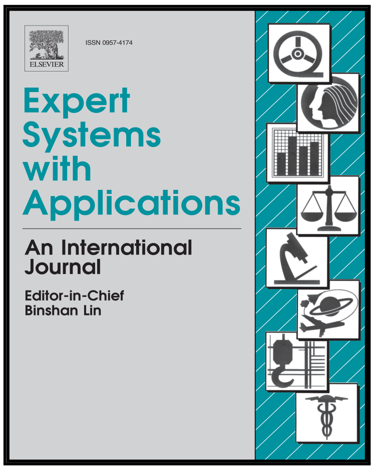

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Highlights

- Define a new technical indicator for measuring the trend of the fuzzy time series
- Introduce a new weighted fuzzy-trend time series method to forecast stock indices
- Compare ex-post performances of weighted FTS methods using stock market indices
- Assess statistical significance of ex-post forecast accuracy for weighted FTS methods

Improving stock index forecasts by using a new weighted fuzzy-trend time series method

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Abstract

We propose using new weighted operators in fuzzy time series to forecast the future performance of stock market indices. Based on the chronological sequence of weights associated with the original fuzzy logical relationships, we define both chronological-order and trend-order weights, and incorporate our proposals for the ex-post forecast into the classical modeling approach of fuzzy time series. These modifications for the assignation of weights affect the forecasting process, because we use jumps as technical indicators to predict stock trends, and additionally, they provide a trapezoidal fuzzy number as a forecast of the future performance of the stock index value. Working with trapezoidal fuzzy numbers allows us to analyze both the expected value and the ambiguity of the future behavior of the stock index, using a possibilistic interval-valued mean approach. Therefore, using fuzzy logic more useful information is provided to the decision analyst, which should be appropriate in a financial context.

We analyze the effectiveness of our approach with respect to other weighted fuzzy time series methods using trading data sets from the Taiwan Stock Index (TAIEX), the Japanese NIKKEI Index, the German Stock Index (DAX) and the Spanish Stock Index (IBEX35). The comparative results indicate the better accuracy of our procedure for point-wise one-step ahead forecasts.

Keywords: Fuzzy time series, Forecasting, Trend analysis, Stock Market Indices, Fuzzy numbers

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1. Introduction

Since the fuzzy modeling proposal for time series by Song and Chissom in 1993, many different models, approaches and procedures have been developed for suitably managing forecasting in a fuzzy context. The major points in their modeling approach are related to the partitioning of the universe of discourse, the establishment of fuzzy relationships from the fuzzy time series and the process of forecasting and defuzzification of the outputs. These authors introduced both time-variant and time-invariant models to forecast the enrollment of students at Alabama University (Song and Chissom, 1993a,b, 1994). In a financial context, fuzzy approaches for solving time series problems have been recently applied in stock index forecasting, for modeling business cycles and other financial optimization problems (Chen et al., 2012, Hung and Lin, 2013, Wang et al., 2014, Diaz et al., 2015, Zhou et al., 2015). Additionally, some authors combine fuzzy and non-fuzzy time series forecasting strategies with heuristic optimization methods in order to provide accurate forecasts for stock market indices (Chen et al., 2013, Li and Chiang, 2013, Wei, 2013, Chen et al., 2014, Askari and Montazerin, 2015). Recently, other approaches based on neural networks and heuristic procedures have been also applied to forecast economic and financial time series (Lahrimi, 2016a,b).

Concerning the basic steps of the fuzzy time series methods, for the partitioning of the universe of discourse, various proposals to determine suitable intervals have been considered. It has been established that the determination of the effective length of intervals improves forecasting results in fuzzy time series (Huarng, 2001a, Wang et al., 2013). To state appropriate fuzzy relationships is also critical in fuzzy time series, and a very interesting approach is proposed by Yu (2005), where recurrent fuzzy relationships for the assignation of weights of each individual relationship are used, which outperforms forecasting. However, with respect to forecasting proposals (both ex-ante and ex-post forecasts) many authors follow the scheme originally suggested by Song and Chissom (see, for instance, Wang et al. (2014) and references therein). So, these FTS methods look for point forecasts, without taking into account the implicit uncertainty in the ex-post forecasts.

It is well-known that stock market indices indicate how the stocks represented in the corresponding index have changed their value over a timewindow; that is, their behavior could be analyzed like the performance of a given portfolio. We can find several suitable Soft Computing approaches for approximating the uncertainty of future returns on assets, and modeling the beliefs and the imprecise knowledge in the fuzzy portfolio selection problem (Inuiguchi and Ramík, 2000, Carlsson et al., 2002, León et al., 2004, Wang et al., 2011, Vercher and Bermúdez, 2015, Kocadagli and Keskin, 2015).

In previous works, we have dealt with LR-type fuzzy numbers as useful approximations of the uncertainty of portfolio returns, for which possibilistic moments allow us to calculate the value, the ambiguity and some significant properties of fuzzy numbers (Vercher et al., 2007, Bermúdez et al., 2012, Vercher and Bermúdez, 2013). In this paper, we propose to analyze and predict the future performance of the stock market indices using trapezoidal fuzzy numbers. Other authors have also used trapezoidal fuzzy numbers as representation of the linguistic variables involved in the fuzzy time series modeling (Liu, 2007, Lu et al., 2014).

The main question concerning to forecast stock market indices is related to the trend changes, it is then important to measure the index fluctuations in order to provide accurate forecasts based on stock trends. Recently, several hybrid approaches for fuzzy time series have been developed for managing this volatility (Chen and Chen, 2015, Peng et al., 2015). In order to efficiently manage it, Chen and Chen (2015) introduce covariates in the FTS analysis, however to decide what covariate must be used for any stock market index should require a more extensive research. On the other hand, trend forecasting has been used for enhancing stability of the time series, however it has been usually analyzed assuming stationary conditions of the time series in order to obtain accurate forecasts.

The goal of the present research is to design a new weighted fuzzy-trend time series modeling approach for providing fuzzy numbers as forecasts of future performance of stock indices, using historical data sets. We define a new technical indicator for measuring the fuzzy trend of the time series, and use heuristic rules for assigning weights in the forecasting step, which provides fuzzy forecast outputs. In our opinion, this information will be useful to the decision maker, because it will make possible to analyze the ex-post forecast of the stock market index value from a fuzzy point of view (Dubois and Prade, 1987, Delgado et al., 1998). Throughout the paper, we will show that fuzzy-trend analysis provides useful information about the changes among the fuzzy sets that compone the universe of discourse of the fuzzy time series.

The remaining content of this paper is organized as follows. Section 2 provides a review of fuzzy time series definitions and methods. In Section 3 we introduce our proposals for weighting the frequency of one-to-one fuzzy relationships in a fuzzy-trend time series scheme, and use them for improving the one-step ahead fuzzy forecasts. Finally, in Section 4 we analyze the comparative performance of classical weighted fuzzy time series methods with respect to the proposed modifications, by using current trading data sets from several stock indices (the Taiwan Stock Index (TAIEX), the German Stock DAX 30 Index, the Japanese NIKKEI225 Index and IBEX35, the Index of the Spanish Stock Market). Conclusions are presented in Section 5.

2. Fuzzy Time Series

Since classical time series forecasting methods cannot deal with problems in which the values of time series are linguistic terms represented by fuzzy sets, Song and Chissom (1993a) developed fuzzy time series to overcome this drawback. Let us briefly review some definitions of fuzzy time series (FTS).

Definition 1. Let U be the universe of discourse. A fuzzy set A of the universe of discourse U can be defined by its membership function, $\mu_A : U \to [0,1]$, $\mu_A(u)$ being the degree of membership of the element $u \in U$ in the fuzzy set A.

Definition 2. Let $\mathcal{A}(t)$ be a collection of fuzzy sets of the universe of discourse $U(t)$, $t = \ldots, 0, 1, 2, \ldots$. A fuzzy time series $F(t)$ $t = 1, \ldots, N$ is a realization of a dynamic process in which $F(t) \in \mathcal{A}(t)$.

Then, fuzzy relationships between two consecutive observations can be defined.

Definition 3. Let $F(t-1) = A_i$ and $F(t) = A_j$, for $i, j = 1, 2, ..., N$. The relationship between two consecutive observations, $F(t-1)$ and $F(t)$ is denoted by $F(t-1) \to F(t)$ or by $A_i \to A_j$, where A_i is called the left-hand side (LHS) and A_j the right-hand side (RHS) of the fuzzy logical relationship (FLR).

Definition 4. Suppose that there are several fuzzy logical relationships with the same left-hand side, that is, $A_i \to A_{j1}$, $A_i \to A_{j2}$, and so on, they can be grouped into a fuzzy logical group (FLG) by putting all their right-hand sides together as the RHS of the fuzzy logical group, such as $A_i \to A_{j1}, A_{j2}, \ldots$.

The above definitions of fuzzy logical relationships play a crucial role in weighted fuzzy time series methods, which incorporate the chronological ordering on FLG and/or the frequency of fuzzy relationships on FLR in order to assign suitable weights for the forecast (Huarng, 2001b, Yu, 2005, Cheng et al., 2008, Lee et al., 2009, Rubio et al., 2016).

Let us now briefly recall some definitions and results which will be used in what follows.

Definition 5. A fuzzy number A is said to be a trapezoidal fuzzy number, $A = (a_l, a_1, a_2, a_u)$, if its membership function has the following form:

$$\mu_A(y) = \begin{cases} \frac{y-a_l}{a_1-a_l} & \text{if } a_l \leq y \leq a_1 \\ 1 & \text{if } a_1 \leq y \leq a_2 \\ \frac{a_u-y}{a_u-a_2} & \text{if } a_2 \leq y \leq a_u \\ 0 & \text{otherwise} \end{cases}$$

where $[a_1, a_2]$ is the core of A, and the support of A, $supp(A) = \{y : \mu_A(y) \geq 0\}$, is $[a_l, a_u]$.

The aggregation of positive linear combinations of trapezoidal fuzzy numbers can be dealt with using Zadeh's extension principle Zadeh (1965).

Proposition 1. Let $A = (a_l, a_1, a_2, a_u)$ and $B = (b_l, b_1, b_2, b_u)$ be trapezoidal fuzzy numbers and let λ be a real number. Then,

(1) $A \oplus B = (a_l + b_l, a_1 + b_1, a_2 + b_2, a_u + b_u)$

(2) $\lambda A = \begin{cases} (\lambda a_l, \lambda a_1, \lambda a_2, \lambda a_u) & \lambda \geq 0 \\ (\lambda a_u, \lambda a_2, \lambda a_1, \lambda a_l) & \lambda \leq 0 \end{cases}$

where the addition and multiplication by a scalar is defined by the sup-min extension principle.

Trapezoidal numbers are LR-fuzzy numbers with linear reference functions, and they have been extensively used in fuzzy modeling processes at many areas of decision making. In addition, they have the interesting property that every linear aggregation of trapezoidal fuzzy numbers leads to a trapezoidal one. However, the aggregation of LR-fuzzy numbers with different shapes must be analyzed through the α-cuts of their corresponding membership functions (Dubois and Prade, 1988), and it is not ensured that leads to an LR-fuzzy number.

2.1. Fuzzy time series forecasting methods

Let us introduce the basic steps of the forecasting process in fuzzy time series following Chen (1996) proposal:

- *Step 1. Partitioning the universe of discourse.* This includes the definition of the universe of discourse U and the number and length of the intervals.

- *Step 2. Defining the fuzzy sets on U and fuzzifying the historical data.*
- *Step 3. Establishing fuzzy logical relationships (FLR) from the fuzzy time series.* These relationships are based on the historical data and make it possible to derive fuzzy logical groups (FLG).
- *Step 4. Calculating the forecast values.* Locate the linguistic value of the last observed data point (LHS) and use the midpoint of each linguistic value of its fuzzy logical relationship (FLR) on the right-hand side for calculating the forecast: the arithmetic mean of all these midpoints.

Many authors apply the above 4 steps in their FTS models, using intervals of the same length like in Chen (1996), where the universe of discourse U is defined from min and max data values: x_{min} and x_{max}, respectively (obtained among all historical data x_i, $i = 1, ..., N$); in such a way that $U = [x_{min} - d_1, x_{max} + d_2]$, d_1 and d_2 being two suitable positive numbers. Once the length of the universe of discourse is determined, U can be partitioned into several equal length intervals u_j (Chen's proposal involves seven intervals). Each observation x_i in the training data set has a related fuzzy set (linguistic value), which is the fuzzy set with the greatest membership value at x_i; and usually, if the observed value is located in the range u_j, its related fuzzy set is A_j. Huarng (2001a) observed that this length affects the accuracy of the forecasting results, so he proposed distribution and average-based length methods for determining the length of intervals. Applying the above methods, an appropriate length l is determined taking into account the observed data according to a heuristic rule, and the number of intervals m is computed. So, following the Huarng proposals there are m intervals and m fuzzy sets (u_j and A_j, resp.), $j \in \{1, 2, ..., m\}$. Once the historical data have been fuzzified, the fuzzy logical relationships are stated using Defs. 3 and 4. Both Chen's and Huarng's fuzzy time series methods use then the same definitions in the intermediate steps and the same rules for calculating the crisp forecasts, which are the arithmetic mean of related midpoints (Step 4).

Concerning Step 2, Liu (2007) FTS forecasting method also maintains the partitioning of the universe of discourse suggested by Huarng (2001a), although it builds A_i as symmetrical trapezoidal fuzzy numbers, which have the interval u_i as their core, and whose right and left spreads have the same length than the length of u_i. As usual, each observed piece of data is fuzzified to A_k if the maximum degree of membership of data is in A_k. Liu's TFS method calculates the forecast outputs as trapezoidal fuzzy numbers using the FLG relationships (Def. 4), but his method assigns the same weight to all the fuzzy elements on the left-hand side of the FLG. Using trapezoidal fuzzy numbers for representing the linguistic variables in FTS methods has been also applied in Lu et al. (2014), where granule information provides the partition of the universe of discourse into different time-windows of unequal length, where the data are used to define suitable trapezoidal fuzzy sets.

Other authors have also introduced alternative procedures to provide a partition of the universe of discourse into intervals of unequal length (Wang et al., 2013, 2014); their proposals for Step 1 are based on fuzzy clustering techniques and information granules and they show that these new approaches can improve on forecast accuracy. Although, Wang et al. (2013) use both the Chen's and Lee et al proposals for assigning weights and calculating forecasts (Chen, 1996, Lee et al., 2009).

On their hand, Yu (2005) modifies Step 3, establishing fuzzy logical relationships which incorporate chronologically-determined weights in order to reflect recurrence; Yu's FTS method also uses its particular weighted average in the defuzzification process (in Step 4). Following this research line, Cheng et al. (2008) propose weighting the relationships among the different fuzzy sets (based on frequency), and Lee et al. (2009) present other modified weighted versions of Yu's FTS method.

Recently, Singh (2015) proposes a double partition of the universe of discourse using the arithmetic mean as the midpoint of these two intervals, and assigns weights on the FLR using the index of its right-hand side, which could be considered as a measure of trend, but only for the bullish cases nor for bearish ones. Finally, for the forecast output calculates the arithmetic mean of the data set in the last observed interval, in such a way that the ex-post forecast will remain into the universe of discourse. On their hand, Chen and Chen (2015) develop a FTS model based on two-factors and fuzzy relationships of second order, using a fixed number of intervals (100 equal length intervals) while assigning three kinds of trends (lower, equal or upper) and chronological weights for these trends. In contrast, our fuzzy-trend time series method uses a double set of weights based on the fuzzy logical relationships, for measuring both the frequency and the trend of the time series data, and determines dynamically the number of intervals for the partition of the universe of discourse, based on the variability of the raw data.

3. A new weighted fuzzy-trend time series method

We propose several modifications of the standard FTS approaches following the flowchart shown in Fig. 1. In our proposal, the universe of discourse $U = [x_{min} - d_1, x_{max} + d_2]$ is partitioned into m intervals $u_i = [u_{i1}, u_{i2}]$ of equal length (like was suggested by Huarng (2001a)), but the length of the intervals $Len = u_{i2} - u_{i1}$ is evaluated from the standard deviation of the observed historical data in U. In this paper, we decide to fit $Len = \frac{SD(data)}{10}$, because this value has working well for all our data sets in the numerical experiment. But, more research is needed to establish heuristic rules based on the sample size N of data set, the amplitude of U and the variability of observed data.

In addition, we define the membership function of every linguistic value A_i as a trapezoidal fuzzy number at Step 2. So, there are m intervals of length $Len = l$, and m trapezoidal fuzzy numbers A_i, whose core coincides with u_i and with $supp(A_i) = [u_{i1} - \frac{l}{2}, u_{i2} + \frac{l}{2}]$. Note that these fuzzy numbers are defined in a different way that in Liu (2007), since the left and right spreads have the semi-length of the core. Finally, we apply different rules to build the fuzzy forecast outputs at Step 4.

Let us introduce a new indicator for measuring the relationship between two consecutive elements of the fuzzy time series.

Definition 6. Let $F(t-1) = A_i$ and $F(t) = A_j$, for $i, j = 1, 2, ..., m$, that is $A_i \to A_j$. We denote by $k = j - i$ the jump associated to this fuzzy logical relationship. The size of the jump then fulfills that $-m + 1 \leq k \leq m - 1$.

Once the jump associated to every fuzzy logical relationships (FLR) has been evaluated (which provide positive, negative or null relative positioning), we can use this information to measure the trend between consecutive linguistic variables. The quality of this information is highly related to the number of intervals that compose the universe of discourse, if there is a little number of intervals this measure could be less informative. However, the main interesting counterpart of using jumps in FTS is that it allows to introduce suitable one-step ahead forecasts even when the last observation is at the endpoints of the universe of discourse. This casuistry is not usually provided in most FTS methods, which only build ex-post forecasts inside the universe of discourse.

Concerning the heuristic rules used for assigning weights, our procedure firstly uses the fuzzy logical relationships (FLR) of the original fuzzy time series, simultaneously taking into account the frequency and temporal posi-

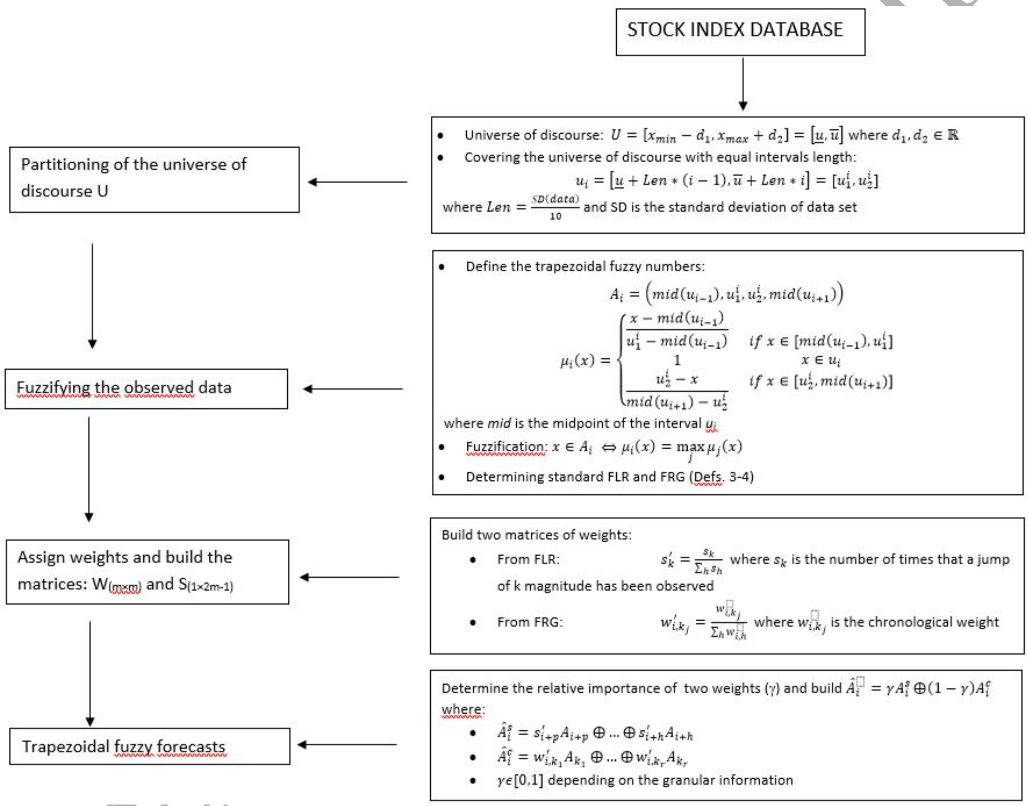

Figure 1: Flowchart of our weighted fuzzy-trend time series procedure

tion of each one-to-one fuzzy logical relationship, and afterwards it considers fuzzy logical groups (FLG) relationships for normalizing weights. Our fuzzy-trend forecasting strategy uses the same FLR basis than the classical FTS methods, because these original weights allows to measure the chronological trend.

Let us introduce some notation for representing the modified weights of our proposal.

3.1. Modified weighting rules

Let us suppose that we have observed the data $Y(t) = x_t$, for $t = 1, 2, ..., N$, which belong to the universe of discourse $U = \{u_1, u_2, ..., u_m\}$. Let $x_{t_0} \in u_p$ and $x_{t_0+1} \in u_r$, then $F(t_0) = A_p$ and $F(t_0 + 1) = A_r$ providing the $A_p \longrightarrow A_r$ fuzzy logical relationship, which will have a chronological weight associated with it, $\beta^{t_0}_{i,j} = t_0$ if $i = p$ and $j = r$ and zero otherwise.

(i) With respect to the weights assigned to the LHS elements of the fuzzy logical relationship groups (FRG), the procedure takes into account the weights in the FLR, $\beta^t_{i,j}$, which incorporate the frequency and the time in which this relationship has been observed, $t = 1, 2, ..., N$.

Suppose that A_i goes to many $A_{k1}, A_{k2}, ..., A_{kr}$, then the procedure calculates the weights $w_{i,h} = \sum_t \beta^t_{i,h}$, for $h = k1, k2, ..., kr$, and it assigns to $A_i \longrightarrow A_{kj}$ the normalized weight $w'_{i,kj}$, which is calculated as follows:

$$w'_{i,kj} = \frac{w_{i,kj}}{\sum_h w_{i,h}}, h = k1, k2, ..., kr$$

Note that the original weights take into account all the FLR in the fuzzy time series, but the normalizing process only considers the observed linguistic variables in the fuzzy logical group relationship.

(ii) Concerning the weights assigned to measure the chronological trend in the FTS, let us denote by $k = r - p$ the jump between two temporarily consecutive fuzzy linguistic variables, $A_p \longrightarrow A_r$, for $-m + 1 \leq k \leq m - 1$. So, we can calculate a vector of weights S, which measures the frequency of every jump in the observed fuzzy time series.

Let us denote by s_k the number of times that a jump of k magnitude has been observed, then $s_k = \sum_t \sum_p \beta^t_{p,p+k}$. The normalized trend weights s'_k take into account all the observed jumps in the FTS:

$$s'_k = \frac{s_k}{\sum_h s_h}, -m + 1 \leq k, h \leq m - 1$$

and the vector of weights is:

$$S = [s'_{-m+1}, ..., s'_{-1}, s'_0, s_1, ..., s'_{m-1}]$$

In summary, the set of chronological weights based on FLG try to incorporate information from identical patterns observed in the FTS, so they could be useful when the time series does not show a clear trend. On the other hand, the set of weights based on jumps try to learn from affine patterns -specifically, identical patterns after a translation on the scale of the universe of discourse-, and they could be very useful when the time series has a clear trend because, in that case, the fuzzy time series presents few identical patterns but a lot of affine patterns.

In the forecasting step our procedure can either alternatively or jointly use these two chronological weights, $w'_{i,j}$ and S. For the fuzzy forecast, however, we propose to use a linear combination of them in order to incorporate the frequency and the trend of the fuzzy time series into the fuzzy forecast.

3.2. Fuzzy forecast outputs

Let us suppose that $F(N) = A_i$, x_N being the last observed data, then the procedure builds the fuzzy forecast $\tilde{A}_i$ using the following heuristic rules:

1. If the FLR of A_i goes to only one A_k, this means that only one FLR has been observed from A_i. The procedure calculates as forecast:

$$\tilde{A}_i = \gamma \tilde{A}_i^s \oplus (1-\gamma) A_k$$

   where the vector of weights S is used to build the trapezoidal fuzzy number $\tilde{A}_i^s$, as follows:

$$\tilde{A}_i^s = s'_{i+p} A_{i+p} \oplus s'_{i+(p+1)} A_{i+(p+1)} \oplus ... \oplus s'_{i+h} A_{i+h}$$

   for $p = max\{-m+1, 1-i\}$, $h = min\{m-1, m-i\}$, and $\gamma \in (0,1)$ measures the importance given to the trend by the forecaster.
2. If the FLR of A_i goes to many $A_{k1}, A_{k2}, ..., A_{kr}$, it means that an FLG has been observed. The procedure builds a trapezoidal fuzzy number with a suitable weighted combination of $A_{k1}, A_{k2}, ..., A_{kr}$ and the weighted trend around A_i, as follows:

$$\tilde{A}_i = \gamma \tilde{A}_i^s \oplus (1-\gamma) \tilde{A}_i^c$$

   where $\tilde{A}_i^c = w'_{i,k1} A_{k1} \oplus w'_{i,k2} A_{k2} \oplus ... \oplus w'_{i,kr} A_{kr}$ is the forecast based on this FLG relationship.

3. If the FLR of A_i is empty, no FLR has been observed going from A_i. The fuzzy forecast will use the vector of weighted trend S to build the trapezoidal fuzzy number $\tilde{A}_i^s$, and then $\tilde{A}_i = \tilde{A}_i^s$.

Note that, if nonsymmetric trapezoidal fuzzy numbers were used to represent A_i, $i = 1, ..., m$ (Rubio et al., 2016), or following the clustering approaches suggested in Wang et al. (2013, 2014), the core and spreads of the fuzzy forecast $\tilde{A}_i$ will also be affected by this asymmetry. On the other hand, for the sole purpose of comparing our weighted fuzzy-trend approach with some classical FTS forecasting proposals, the procedure also calculates as a point-wise forecast the possibilistic mean value of the forecast trapezoidal fuzzy number $\tilde{A}_i$. However, our procedure needs to determine the best value for the parameter γ, which measures the importance of the fuzzy-trend in the time series (when the value of γ increases, the relevance of the chronological fuzzy-trend weights increases with respect to the FLG weights). This optimal value is established working with the training data set of the time series, following this heuristic rule: (i) repeatedly apply our fuzzy-trend FTS method for $\gamma = \{0, 0.1, 0.2, ..., 1\}$ and measure the averaged RMSE of the fitting one-step ahead errors; (ii) select the γ-value associated to the minimum RMSE, and use it for build the ex-post fuzzy forecasts.

In summary, our procedure returns a trapezoidal number $\tilde{A}_i$ as the fuzzy forecast, then value, ambiguity or any other suitable characteristic can be calculated in order to provide either single value or interval forecasts (Dubois and Prade, 1987, Delgado et al., 1998, Fullér and Majlender, 2003). By construction, the fuzzy forecast $\tilde{A}_i$ preserves the characteristics of the original trapezoidal fuzzy numbers (core and spreads amplitudes), because it has been obtained as convex linear combination of the weighted trend, $\tilde{A}_i^s$, and through the weighted combination of RHS trapezoidal fuzzy numbers, $\tilde{A}_i^c$.

3.3. One-step ahead forecast: a simple numerical example

Let us present an example of the performance of our method using 511 daily quotes of IBEX35 Stock Index. There, the universe of discourse has been approximated by $U = [6600, 12000]$, where $x_{min} = 7553$, $x_{max} = 11188$, $d_1 = 953$ and $d_2 = 812$. Since the standard deviation is $SD = 1029.4$, the procedure works with 52 equal intervals of length $l = 103$, which cover the universe of discourse. Analogously, the procedure builds the symmetric trapezoidal fuzzy numbers A_i, $i = 1, ..., 52$. Fig. 2 shows the observed IBEX35 quotations, both for training and testing data sets.

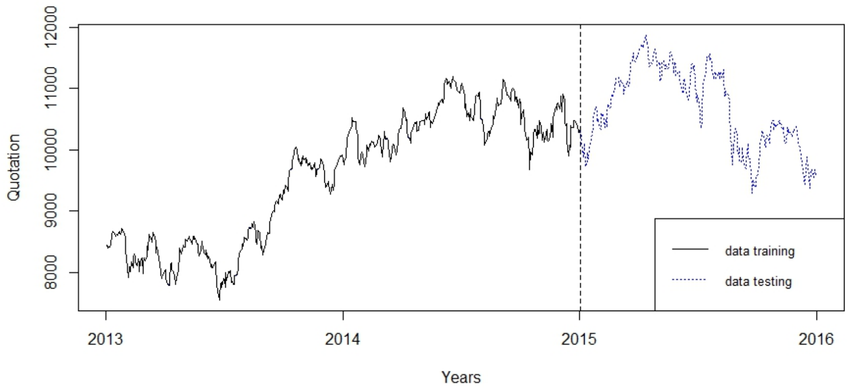

Figure 2: Trace of the time series of daily quotes of IBEX35, from January 2013 to December 2015.

Using the fuzzy logical relationship information, our procedure determines the γ-value which minimizes the RMSE (root of the mean squared error) of the one-step ahead fitting errors. So, applying the previous heuristic rule for γ-values, that is increasing it 0.1 points, the optimal fit is attained for γ=0.9 (RMSE=29.39), which is used for building the one-step ahead fuzzy forecast. Table 1 shows the fitting errors using other values of γ, showing the increasing influence of the trend.

Table 1: Fitting errors of the training data set using different γ-values.

	γ=0	γ=0.1	γ=0.3	γ=0.5	γ=0.7	γ=0.8	γ=0.9	γ=1
RMSE	40.95	38.89	35.20	32.24	31.11	29.66	29.39	29.43

Let us explicitly calculate the first fuzzy forecast, being x_{511} the final observation in the training set. x_{511} belongs to the fuzzy set A_{37}, whose associated right-hand size of the FRG is $A_{35}, A_{36}, A_{37}, A_{38}, A_{39}$, with a vector of chronological weights $w'_{37,k} = [0.14, 0.28, 0.27, 0.28, 0.03]$. So, the procedure returns the fuzzy number $\tilde{A}^c_{37} = (10231.5, 10283, 10386, 10437.4)$. On the other hand, the vector of positive weights associated with jumps is $S = [s'_{-4}, ..., s'_3]$=[.005,.016,.056,.216,.376,.250,.067,.014], being negligible the

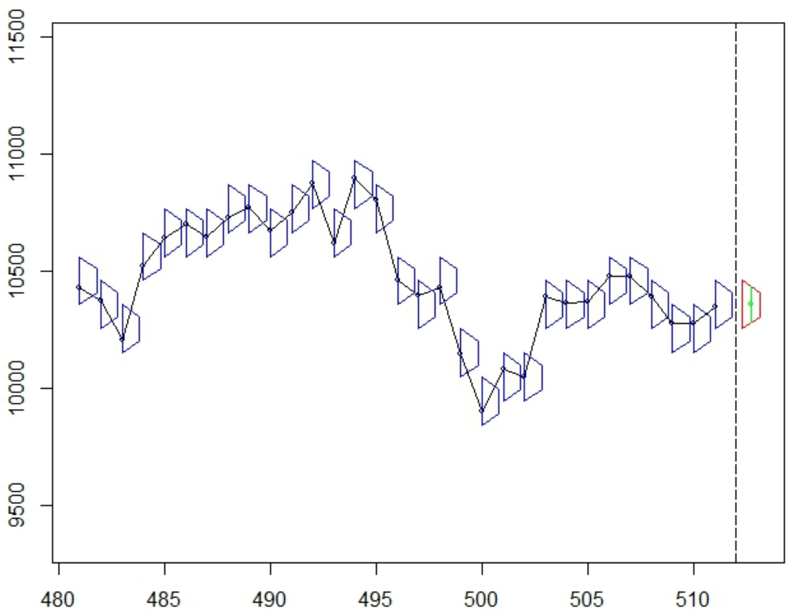

Figure 3: Partial representation of the fuzzy time series of daily quotes of IBEX35 and the first one-step ahead fuzzy forecast.

weights associated to the other jump sizes. Consequently, the procedure calculates the trapezoidal fuzzy number, $\tilde{A}^s_{37}$, based on the vector of weights S, as $\tilde{A}^s_{37} = \bigoplus_{h=-4}^{3} s'_h A_{37+h} = (10258, 10309.5, 10412.5, 10464)$. Finally, the one-step fuzzy forecast is $\tilde{A}_{37} = 0.9\tilde{A}^s_{37} \oplus 0.1\tilde{A}^c_{37}$. The point-wise forecast, $\tilde{x}_{512}$, will be the middle point of the interval-valued mean of $\tilde{A}_{37}$, whose one-step forecasting error is $|\tilde{x}_{512} - x_{512}| = 365.03$.

Figure 3 shows the fuzzy time series of the last observed daily quotes of the training period of this example, at the left hand-side. At the right-hand side, the plot corresponds to the fuzzy forecast $\tilde{A}_{37}$ and the point-wise forecast $\tilde{x}_{512}$.

4. Numerical experiment and comparisons

In order to check the predictive accuracy of the FTS method described in Section 3 we use historical data sets of four stock indices (IBEX35, Nikkei 225, DAX 30 and TAIEX), obtained at https://www.google.com/finance and TAIEX web site, respectively. For every aforementioned stock market index, we work with daily values collected over three years from January 2013 to December 2015. Data from the first two years are used for training and data from the year 2015 for testing. Note that sample sizes can be different depending on the opening days of every stock market and country.

Let us introduce our numerical experiment. We apply a weekly rolling horizon strategy for 5 daily consecutive quotes of the testing data set (as a representative of one standard week); in such a way that the weights provided in the training period have been maintained for each run; however, the daily one-step ahead fuzzy forecast depends on the current final observation. The rolling horizon strategy preserves the size of each training set by eliminating their first 5 observations and by including the last observed five weekly quotes. The fuzzy logical relationships for the new training set and the FLG are then reevaluated and, also both matrices of weights. This experiment is repeated for each run for the corresponding weekly periods of the 2015 data set, respectively. Table 2 shows sample sizes and the computational time devoted by our fuzzy-trend time series method for solving the numerical experiments. In our computational tests, once the stock database is provided our procedure gives the fuzzy forecast in 0.72 seconds on average for each run.

Table 2: Experimental data sizes and computational times of the proposed fuzzy-trend time series method.

	IBEX35	NIKKEI225	DAX30	TAIEX
Training data	511	502	508	494
Testing data	255	245	250	240
Runs	51	49	50	48
Total time (seconds)	45.41	31.05	39.06	26.97
Averaged time	0.89	0.63	0.78	0.56

The performance of our weighted fuzzy-trend time series method is compared with Yu's and Cheng et al.'s weighted methods (Yu, 2005, Cheng et al.,

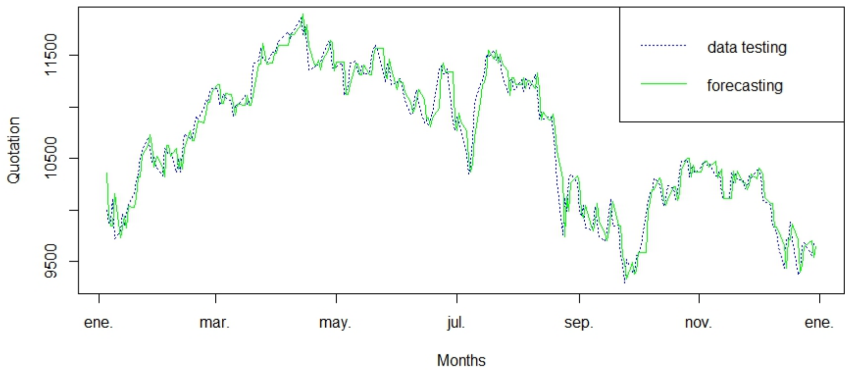

Figure 4: Jointly trace of the time series of daily quotes of IBEX35 and forecasts provided by our fuzzy-trend FTS method, from January 2015 to December 2015.

2008), and Wang et al. (2013) proposal using the aforementioned data sets. Note that Yu (2005) used the Huarng proposal for defining and partitioning of the universe of discourse, while Cheng et al. (2008) and Wang et al. (2013) use a different strategy for the partitioning in Step 1. All the above methods and our fuzzy-trend proposal have been implemented on a personal computer using the R language (http://www.r-project.org/), following the instructions given by the authors. To analyze the performance accuracy of the different FTS methods we use the RMSE and the MAD (mean absolute deviation) of the ex-post forecast errors, considering blocks of five opening days, although they were not necessarily the quotes of one standard week.

4.1. Forecasting IBEX35

In the first experiment, daily values of the Spanish IBEX35 Stock Index are considered, and the aforementioned weighted FTS methods are applied for the 255 daily quotes observed at year 2015. Following the rolling horizon strategy, the experiment is repeated 51 times for the weekly periods of that year. Fig. 4 shows the time series of the daily observations and the daily forecasts provided by our fuzzy-trend procedure for the year 2015, using γ=0.9.

The forecasting errors for the day of each week, $e_i = \tilde{x}_i - x_i$, have been calculated. The mean of their absolute values, $\text{MAD} = \sum_i |e_i|/5$, and the root of the mean of the squared errors, $\text{RMSE} = \sqrt{\sum_i e_i^2/5}$, for $i = 512, ..., 516$

at the first period are repeatedly evaluated for the next 50 weekly periods. Finally, the averaged RMSE and MAD can be obtained (see the last column of Table 3).

The accuracy of our forecasting FTS method is compared with those obtained by applying Yu's, Cheng at al. and Wang et al. weighted FTS's modeling approaches to forecast IBEX35, under the same experimental conditions. Table 3 shows the obtained results; our FTS proposal getting higher forecasting accuracy than the other weighted methods. The method proposed in Wang et al (2013) is the one which provides the worst results; this may be because they propose a partition of U in only seven intervals. This partitioning could be appropriate in small series o time series with low correlation, but highly autocorrelated long series like the ones that have been considered in this paper seem to need a finer partition.

Table 3: Mean forecast errors for different FTS models, with 2015 IBEX35 data set.

	Yu(2005) model	Cheng et al.(2008) model	Wang et al.(2013) model	Fuzzy-trend model γ=0.9
MAD	116.35	119.29	394.19	110.40
RMSE	145.49	144.55	429.27	134.27

In order to analyze the performance of the forecasts errors achieved by each method, a statistical analysis was carried out using the R language. Table 4 shows the adjusted p-values of the pairwise statistical comparison of the RMSE forecasts errors, which was performed through the paired t-test, adjusting the p-values using the Holm method (taking into account the multiple comparison problem).

The statistical analysis reveals that there are significant differences between the mean of the RMSE attained by our proposal and the other forecasting methods, in favor of our proposal. However, there is no statistical difference between the mean of the RMSE forecast errors of Yu's and Cheng et al. FTS methods, while the Wang et al. (2013) model presents significant differences with all the other weighted FTS methods.

Table 4: Adjusted p-values of the pairwise comparison of the RMSE forecast errors obtained for different FTS models, with 2015 IBEX35 data set.

	Yu(2005) model	Cheng et al.(2008) model	Wang et al.(2013) model
Cheng et al model	0.84		
Wang et al model	3.3e-11	2.3e-11	
Fuzzy-trend model	0.02	0.02	2.1e-11

4.2. Forecasting Nikkei 225

Let us consider the Nikkei225 Stock Index data set. Fig. 5 shows this time series. Note that in some occasions the last observation could be situated at the endpoint of the universe of discourse, so it is very important to consider a measure of fuzzy-trend in order to build the ex-post forecasts. Based on the training period data set the procedure establishes the universe of discourse, $U = [9500, 19000]$, dealing with 61 equal intervals, with $l = 156$ points. The fuzzy logical relationship information allows us to determine the γ-value which minimizes the RMSE of the one-step ahead fitting errors, that is γ=1 (RMSE=46.90). The experiment was repeated for finer partitions, and finally we obtain a different optimal value for γ=0.98, with RMSE=46.88. These results are neither statistically nor numerically significant. Additionally, we analyze this overfitting for the γ-value and it doesn't provide better results in the forecasting phase (RMSE=220.24 instead of 220.13, for γ=1).

Therefore, since γ=1 our FTS procedure will only work with the fuzzy-trend information provided by the weights associated to the jumps. Let us show this performance for the last observation $x_{502} = 17409$, which is related to the fuzzy set A_{51}. The procedure obtains the vector of weights $S = [s'_{-8}, ..., s'_5]$, and provides the fuzzy forecast $\tilde{A}^s_{51} = (17233, 17311, 17467, 17545)$. Finally, the point-wise forecast is $\tilde{x}_{503} = 17389$, and the one-step ahead prediction error is $|\tilde{x}_{503} - x_{503}| = 505.7$.

In this experiment, the rolling horizon experiment deals with 49 weekly periods of 5 consecutive daily quotes. Tables 5 and 7 show the averaged MAD and RMSE forecasting errors for these 49 periods, and the adjusted p-values of the pairwise comparison of the RMSE forecasts errors through the paired t-test using the Holm method, respectively.

Our proposal again attains better forecasting accuracy than the other

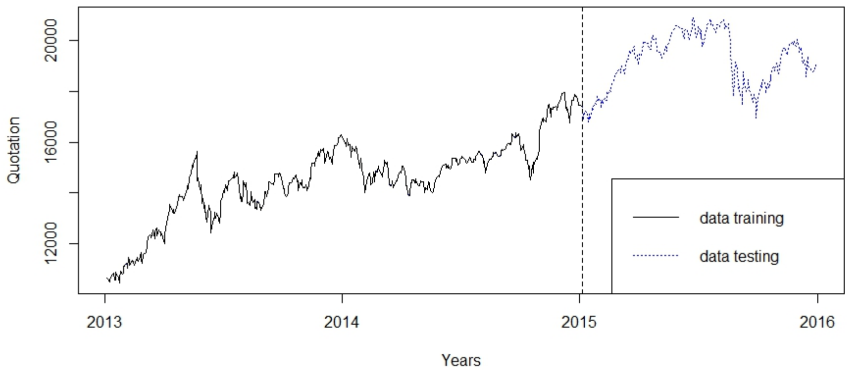

Figure 5: Trace of the time series of daily quotes of Japanese Nikkei 225, from January 2013 to December 2015.

Table 5: Mean forecast errors of the Nikkei 225 Index, with data set of the year 2015 and for different FTS models.

	Yu(2005) model	Cheng et al.(2008) model	Wang et al.(2013) model	Fuzzy-trend model γ=1
MAD	199.22	221.05	445.84	182.33
RMSE	245.16	277.02	510.44	220.13

models, for both forecasting errors. The statistical analysis revealed that there are significant differences between the mean of the RMSE attained by our proposal and the other forecasting methods, in favor of our proposal, and the same significant evidence with respect to the performance of the other FTS metods.

4.3. Comparative prediction results for DAX30 and TAIEX indices

In this section, the FTS procedures deal with data sets from the German and Taiwanese stock markets, whose temporal behavior during the considered period seems quite different from the previous ones. Figs. 6 and 7 show the traces of the daily quotes for the years 2013, 2014 and 2015, where the presence of positive and negative trend is reflected.

Firstly, our procedure calculates the best value of γ for the 508 quotes of

Table 6: Adjusted p-values of the pairwise comparison of the RMSE forecast errors obtained for different FTS models, with 2015 Nikkei data set.

	Yu(2005) model	Cheng et al.(2008) model	Wang et al.(2013) model
Cheng et al model	0.063		
Wang et al model	1.0e-07	5.2e-0.6	
Fuzzy-trend model	0.043	2.9e-05	8.1e-09

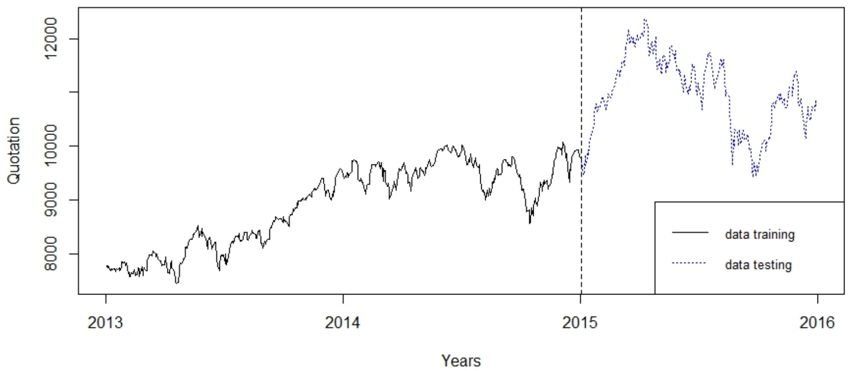

Figure 6: Trace of the time series of daily quotes of DAX30.

the DAX30 training data set, $\gamma = 1$, and for the 494 quotes of TAIEX which is γ=0.9. Then, it applies the numerical experiment for both stock indices data set, in which the FLR and FLG have been redefined every time that a new weekly data set is observed, allowing to incorporate recent granular information. The rolling horizon has been applied for 50 and 48 weekly periods, respectively.

Fig. 8 shows the trace of the daily forecasts attained using our weighted fuzzy-trend method jointly with the observed daily quotations of the TAIEX during the year 2015.

Finally, tables 7 and 8 show the averaged (MAD and RMSE) weekly forecast errors attained for the aforementioned weighted TFS methods. These comparative results are also consistent with the best performance of our pro-

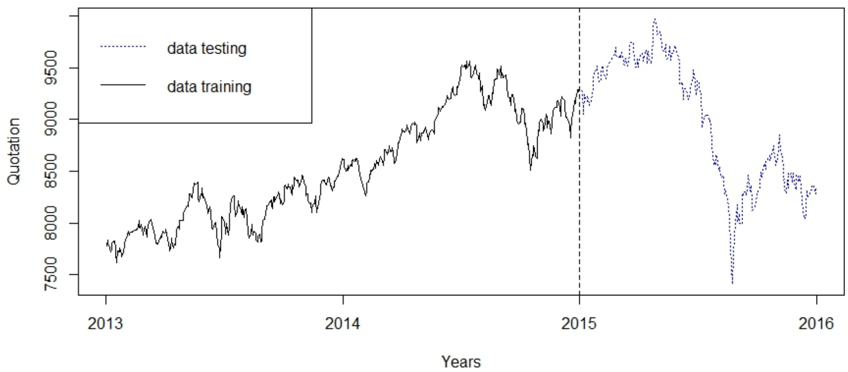

Figure 7: Trace of the time series of daily quotes of TAIEX.

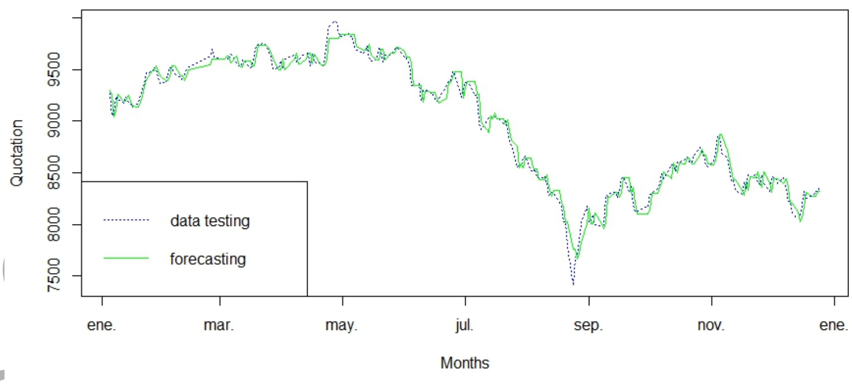

Figure 8: Traces of the time series of daily quotes of TAIEX and forecasts, from January 2015 to December 2015.

posed fuzzy-trend FTS method, at least with respect to forecasting accuracy.

Table 7: Mean forecast errors of the German DAX 30 Index, with data set of the year 2015 and for different FTS models.

	Yu(2005) model	Cheng et al.(2008) model	Wang et al.(2013) model	Fuzzy-trend model γ=1
MAD	145.87	139.98	343.25	126.34
RMSE	172.69	170.56	376.80	153.15

Table 8: Mean forecast errors of the TAIEX Index, with data set of the year 2015 and for different FTS models.

	Yu(2005) model	Cheng et al.(2008) model	Wang et al.(2013) model	Fuzzy-trend model γ=0.9
MAD	71.84	70.59	198.93	65.36
RMSE	87.56	87.39	220.83	80.58

Concerning the statistical significant of the pairwise comparison of the averaged RMSE one-step ahead forecast error with the t-test, lowest p-values are associated to Wang et al. (2013) FTS method, for every data set. Additionally, the greatest p-value attained for our proposal in comparison with the others two weighted FTS methods has been 0.027, while there are not significant differences between Yu's and Cheng et al.'s FTS methods.

We have found analogous results when averaged ex-post forecast errors using MAD have been compared. The statistical differences and homogeneities follow the same scheme than averaged RMSE.

5. Conclusions

In this paper, we introduce a forecasting method based on a convex linear combination of two new weighting operators for fuzzy time series forecasting. We propose using a set of weights obtained from the chronological sequence of fuzzy logical relationships, using information provided by one-to-one FLR.

DAX\TAIEX	Yu(2005) model	Cheng et al.(2008) model	Wang et al.(2013) model	Fuzzy-trend model
Yu'05		0.959	1.5e-09	0.027
Cheng et al'08	0.790		8.3e-10	0.011
Wang et al'13	1.1e-07	4.0e-07		8.3e-10
Fuzzy-trend	0.003	0.011	1.0e-08	

Table 9: Adjusted p-values of the pairwise comparison of the one-step ahead forecast errors obtained in the numerical experiment. The lower submatrix contains the p-values for the pairwise comparisons of RMSE for DAX30 data set and the upper submatrix for those RMSE for the TAIEX data set.

Additionally, in order to consider the possible trend of the fuzzy time series, we introduce a set of weights which measure the relative frequency and magnitude of the jumps observed in the fuzzy time series. Our method also provides trapezoidal fuzzy numbers as ex-post fuzzy forecasts.

Our approach has been tested using historical time series data from four stock market indices (Spanish IBEX35, Japanese NIKKEI 225, German DAX30 and the Taiwanese TAIEX). We analyze the performance of our proposed fuzzy-trend forecast method with respect to the weights involved in the linear combination. In the numerical experiments, we compare our proposal with three well-known weighted fuzzy time series methods, and higher forecasting accuracy is obtained with our fuzzy-trend proposal in all cases.

Statistical pairwise comparison have been used for a weekly rolling horizon strategy, using daily quotes of the four indices during the year 2005, showing significant differences of the averaged RMSE forecast errors of our method with respect the other analyzed FTS methods.

Future studies could focus on the incorporation of other strategies for the partitioning of the universe of discourse, considering unequal intervals, and applying LR-fuzzy numbers as representative of the linguistic variables involved in the fuzzy time series.

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