Project

ICCS315: Applied Algorithms

Thanatad Anukoolprasert, Yuttasart Viratpan

April 15, 2023

Objective

The objective of this project is to benchmark two different type of data structures and compare different implementation of each type as well as their theoretical running time. The first type is resizable array where the following data structures are of interest

- Hashed Array Tree (Sitarski, 1996)
- Brodnik's resizable array non-superblock version (Brodnik et al., 1999)
- Brodnik's resizeable array superblock version (Brodnik et al., 1999)

The second type is hash table with 3 different collision resolution schemes, namely

- Chaining
- Open addressing
- Cuckoo hashing (Rasmus Pagh & Flemming Rodler., 2004)

For a lack of a better name, we are going to called both the data structures from Brodnik et al 1999, Hashed Array Tree (HAT). Not only that we are going to called Brodnik's HAT with non-superblock version *Brodnik's HAT A* and the superblock version *Brodnik's HAT B*.

The dimensions of interest are the following

- Append latency
- Access latency
- Scan throughput
- Overall throughput

In addition, for hash table we also going to look at deletion time.

Method

We are going the implement the data structures above and benchmark them. Our implementation can be found here: https://github.com/thanatadcs/apal-project. We are going the measure running time in cycles using *rdtsc* assembly instruction in x86 so that we are able to time quick operation like append with accurary.

Remark on implementation

The implementation of *get* function of Brodnik's HAT B is different from *locate* function in the original paper. Namely, we cannot implement part of *locate* function since there might be something wrong with the pseudocode in the paper, specifically $p = 2^k - 1$ mentioned in the paper does not represent the number of data blocks before the k-th superblock as claimed, but it represent the number of elements before the k-th superblock instead. For this reason I have to implement my own way to get the number of data blocks before the k-th superblock. This can effect the running time of the *get* function for Brodnik's HAT B. More information can be found in brodnik-hat-b.cpp.

For Brodnik's HAT A and B, background-rebuilding optimization are also implemented so that we get $O(1)$ worst case append time. Brodnik's HAT B also utilized BSR assembly instruction in x86 to find the superblock number.

Results and Discussion

Hashed Array Tree

All the number presented has the unit in cycles.

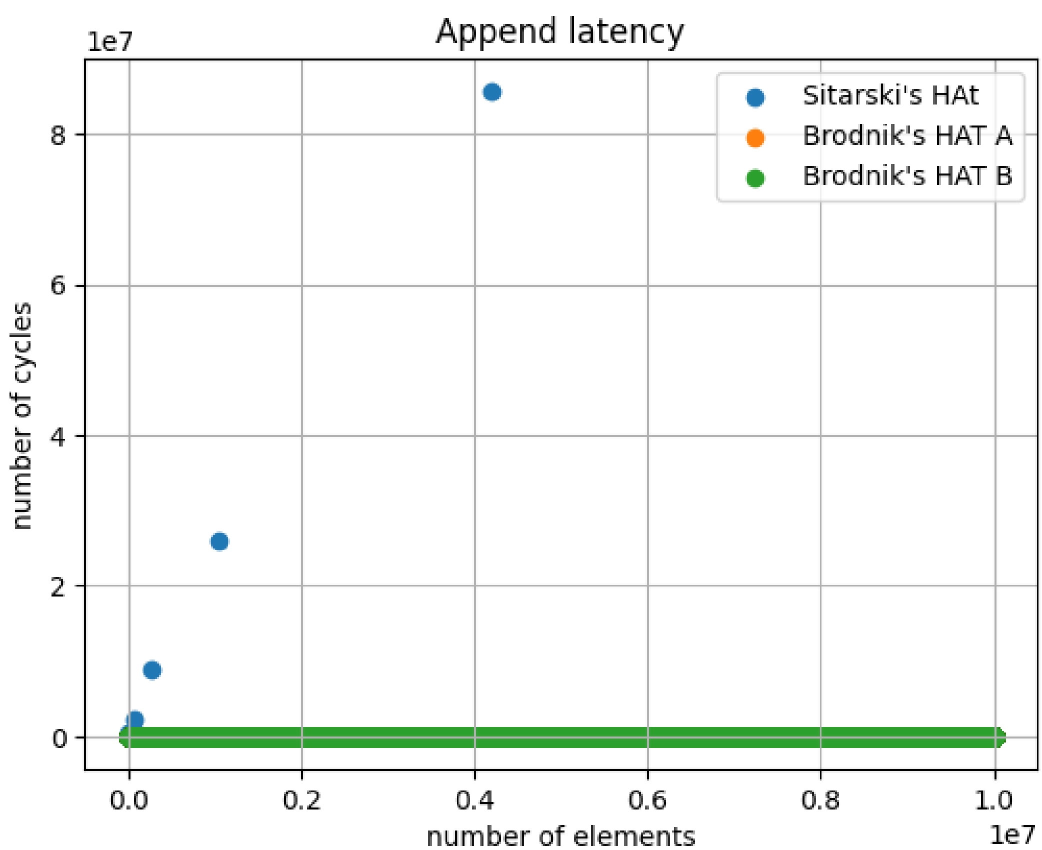
Append latency

10 millions elements are appended and each append are timed, here are the statistics

	Sitarski's HAT	Brodnik's HAT A	Brodnik's HAT B
mean	59.7470848	48.3077336	46.6222048
standard deviation	28458.271801239895	173.0441420332727	227.73642714240262

For append latency, the ranking from best to worst running time is Brodnik's HAT B, Brodnik's HAT A, and Sitarski's HAT. This makes sense since Sitarski's HAT have to copy not only the pointer, but also all elements every time it's expand as compare to Brodnik's HAT A and B where only pointer need to be copy and since we do not need to copy the elements, background-rebuilding is possible which make this even better and in fact constant in term of append time.

In another aspect, our results also follow the theoretical running time very well. For Sitarski's HAT, in theory append time is $O(1)$ amortized, so we can see that the mean is that not far off from Brodnik's A and B, but since this is amortized cost, the standard deviation can be very high since there might be spike in running time once in a while. Brodnik's A and B has $O(1)$ worst case append time so the standard deviation of both of these is very low. Here is a the plot for append time



which demostrated how the append time for Brodnik's A and B are constant.

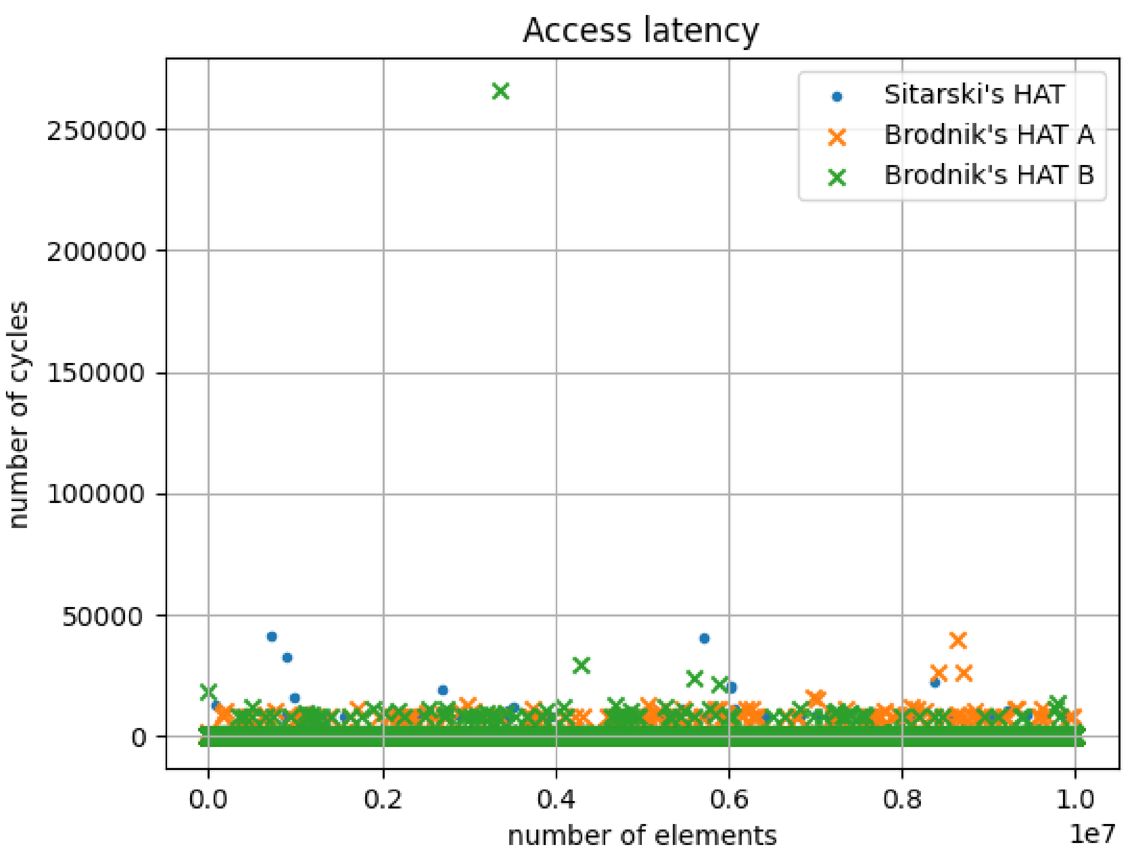
Access latency

10 millions elements are appended and all 10 millions elements get access through *get* function. Each access is timed, here are the statistics

	Sitarski's HAT	Brodnik's HAT A	Brodnik's HAT B
mean	46.9125558	71.8199474	68.7515222
standard deviation	31.649467204234057	35.97450913996233	90.24513109294548

Here the ranking from best to worst are Sitarski's HAT , Brodnik's HAT B, and Brodnik's HAT A. Sitarski's HAT require little computation (a little bit of bitwise operation) to get to the index so it is the fastest. Brodnik's B is better than Brodnik's HAT A in their mean by a little bit which is a surprise since Brodnik's HAT B does not have any complex operation like square root. In term of standard deviation, Brodnik's HAT A clearly out perform Brodnik's HAT B since the running time is more consistant.

In theory, Sitarski's HAT has $O(1)$ access time so it is not a surprise that it is very fast with low standard deviation. Brodnik's HAT B supposed far to be better than Brodnik's HAT A, since Brodnik's HAT B promise $O(1)$ access time where Brodnik's HAT A does not since it has to compute square root, but our results does not goes that way since Brodnik's HAT B and A relatively has similar running time in term of mean, but for Brodnik's HAT A clearly has a more consistant result. Here is the graph for access time



which show how the access time for all 3 HATs looks relatively constant (except that Brodnik's HAT A access is not constant in theory, but is low enough to look like so).

Scan throughput

1000 elements are appended and all the 1000 elements are accessed. The total time used to access all elements is measured and we do this for 1000 times. Here are the statistics.

	Sitarski's HAT	Brodnik's HAT A	Brodnik's HAT B
mean	16439.928	38772.878	53974.224
standard deviation	1509.39901776038	3611.297266234946	2422.0789396351224

For scan throughput, ranking from best to worst is Sitarski's HAT, Brodnik's A, and Brodnik's B. Again similar to our access time, Sitarski's HAT perform the best. Brodnik's HAT A beat Brodnik's HAT B this time which is a surprise. Even though the access time for Brodnik's HAT A is not constant, but it is still relatively very low so it might also occur that Brodnik's HAT B access time get out performed by Brodnik's HAT A because of overhead.

In term of theory, Sitarski follows the theoretical running time very well with constant low access time (because of simple bitwise). But for Brodnik's HAT A, even though in theory, we would say that it is worse than Brodnik's HAT B, but our results has shown that it can be better.

Of course, it might also means that our Brodnik's HAT B is not optimized enough and we may have to do additional book keeping to make this more optimized. Another factor can also stem from the fact that our implementation is a bit different from that of the paper, in this case we might as well trying to find a better a better and faster way to compute the number of data blocks before a given superblock.

Overall throughput

1000 elements are appended. The total time used to append all 1000 elements is measured and we do this for 1000 times. Here are the statistics

	Sitarski's HAT	Brodnik's HAT A	Brodnik's HAT B
mean	31624.766	26366.932	28797.736
standard deviation	1774.3194777840883	1691.4454491280528	2090.277559154286

For overall throughput, ranking from best to worst is Brodnik's HAT A, Brodnik's HAT B, and Sitarski's HAT. All three HATs does not have that far off running time in terms of mean and they have relatively the same standard deviation.

This is according to the theory since all of the have $O(1)$ append time (Sitarski's is amortized), the overall throughput average out some bad running time for Sitarski's HAT, but still it cannot beat Brodnik's HAT A and B since both of these are constant worst case append time.

Hash table

The test case are seperate into two sections. The test case where there is no collision, to just test the implementation. And the test case where the collision during insertion happened. To test the collision resolution scheme and how it affects the performance The methods and keywords glossary will be describe below:

- size - The size of the initial hash table. It's also the number of the element that would (eventually) be in the table
- insert - The method insert inserts elements to fill up the table to the brim. When it's in the context of collision case, it's an re-insertion of the element in attempt to collide with all index of table. (Note: the implementation of cuckoo hashing use simple hash function randomization)
- Search - The method search retrieve all the inserted elements sequentially. Does not try to search the key that wasn't inserted. In the context of collision case, it search for every key again after the collision occurs
- Delete - The method delete remove the key that is in the table. Does not try to delete the key that wasn't inserted. In the context of collision case, it deletes every element in the table after the collision insertion happened.
- Scan - The method scan sequentially access all the key in the table. In the context of collision case, it scans through the table just like normal, but with maybe new value after collision

Note: The numbers on the table are in the unit of cpu cycles. To convert it to seconds, divide those numbers by 2.6 GHz

No collision case

Chaining scheme

	Insert	Search	Delete	Scan
size: 1	930	320	3,486	670
size: 100	143,110	13,354	40,620	26,080
size: 100K	134,573,520	12,750,922	38,332,107	24,604,098
size: 10M	14,144,086,730	1,319,286,013	4,042,035,833	2,517,830,762

For Chaining scheme, the theoretical time for all operations largely depends on the length of the chains on each indexes. The expected chains length is $O\left(\frac{n}{m}\right)$ where n is the number of keys and m is the number of slots. The maximum number of chains is $O\left(\frac{\log(n)}{\log(\log(n))}\right)$ where n, again, is the number of keys. Looking at the table, we see that as the runtime grows proportionally to the size, at the almost the same rate on how the size grows. The most notable thing are that:

- Insert method comes at the last place out of the three schemes. Weirdly it made a huge jump on the runtime from size of one to size of one-hundred. Most likely due the high amount of time of resizing, due to resize factor being multiple of two
- Search method comes at the second place. This is most like due to the algorithms being un-optimize, as the hash function is called twice during the search
- Detele method took the longest out of the three schemes. Most likely due to freeing the LinkedList structure
- Scan method took the longest compared to other two schemes. Since we're keeping the elements as a linked list. Extra time to fetch-execute cycle.

Open Addressing scheme

	Insert	Search	Delete	Scan
size: 1	3,654	380	330	104
size: 100	61,856	24,550	24,982	836
size: 100K	54,732,644	24,174,556	26,079,140	684,162
size: 10M	5,143,961,418	2,280,300,642	2,400,285,752	67,605,464

In Open Addressing scheme, the theoretical time depends on how good the hash function is. If the hash function keeps hashing the new key to already occupied index, then the code will take longer to do the operations. So the runtime is proportional the number of (linear) probing. The most notable thing are that:

- Insert method is the slowest at the start out of the three scheme, but it scales better than chaining, placing it as number two out of the three scheme. This is most likely due to implementation design, as an enums were kept for other methods usage
- Search method comes at the last place. This is most like due to the algorithms being un-optimize, as the hash function is called twice during the search
- Detele method placed second out of the three schemes. Expectedly should be placed first, due to usage of enums. If we were to remove element, it will just be marked empty. So the bottleneck is most likely the fact that the hash function was called twice to find the key
- Scan method placed first. Yay

Cuckoo Hashing scheme

	Insert	Search	Delete	Scan
size: 1	158	186	114	556
size: 100	8,548	5,566	6,712	9,210
size: 100K	8,384,710	6,419,210	7,441,992	10,242,288
size: 10M	656,286,450	450,796,896	595,650,074	918,961,414

In Cuckoo Hashing scheme, the theoretical time are:

- Insert - $E[O(1)]$
- Search - $O(1)$ worst case
- Detele - $E[O(1)]$
- Scan - $O(n)$ on average

The most notable thing are that:

- Insert method is the fastest.
- Search method comes at the first place.
- Detele method placed first
- Scan method placed second. This is most likely due to maintaining two hash tables.

The three methods Insert, Search and Delete should have similar runtime like open addressing (since cuckoo hashing is a type of open addressing method). But the reason that the results comes about is most likely due to less bottleneck unlike open addressing scheme.

Collision for each and every insert

Chaining scheme

	Insert	Search	Delete	Scan
size: 1	6,436	618	1,818	942
size: 100	12,638	12,444	36,630	23,882
size: 100K	12,805,934	12,400,152	36,475,868	23,906,203
size: 10M	1,381,820,700	1,278,416,286	3,839,070,001	2,430,914,352

The most notable thing are that:

- Insert method is the fastest out of the three scheme. Since you can just keep connecting the linked list node
- Search method comes at the second place. This theoretically should be slower than open addressing on worst case scenario. But practically, time spent probing took longer than just find the linked list.
- Detele method placed last out of the three schemes. Since we have to go through linked list to delete it. And then we have to reorganize the linked list as well
- Scan method placed last. Because this is resizable, It has n elements to go through (which include null indexes as well), plus the linked list in each indexes.

For chaining, this approach sounds very memory expensive. Since the pointer to each linked list needs to be remembered.

Open Addressing scheme

	Insert	Search	Delete	Scan
size: 1	2,602	514	354	120
size: 100	23,580	21,490	22,120	814
size: 100K	24,796,556	22,563,490	23,658,006	666,302
size: 10M	2,474,628,412	2,260,518,832	2,360,901,784	66,875,470

The most notable thing are that:

- Insert method is the slowest.
- Search method comes at the last place.
- Delete method placed second.
- Scan method placed first.

The more the probing, the longer the operations. This applies to Insert, Search and Delete

Cuckoo Hashing scheme

	Insert	Search	Delete	Scan
size: 1	346	208	172	556
size: 100	183,484	8,524	8,986	9,048
size: 100K	107,326,484	9,201,168	8,696,178	9,369,126
size: 10M	1,572,211,006	513,099,226	598,603,314	921,694,710

The most notable thing are that:

- Insert method is the second fastest.
- Search method comes at the first place.
- Delete method placed first.
- Scan method placed second.
- Notice that collision, the load on insert highly increases, but at a trade for faster search and delete comparing to other scheme

From the looks of it, Cuckoo Hashing seems to be the most generally speedy collision resolution scheme for hash table. However, further testing/test cases needed to be done. As well as the optimization for all the scheme for a fairer test.

Conclusion

We have implemented and benchmark 3 versions of HATs and 3 versions of hash table.

For HATs, we have found that Brodnik's HAT A and B does very well for append/overall throughput when compare to Sitarski's HAT this is partly due to the background-rebuilding optimization that has been implement for both of the former and the fact that the former does not copy over all the elements when expanding. For access/scan throughput, Sitarski's outperformed Brodnik's HAT A and B, this is because the former does only simple bitwise operation to figure out the index, but Brodnik's HAT A is more complex which involves square root which does not run in constant time and Brodnik's HAT B even though use some bitwise operation and BSR x86 assembly directly, but the arithmetics required can be heavy enough to weight it down, additional book keeping might be needed to make this better. Also noted that the modification from the original paper for the function used to access might also affect the running time.

Also note that Brodnik's HAT A performed better than Brodnik's HAT B for scan throughput, even though Brodnik's HAT B promise better theoretical constant running time, but in this case our implementation might introduce some overhead to make this worse than the Brodnik's HAT A which does not have constant access, but access time can be low enough to beat the overhead for Brodnik's HAT B. This does show that even though the theory says that Brodnik's HAT B is better than Brodnik's HAT A, but our result show that this is not the case in practice. If we are taking in to account the ease of implementation for Brodnik's HAT A and our results for its performence, Brodnik's HAT A is better than Brodnik's HAT B in this case.

For Hash tables' collision scheme, we've look over how each of the hashing scheme are compared to each other in a non-collision case and full collision case. In general the cuckoo hashing scheme seems to be very well rounded, placing generally first in both categories.