

HW 4

$$\text{Info}(D) = I(8,4) = -\frac{8}{12} \log_2\left(\frac{8}{12}\right) - \frac{4}{12} \log_2\left(\frac{4}{12}\right) = 0.9183$$

∴ dimensions split

$$\text{Gain}(\text{age}) = 0.1804 \rightarrow \text{don't Gain}(\text{age})$$

$$\text{Gain}(\text{credit_rating}) = 0.1686$$

$$\text{Gain}(\text{student}) = 0.0977$$

$$\text{Gain}(\text{income}) = 0.0546$$

age	income	student	credit_rating	buys computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31...40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
>40	medium	no	excellent	no

age	P _i	n _i	I(P _i , n _i)
<=30	2	2	1
31...40	3	0	0
>40	3	2	0.971

$$I(2,2) = -\frac{2}{4} \log_2\left(\frac{2}{4}\right) - \frac{2}{4} \log_2\left(\frac{2}{4}\right) = 1$$

$$I(3,0) = -\frac{3}{3} \log_2\left(\frac{3}{3}\right) - \frac{0}{3} \log_2\left(\frac{0}{3}\right) = 0$$

$$I(3,2) = -\frac{3}{5} \log_2\left(\frac{3}{5}\right) - \frac{2}{5} \log_2\left(\frac{2}{5}\right) = 0.971$$

$$\begin{aligned} \text{Info}_{\text{age}}(D) &= \frac{4}{12} I(2,2) + \frac{3}{12} I(3,0) + \frac{5}{12} I(3,2) \\ &= \frac{4}{12} \cdot 1 + \frac{3}{12} \cdot 0 + \frac{5}{12} \cdot 0.971 \\ &= 0.7379 \end{aligned}$$

$$\begin{aligned} \text{Gain}(\text{age}) &= \text{Info}(D) - \text{Info}_{\text{age}}(D) \\ &= 0.9183 - 0.7379 \\ &= 0.1804 \end{aligned}$$

income	P _i	n _i	I(P _i , n _i)
High	2	2	1
Medium	4	1	0.7219
low	2	1	0.9183

$$I(2,2) = -\frac{2}{4} \log_2\left(\frac{2}{4}\right) - \frac{2}{4} \log_2\left(\frac{2}{4}\right) = 1$$

$$I(4,1) = -\frac{4}{5} \log_2\left(\frac{4}{5}\right) - \frac{1}{5} \log_2\left(\frac{1}{5}\right) = 0.7219$$

$$I(2,1) = -\frac{2}{3} \log_2\left(\frac{2}{3}\right) - \frac{1}{3} \log_2\left(\frac{1}{3}\right) = 0.9183$$

$$\begin{aligned} \text{Info}_{\text{income}}(D) &= \frac{4}{12} I(2,2) + \frac{5}{12} I(4,1) + \frac{3}{12} I(2,1) \\ &= \frac{4}{12} \cdot 1 + \frac{5}{12} \cdot 0.7219 + \frac{3}{12} \cdot 0.9183 \\ &= 0.8637 \end{aligned}$$

$$\begin{aligned} \text{Gain}(\text{income}) &= \text{Info}(D) - \text{Info}_{\text{income}}(D) \\ &= 0.9183 - 0.8637 \\ &= 0.0546 \end{aligned}$$

student	P _i	n _i	I(P _i , n _i)
Yes	5	1	0.65
No	3	3	1

$$I(5,1) = -\frac{5}{6} \log_2\left(\frac{5}{6}\right) - \frac{1}{6} \log_2\left(\frac{1}{6}\right) = 0.6500$$

$$I(3,3) = -\frac{3}{6} \log_2\left(\frac{1}{6}\right) - \frac{3}{6} \log_2\left(\frac{3}{6}\right) = 1$$

$$\begin{aligned} \text{Info}_{\text{student}}(D) &= \frac{6}{12} I(5,1) + \frac{6}{12} I(3,3) \\ &= \frac{6}{12} \cdot 0.6500 + \frac{6}{12} \cdot 1 \\ &= 0.825 \end{aligned}$$

$$\begin{aligned} \text{Gain}(\text{student}) &= \text{Info}(D) - \text{Info}_{\text{student}}(D) \\ &= 0.9183 - 0.825 \\ &= 0.0933 \end{aligned}$$

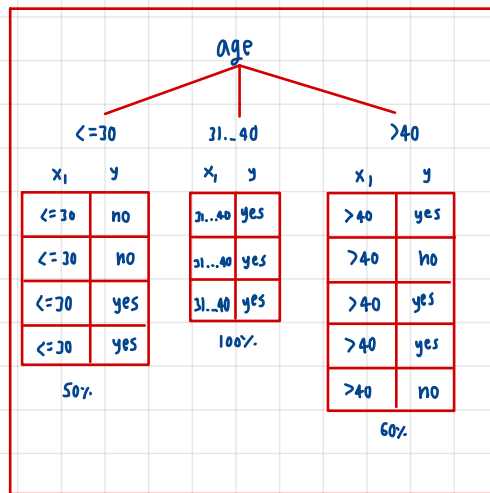
Credit_rating	P _i	n _i	I(P _i , n _i)
fair	6	1	0.5917
excellent	2	3	0.9183

$$I(6,1) = -\frac{6}{7} \log_2\left(\frac{6}{7}\right) - \frac{1}{7} \log_2\left(\frac{1}{7}\right) = 0.5917$$

$$I(2,3) = -\frac{2}{5} \log_2\left(\frac{2}{5}\right) - \frac{3}{5} \log_2\left(\frac{3}{5}\right) = 0.9183$$

$$\begin{aligned} \text{Info}_{\text{credit_rating}}(D) &= \frac{7}{12} I(6,1) + \frac{5}{12} I(2,3) \\ &= \frac{7}{12} \cdot 0.5917 + \frac{5}{12} \cdot 0.9183 \\ &= 0.7437 \end{aligned}$$

$$\begin{aligned} \text{Gain}(\text{credit_rating}) &= \text{Info}(D) - \text{Info}_{\text{credit_rating}}(D) \\ &= 0.9183 - 0.7437 \\ &= 0.1686 \end{aligned}$$



age ≤ 30

x ₂	x ₃	x ₄	y
income	student	credit_rating	buys_computer
high	no	fair	no
high	no	excellent	no
low	yes	fair	yes
medium	yes	excellent	yes

$$Info(D, \leq 30) = I(2,2) = -\frac{2}{4} \log_2\left(\frac{2}{4}\right) - \frac{2}{4} \log_2\left(\frac{2}{4}\right) = 1$$

income	p _i	n _i	I(p _i , n _i)
High	0	2	0
Medium	1	0	0
low	1	0	0

$$I(0,2) = -\frac{0}{2} \log_2\left(\frac{0}{2}\right) - \frac{2}{2} \log_2\left(\frac{2}{2}\right) = 0$$

$$I(1,0) = -\frac{1}{1} \log_2\left(\frac{1}{1}\right) - \frac{0}{1} \log_2\left(\frac{0}{1}\right) = 0$$

$$I(1,0) = -\frac{1}{1} \log_2\left(\frac{1}{1}\right) - \frac{0}{1} \log_2\left(\frac{0}{1}\right) = 0$$

student	p _i	n _i	I(p _i , n _i)
Yes	2	0	0
No	0	2	0

$$I(2,0) = -\frac{2}{2} \log_2\left(\frac{2}{2}\right) - \frac{0}{2} \log_2\left(\frac{0}{2}\right) = 0$$

$$I(0,2) = -\frac{0}{2} \log_2\left(\frac{0}{2}\right) - \frac{2}{2} \log_2\left(\frac{2}{2}\right) = 0$$

$$Info_{student}(D, \leq 30) = \frac{2}{4} I(2,0) + \frac{2}{4} I(0,2) = \frac{2}{4} \cdot 0 + \frac{2}{4} \cdot 0 = 0$$

$$Gain_{student} = Info(D, \leq 30) - Info_{student}(D, \leq 30) = 1 - 0 = 1$$

Credit_rating	p _i	n _i	I(p _i , n _i)
fair	1	1	1
excellent	1	1	1

$$I(1,1) = -\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) = 1$$

$$I(1,1) = -\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) = 1$$

$$Info_{credit_rating}(D, \leq 30) = \frac{2}{4} I(1,1) + \frac{2}{4} I(1,1) = \frac{2}{4} + \frac{2}{4} = 1$$

$$Gain_{credit_rating} = Info(D, \leq 30) - Info_{credit_rating}(D, \leq 30) = 1 - 1 = 0$$

age ≤ 30 100% Gain (income), Gain (student)
age > 30 100% Gain (credit_rating)

$$Info_{income}(D, \leq 30) = \frac{2}{4} I(0,2) + \frac{1}{4} I(1,0) + \frac{1}{4} I(1,0) = \frac{2}{4} \cdot 0 + \frac{1}{4} \cdot 0 + \frac{1}{4} \cdot 0 = 0$$

$$Gain_{income} = Info(D, \leq 30) - Info_{income}(D, \leq 30) = 1 - 0 = 1$$

x ₂	x ₃	x ₄	y
income	student	credit_rating	buys_computer
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

$$Info(D, > 40) = I(3,2) = -\frac{3}{5} \log_2\left(\frac{3}{5}\right) - \frac{2}{5} \log_2\left(\frac{2}{5}\right) = 0.971$$

income	p _i	n _i	I(p _i , n _i)
medium	2	1	0.971
low	1	1	1

$$I(2,1) = -\frac{2}{3} \log_2\left(\frac{2}{3}\right) - \frac{1}{3} \log_2\left(\frac{1}{3}\right) = 0.9183$$

$$I(1,1) = -\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) = 1$$

$$Info_{income}(D, > 40) = \frac{3}{5} I(2,1) + \frac{2}{5} I(1,1) = \frac{3}{5} \cdot 0.9183 + \frac{2}{5} \cdot 1 = 0.9826$$

$$Gain_{income} = Info(D, > 40) - Info_{income}(D, > 40) = 0.971 - 0.9826 = 0.0827$$

student	p _i	n _i	I(p _i , n _i)
Yes	2	1	0.9183
No	1	1	1

$$I(2,1) = -\frac{2}{3} \log_2\left(\frac{2}{3}\right) - \frac{1}{3} \log_2\left(\frac{1}{3}\right) = 0.9183$$

$$I(1,1) = -\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) = 1$$

$$Info_{student}(D, > 40) = \frac{2}{5} I(2,1) + \frac{3}{5} I(1,1) = \frac{2}{5} \cdot 0.9183 + \frac{3}{5} \cdot 1 = 0.9826$$

$$Gain_{student} = Info(D, > 40) - Info_{student}(D, > 40) = 0.971 - 0.9826 = 0.0827$$

Credit_rating	p _i	n _i	I(p _i , n _i)
fair	3	0	0
excellent	0	2	0

$$I(3,0) = -\frac{3}{3} \log_2\left(\frac{3}{3}\right) - \frac{0}{3} \log_2\left(\frac{0}{3}\right) = 0$$

$$I(0,2) = -\frac{0}{2} \log_2\left(\frac{0}{2}\right) - \frac{2}{2} \log_2\left(\frac{2}{2}\right) = 0$$

$$Info_{credit_rating}(D, > 40) = \frac{3}{5} I(3,0) + \frac{2}{5} I(0,2) = \frac{3}{5} \cdot 0 + \frac{2}{5} \cdot 0 = 0$$

$$Gain_{credit_rating} = Info(D, > 40) - Info_{credit_rating}(D, > 40) = 0.971 - 0 = 0.971$$

