Image Processing and Computer Vision (MPHY39600/CS35600) (Kenji Suzuki)

Problem Set 11 (Due: the class after the next class)

Solutions should include relevant images and original code (written in your favorite computer language, e.g., C, C++, Matlab, IDL, etc.) of the algorithms developed, along with any discussion requested. All the images are on the Chalk website at http://chalk.uchicago.edu and in the uncompressed TIFF format. <u>The use of a library for PCA is allowed.</u>

1. Assume that the dimension of the feature space is *N* and the number of samples (patterns) is *P*. Prove that the set of the first principal components, $\left\{a_1^{(\mu)}; \mu = 1, \ldots, P\right\}$, has the maximum variance among any of the other sets of principal components $\left\{a_i\right\}, i = 2, \ldots, N$. This indicates that the first principal axis, ϕ_1, most effectively summarizes the input patterns. Explain why.
2. Using the following trick <u>or reducing the size of the original images so that PCA calculation will end in a reasonable time</u>, calculate and display the first 8 eigenfaces (i.e., the principal axes $\left\{\phi^{(\mu)}; \mu = 1, \ldots, 8\right\}$) from the 32 images, biq01.tif, …, biq32.tif. Also, calculate the first 8 approximation faces for the first person's face, biq01.tif: $\tilde{x}^{(1)} = \sum_{i=1}^{D} a_i^{(1)} \phi^{(i)}, 1 \leq D \leq 8$.

   "Turk-Pentland trick": Suppose that $x^{(\mu)}$ are feature vectors for a sample μ. If $\widehat{\phi}$ is an eigenvector of $\widehat{C} = X^T X$, where $X = \left(x^{(1)} \mid x^{(2)} \mid, \ldots, \mid x^{(P)}\right)$, with an eigenvalue of $\widehat{\lambda}$, then $\phi = X\widehat{\phi}$ is an eigenvector of the covariance matrix $C = XX^T$ with the same eigenvalue. More precisely, suppose that $\widehat{C}\widehat{\phi}^{(\mu)} = \widehat{\lambda}_\mu \widehat{\phi}^{(\mu)}, \mu = 1, \ldots, P$. If we denote $\phi^{(\mu)} = \frac{1}{\sqrt{\lambda_\mu}} X\widehat{\phi}^{(\mu)}, \lambda_\mu = \widehat{\lambda}_\mu$, then $C\phi^{(\mu)} = \lambda_\mu \phi^{(\mu)}$.

   Ref. M. Turk and A. Pentland, "Eigenfaces for Recognition," *Journal of Cognitive Neuroscience*, vol. 3, no. 1, pp. 71-86, 1991.