1 Problem 3

1.1 Part a

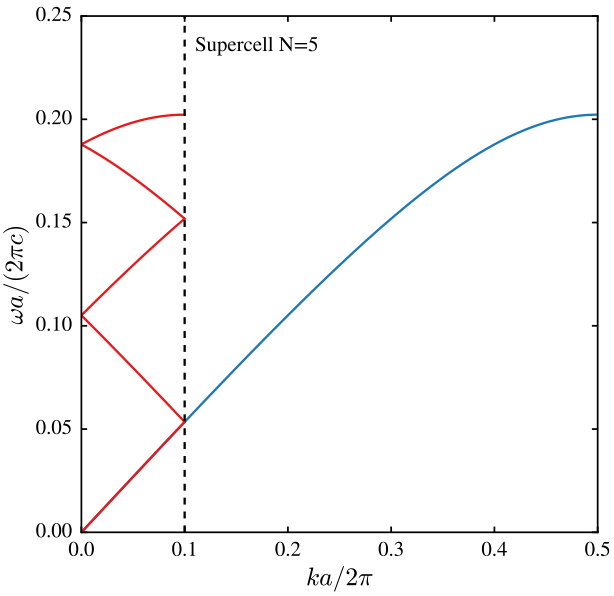

Figure 1: The band diagram $\omega(k)$ for a periodic structure of period a. The blue curve represents the band calculated in MPB with a single period geometry. The red band represents the band for the same geometry, but repeated in an $N = 5$ supercell. As expected, the frequencies fold over when they reach $k = 2\pi N/a$.

1.2 Part b

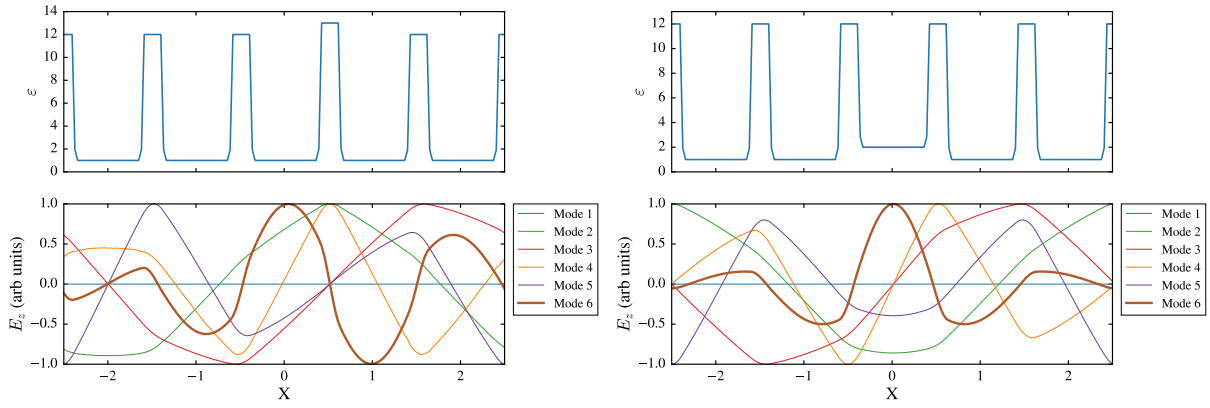

Figure 2: E_z-fields for different defect modes. (Left) defect in ε_1 (high) - Modes 2 and 4 are even; 3,5 are odd. Mode 6 is odd and decays (it is a localized mode). (Right) defect in ε_2 (low value) - Modes 2 and 5 are even; 3 and 4 are odd; again mode 6 is a localized mode (even) that decays away from the defect.

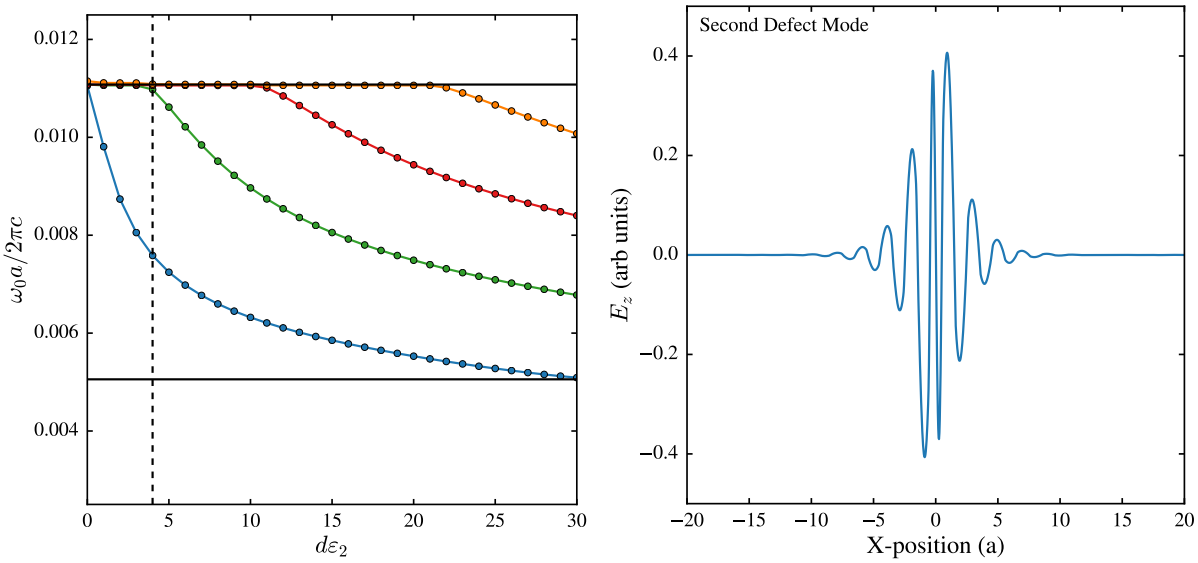

Figure 3: (Left) Defect mode frequencies (at $k = 0$) within the gap as ε_2 is increased by $d\varepsilon_2$. Solid black lines on the top and bottom indicate the band gap edges for the unperturbed 1d-crystal. A second defect appears when $d\varepsilon_2 \approx 4$. (Right) The (localized) electric field for the second defect mode when $d\varepsilon_2 = 5$.

1.3 Part c

1.4 Part d

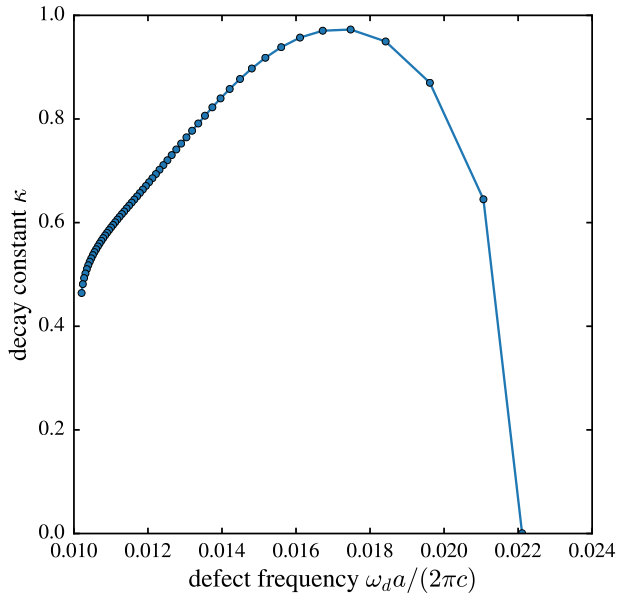

Figure 4: Decay constant calculated by fitting the electric field to a cosine oscillation bounded by a decaying exponential. The fit was calculated at each value of $d\varepsilon_2$, and is plotted against the defect frequency (for the first mode). This plot reveals the most highly localized mode occurs at the center of the gap.