Identity between nested sum and power sum : Personal rediscovery

While reviewing summation techniques, I noticed identity of $\sum_{k=1}^{n} k^{2} = \sum_{k=1}^{n}\sum_{j=k}^{n} j$ by thinking in geometrically like form of

$\sum_{k=1}^{n}\sum_{j=k}^{n} j$ =

1	2	3	4	5	…	n	+
	2	3	4	5	…	n	+
		3	4	5	…	n	+
			4	5	…	.	+
				5	…	.	+
					…	.	+
						n	

Which equal to 1*1 + 2*2 + 3*3 + 4*4 … n*n = $\sum_{k=1}^{n} k^{2}$

As I found this I realized that this identity extend to other summation of higher power and can be generalized in the form

$\sum_{k=1}^{n}\sum_{j=k}^{n} j^{m} = \sum_{j=1}^{n} k^{m+1}$ for n, m ∈ Z^{+}

$\sum_{k=1}^{n}\sum_{j=k}^{n} j^{m}$ =

1^{m}	2^{m}	3^{m}	4^{m}	5^{m}	…	n^{m}	+
	2^{m}	3^{m}	4^{m}	5^{m}	…	n^{m}	+
		3^{m}	4^{m}	5^{m}	…	n^{m}	+
			4^{m}	5^{m}	…	n^{m}	+
				5^{m}	…	n^{m}	+
					…	n^{m}	+
						n^{m}	

Which for each column equal $j^{m} \times j = j^{m+1}$ and the summation ultimately equal to $\sum_{j=1}^{n} k^{m+1}$

Which is the form that I never realized before although maybe discovered before.

Overall I think it has potential for recursiveness and the logic behind it could easily be understood which might have some potential for teaching or using in computer algorithms. Although the Faulhaber equation still would be easier used in computer algorithms.

If anyone has seen this identity in the literature, I'd love to know where it appears.

Thanapong Sereerat
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