```
> #### sequential shooting ####
> V010:=a:
  V001:=-b:
  eq1:=V111=p1*V010+(1-p1)*V211;
  eq2:=V211=p2*V001+(1-p2)*V111;
  SOL:=solve({eq1,eq2},{V111,V211}):
  V1_111:=rhs(SOL[1]);
  V1_211:=rhs(SOL[2]);
> V010:=a:
  V001:=-b:
  eq1:=V111=x1*p1*V010+(x1*(1-p1)+(1-x1))*V211;
  eq2:=V211=x2*p2*V001+(x2*(1-p2)+(1-x2))*V111;
  SOL:=solve({eq1,eq2},{V111,V211}):
  V1_111:=rhs(SOL[1]);
  V1_211:=rhs(SOL[2]);
> V010:=-b:
  V001:=a:
  eq1:=V111=p1*V010+(1-p1)*V211;
  eq2:=V211=p2*V001+(1-p2)*V111;
  SOL:=solve({eq1,eq2},{V111,V211}):
  V2_111:=rhs(SOL[1]);
  V2_211:=rhs(SOL[2]);
> V010:=-b:
  V001:=a:
  eq1:=V111=x1*p1*V010+(x1*(1-p1)+(1-x1))*V211;
  eq2:=V211=x2*p2*V001+(x2*(1-p2)+(1-x2))*V111;
  SOL:=solve({eq1,eq2},{V111,V211}):
  V2_111:=rhs(SOL[1]);
  V2_211:=rhs(SOL[2]);
> #### parallel shooting ####
> V10:=a:
  V01:=-b:
  V00:=a-b:
  eq1:=V11=p1*p2*V00+p1*(1-p2)*V10+(1-p1)*p2*V01+(1-p1)*(1-p2)*V11;
  V1_11:=solve(eq1,V11);
```

$$eq1 := V11 = p1\,p2\,(a-b) + p1\,(1-p2)\,a - (1-p1)\,p2\,b + (1-p1)\,(1-p2)\,V11$$

$$V1\_11 := -\frac{p1\,a - p2\,b}{p1\,p2 - p1 - p2} \qquad (1)$$

```
> V10:=a:
  V01:=-b:
  V00:=a-b:
  eq1:=V11=x1*p1*x2*p2*V00+x1*p1*((1-x2)+x2*(1-p2))*V10+((1-x1)+x1*
  (1-p1))*x2*p2*V01+
  (x1*(1-p1)*x2*(1-p2)+(1-x1)*x2*(1-p2)+x1*(1-p1)*(1-x2)+(1-x1)*(1-
  x2))*V11;
  V1_11:=solve(eq1,V11);
```

$$eq1 := V11 = x1\,p1\,x2\,p2\,(a-b) + x1\,p1\,(x2\,(1-p2) + 1 - x2)\,a - (x1\,(1-p1) + 1 - x1)\,x2\,p2\,b + (x1\,(1-p1)\,x2\,(1-p2) + (1-x1)\,x2\,(1-p2) + x1\,(1-p1)\,(1-x2) + (1-x1)\,(1-x2))\,V11$$

$$V1\_11 := -\frac{x1\,p1\,a - x2\,p2\,b}{p1\,p2\,x1\,x2 - x1\,p1 - x2\,p2} \qquad (2)$$

```
> V10:=-b:
  V01:=a:
  V00:=a-b:
  eq1:=V11=x1*p1*x2*p2*V00+x1*p1*((1-x2)+x2*(1-p2))*V10+((1-x1)+x1*
  (1-p1))*x2*p2*V01+
  (x1*(1-p1)*x2*(1-p2)+(1-x1)*x2*(1-p2)+x1*(1-p1)*(1-x2)+(1-x1)*(1-
  x2))*V11;
  V2_11:=solve(eq1,V11);
```

$$eq1 := V11 = x1\,p1\,x2\,p2\,(a-b) - x1\,p1\,(x2\,(1-p2)+1-x2)\,b + (x1\,(1-p1)+1 - x1)\,x2\,p2\,a + (x1\,(1-p1)\,x2\,(1-p2) + (1-x1)\,x2\,(1-p2) + x1\,(1-p1)\,(1-x2) + (1-x1)\,(1-x2))\,V11$$

$$V2\_11 := -\frac{x2\,p2\,a - x1\,p1\,b}{p1\,p2\,x1\,x2 - x1\,p1 - x2\,p2} \qquad \textbf{(3)}$$

```
>
```