Binary Trees

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Introduction

Binary Tree is a special tree in which each node can have maximum of two child nodes.

Properties of Binary Tree

1. The maximum number of nodes at level L = (2)^L. Level of root is 0.

2. The maximum number of nodes in tree of height H = ((2)^H) - 1. Height of single node tree is 1.

3. In a Binary Tree with N nodes, minimum possible height or the minimum number of levels is Log2(N+1).

4. A Binary Tree with L leaves has at least Log2(L) + 1.

5. In a Binary Tree where every node has 0 or 2 children, the number of leaf nodes is always one more than nodes with two children.

Binary Tree Types

Full Binary Tree

A Binary Tree is a full binary tree if every node has 0 or 2 children. In other words, except leaf nodes all other nodes has two children.

Complete Binary Tree

A Binary Tree is a Complete Binary Tree if all the levels are completely filled except possibly the last level and the last level has all keys as left as possible. Eg: Binary Heap

Perfect Binary Tree

A Binary tree is a Perfect Binary Tree in which all the internal nodes have two children and all leaf nodes are at the same level.

Balanced Binary Tree

A binary tree is balanced if the height of the tree is O(Log n) where n is the number of nodes. Balanced Binary Search trees are performance-wise good as they provide O(log n) time for search, insert and delete.

Example

AVL tree maintains O(Log n) height by making sure that the difference between the heights of the left and right subtrees is at most 1.

Red-Black trees maintain O(Log n) height by making sure that the number of Black nodes on every root to leaf paths is the same and there are no adjacent red nodes.

Degenerate or Pathological Tree

A Tree where every internal node has one child. Such trees are performance-wise same as linked list.

Enumeration of Binary Tree

A Binary Tree is considered labelled if every node is assigned a Label and as unlabelled if nodes are not assigned any label.

The number of different unlabelled binary trees that can be formed with N nodes are:

$$T(N) = (2N)! / ((N+1)! * N!)$$

This is basically representing n'th Catalan Numbers.

The number of labelled binary trees that can be formed with N nodes are:

$$T(N) = [(2N!) / ((N+1)! * N!)] * N!$$