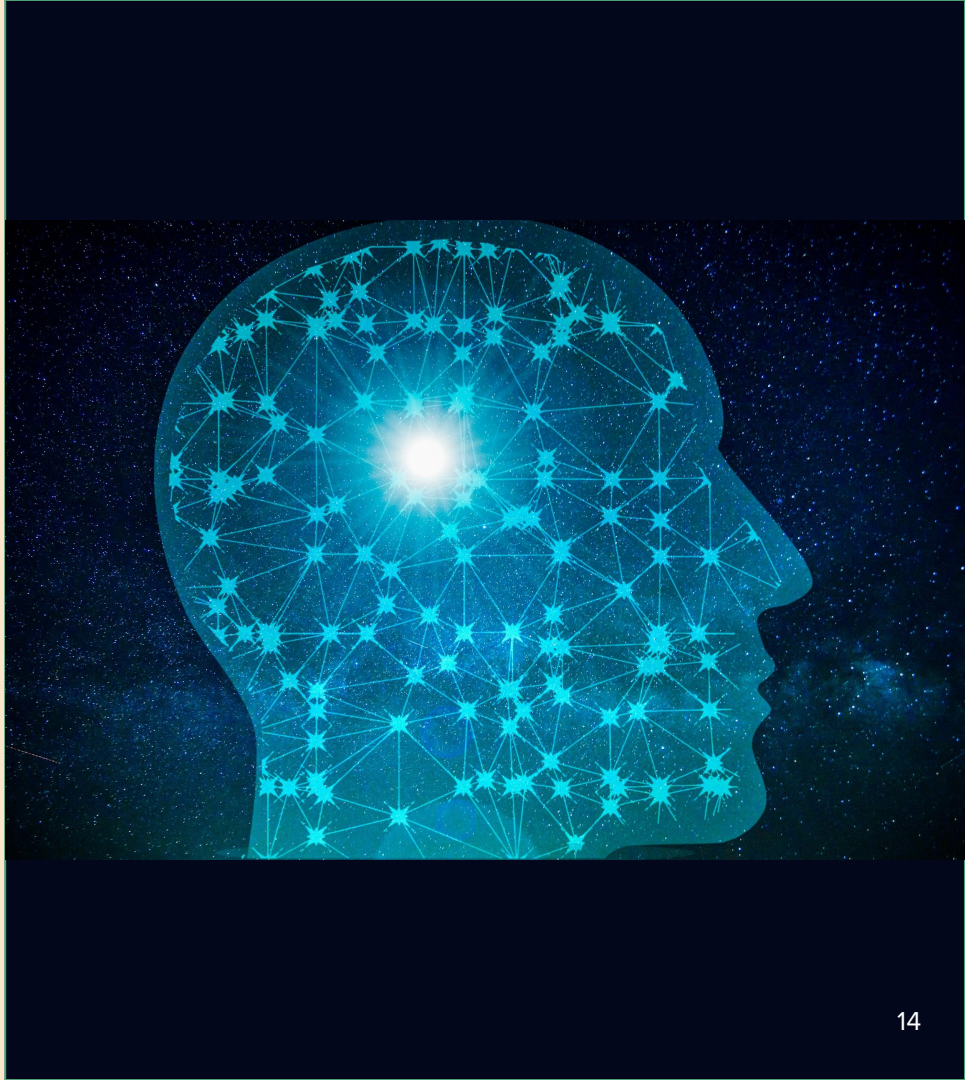


Good and bad models

$$h(x) = \theta_0 + \theta_1 x$$



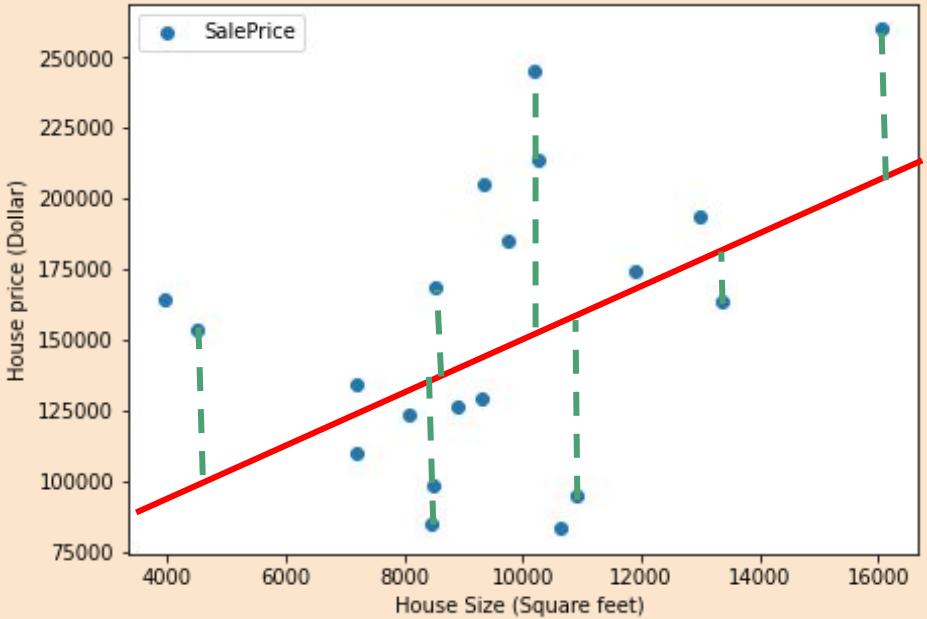
Cost Function



Cost function

Cost function (loss function) is a measure of whether a model is a good fit to the data.

For example, a famous cost function is called ‘squared error’ function that takes the difference between what you predict and the actual data value and square it.



$$E = \sum_i (h(x_i) - y_i)^2$$
$$= Cost\ function$$

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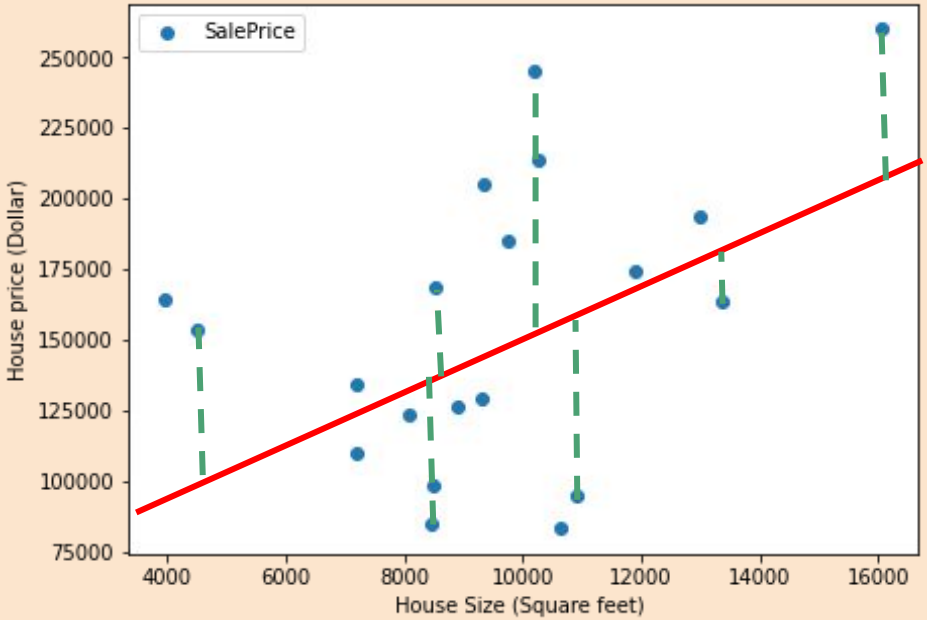

$$E = \sum_i (h(x_i) - y_i)^2$$
$$= Cost\ function$$

Low cost function ➔ good model

High cost function ➔ bad model

Cost function

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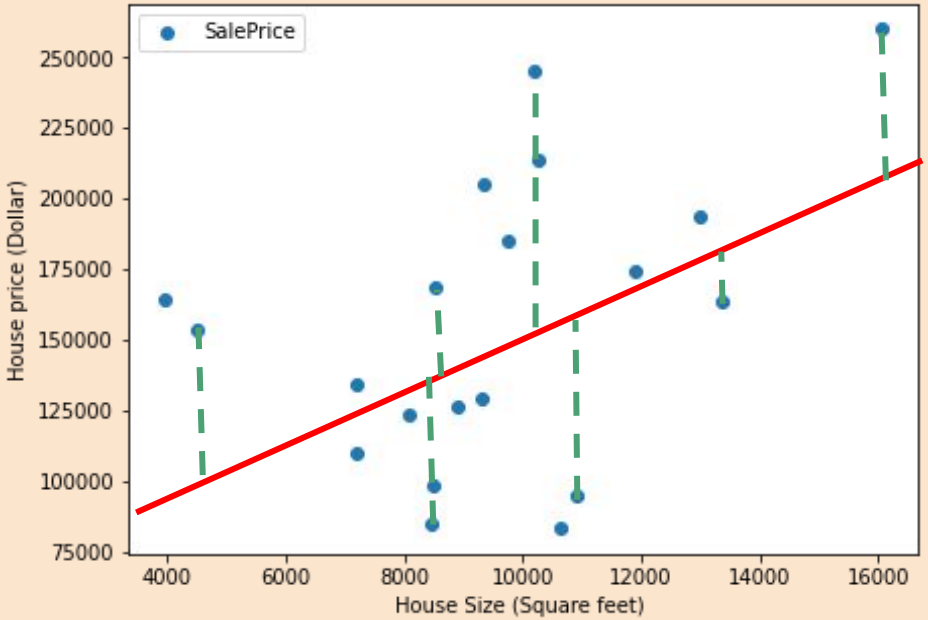
For example, a famous cost function is called ‘squared error’ function that takes the difference between what you predict and the actual data value and square it.



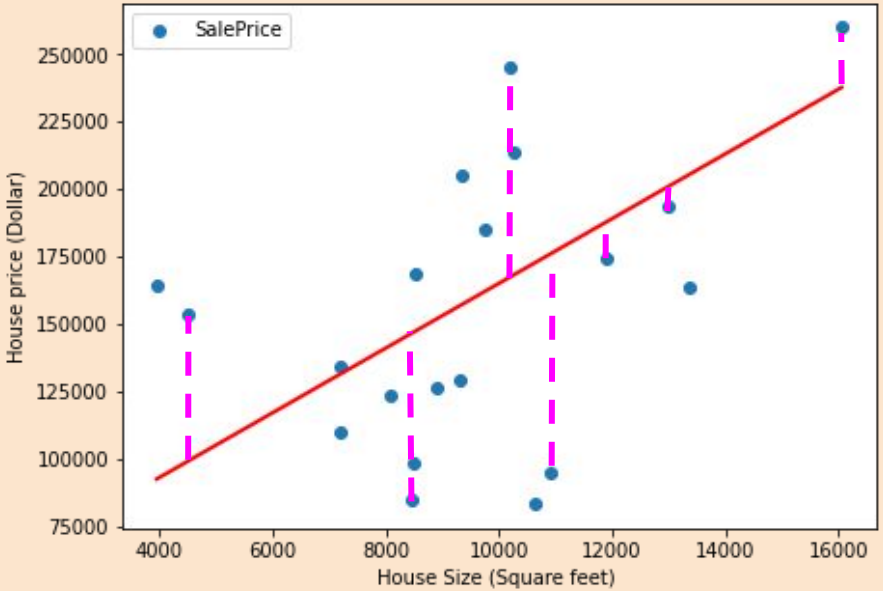
$$Root\ mean\ square\ error\ (RMSE)$$

$$Cost(\theta) = \sqrt{\frac{1}{m}\sum_i (h(x_i) - y_i)^2}$$

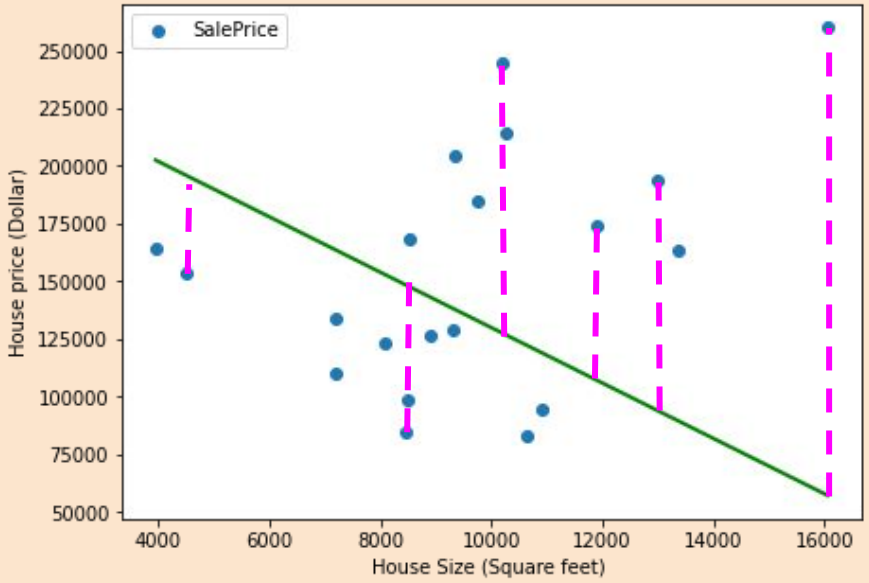
m : # of sample in the dataset

Lowering cost function

$$Model : h(x_i) = \theta_0 + \theta_1 x_i$$



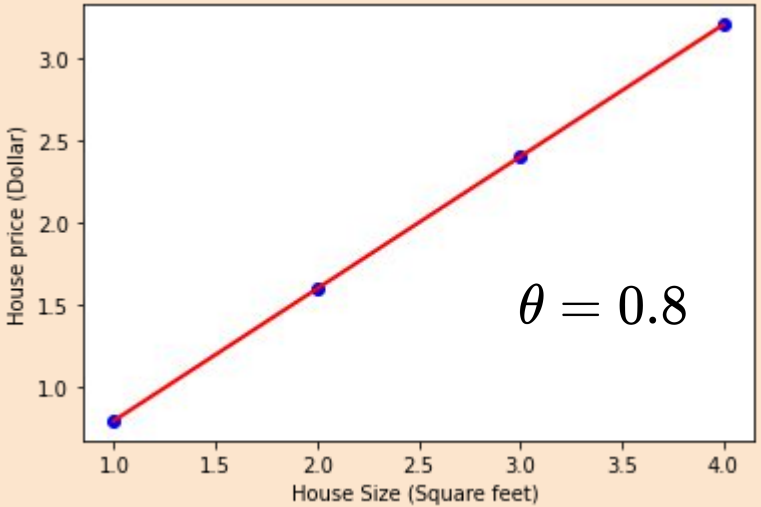
$$\theta_0 = 45000$$
$$\theta_1 = 12$$



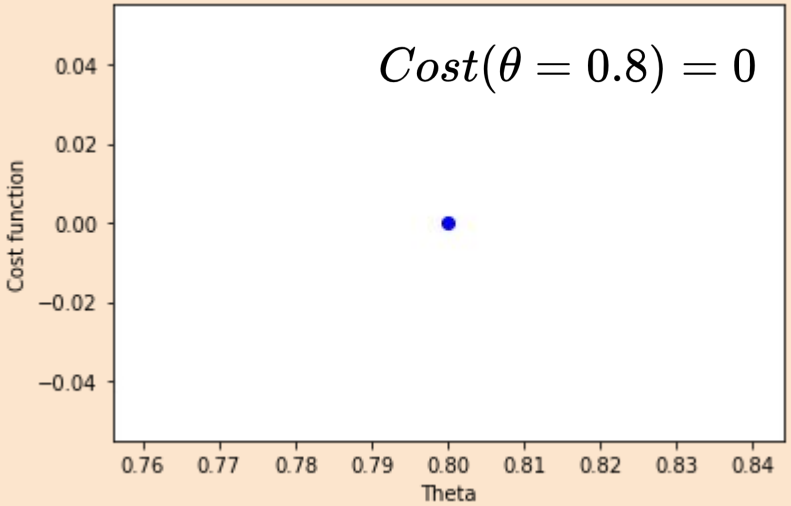
$$\theta_0 = 250000$$
$$\theta_1 = -12$$

Minimizing cost function

$Model : h(x) = \theta x$



$Cost(\theta)$



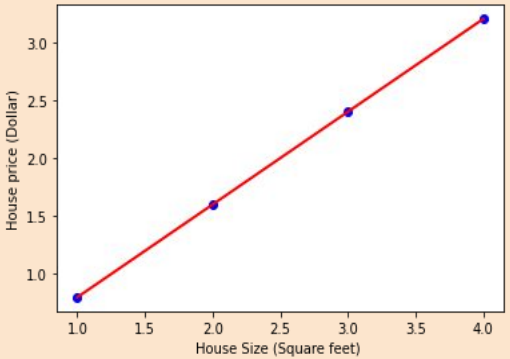
$x = 1, 2, 3, 4$

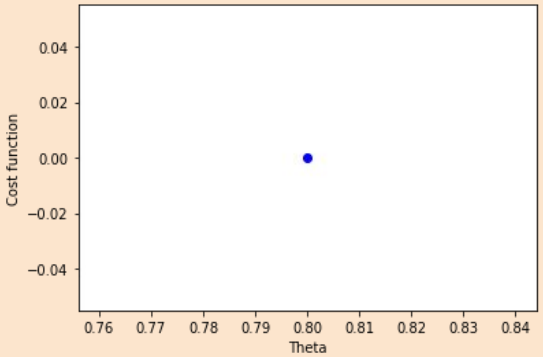
$y = 0.8, 1.6, 2.4, 3.2$

Minimizing cost function

$\theta = 0.8$

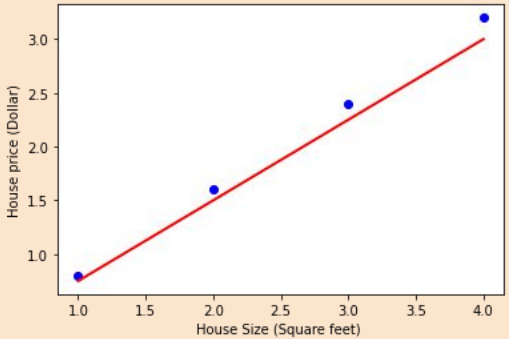
$Cost(\theta = 0.8) = 0$

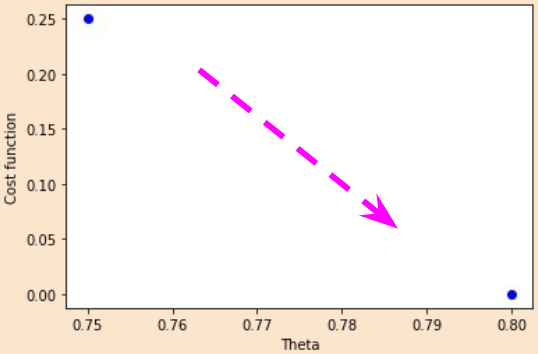




$\theta = 0.75$

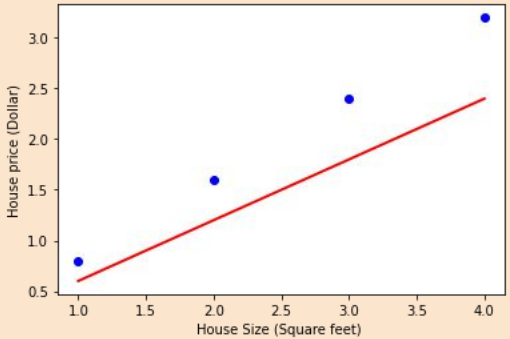
$Cost(\theta = 0.75) = 0.25$

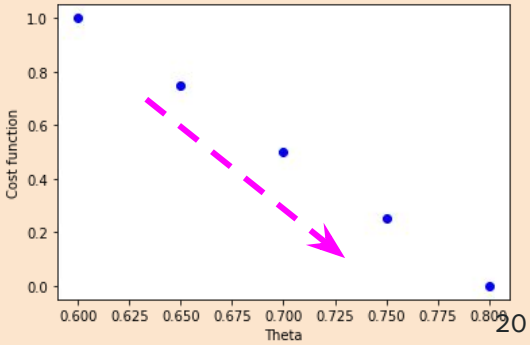




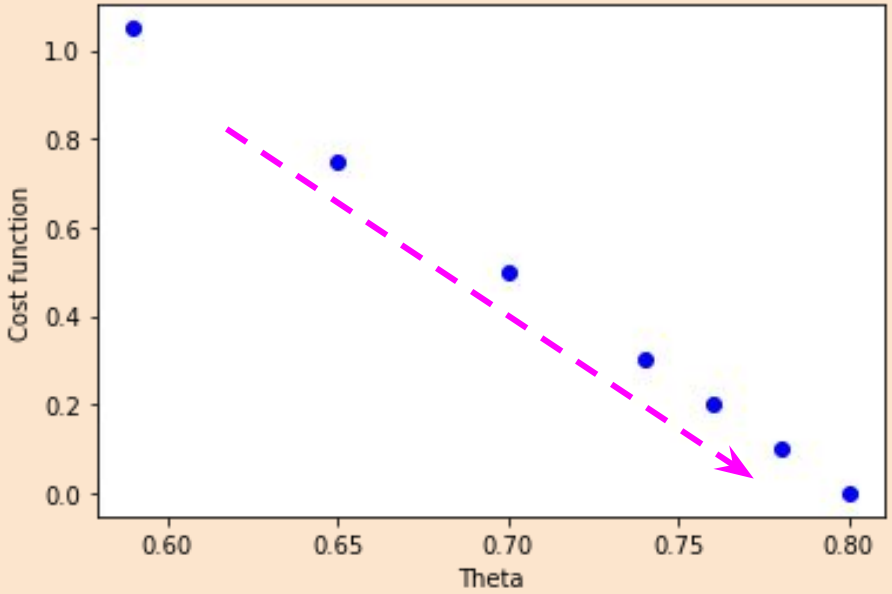
$\theta = 0.6$

$Cost(\theta = 0.6) = 1$



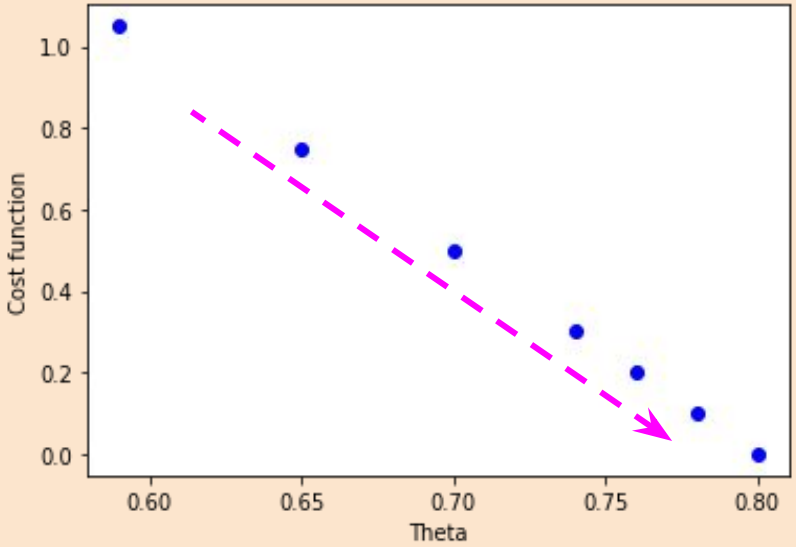


Minimizing cost function



- Best-fitted model has lowest cost function.
- Training machine learning models = minimizing cost function
- Often accomplished by using ‘optimization algorithms’

Minimizing cost function



$$Model : h(x_i) = \theta x_i$$

$$Parameter \ : \ \theta$$

$$Cost\ function \ : \ Cost(\theta) = \sqrt{\frac{1}{m}\sum_i (h(x_i) - y_i)^2}$$

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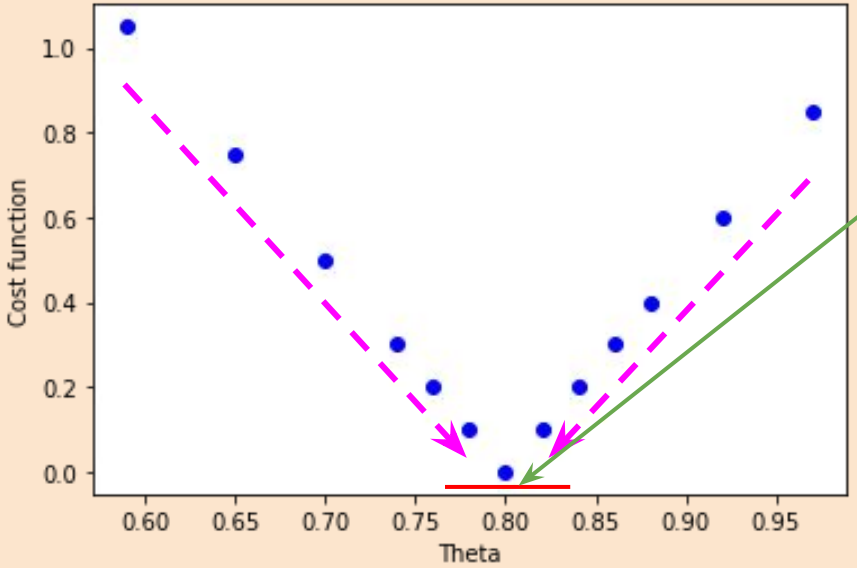
Minimizing cost function



$$\frac{\partial Cost(\theta)}{\partial(\theta)} = 0$$

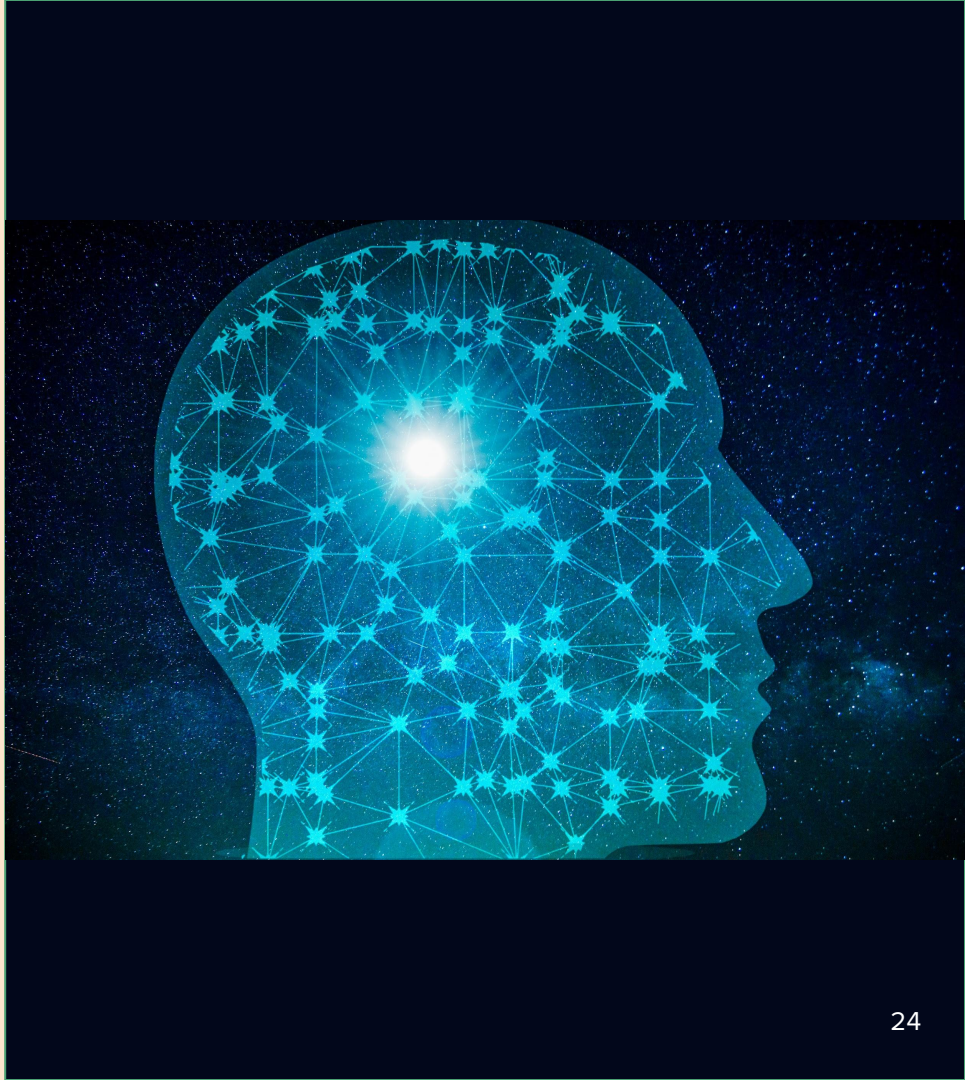
$$Cost(\theta) = \sum_i (h(x_i) - y_i)^2$$

$$= \sum_i (\theta x_i - y_i)^2$$

$$\frac{\partial Cost(\theta)}{\partial(\theta)} = \sum_i 2(\theta x_i - y_i)x_i = 0$$

$$\theta = 0.8$$

Over Fitting

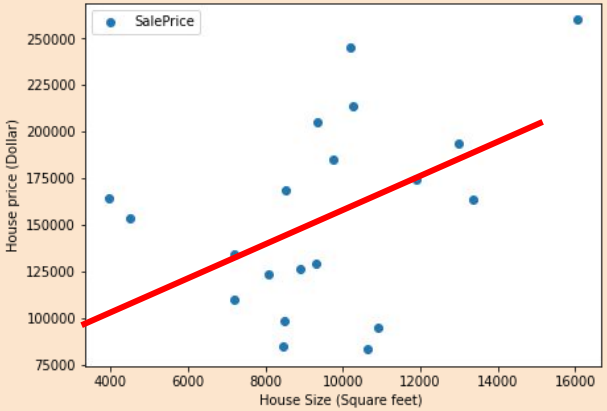


Overfitting

In machine learning, we need to minimize cost function, however sometimes, we have the opposite problem when the cost function is too low.

Underfit

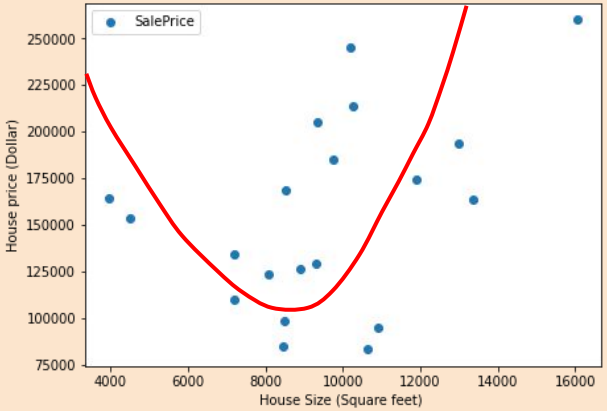
$Cost = 0.5$



$h(x) = \theta_0 + \theta_1 x$

Right

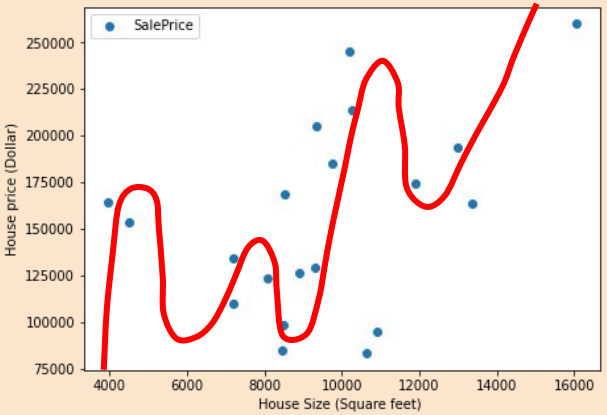
$Cost = 0.04$



$h(x) = \theta_0 + \theta_1 x + \theta_2 x^2$

Overfit

$Cost = 0.01$



$h(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \ldots + \theta_9 x^9$

Overfitting

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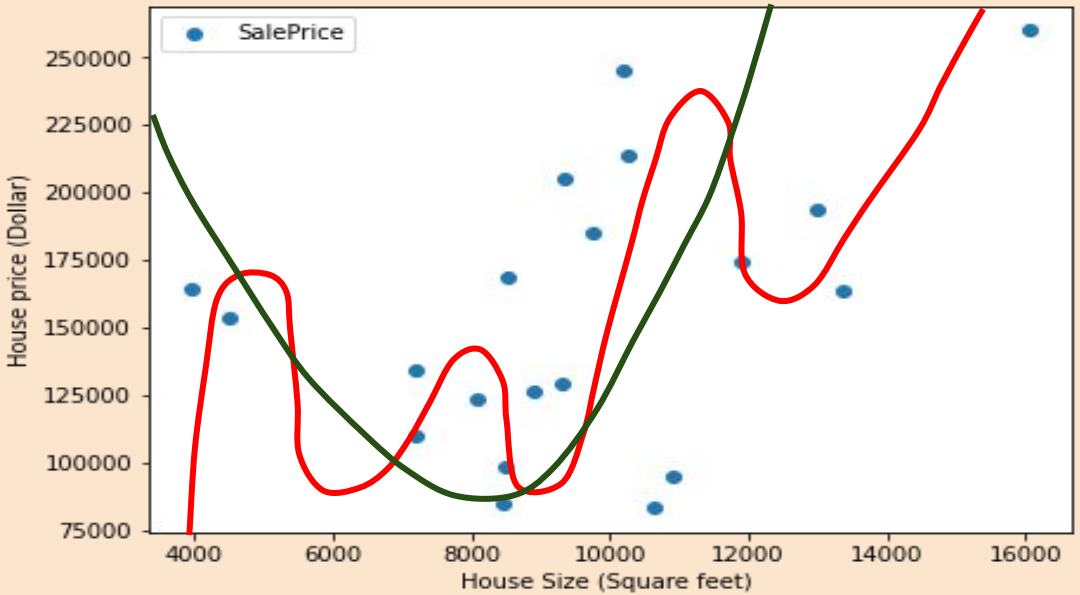

- Training data: good predictions
- Data outside training set: bad predictions

Overfitting

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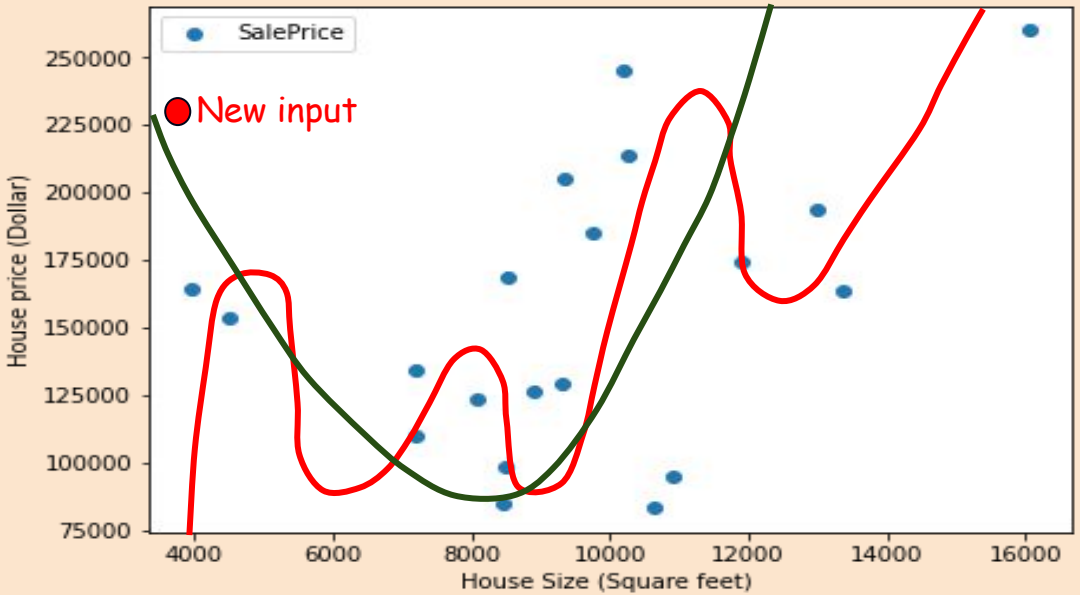

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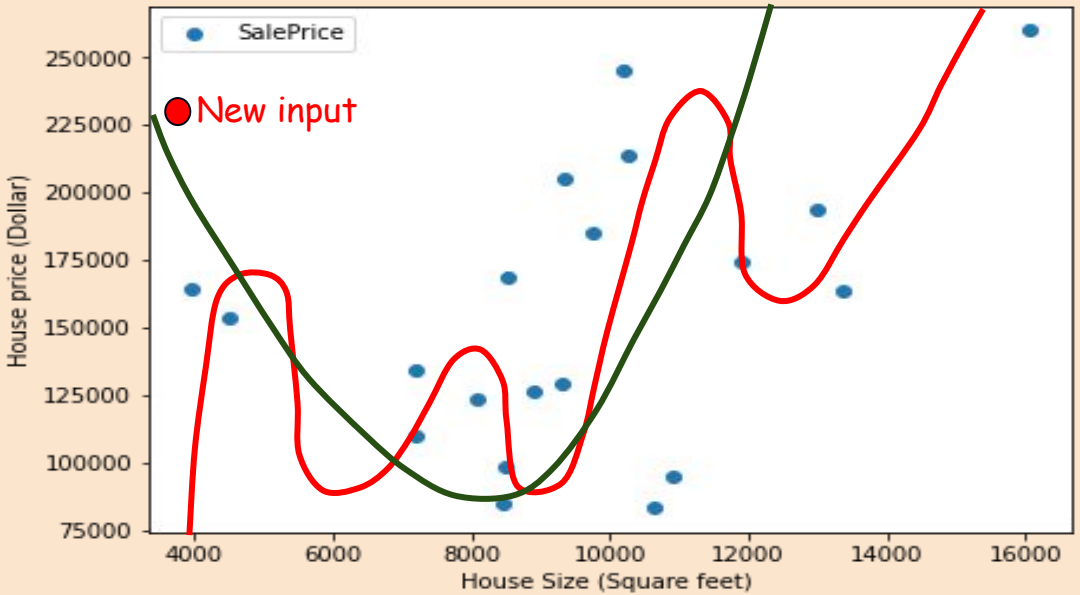
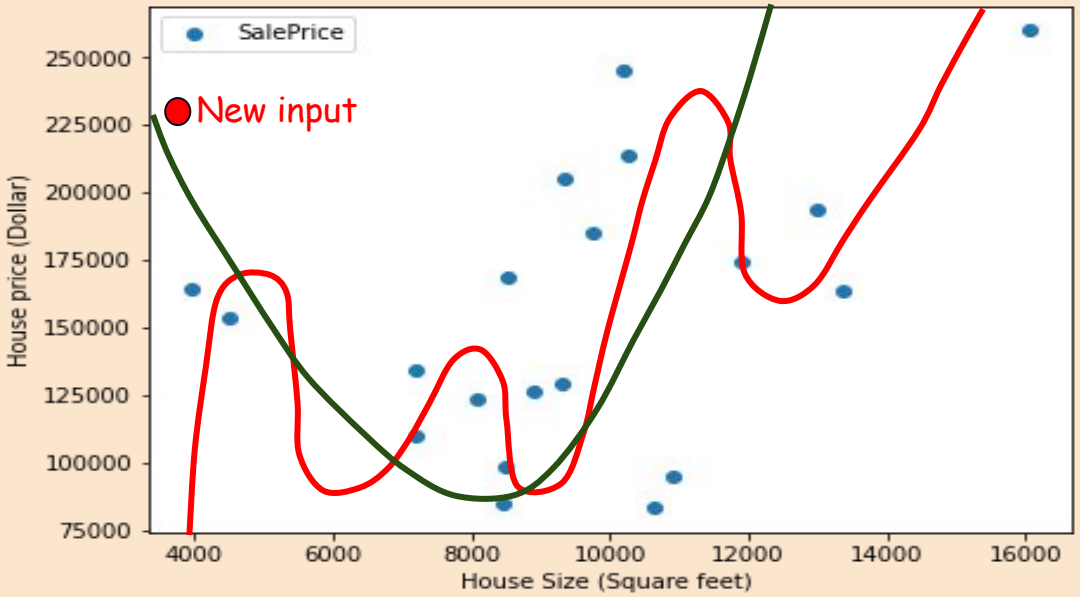


Overfitting model

- Signal to noise
- Memorize noize

Overfitting

In machine learning, we need to minimize cost function, however sometimes, we have the opposite problem when the cost function is too low.



Fact about over fitting

- The more parameters you have, the more likely your model will overfit.
- The more data you have (compared to parameters) the less likely your model will overfit, because noise will be more likely to average out.

Overfitting

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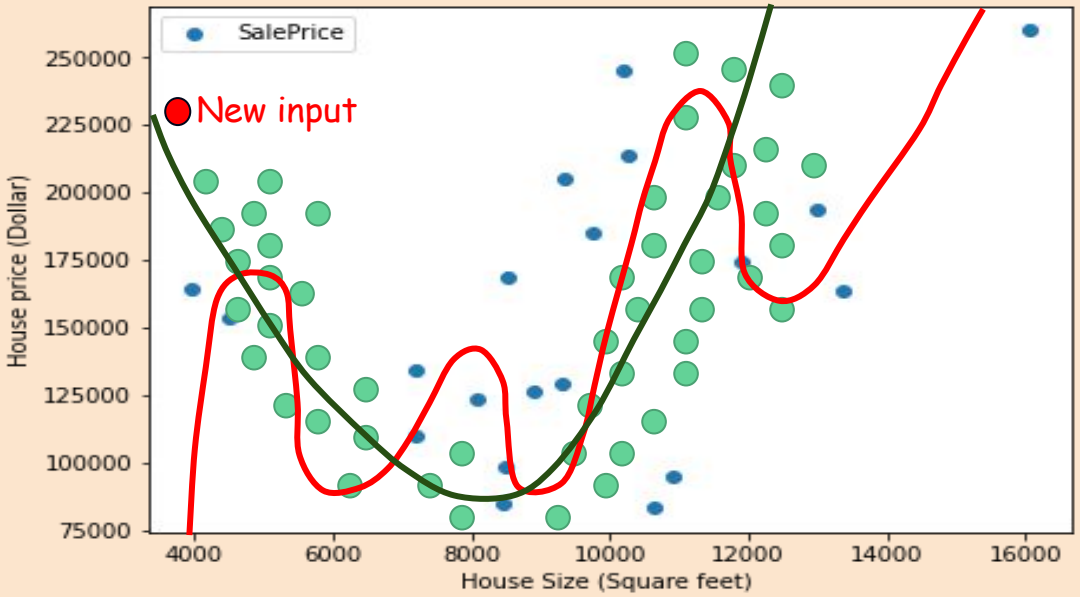

Fact about over fitting

- The more parameters you have, the more likely your model will overfit.
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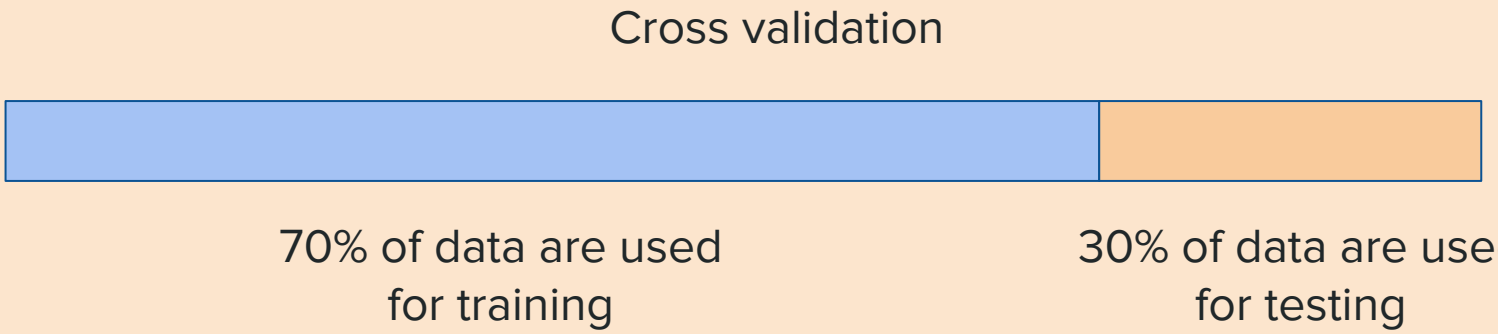
Avoid overfitting

Cross-validate your models: train models with one set of data (training set) and test them with another set of data (test set).

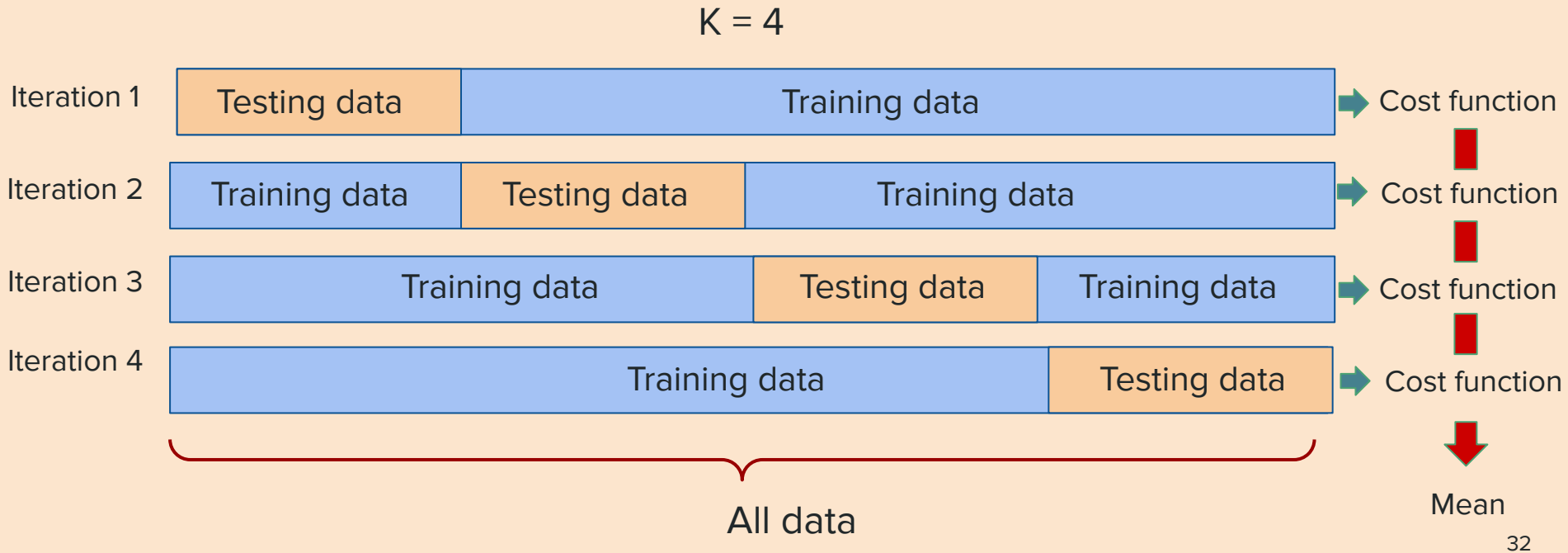


Overfitting model

- Train data → low cost but test data → high cost

Avoid overfitting

- F-fold cross validation: divide data into K chunks.
- Fit the model K times, each time use one chunk as test data, the rest as training data.



Overfitting

- Get more data
- Reduce the number of features (select only a few features that are more powerful)
- Reduce the number of parameters (using an algorithm to remove parameters that not necessary)
- Constrain the parameter set in the optimization process (regularization)

Regularization

$$Cost(\theta_0, \theta_1, \ldots, \theta_j) = \frac{1}{m} \sum_i (h(x_i) - y_i)^2 + \lambda \sqrt{\sum_j ||\theta_j||^2}$$

- Regularization is a mathematical way to reduce overfitting automatically, by limiting the influence of each feature.
- To minimize this cost function, you have to minimize both first and second terms.
- Minimize the second term means less overfitting.

Regularization

Error term

Regularization term

$$Cost(\theta_0, \theta_1, \ldots, \theta_j) = \frac{1}{m} \sum_i (h(x_i) - y_i)^2 + \lambda \sqrt{\sum_j ||\theta_j||^2}$$

The norm of cost function

The norm of parameter vector

- Regularization is a mathematical way to reduce overfitting automatically, by limiting the influence of each feature.
- To minimize this cost function, you have to minimize both first and second terms.
- Minimize the second term means less overfitting.

Regularization

Error term

Regularization term

$$Cost(\theta_0, \theta_1, \ldots, \theta_j) = \frac{1}{m} \sum_i (h(x_i) - y_i)^2 + \lambda \sqrt{\sum_j ||\theta_j||^2}$$

The norm of cost function

The norm of parameter vector

- Lambda is called ‘regularization parameter’
- Because we adjust lambda to adjust the weight of the two cost function terms
- High lambda: severely limit parameter size
- Low lambda: allow parameters to scale up more freely, give more importance to lowering error.

Regularization

Error term

Regularization term

$$Cost(\theta_0, \theta_1, \ldots, \theta_j) = \frac{1}{m} \sum_i (h(x_i) - y_i)^2 + \lambda \sqrt{\sum_j ||\theta_j||^2}$$

The norm of cost function

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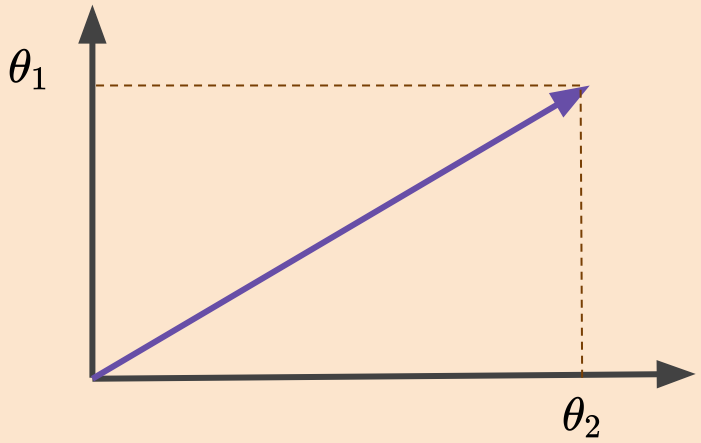
Example

Model overfit at lambda = 0.0001 ➔ adjust lambda =0.0005 ➔ less overfitting

Regularization

$$Cost(\theta_0, \theta_1, \ldots, \theta_j) = \frac{1}{m} \sum_i (h(x_i) - y_i)^2 + \lambda \sqrt{\sum_j ||\theta_j||^2}$$

The norm of parameter vector

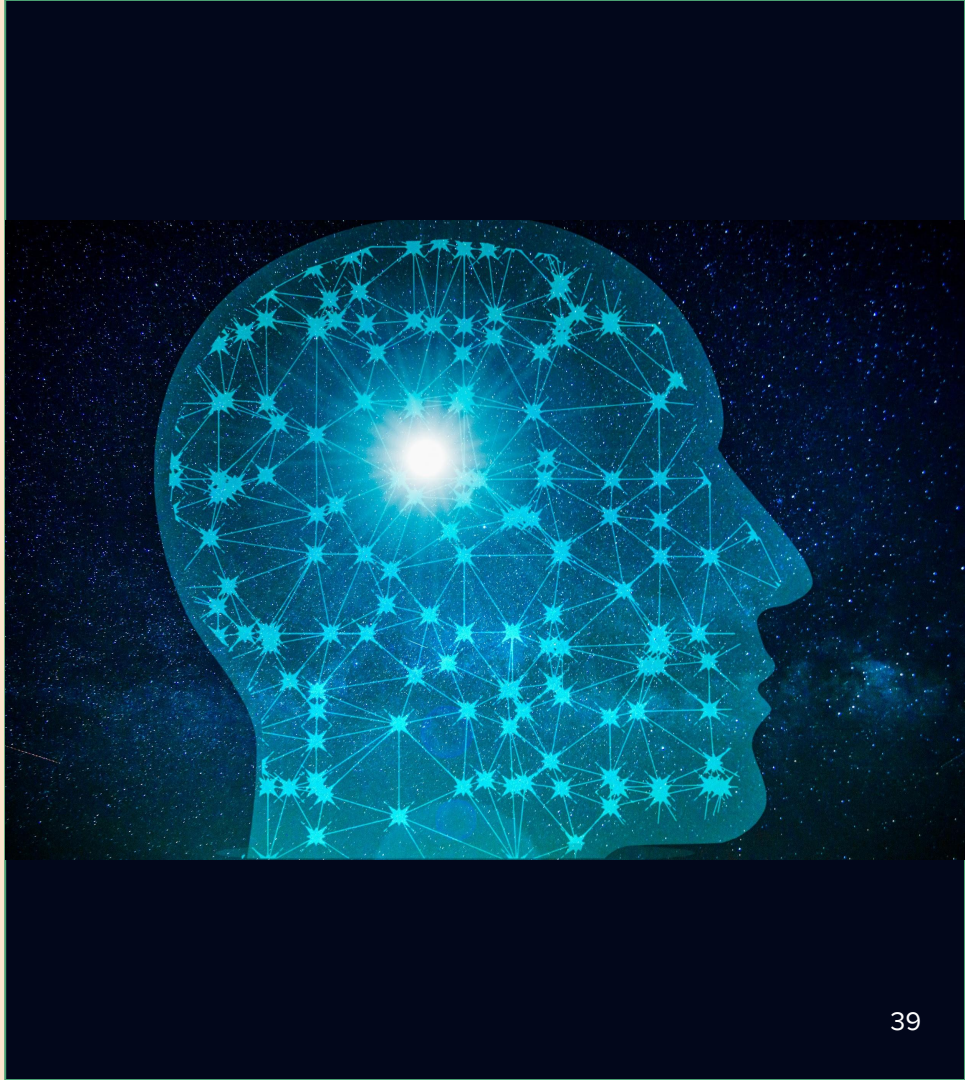


Euclidean norm

$$norm = \sqrt{\theta_1^2 + \theta_2^2}$$

$$= magnitude\ of\ < \theta_1, \theta_2 >$$

Linear Regression



Linear regression model

Output

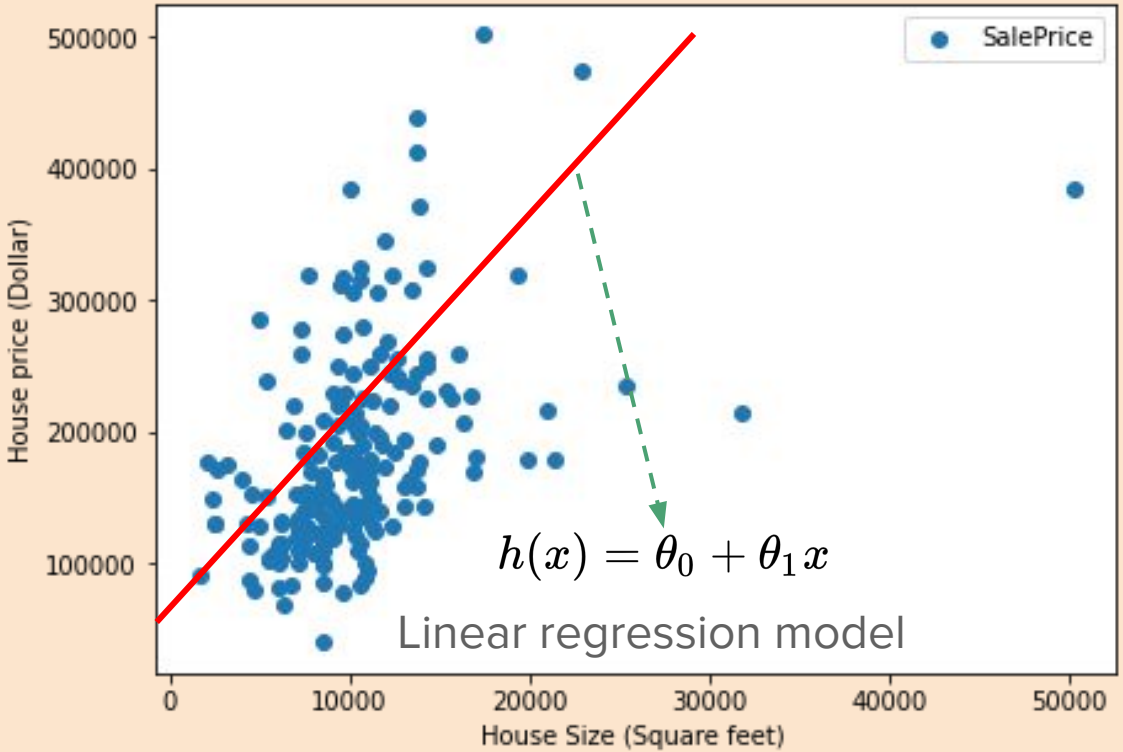
Input

$$h(x) = \theta_0 + \theta_1 x$$

Parameters of the model

$\theta_0 = 50000$

$\theta_1 = 15$



Linear regression model

$$h(\vec{x}) = \underbrace{\theta_0 + \theta_1 x_1 + \ldots + \theta_n x_n}_{\text{Regression}}$$

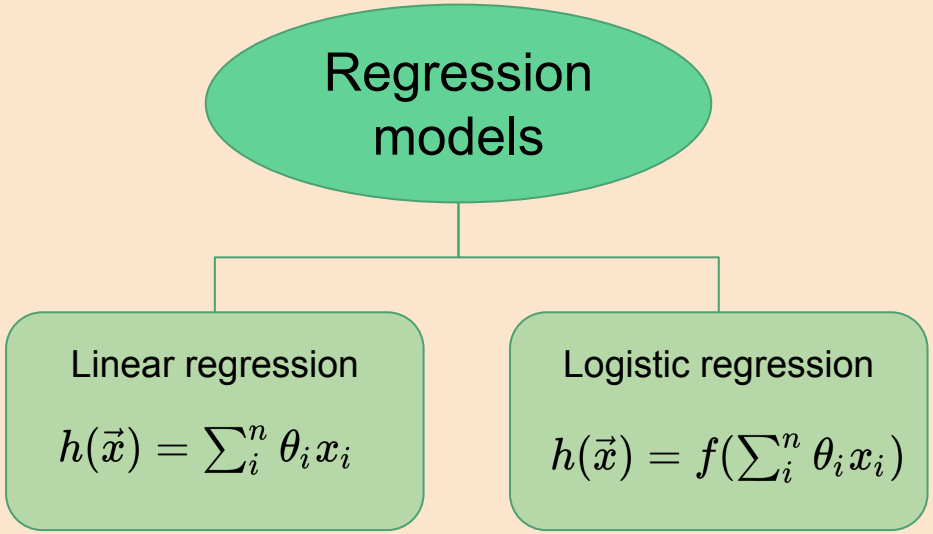
$$= \sum_i^n \theta_i x_i = \vec{\theta}^T \vec{x}$$

where $\vec{x} = [1, x_1, x_2, \ldots, x_n]^T$

$\vec{\theta} = [\theta_0, \theta_1, \ldots, \theta_n]^T$

n is the number of features

$\vec{x}, \vec{\theta} \in \Re^{n+1 \times n}$



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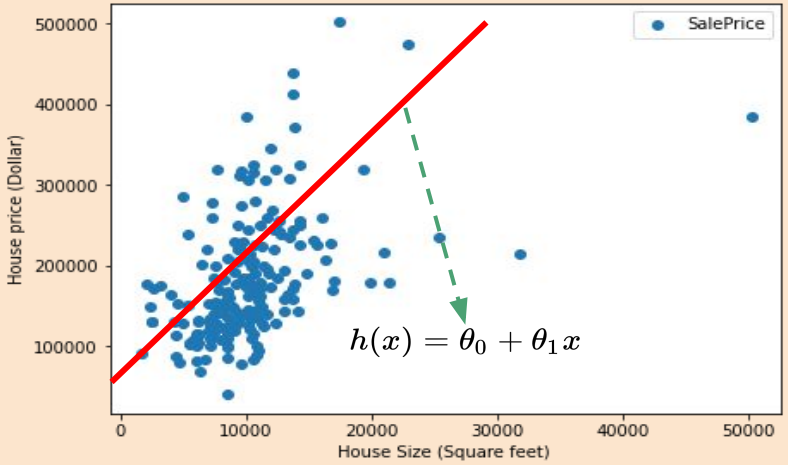

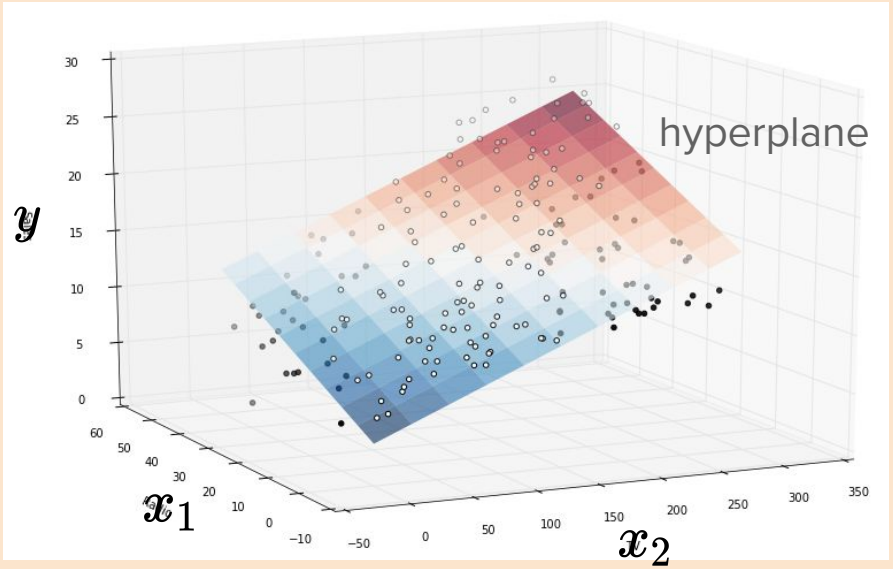

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Linear regression model

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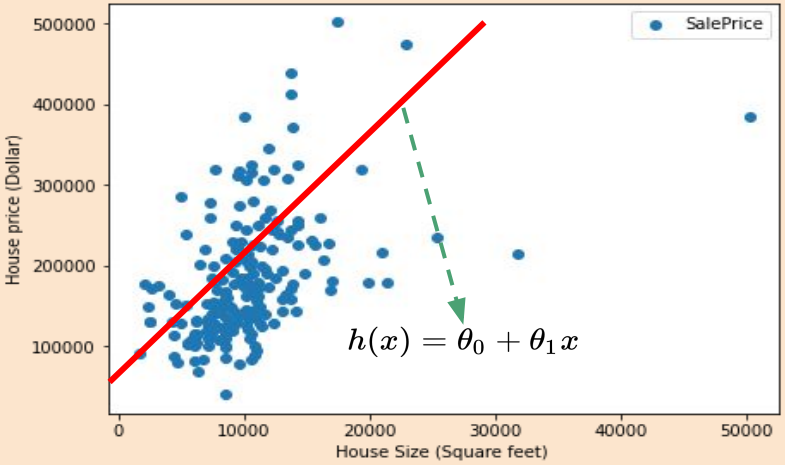
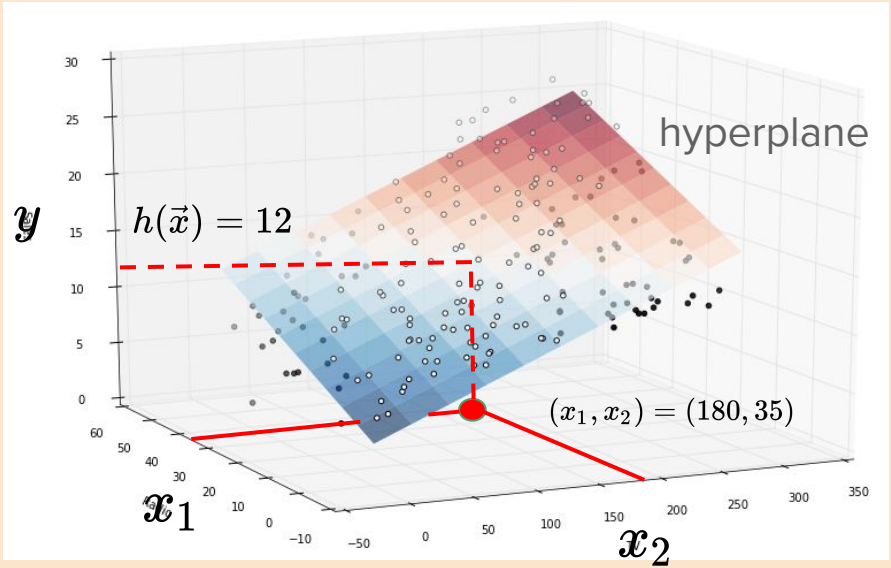
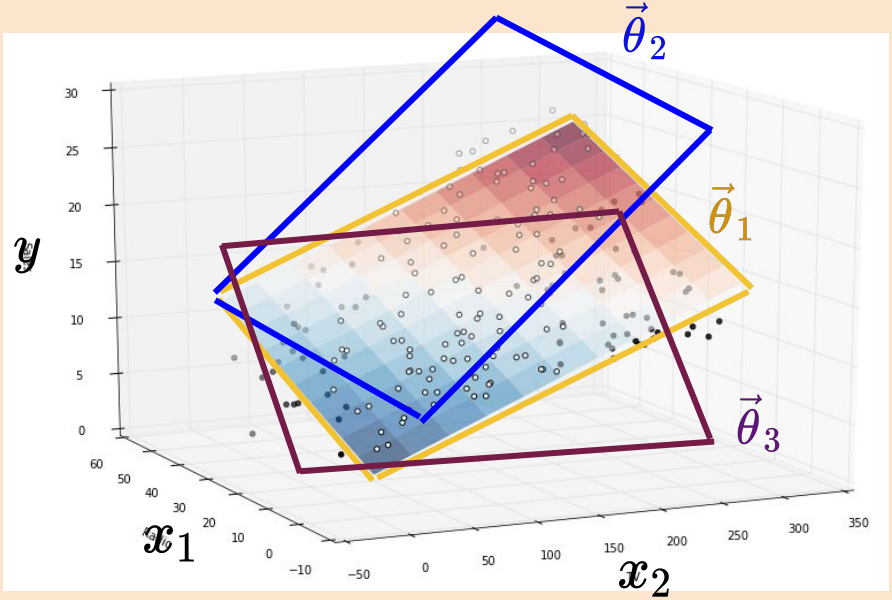




$$h(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$

Linear regression model

- Cost function: Mean square error (MSE), Root mean square error (RMSE)
- Find $\vec{\theta}$ to minimize cost function



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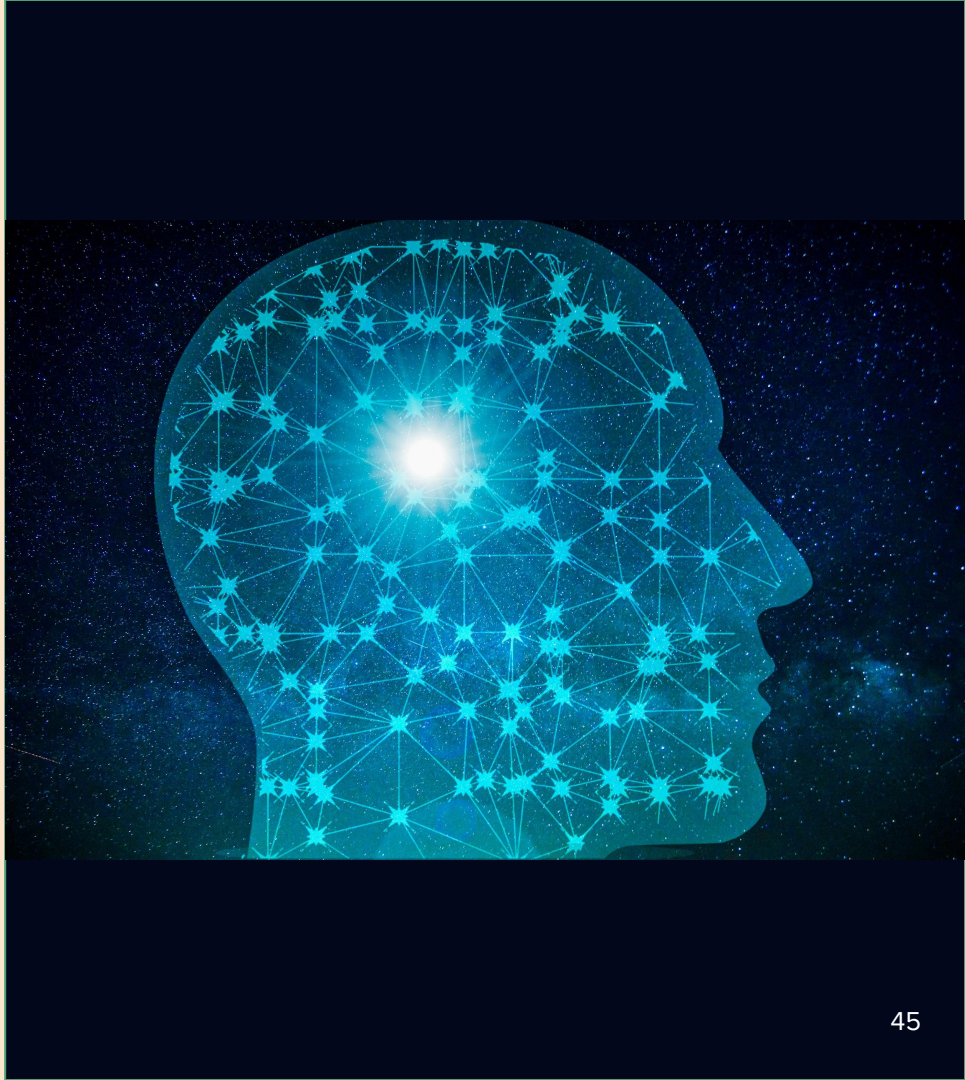
Regression Performance



Regression performance

- R-squared: how close the data are to the fitted regression line.
- R-squared = the percentage of the response variable variation that is explained by a linear model.

R-squared = Explained variation / Total variation

Regression performance

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- R-squared = the percentage of the response variable variation that is explained by a linear model.

$$Explained\ variation = Total\ variance - \sum_i (y_i - h(\vec{x}_i))^2$$

(*Total variation*)

R-squared = Explained variation / Total variation

$$Total\ variation = Variance\ of\ y = s^2 = \sum_i (y_i - \bar{y})^2$$

Regression performance

- R-squared: how close the data are to the fitted regression line.
- R-squared = the percentage of the response variable variation that is explained by a linear model.

R-squared = Explained variation / Total variation

- R-squared → 0 (model explains none of the variability of data) → bad model
- R-squared → 1 (model explains all of the variability of data) → good model

Regression performance

Mean squared error	$\text{MSE} = \frac{1}{n}\sum_{t=1}^{n} e_t^2$
Root mean squared error	$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n} e_t^2}$
Mean absolute error	$\text{MAE} = \frac{1}{n}\sum_{t=1}^{n} \lvert e_t \rvert$
Mean absolute percentage error	$\text{MAPE} = \frac{100\%}{n}\sum_{t=1}^{n} \left\lvert \frac{e_t}{y_t} \right\rvert$

where $e_t = (y_t - \hat{y}_t)$

Regression remark

- We use mean square error or root mean square error as a cost function of linear regression.
- Remember that linear regression model assume all features and labels are **normally distributed**.
- All features should have **equal variance** and are **not correlated** it each other.

Regression remark

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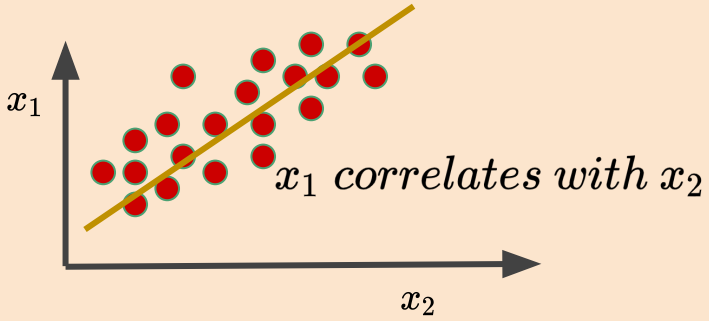
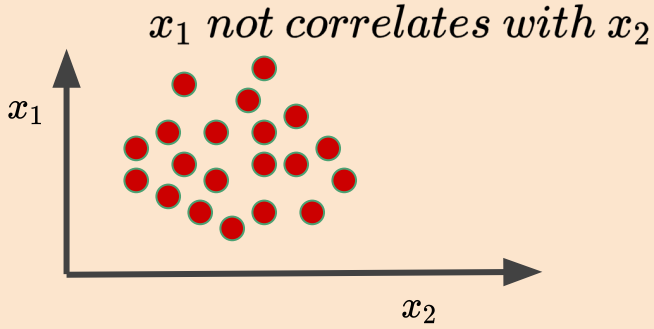
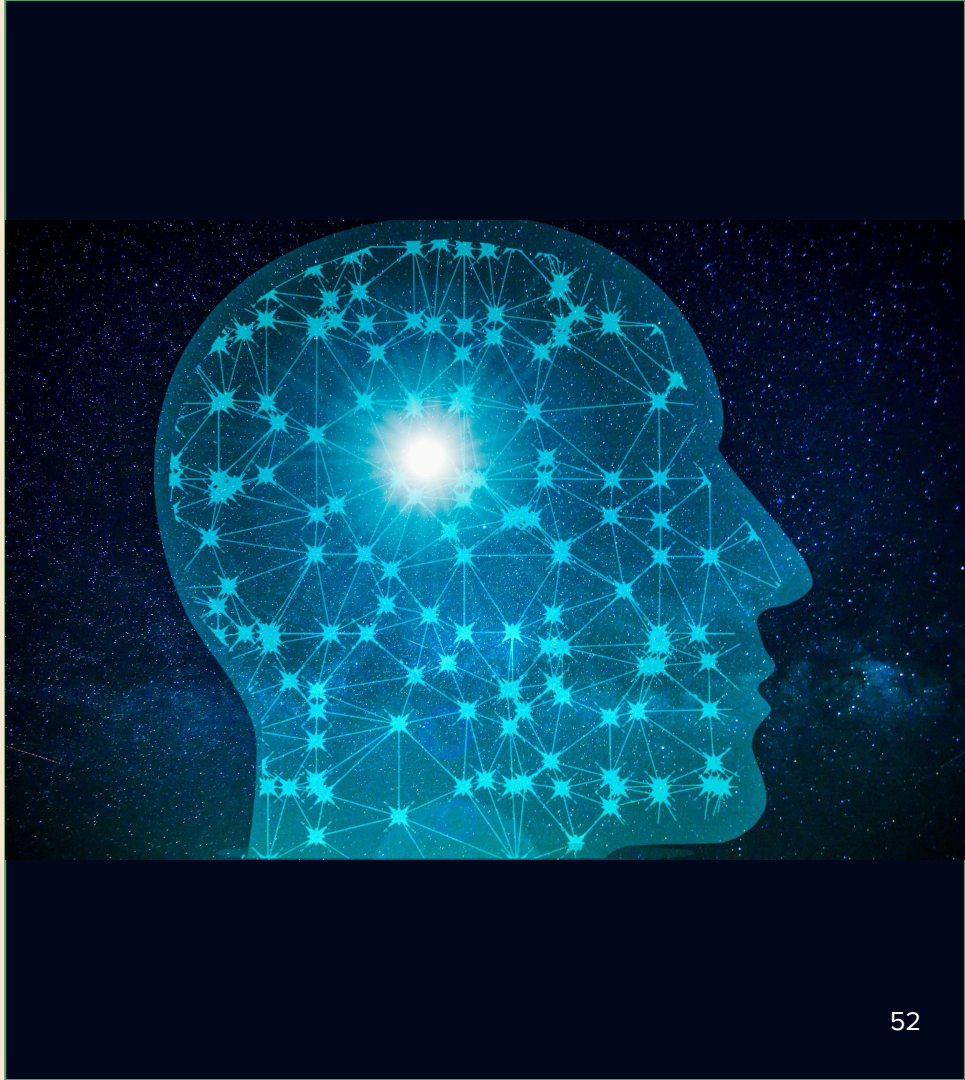




Logistic Regression



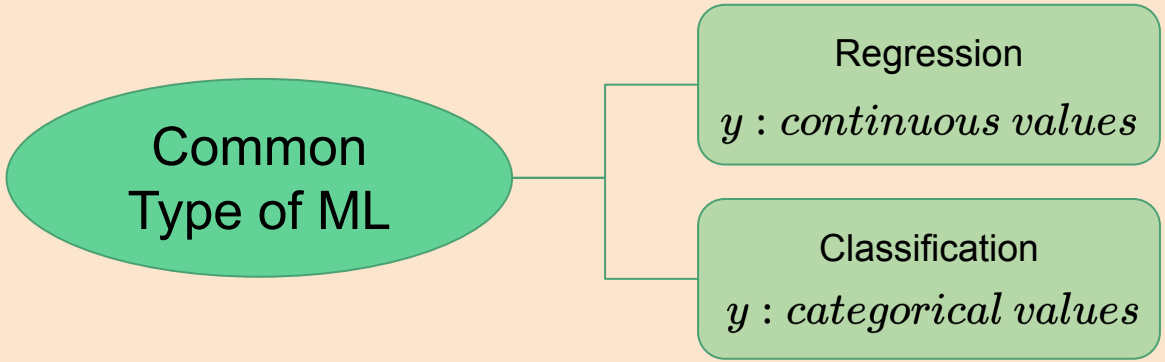
Logistic regression model

Logistic regression is similar to linear regression, however it is used for classification problem.

$$h(\vec{x}) = f(\theta_0 + \theta_1 x_1 + \ldots + \theta_n x_n)$$

where $\vec{x} = [1, x_1, x_2, \ldots, x_n]^T$ $\vec{\theta} = [\theta_0, \theta_1, \ldots, \theta_n]^T$ n is the number of features

$\vec{x}, \vec{\theta} \in \Re^{n+1 \times n}$



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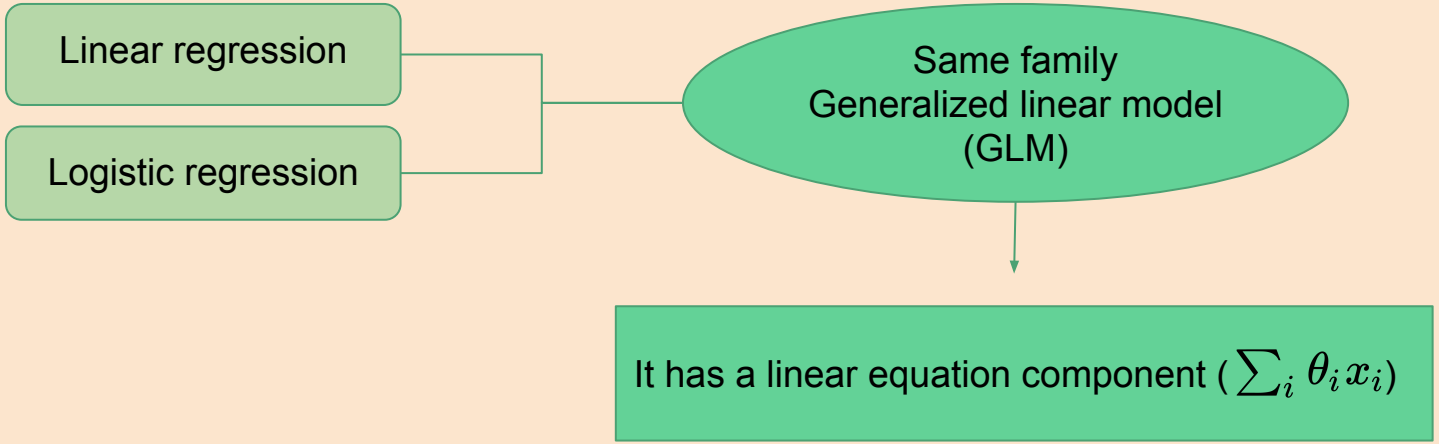


Logistic regression model

Logistic regression is similar to linear regression, however it is used for classification problem.

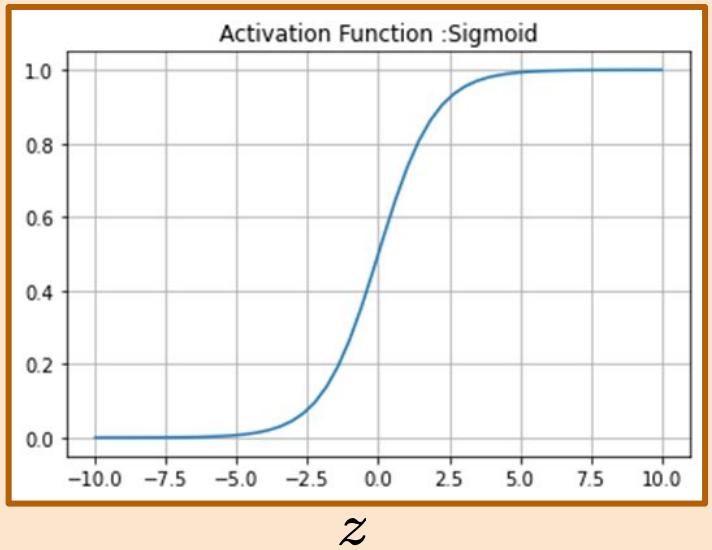
- Linear equation: $\sum_i \theta_i x_i = z$
- Every model in GLM: $h(\vec{x}) = f(z)$
- Linear regression model: $h(\vec{x}) = f(z) = f(\sum_i \theta_i x_i)$

  let $f(z) = z$ then $h(\vec{x}) = \sum_i \theta_i x_i$
- Logistic regression model: $f(z) = ?$ ➜ Logistic function

Logistic regression model

Logistic regression is similar to linear regression, however it is used for classification problem.

$$f(z) = h(\vec{x}) = f(\theta_0 + \theta_1 x_1 + \ldots + \theta_n x_n)$$



$$f(z) = \frac{1}{1+e^{-z}}$$

- Domain: $z \in \Re$
- Range: $f(z) \in (0, 1)$
- $z \to -\infty$ then $f(z) \to 0$
- $z \to \infty$ then $f(z) \to 1$

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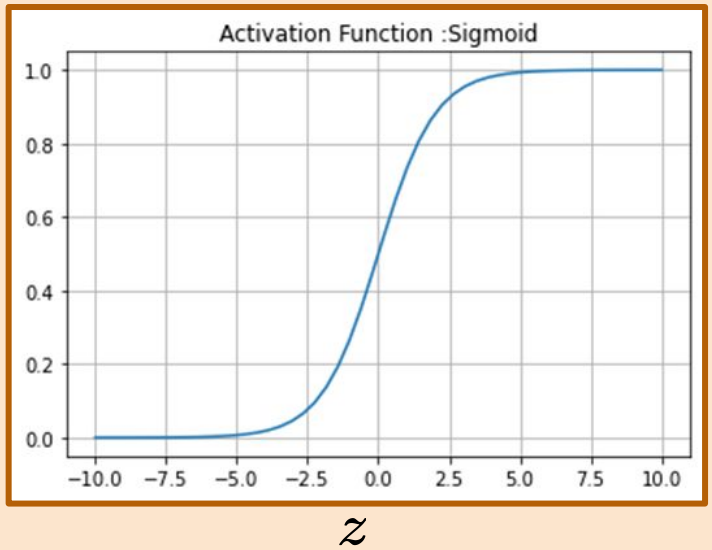

Probability

Sample 1	$x_1^{(1)}, x_2^{(1)}, \ldots, x_n^{(1)}$	...	$h(\vec{x}^{(1)})$
Sample 2	$x_1^{(2)}, x_2^{(2)}, \ldots, x_n^{(2)}$	...	$h(\vec{x}^{(2)})$
...	...	...	...
Sample *i*	$x_1^{(i)}, x_2^{(i)}, \ldots, x_n^{(i)}$	...	$h(\vec{x}^{(i)})$

$if\ h(\vec{x}^{(i)}) = 0.8\ :$

$Sample\ i \equiv\ Class\ 1\ : 80\%\ or\ 0.8$

$Sample\ i \equiv\ Class\ 0\ : 20\%\ or\ 0.2$

Logistic regression model

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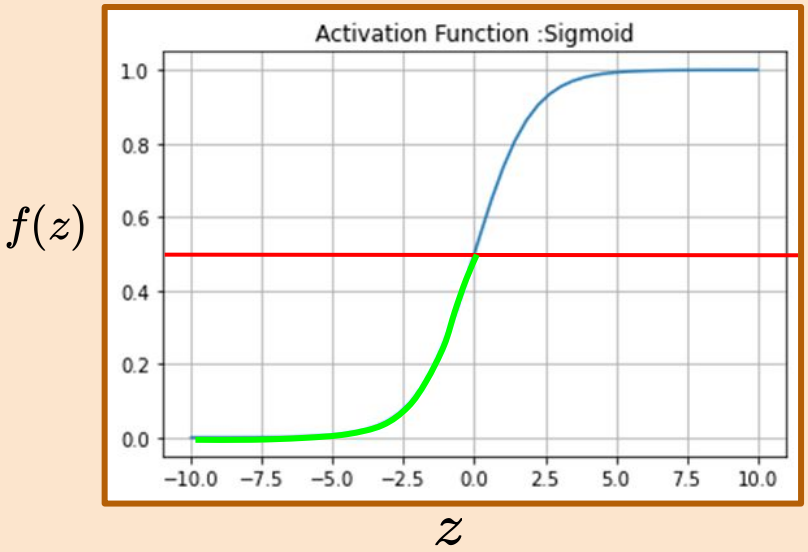

Example

Sample *i*	x_1^i	x_2^i	...	$h(\vec{x}^{(i)})$	y
1	3	A	...	0.6	1
2	6	A	...	0.3	0
3	12	B	...	0.5	1

$$Dicision\ rule:$$

$$y = \begin{cases} 1, & \text{if } f(z) = h(\vec{x}^{(i)}) \geq 0.5 \text{ or } z \geq 0 \\ 0, & \text{if } f(z) = h(\vec{x}^{(i)}) < 0.5 \text{ or } z < 0 \end{cases}$$

A simple example

Let's look at a simple logistic regression procedure.

Sample i	age $x_1^{(i)}$	income $x_2^{(i)}$	dependents $x_3^{(i)}$	spending $x_4^{(i)}$	history y
1	40	50	0	30	1
2	25	40	2	35	1
3	18	10	0	12	0
4	34	22	1	10	1
	55	**65**	**4**	**60**	**?**

A simple example

We first to do preprocessing, such as normalization and standardization

i	$x_1^{(i)}$	$x_2^{(i)}$	$x_3^{(i)}$	$x_4^{(i)}$	y
1	40	50	0	30	1
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A simple example

We first to do preprocessing, such as normalization and standardization

$$new\ x_j^{(i)} = \frac{x_j^{(i)}}{max(x_j^{(i)} \mid i{=}1,2,3,4\ and\ j{=}1,2,3,4)}$$

i	$x_1^{(i)}$	$x_2^{(i)}$	$x_3^{(i)}$	$x_4^{(i)}$	y
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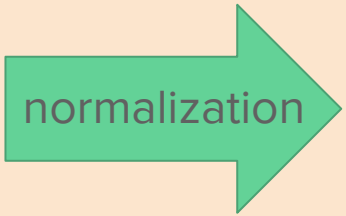

i	$x_1^{(i)}$	$x_2^{(i)}$	$x_3^{(i)}$	$x_4^{(i)}$	y
1	0.44	0.63	0	0.6	1
2	0.28	0.5	0.5	0.7	1
3	0.2	0.13	0	0.24	0
4	0.38	0.28	0.25	0.2	1

$$max(x_j^{(i)} \mid i = 1,2,3,4\ and\ j = 1,2,3,4) = 91$$

A simple example

Then, fit the logistic regression to the data. Suppose after fitting, here are the wight numbers.

$$h(\vec{x}) = f(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4)$$

$$= f(0 + 0.7x_1 + 0.6x_2 - 0.1x_3 - 0.2x_4)$$

A simple example

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Let make prediction for a single customer.

j	Raw data	Normalization	θ_j	$\theta_j x_j$
age x_1	40.00	0.44	0.7	0.31
income x_2	57.00	0.63	0.6	0.38
dependent x_3	0.00	0.00	-0.1	0.00
spending x_4	55	0.60	-0.2	-0.12
			Sum:	0.57
			$h(\vec{x})$	0.64

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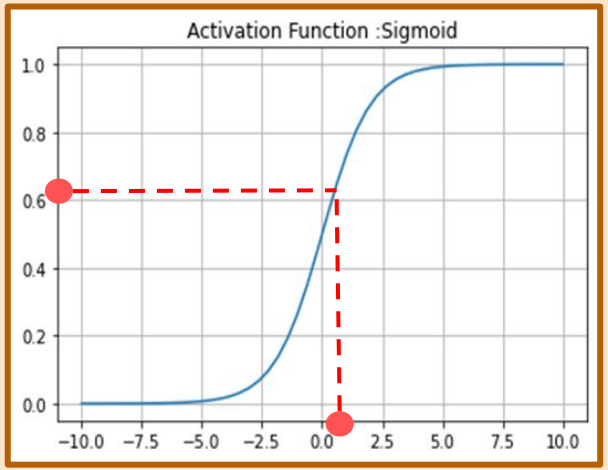

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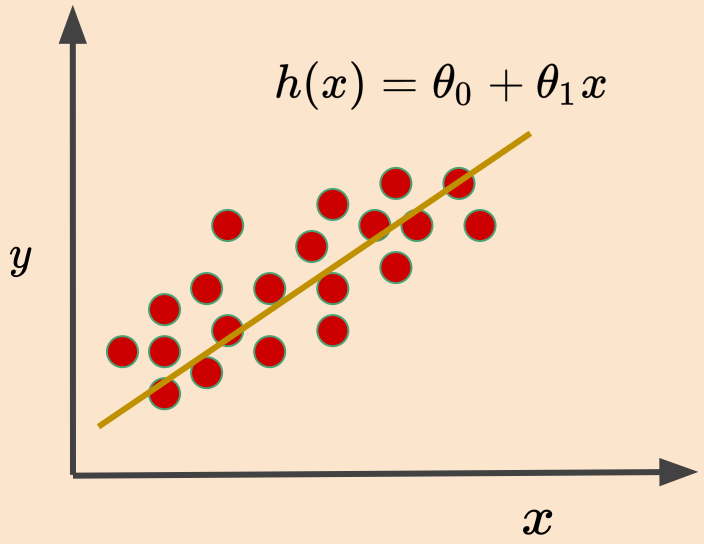
$h(\vec{x})$ Indicates the probability of customer being good.

$$Dicision\ rule:$$

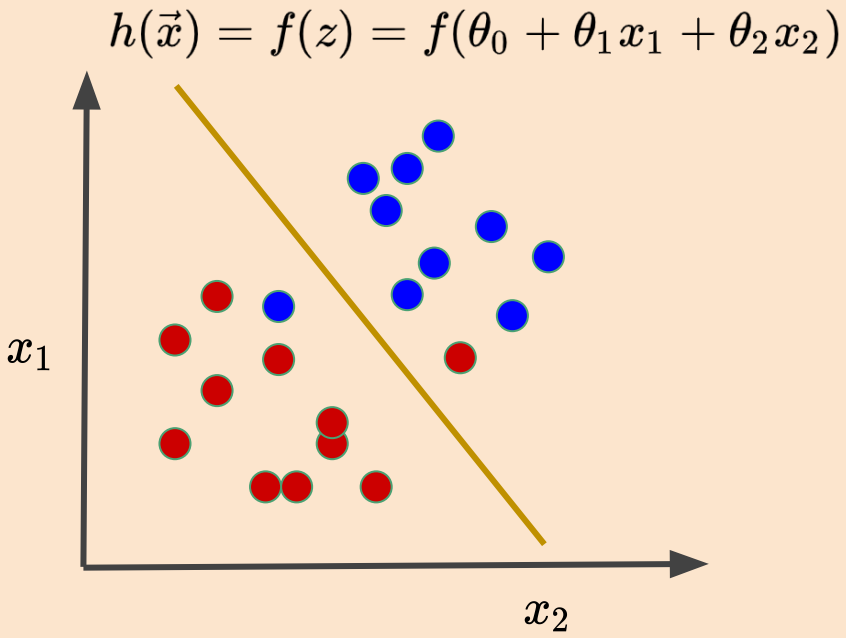
$$y = \begin{cases} 1, & \text{if } h(\vec{x}^{(i)}) \geq 0.5 \\ 0, & \text{if } h(\vec{x}^{(i)}) < 0.5 \end{cases}$$

➜ This is a good customer !

A simple example



Linear regression



Logistic regression

A simple example

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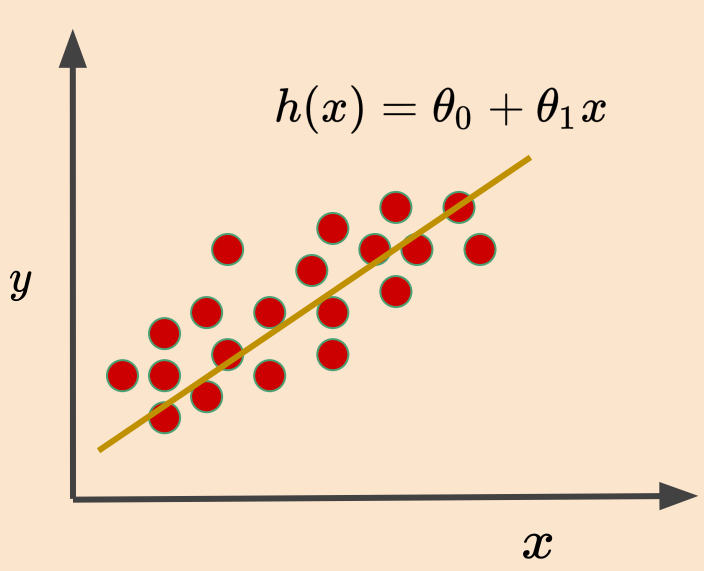

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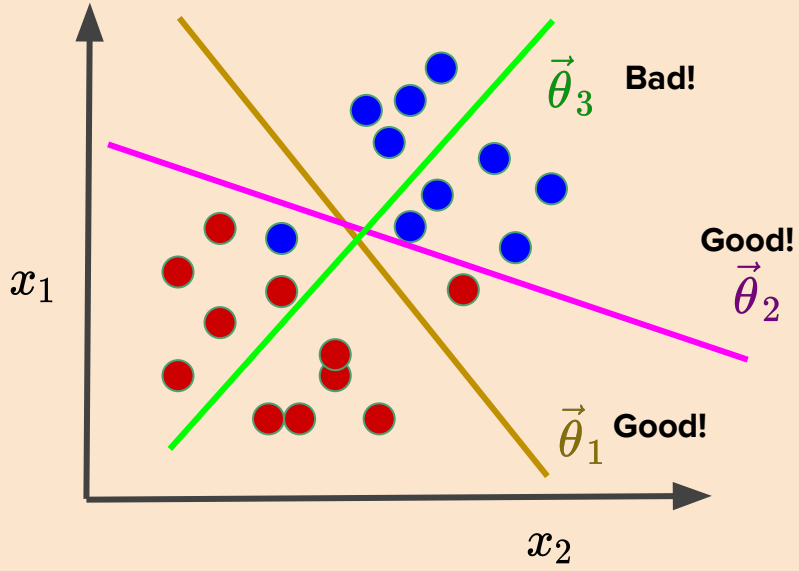


Logistic regression

Classification cost function

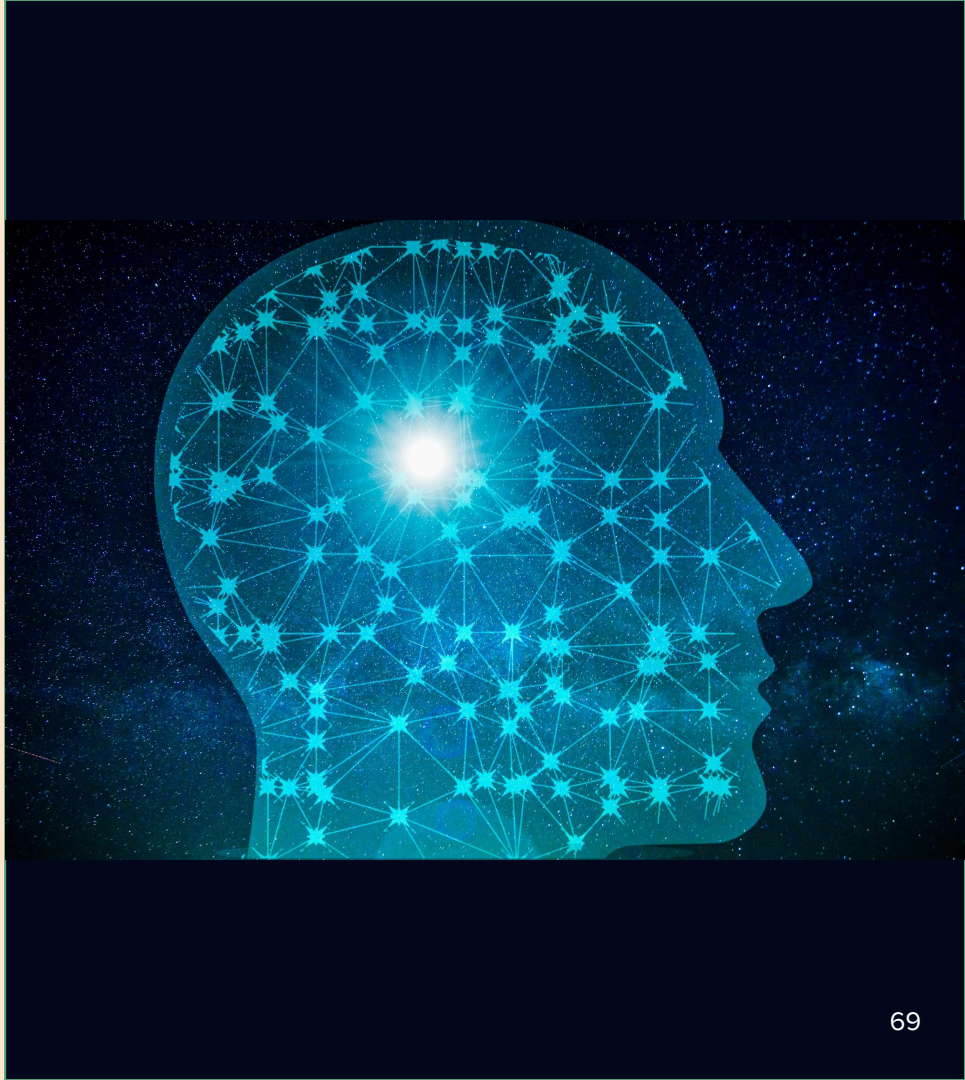
Cost function: sum of errors from classifying sample *i* : $\sum_i E_i$

$$E_i = -(y^{(i)} log(h(\vec{x}^{(i)})) + (1 - y^{(i)}) log(1 - h(\vec{x}^{(i)})))$$

The cost function is ‘cross entropy’ error, defining how actual classes match your class predictions.

$h(\vec{x}^{(i)})$	y	**Cost**
0.8	1	0.10
0.1	1	1.00
0.1	0	0.05

Classification Performance



Classification Performance

- Log loss (‘cross entropy’ error)
- Accuracy
- Precision
- Recall
- F1-score

Classification Performance

- Log loss (‘cross entropy’ error)
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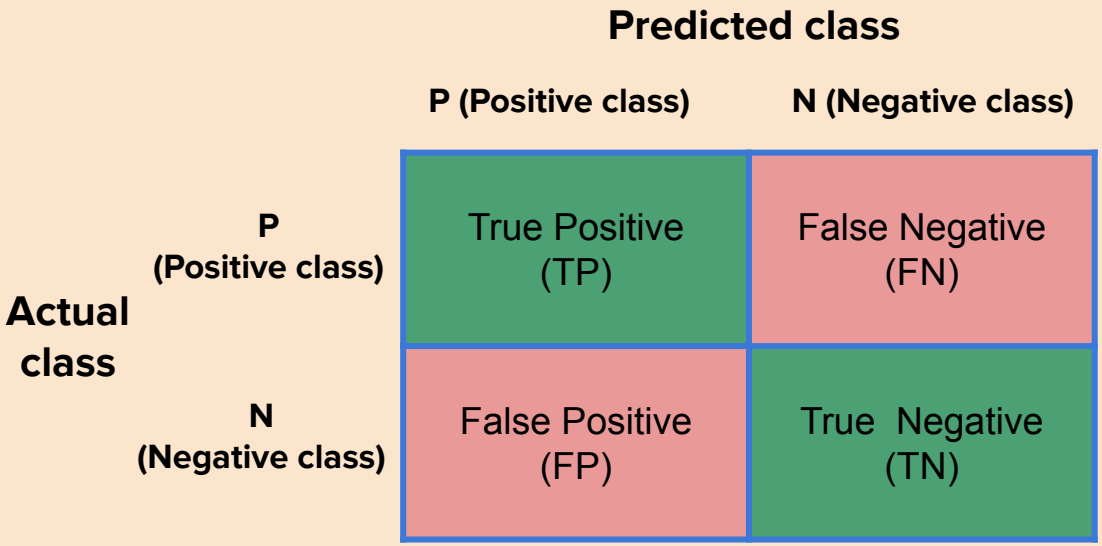
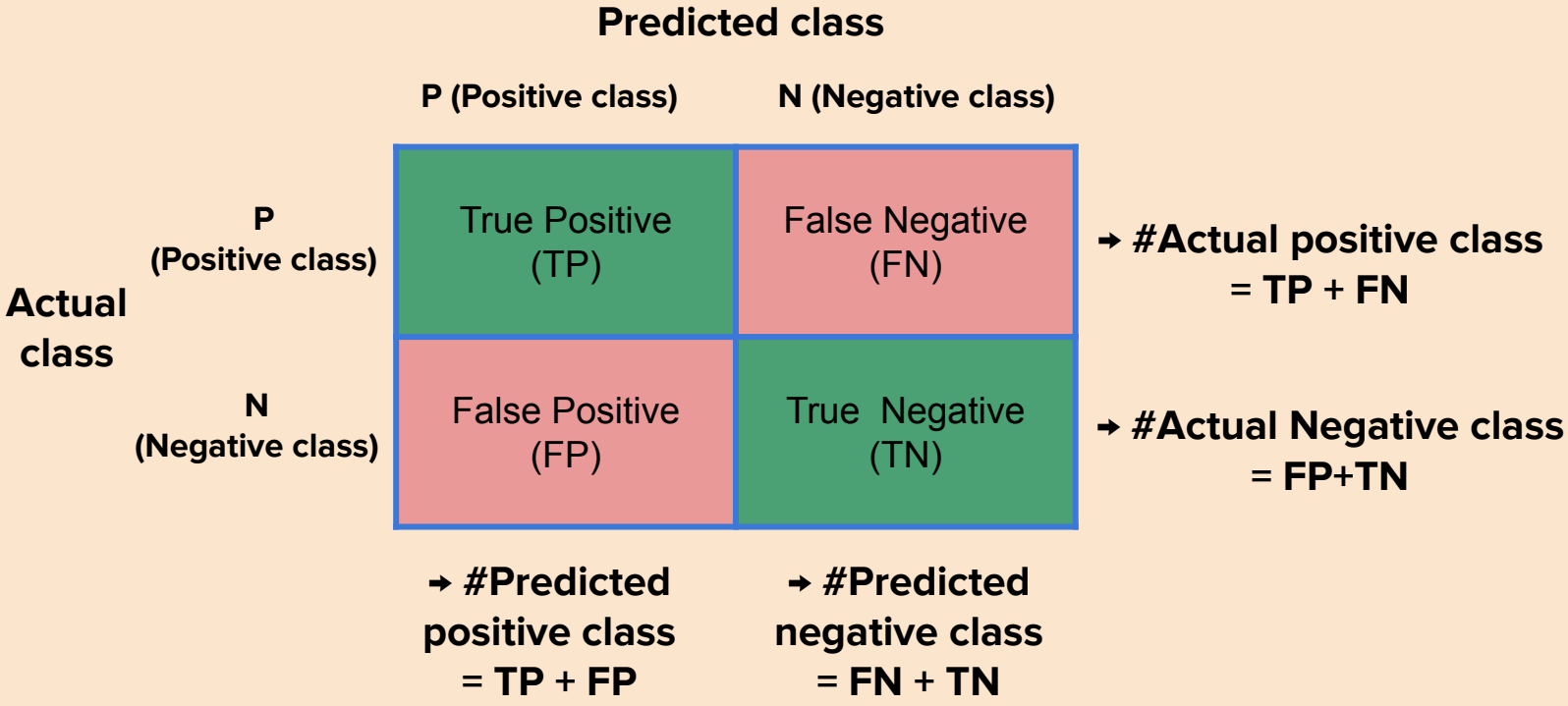
Confusion Matrix



$$P \equiv Positive\ class \equiv label\ 1$$
$$N \equiv Negative\ class \equiv label\ 0$$

Classification Performance

Confusion Matrix



Classification Performance

Confusion Matrix

Actual class \ Predicted class	P (Positive class)	N (Negative class)
P (Positive class)	True Positive (TP)	False Negative (FN)
N (Negative class)	False Positive (FP)	True Negative (TN)

$$Accuracy = \frac{TP+TN}{TP+FN+FP+TN}$$

$$Precision = \frac{TP}{TP+FP}$$

$$Recall = \frac{TP}{TP+FN}$$

$$F1 - score = 2 \times \frac{Precision \times Recall}{Precision+Recall}$$

Classification Performance

Example 1

Actual class \ Predicted class	P (Positive class)	N (Negative class)
P (Positive class)	70 (TP)	30 (FN)
N (Negative class)	35 (FP)	65 (TN)

$$Accuracy = \frac{70+65}{70+30+35+65} = 0.675$$

$$Precision = \frac{70}{70+35} = 0.67$$

$$Recall = \frac{70}{70+30} = 0.7$$

$$F1 - score = 2 \times \frac{0.67 \times 0.7}{0.67+0.7} = 0.685$$

- # Actual positive class: 100
- # Actual negative class: 100

Classification Performance

Example 2

		Predicted class	
		P (Positive class)	**N (Negative class)**
Actual class	**P (Positive class)**	0 (TP)	5 (FN)
	N (Negative class)	5 (FP)	995 (TN)

$$Accuracy = \frac{0+995}{0+5+5+995} = 0.99$$

$$Precision = \frac{0}{0+5} = 0$$

$$Recall = \frac{0}{0+5} = 0$$

$$F1 - score = 2 \times \frac{0 \times 0}{0+0} = 0$$

- # Actual positive class: 5
- # Actual negative class: 1000

Good luck 😉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