

MENG SUN

BOGOLONS

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1

Basic properties

In what follows, we have $\hbar = k_B = 1$.

1.1 Bogoliubov coefficient

Bogoliubov coefficient is defined in ¹

$$u_{\mathbf{p}}^2 = 1 + v_{\mathbf{p}}^2 = \frac{1}{2} \left(1 + \left[1 + \left(\frac{Ms^2}{\omega_{\mathbf{p}}} \right)^2 \right]^{1/2} \right), \quad (1.1)$$

$$u_{\mathbf{p}}v_{\mathbf{p}} = -\frac{Ms^2}{2\omega_{\mathbf{p}}}, \quad (1.2)$$

where the M is the effective mass of the condensed particle; s is the sound velocity of bogolons and $s = \sqrt{\frac{\kappa n_c}{M}}$; the n_c is the density of condensed particle; κ is the interaction strength; and the dispersion of the bogolons are²

$$\omega_{\mathbf{p}} = s\mathbf{p}\sqrt{1 + \mathbf{p}^2\zeta_h^2}. \quad (1.3)$$

The healing length is defined as $\zeta_h = \frac{1}{2Ms}$.

1.2 Green's function

Bogolons are bosons. The Green's function is defined like phonons, First, introducing the following operators

$$A_{\mathbf{q}} = u_{\mathbf{q}}b_{\mathbf{p}} + v_{\mathbf{q}}b_{-\mathbf{q}}^{\dagger} \quad (1.4)$$

unlike the phonons where $u_{\mathbf{q}} = v_{\mathbf{q}} = 1$, we do **not** have $A_{\mathbf{q}}^{\dagger} = A_{-\mathbf{q}}$. These corresponding operators are

$$A_{-\mathbf{q}} = u_{\mathbf{q}}b_{-\mathbf{q}} + v_{\mathbf{q}}b_{\mathbf{q}}^{\dagger} \quad (1.5)$$

$$A_{\mathbf{q}}^{\dagger} = u_{\mathbf{q}}b_{\mathbf{q}}^{\dagger} + v_{\mathbf{q}}b_{-\mathbf{q}} \quad (1.6)$$

where we see from (1.1), $u_{\mathbf{p}}$ and $v_{\mathbf{p}}$ are only magnitude depended.

¹ S. Giorgini. Damping in dilute Bose gases: A mean-field approach. *Phys. Rev. A - At. Mol. Opt. Phys.*, 57(4):2949–2957, 1998

² Usually, if we consider the exciton condensation, κ is exciton-exciton interaction, for indirect exciton, the result is $\kappa = \frac{e_0^2 d}{\epsilon}$.

From here, we have two definitions of the Green's function

$$\mathcal{F}(\mathbf{q}, \tau) = -\langle TA_{\mathbf{q}}(\tau)A_{-\mathbf{q}} \rangle \quad (1.7)$$

The corresponding Green's function in Matsubara frequency is

$$\mathcal{D}(\mathbf{q}, i\omega_n) = u_{\mathbf{q}}v_{\mathbf{q}} \left[\frac{1}{i\omega_n - \omega_{\mathbf{q}}} - \frac{1}{i\omega_n + \omega_{\mathbf{q}}} \right] = \frac{2u_{\mathbf{q}}v_{\mathbf{q}}\omega_{\mathbf{q}}}{(i\omega_n)^2 - \omega_{\mathbf{q}}^2} \quad (1.8)$$

However, if we consider this

$$\mathcal{G}(\mathbf{q}, \tau) = -\langle TA_{\mathbf{q}}(\tau)A_{\mathbf{q}}^{\dagger} \rangle \quad (1.9)$$

The corresponding result is

$$\mathcal{G}(\mathbf{q}, i\omega_n) = \frac{u_{\mathbf{q}}^2}{i\omega_n - \omega_{\mathbf{q}}} - \frac{v_{\mathbf{q}}^2}{i\omega_n + \omega_{\mathbf{q}}} \quad (1.10)$$

With another notation, we found

$$\mathcal{G}'(\mathbf{q}, \tau) = -\langle TA_{\mathbf{q}}^{\dagger}(\tau)A_{\mathbf{q}} \rangle \quad (1.11)$$

the result is

$$\mathcal{G}'(\mathbf{p}, i\omega_n) = \frac{v_{\mathbf{q}}^2}{i\omega_n - \omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}^2}{i\omega_n + \omega_{\mathbf{q}}} \quad (1.12)$$

We can make a table for the result of the Green's function in Matsubara frequency

$\int d\tau e^{i\omega_n \tau}$	$-\langle TA_{\mathbf{q}}(\tau) \cdot$	$-\langle TA_{\mathbf{q}}^{\dagger}(\tau) \cdot$	$-\langle TA_{-\mathbf{q}}(\tau) \cdot$	$-\langle TA_{-\mathbf{q}}^{\dagger}(\tau) \cdot$	\times
	0	$\frac{v_{\mathbf{q}}^2}{i\omega_n - \omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}^2}{i\omega_n + \omega_{\mathbf{q}}}$	$\frac{u_{\mathbf{q}}v_{\mathbf{q}}}{i\omega_n - \omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}v_{\mathbf{q}}}{i\omega_n + \omega_{\mathbf{q}}}$	$\frac{v_{\mathbf{q}}^2}{i\omega_n - \omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}^2}{i\omega_n + \omega_{\mathbf{q}}}$	$A_{\mathbf{q}} \rangle$
	$\frac{u_{\mathbf{q}}^2}{i\omega_n - \omega_{\mathbf{q}}} - \frac{v_{\mathbf{q}}^2}{i\omega_n + \omega_{\mathbf{q}}}$	0	$\frac{u_{\mathbf{q}}^2}{i\omega_n - \omega_{\mathbf{q}}} - \frac{v_{\mathbf{q}}^2}{i\omega_n + \omega_{\mathbf{q}}}$	$\frac{u_{\mathbf{q}}v_{\mathbf{q}}}{i\omega_n - \omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}v_{\mathbf{q}}}{i\omega_n + \omega_{\mathbf{q}}}$	$A_{\mathbf{q}}^{\dagger} \rangle$
	$\frac{u_{\mathbf{q}}v_{\mathbf{q}}}{i\omega_n - \omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}v_{\mathbf{q}}}{i\omega_n + \omega_{\mathbf{q}}}$	$\frac{v_{\mathbf{q}}^2}{i\omega_n - \omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}^2}{i\omega_n + \omega_{\mathbf{q}}}$	0	$\frac{v_{\mathbf{q}}^2}{i\omega_n - \omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}^2}{i\omega_n + \omega_{\mathbf{q}}}$	$A_{-\mathbf{q}} \rangle$
	$\frac{u_{\mathbf{q}}^2}{i\omega_n - \omega_{\mathbf{q}}} - \frac{v_{\mathbf{q}}^2}{i\omega_n + \omega_{\mathbf{q}}}$	$\frac{u_{\mathbf{q}}v_{\mathbf{q}}}{i\omega_n - \omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}v_{\mathbf{q}}}{i\omega_n + \omega_{\mathbf{q}}}$	$\frac{u_{\mathbf{q}}^2}{i\omega_n - \omega_{\mathbf{q}}} - \frac{v_{\mathbf{q}}^2}{i\omega_n + \omega_{\mathbf{q}}}$	0	$A_{-\mathbf{q}}^{\dagger} \rangle$

By looking the (1.10) and (1.12), we realized that this two Green's function are identical

$$\mathcal{G}(\mathbf{q}, i\omega_n) = \mathcal{G}'(\mathbf{q}, -i\omega_n). \quad (1.13)$$

Using the (1.1), with some calculation we can write down the Green's function in a matrix form ³

$$\hat{\mathcal{G}} = \begin{pmatrix} \mathcal{G} & \mathcal{F} \\ \mathcal{F} & \mathcal{G}' \end{pmatrix} \quad (1.14)$$

Another method to calculate the Green's function is based on the the discussion in ⁴ The result is given as the retarded Green's

³ R. S. Christensen, J. Levinsen, and G. M. Bruun. Quasiparticle Properties of a Mobile Impurity in a Bose-Einstein Condensate. *Phys. Rev. Lett.*, 115(16):1-5, 2015

⁴ S. Giorgini. Damping in dilute Bose gases: A mean-field approach. *Phys. Rev. A - At. Mol. Opt. Phys.*, 57(4):2949-2957, 1998; and V. M. Kovalev and A. V. Chaplik. Lifetime of quasiparticles in a hybrid electron-exciton system. *JETP Lett.*, 98(6):331-334, 2013

function

$$\hat{\mathcal{G}}_{ret} = \begin{pmatrix} \frac{E+(q^2/2M)+\kappa n_c}{E^2-\omega_{\mathbf{q}}^2+i\delta} & \frac{-\kappa n_c}{E^2-\omega_{\mathbf{q}}^2+i\delta} \\ \frac{-\kappa n_c}{E^2-\omega_{\mathbf{q}}^2+i\delta} & \frac{-E+(q^2/2M)+\kappa n_c}{E^2-\omega_{\mathbf{q}}^2+i\delta} \end{pmatrix} \quad (1.15)$$

This result is given by the following approximation:

$$\begin{aligned} & \frac{u_{\mathbf{q}}^2}{i\omega_n + \omega_{\mathbf{q}}} - \frac{v_{\mathbf{q}}^2}{i\omega_n + \omega} \\ = & \frac{(u_{\mathbf{q}}^2 - v_{\mathbf{q}}^2)i\omega_n + (v_{\mathbf{q}}^2 + u_{\mathbf{p}}^2)\omega_{\mathbf{q}}}{(i\omega_n)^2 - (\omega_{\mathbf{q}})^2} \\ = & \frac{E + i\delta + (v_{\mathbf{q}}^2 + u_{\mathbf{q}}^2)\omega_{\mathbf{q}}}{E^2 + 2i\delta E - \delta^2 - \omega_{\mathbf{q}}^2} \\ \approx & \frac{E + (u_{\mathbf{q}}^2 + v_{\mathbf{q}}^2)\omega_{\mathbf{q}}}{E^2 - \omega^2 + i\delta \text{sign}(E)} \end{aligned} \quad (1.16)$$

where we first apply the analytic continuous: $i\omega_n \rightarrow E + i\delta$ and assume $\delta E \rightarrow 0$. Further, for the rest part we consider

$$\begin{aligned} u_{\mathbf{q}}^2 &= \frac{1}{2} \left(1 + \frac{Ms^2}{\omega_{\mathbf{q}}} \sqrt{1 + \frac{\omega_{\mathbf{p}}^2}{M^2 s^4}} \right) \\ &\approx \frac{1}{2} \left(1 + \frac{Ms^2}{\omega_{\mathbf{q}}} \left(1 + \frac{\omega_{\mathbf{q}}^2}{2M^2 s^4} \right) \right) \\ &= \frac{1}{2} \left(1 + \frac{Ms^2}{\omega_{\mathbf{q}}} + \frac{\omega_{\mathbf{q}}}{2Ms^2} \right) \end{aligned} \quad (1.17)$$

$$v_{\mathbf{q}}^2 = \frac{1}{2} \left(\frac{Ms^2}{\omega_{\mathbf{q}}} + \frac{\omega_{\mathbf{q}}}{2Ms^2} - 1 \right) \quad (1.18)$$

we can get the final result of the first element in (1.15).

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Appendix

2.1 Fourier transform pairs

For a 3D functions, the Fourier transformation define

$$f(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \iiint E(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}} d\mathbf{k} \quad (2.1)$$

and the inverse functions define

$$E(\mathbf{k}) = \frac{1}{(2\pi)^{3/2}} \iiint f(\mathbf{r}) e^{-i\mathbf{k} \cdot \mathbf{r}} d\mathbf{r} \quad (2.2)$$

in the function are **spherically symmetric**, we have

$$f(r) = \sqrt{\frac{2}{\pi}} \frac{1}{r} \int_0^\infty E(k) \sin(kr) k dk \quad (2.3)$$

and

$$E(k) = \sqrt{\frac{2}{\pi}} \frac{1}{k} \int_0^\infty f(r) \sin(kr) r dr \quad (2.4)$$

For different integral function we have the following identity

$$e^{-\alpha r} \Leftrightarrow \sqrt{\frac{2}{\pi}} \frac{2\alpha}{(\alpha^2 + k^2)^2} \quad (2.5)$$

$$\frac{e^{-\alpha r}}{r} \Leftrightarrow \sqrt{\frac{2}{\pi}} \frac{1}{\alpha^2 + k^2} \quad (2.6)$$

$$1 \Leftrightarrow (2\pi)^{3/2} \delta(\mathbf{k}) \quad (2.7)$$

For 2D function, the Fourier transformation

$$f(\mathbf{r}) = \frac{1}{2\pi} \iint E(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}} d\mathbf{k} \quad (2.8)$$

and the inverse Fourier transformation

$$E(\mathbf{k}) = \frac{1}{2\pi} \iint f(\mathbf{r}) e^{-i\mathbf{k} \cdot \mathbf{r}} d\mathbf{r} \quad (2.9)$$

An important pair is

$$\frac{1}{r} \Leftrightarrow \frac{1}{k} \quad (2.10)$$

If we consider the form, $\frac{1}{\mathbf{k}^2}$

$$\begin{aligned}
 & \frac{1}{2\pi} \iint \frac{1}{\mathbf{k}^2} e^{i\mathbf{k} \cdot \mathbf{r}} d\mathbf{k} \\
 = & \int_0^{2\pi} \int_0^\infty \frac{e^{ikr \cos \theta}}{k} dk d\theta \\
 = & \int_0^\infty \frac{J_0(kr)}{k} dk
 \end{aligned} \tag{2.11}$$

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