MENG SUN

BOGOLONS

Contents

1

Basic properties

In what follows, we have $\hbar = k_B = 1$.

1.1 Bogoliubov coefficient

Bogoliubov coefficient is defined in [1]

[1] S. Giorgini. Damping in dilute Bose gases: A mean-field approach. *Phys. Rev. A - At. Mol. Opt. Phys.*, 57(4):2949–2957, 1998

$$u_{\mathbf{p}}^2 = 1 + v_{\mathbf{p}}^2 = \frac{1}{2}\left(1 + \left[1 + \left(\frac{Ms^2}{\omega_{\mathbf{p}}}\right)^2\right]^{1/2}\right), \tag{1.1}$$

$$u_{\mathbf{p}} v_{\mathbf{p}} = -\frac{Ms^2}{2\omega_{\mathbf{p}}}, \tag{1.2}$$

where the M is the effective mass of the condensed particle; s is the sound velocity of bogolons and $s = \sqrt{\frac{\kappa n_c}{M}}$; the n_c is the density of condensed particle; κ is the interaction strength; and the dispersion of the bogolons are[2]

[2] Usually, if we consider the exciton condensation, κ is exciton-exciton interaction, for indirect exciton, the result is $\kappa = \frac{e_0^2 d}{\epsilon}$.

$$\omega_{\mathbf{p}} = s\mathbf{p}\sqrt{1 + \mathbf{p}^2 \xi_h^2}. \tag{1.3}$$

The healing length is defined as $\xi_h = \frac{1}{2Ms}$.

1.2 Green's function

Bogolons are bosons. The Green's function is defined like phonons, First, introducing the following operators

$$A_{\mathbf{q}} = u_{\mathbf{q}} b_{\mathbf{p}} + v_{\mathbf{q}} b_{-\mathbf{q}}^{\dagger} \tag{1.4}$$

unlike the phonons where $u_{\mathbf{q}} = v_{\mathbf{q}} = 1$, we do **not** have $A_{\mathbf{q}}^{\dagger} = A_{-\mathbf{q}}$. These corresponding operators are

$$A_{-\mathbf{q}} = u_{\mathbf{q}} b_{-\mathbf{q}} + v_{\mathbf{q}} b_{\mathbf{q}}^{\dagger} \tag{1.5}$$

$$A_{\mathbf{q}}^{\dagger} = u_{\mathbf{q}} b_{\mathbf{q}}^{\dagger} + v_{\mathbf{q}} b_{-\mathbf{q}} \tag{1.6}$$

where we see from (1.1), $u_{\mathbf{p}}$ and $v_{\mathbf{p}}$ are only magnitude depended.

From here, we have two definitions of the Green's function

$$\mathcal{F}(\mathbf{q},\tau) = -\langle T A_{\mathbf{q}}(\tau) A_{-\mathbf{q}} \rangle \tag{1.7}$$

The corresponding Green's function in Matsubara frequency is

$$\mathcal{D}(\mathbf{q}, i\omega_n) = u_{\mathbf{q}} v_{\mathbf{q}} \left[\frac{1}{i\omega_n - \omega_{\mathbf{q}}} - \frac{1}{i\omega_n + \omega_{\mathbf{q}}} \right] = \frac{2 u_{\mathbf{q}} v_{\mathbf{q}} \omega_{\mathbf{q}}}{(i\omega_n)^2 - \omega_{\mathbf{q}}^2} \tag{1.8}$$

However, if we consider this

$$\mathcal{G}(\mathbf{q},\tau) = -\langle T A_{\mathbf{q}}(\tau) A^{\dagger}_{\mathbf{q}} \rangle \tag{1.9}$$

The corresponding result is

$$\mathcal{G}(\mathbf{q}, i\omega_n) = \frac{u_{\mathbf{q}}^2}{i\omega_n - \omega_{\mathbf{q}}} - \frac{v_{\mathbf{q}}^2}{i\omega_n + \omega_{\mathbf{q}}} \tag{1.10}$$

With another notation, we found

$$\mathcal{G}'(\mathbf{q},\tau) = -\langle T A^{\dagger}_{\mathbf{q}}(\tau) A_{\mathbf{q}} \rangle \tag{1.11}$$

the result is

$$\mathcal{G}'(\mathbf{p}, i\omega_n) = \frac{v_{\mathbf{q}}^2}{i\omega_n - \omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}^2}{i\omega_n + \omega_{\mathbf{q}}} \tag{1.12}$$

We can make a table for the result of the Green's function in Matsubara frequency

$\int d\tau e^{i\omega_n \tau}$	$-\langle T A_{\mathbf{q}}(\tau)\cdot$	$-\langle T A^{\dagger}_{\mathbf{q}}(\tau)\cdot$	$-\langle T A_{-\mathbf{q}}(\tau)\cdot$	$-\langle T A^{\dagger}_{-\mathbf{q}}(\tau)\cdot$	$\times$
	0	$\frac{v_{\mathbf{q}}^2}{i\omega_n-\omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}^2}{i\omega_n+\omega_{\mathbf{q}}}$	$\frac{u_{\mathbf{q}}v_{\mathbf{q}}}{i\omega_n-\omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}v_{\mathbf{q}}}{i\omega_n+\omega_{\mathbf{q}}}$	$\frac{v_{\mathbf{q}}^2}{i\omega_n-\omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}^2}{i\omega_n+\omega_{\mathbf{q}}}$	$A_{\mathbf{q}}\rangle$
	$\frac{u_{\mathbf{q}}^2}{i\omega_n-\omega_{\mathbf{q}}} - \frac{v_{\mathbf{q}}^2}{i\omega_n+\omega_{\mathbf{q}}}$	0	$\frac{u_{\mathbf{q}}^2}{i\omega_n-\omega_{\mathbf{q}}} - \frac{v_{\mathbf{q}}^2}{i\omega_n+\omega_{\mathbf{q}}}$	$\frac{u_{\mathbf{q}}v_{\mathbf{q}}}{i\omega_n-\omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}v_{\mathbf{q}}}{i\omega_n+\omega_{\mathbf{q}}}$	$A^{\dagger}_{\mathbf{q}}\rangle$
	$\frac{u_{\mathbf{q}}v_{\mathbf{q}}}{i\omega_n-\omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}v_{\mathbf{q}}}{i\omega_n+\omega_{\mathbf{q}}}$	$\frac{v_{\mathbf{q}}^2}{i\omega_n-\omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}^2}{i\omega_n+\omega_{\mathbf{q}}}$	0	$\frac{v_{\mathbf{q}}^2}{i\omega_n-\omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}^2}{i\omega_n+\omega_{\mathbf{q}}}$	$A_{-\mathbf{q}}\rangle$
	$\frac{u_{\mathbf{q}}^2}{i\omega_n-\omega_{\mathbf{q}}} - \frac{v_{\mathbf{q}}^2}{i\omega_n+\omega_{\mathbf{q}}}$	$\frac{u_{\mathbf{q}}v_{\mathbf{q}}}{i\omega_n-\omega_{\mathbf{q}}} - \frac{u_{\mathbf{q}}v_{\mathbf{q}}}{i\omega_n+\omega_{\mathbf{q}}}$	$\frac{u_{\mathbf{q}}^2}{i\omega_n-\omega_{\mathbf{q}}} - \frac{v_{\mathbf{q}}^2}{i\omega_n+\omega_{\mathbf{q}}}$	0	$A^{\dagger}_{-\mathbf{q}}\rangle$

By looking the (1.10) and (1.12), we realized that this two Green's function are identical

$$\mathcal{G}(\mathbf{q}, i\omega_n) = \mathcal{G}'(\mathbf{q}, -i\omega_n). \tag{1.13}$$

Using the (1.1), with some calculation we can write down the Green's function in a matrix form [3]

$$\hat{\mathcal{G}} = \begin{pmatrix} \mathcal{G} & \mathcal{F} \\ \mathcal{F} & \mathcal{G}' \end{pmatrix} \tag{1.14}$$

Another method to calculate the Green's function is based on the the discussion in [4,5]. The result is given as the retarded Green's function

$$\hat{\mathcal{G}}_{ret} = \begin{pmatrix} \frac{E+(q^2/2M)+\kappa n_c}{E^2-\omega_{\mathbf{q}}+i\delta} & \frac{-\kappa n_c}{E^2-\omega_{\mathbf{q}}^2+i\delta} \\ \frac{-\kappa n_c}{E^2-\omega_{\mathbf{q}}^2+i\delta} & \frac{-E+(q^2/2M)+\kappa n_c}{E^2-\omega_{\mathbf{q}}^2+i\delta} \end{pmatrix} \tag{1.15}$$

It is still not clear how to get it.

[3] R. S. Christensen, J. Levinsen, and G. M. Bruun. Quasiparticle Properties of a Mobile Impurity in a Bose-Einstein Condensate. *Phys. Rev. Lett.*, 115(16):1–5, 2015

[4] S. Giorgini. Damping in dilute Bose gases: A mean-field approach. *Phys. Rev. A - At. Mol. Opt. Phys.*, 57(4):2949–2957, 1998

[5] V. M. Kovalev and A. V. Chaplik. Impurity Screening and Surface Acoustic Wave Absorption in a Dipolar Exciton Condensate at Finite Temperatures. oct 2013

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Bibliography

[1] R. S. Christensen, J. Levinsen, and G. M. Bruun. Quasiparticle Properties of a Mobile Impurity in a Bose-Einstein Condensate. *Phys. Rev. Lett.*, 115(16):1–5, 2015.

[2] S. Giorgini. Damping in dilute Bose gases: A mean-field approach. *Phys. Rev. A - At. Mol. Opt. Phys.*, 57(4):2949–2957, 1998.

[3] V. M. Kovalev and A. V. Chaplik. Impurity Screening and Surface Acoustic Wave Absorption in a Dipolar Exciton Condensate at Finite Temperatures. oct 2013.