Comparative Analysis of EM Algorithm and Markov Random Fields for Image Denoising and Inpainting

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Problem Statement

Image restoration remains a fundamental challenge in computational statistics and computer vision. Real-world images are often corrupted by noise or contain missing regions (inpainting). Tough lighting conditions or faulty equipment can often mean high amounts of noise or missing pixels in image stills. This project aims to compare and evaluate two principled statistical approaches for image restoration: Expectation-Maximization (EM) with Gaussian Mixture Models (GMM) and Markov Random Fields (MRFs). While both methods leverage statistical modeling, they represent fundamentally different paradigms—EM operates in a patch-based feature space, while MRF models spatial dependencies directly in the image domain.

Motivation: Understanding the relative strengths, computational requirements, and performance characteristics of these approaches provides valuable insights for selecting appropriate methods in practical applications and demonstrates the versatility of statistical modeling for inverse problems.

Data Sources

We will use standard benchmark datasets to ensure reproducibility and fair comparison:

- Set12 and BSD68 datasets for denoising evaluation
  These are standard grayscale denoising datasets and benchmarks
- MNIST and CIFAR-10 for controlled experiments
  MNIST provides simple high contrast images of handwritten digits. CIFAR-10 consists of 60,000 color images divided into 10 classes.
- Custom synthetic images with known ground truth for method validation
- Real photographs (with real noise) with simulated corruption for qualitative assessment

All datasets are publicly available and require no special permissions. For inpainting, we will simulate missing regions using random masks and structured masks.

Methodology

Approach 1: Expectation-Maximization with Gaussian Mixture Models (EM-GMM)

Core Idea: Model the distribution of clean image patches using a Gaussian Mixture Model, treating the clean patches as latent variables.

Implementation:
- Extract overlapping patches from noisy images
- Learn GMM parameters using EM algorithm
- For denoising: Apply filtering conditioned on patch responsibilities
- For inpainting: Use EM to estimate missing pixels by treating them as additional latent variables
- Reconstruct image by averaging overlapping patch estimates

Key Parameters: Number of mixture components, patch size, noise variance

Approach 2: Markov Random Fields (MRF)

Core Idea: Model the spatial dependencies between pixels using a neighborhood system and solve the restoration problem as maximum a posteriori (MAP) estimation.

Implementation:
- Formulate image as grid-structured MRF with pairwise cliques
- Use Gaussian potentials for continuous-valued images
- For denoising: MAP estimation using gradient-based optimization or graph cuts
- For inpainting: Condition on observed pixels and infer missing ones
- Implement using conjugate gradient descent or iterative conditional modes

Key Parameters: Neighborhood size, potential function parameters, optimization method

Experimental Design

Evaluation Metrics
- Quantitative: PSNR, SSIM, computation time
- Qualitative: Visual inspection of artifact types and structure preservation

Test Scenarios
- Denoising: Gaussian noise, salt-and-pepper noise
- Inpainting: Random missing pixels, structured masks
- Combined: Noisy images with missing regions

Expected Results

We anticipate the following outcomes:

Performance Characterization: EM-GMM will likely excel with textured regions and patch-based regularities, while MRFs will better preserve edges and spatial continuity.

Computational Analysis: EM-GMM will be more computationally intensive during training but efficient during inference. MRFs will show more consistent but potentially slower inference times.

References

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