Advanced fuzzy time series applied to short term load forecasting

Conference Paper · November 2017

DOI: 10.1109/LA-CCI.2017.8285726

5 authors, including:

Guilherme Costa Silva
Federal University of Minas Gerais

Adriano Chaves Lisboa
Gaia

Douglas Alexandre Gomes Vieira
Enacom

Rodney R. Saldanha
Federal University of Minas Gerais

Advanced Fuzzy Time Series Applied to Short Term Load Forecasting

Guilherme Costa Silva*, João L. R. Silva*, Adriano C. Lisboa†, Douglas A. G. Vieira‡, Rodney R. Saldanha§
*Graduate Program in Electrical Engineering - Federal university of Minas Gerais (PPGEE-UFMG)
Av. Antônio Carlos 6627, 31270-901, Belo Horizonte, MG, Brasil
†Gaia Solutions on Demand
Rua Professor José Vieira Mendonça, 770, 31270-901, Belo Horizonte, MG, Brasil
‡ENACOM Handcrafted Technologies
Rua Professor José Vieira Mendonça, 770, 31270-901, Belo Horizonte, MG, Brasil
§ Department of Electrical Engineering - Federal university of Minas Gerais (DEE-UFMG)
Av. Antônio Carlos 6627, 31270-901, Belo Horizonte, MG, Brasil

***Abstract*—This paper proposes the application of advanced fuzzy time series in order to provide short-term load forecasting, which consists of predicting future demands from up to a week. An accurate forecast can influence significantly the availability and reliability of electrical systems. In this work, we tested different fuzzy time series algorithms in order to provide hourly, daily and weekly forecast of the demand in the Polish Electric System. The presented methods have achieved some interesting results to the problem.**

I. Introduction

The load demand information can be considered one of the most important requirements in the operation of power systems. Most operational and even planning problems of power systems rely on these data, since it can determine the proper operation of generators, substations and lines, as well as the planning of new units. The load demand also determines if the available units will handle the demand, according to the sources provided, and the cost of operation.

Since load demand can be modeled as time series data and power systems operations behavior has some tendencies, this information can be predicted, as in Box and Jenkins time series models, for example. This information can provide some good decision models for other power system problems, however, some uncertanties must be handled and, the conventional time series model may not be satisfactory in the analysis.

Fuzzy systems are widely used in literature when a certain degree of uncertainty in application problem is considered. There are several applications in which fuzzy system concepts were implemented with promising results.

Fuzzy time series was introduced by Song and Chissom in 1993 [1] in time series forecasting problems. Some notable advantages of fuzzy time series are their easy implementation by algorithms, which can provide improvememts by increasing forecast accuracy and, since conventional time series may have strong background in mathematics and statistics, in fuzzy time series this need is not implicit.

In this paper, we consider the short term load forecasting (STLF) problem studied in [8]. These parameters are forecasting using the fuzzy time series according to the period of forecasting. The results obtained in this work shows that the use of advanced fuzzy time series (AFTS) may provide better forecast accuracy than some classical models in some cases.

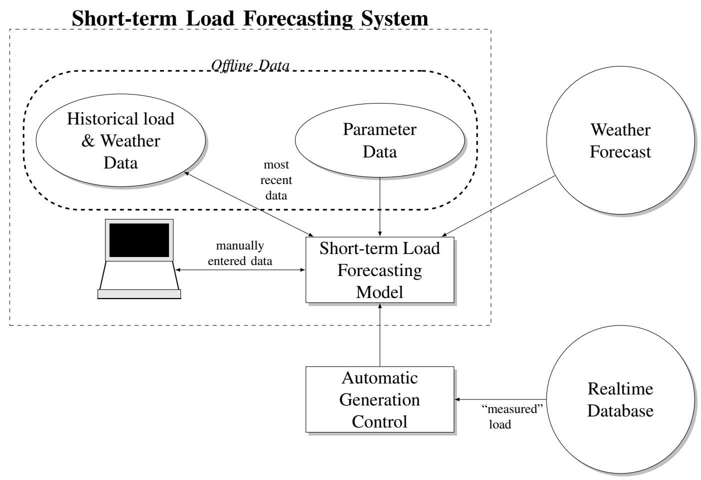

Fig. 1. Input data sources for STLF models, based on [2].

II. Short-term Load Forecasting

The Short term load forecasting problem consists of predicting load demand in a range of few hours ahead up to few weeks ahead. This problem is very important for operations such as unit commitment and economic dispatch, and the planning of electric power systems. When forecast accuracy increases, the power system can operate closer to its optimal point, which affects directly its profitability and stability. Figure 1 illustrates the model, considering input sources.

The sum of all the individual demands at all nodes of the overall power system determines the system load. In fact, the estimation of a system load pattern is a difficult task, considering each customer consumption pattern, which may be random, highly unpredictable, or even subject to atypical factors. However, a distinct consumption pattern resulting from the totality of individual loads can be statistically predicted, according to [2]. Thus, some classes of factors can influence

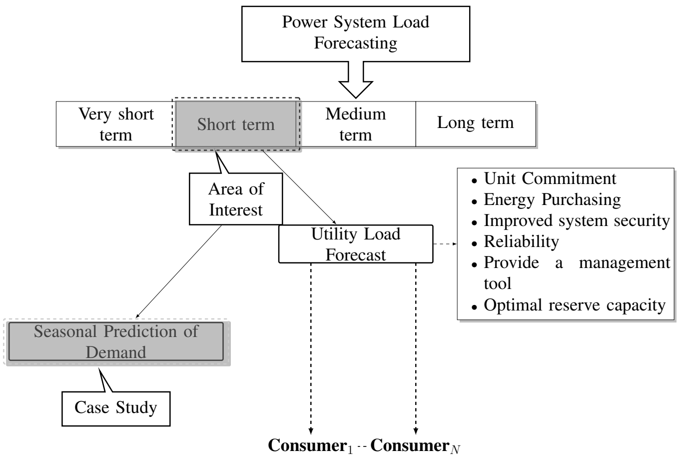

Fig. 2. Types of load forecasting and application of STLF, based on [4].

the system load behavior, such as seasonal, economic, and weather factors, or even random effects. According to [4], there are three objectives for STLR, such as generation scheduling, provide more secure and reliable operation of power plants, and provide economic dispatch and reliability. Figure 2 illustrates types and applications of load forecasting.

Several methods can be employed to solve STLF problems, according to the review in [3], such as statistical methods, autoregressive models, and artificial intelligence methods. The proper development, improvements, and investigation of such mathematical tools can result in more accurate load forecasting techniques.

These problems can also be solved by applying pattern similarity-based methods, as discussed in [5]. This methodology consists of measuring similarities between patterns in order to provide forecasting. Seasonal cycles are also considered in the analysis. Models based on this methodology are discussed in [6].

According to [7], [8], neural networks are the most popular computational intelligence methods applied in STLF considering their many attractive features such as the universal approximation property. However it has some drawbacks as its disruptive and unstable training, and some difficulties regarding the network structure. In addition, there are some other methods listed, such as neuro-fuzzy networks, support vector machines, and artificial immune systems, which can solve the problem. Hybrid approaches such as in [13], which employed Support Vector Regression optimized by evolutionary algorithms, are also considered to provide load forecasting.

Actually, the study in [8] considers simple univariate linear regression models based on patterns of daily cycles for STLF by approximating locally the relationship between input and output patterns according to the neighborhood of the query pattern using linear regression methods. It is pointed that, despite the interesting results, the need to construct new models for different query points is an issue in the approach, making the model highly dependent on model estimation.

The work in [9] employed an ensemble approach based on wavelet transform, extreme learning machine, and partial least squares regression applied to six case studies, considering several aspects of the STLF problem, some of them performing multivariate forecast by considering the temperature.

In this work, we analyze the use of Fuzzy Time Series applied to STLF problems, considering some advantages: the only data required is a range of values used in fuzzification, so that a proper model can be obtained, instead of generating a model by analyzing training data as in pattern recognition approaches. The resulting forecasting model may cope with uncertainties throughout the analyzed data.

III. Fuzzy Time Series

A. *State-of-the-art*

Time series (TS) is a sequence composed of continuous, real-valued elements [10] that correspond to observations made chronologically [11]. Some notable examples of TS are the mean daily temperature of a city, crude oil price per barrel, among others. Time series analysis comprises methods for analyzing data sequentially in order to extract meaningful statistical and relevant information of the data. Time series forecasting consists of using a model to predict future values of the series based on previously observed values. There are many time series-based approaches, such as the Naive model, Mean method, Moving Average Smoothing, Weighted Mean method, Exponential Smoothing, Additive Holt-Winters method, ARMA and ARIMA models, among other models.

Fuzzy set theory, developed by Zadeh in 1965, is widely applied into several subjects, such as decision making, planning, logic, systems theory, artificial intelligence, economics, control theory and so on. Fuzzy time series (FTS) is the combination of fuzzy set theory with time series analysis. The advantage to deal with formal, powerful and quantitative framework to cope with the vagueness of human knowledge as it is expressed by means of natural languages justifies the use of fuzzy sets instead of crispy sets. Some methods are reviewed in [12], including some hybrid or Type-2 Fuzzy-based methods. There are also metrics and databases described in the review. Some notable methods are Chen's method in [14] and Yu's weighted method in [15].

In this work, we focus on some Advanced FTS methods. The methodology around FTS is further discussed, as well as those FTS methods used in this work. Then, the discussed methods are applied to STLF case studies in the next section.

B. *Generalized FTS algorithms*

In general, most FTS forecasting algorithms can be described by its modeling, which is presented below in the following steps.

Step 1. Define the universe of discourse:

$$U = [D_{min} - D_1, D_{max} + D_2] \tag{1}$$

considering historical data, minimum and maximum values from dataset, two numbers $D1$ and $D2$ are chosen to adjust the range of data. The universe of discourse can also be defined from clustering techniques.

Step 2. Partition the universe of discourse defined in (1) into n intervals with equal length:

$$U = [u_1, u_2, \dots, u_n] \tag{2}$$

There are many methods to define the number of intervals, such as by expert knowledge, the Huarng's method in [16], Sturges method or power of p rule [17], or even evolutionary algorithms that can be employed to find the optimum number of intervals.

Step 3. Define fuzzy sets on the universe U according to the n intervals:

$$A_i = f_{Ai}(u_1)/u_1 + f_{Ai}(u_2)/u_2 + \cdots + f_{Ai}(u_n)/u_n \tag{3}$$

where $i = 1, 2, \dots, n$, and A_i can be defined by linguistic values.

Step 4. Fuzzify historical data. In some FTS methods, fuzzy values are assigned automatically by using the n-interval length of U and the distance between m-th data and the mean of n-interval length.

$$\Delta_u = U(2,1) - U(1,1) \tag{4}$$

$$\Delta = \left| D(m) - \frac{\Delta_u}{2} \right| \tag{5}$$

Step 5a. If the model is of first order (one former state), establish fuzzy logical relationships (FLR), which defines the relation between the former states and the state to be forecasted, by determining the sequence of fuzzy sets (A_i) that is equal to 1.

$$\begin{aligned} A1 &\to A1 \\ A1 &\to A2 \\ &\vdots \\ A6 &\to A6 \\ A6 &\to A7 \end{aligned} \tag{6}$$

Step 5b. If the model is of a higher order (multiple former states), Choose the sequence of former states according strategies previously presented. For example:

$$F(t-2) \to F(t-1) \to F(t) \tag{7}$$

Then, establish fuzzy logical relationships (FLR) according the former states presented. In the following example:

$$\begin{aligned} A1, A1 &\to A1 \\ A1, A1 &\to A2 \\ &\vdots \\ A6, A7 &\to A6 \\ A7, A7 &\to A7 \end{aligned} \tag{8}$$

FLR are composed of two former states, for these, define the fuzzy logical relationship group (FLRG) by eliminating the recurrence of fuzzy logical relationship.

$$\begin{aligned} A1, A1 &\to A1, A2 \\ A1, A2 &\to A3 \\ &\vdots \\ A6, A7 &\to A6 \\ A7, A7 &\to A7 \end{aligned} \tag{9}$$

Step 6. Forecast all the right hand side of the fuzzy data in FLR according to the model applied, which consists of finding the $F(i)$ value according to M_i, which is the corresponding midpoint of i-th interval in U, and W_i, a calculated weight for each M_i, when applicable.

C. AFTS models

In this section, we present some recurrent AFTS models in literature. With the exception of the Transformation FTS algorithm, which performs the Box Cox Transformation during the application of the FTS algorithm, all methods presented here follow the steps described in Subsection III-B. All described models were implemented using MATLAB R2016b.

1) Yu's Weighted FTS: The Yu's Weighted FTS algorithm, which is very similar to Chen's method in [14], provides the forecasting based on weight values applied to fuzzy forecasting. A complete description of this algorithm is given in [15].

The algorithm is implemented considering the forecast equation in Step 6 as follows:

$$F(i) = [M_1, M_2, ..., M_i] \times [w_1, w_2, \dots, w_i]^T \tag{10}$$

with the weighted matrix defined by (11), (12) and (13).

$$W(t) = [w_1^{'}, w_2^{'}, \dots, w_k^{'}] \tag{11}$$

$$W(t) = \left[\frac{w_1}{\sum_{h=1}^{k} w_h}, \frac{w_2}{\sum_{h=1}^{k} w_h}, \dots, \frac{w_k}{\sum_{h=1}^{k} w_h} \right] \tag{12}$$

$$\sum_{h=1}^{k} w_h^{'} = 1 \tag{13}$$

2) Exponential FTS: The Exponential FTS algorithm [18] makes some corrections in the weights calculated in Yu's method. This algorithm is implemented considering the forecast equation in Step 6 as follows:

$$W(t) = \left[\frac{1}{\sum_{h=1}^{k} w_h}, \frac{c}{\sum_{h=1}^{k} w_h}, \frac{c^2}{\sum_{h=1}^{k} w_h}, \dots, \frac{c^{k-1}}{\sum_{h=1}^{k} w_h} \right] \tag{14}$$

The adjustment of the weight constant c makes great difference in fuzzy forecast, usually the default c value is 1.2.

3) Transformation FTS: Differently from most FTS algorithms, the Transformation FTS [19] consists of applying the Box Cox transformation to the actual data series and then apply Yu's Weighted FTS algorithm. The transformation aims to remove noisy effects of data and improve forecasting performance.

The steps to implement the Transformation FTS algorithm are presented as follows:

Step 1. Apply Box Cox transformation to the actual data series and then obtain the λ parameter and Z vector:

$$Z_t^\lambda = \begin{cases} \frac{Z_t^\lambda - 1}{\lambda} & \lambda \neq 0 \\ \ln(Z_t) & \lambda = 0 \end{cases} \tag{15}$$

Step 2. Calculate d_S vector according to:

$$d_S = \frac{Z_S^\lambda - Z_{S-1}^\lambda}{Z_S^\lambda} \tag{16}$$

Step 3. Calculate Δ_q as a difference between third and first quartile:

$$\Delta_q = Q_3 - Q_1 \tag{17}$$

Step 4. Calculate l, which is applied to estimate partitions, according to:

$$l = \frac{2 \times \Delta_q}{n^{1/3}} \tag{18}$$

Step 5. Obtain estimated vector $\hat{d}_S$ using Yu's FTS algorithm.

$$\hat{d}_S = [M_1, M_2, \ldots, M_i] \times [w_1, w_2, ..., w_i]^T \tag{19}$$

Step 6. Calculate the estimated $\hat{y}_S$ parameter:

$$\hat{y}_S = \frac{1}{1 - \hat{d}_S} \tag{20}$$

Step 7. Apply the following to the vector Z obtained in (15):

$$\hat{Z}_t^\lambda = \hat{y}_S Z_t^\lambda \tag{21}$$

Step 8. Obtain the forecast data series according to:

$$F(t) = \left(\lambda \hat{Z}_t^\lambda + 1\right)^{1/\lambda} \tag{22}$$

4) High-order FTS: The High-order FTS (or HOFTS) [20] considers the sequence of former states rather than forecasting based on a single state. This can be achieved by applying some of these options below:

- expert knowledge;
- data discovery;
- bruteforce procedure;
- ACF (Auto-correlation function) analysis;
- evolutionary algorithms.

High-order FTS are very useful for seasonal and multi-seasonal series. The algorithm follow the basic steps from subsection III-B, then, the Step 5b is applied. After these steps, the forecast equation in Step 6 is given below:

$$F(i) = \frac{\sum_{j=1}^{k} M_j}{k} \tag{23}$$

5) Interval-based FTS: The Interval-based FTS Algorithm [21] aims to deal with forecasting uncertainty by estimating distributions of possible values instead of a unique point forecast. In this case, the upper and bound limits correspond to the interval and a unique point can be generated by calculating the midpoint between the bounds.

The algorithm follow the basic steps from subsection III-B, with the following modifications in Step 5a:

Step 5c. Each choosen FLR Group will generate intervals $\mathbb{I}^j = [\underline{\mathbb{I}_{min}^j}, \overline{\mathbb{I}_{max}^j}]$ where $\mathbb{I}_{min}^j$ and $\mathbb{I}_{max}^j$ are, respectively, the minimum lower bound and the maximum upper bound of all RHS fuzzy sets related to j.

$$\begin{aligned} \mathbb{I}_{min}^j &= \min\{A_1, A_2, \ldots, A_k\} \\ \mathbb{I}_{max}^j &= \max\{A_1, A_2, \ldots, A_k\} \end{aligned} \tag{24}$$

The Step 6 consists of finding the final forecast interval $\mathbb{I}_f$, calculated as the sum of the FLRGs intervals weighted by the membership value of each FLR Group, as shown in (25).

$$\mathbb{I}_f = \frac{\sum_{j \in A} \mu_j \mathbb{I}^j}{\sum_{j \in A} \mu_j} = \frac{\sum_{j \in A} [\mu_j \underline{\mathbb{I}_{min}^j}, \mu_j \overline{\mathbb{I}_{max}^j}]}{\sum_{j \in A} \mu_j} \tag{25}$$

Considering the following assumption:

$$F(i) \in \mathbb{I}_f \tag{26}$$

a single point can be generated for the forecasting. For this purpose, these intervals can be combined through arithmetic, geometric, or harmonic mean. In this work, the arithmetic mean is considered in terms of simplification.

IV. Case Study

A. Polish Electric System Data

In this section the studied FTS models are tested on STLF problems in the hourly load of the Polish power system from the period of 2002–2004, with its data illustrated in Figure 3. This data was used in [6], [7], [8] as a univariate pattern-based forecasting test since no additional data besides demand are required. For benchmarking purposes, three evaluation cases were proposed in order to forecast data as follows.

1) Evaluation by forecasting data by one hour ahead using time series of the entire dataset;
2) Evaluation by forecasting data per day, hour by hour using time series of the entire dataset except holidays.
3) Evaluation by forecasting data per weekday, hour by hour measuring average of the day and evaluating using time series of the entire dataset except holidays.

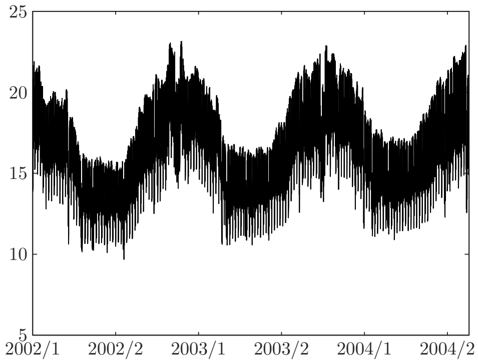

Fig. 3. The hourly electricity demand in Poland in three-year interval, based on [7].

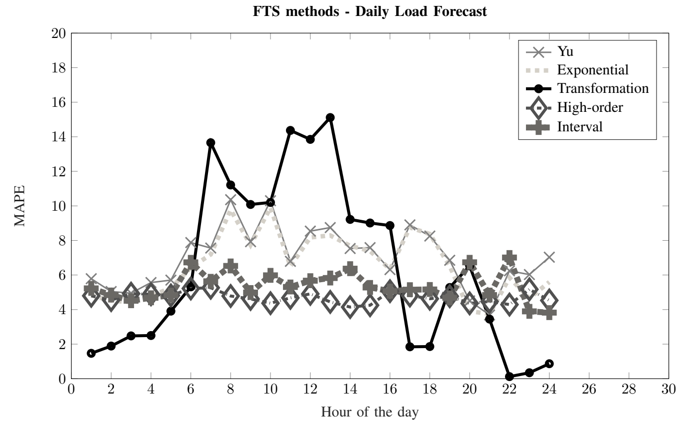

Fig. 4. FTS applied to daily load forecasting per hour.

TABLE I
RESULTS OF FUZZY TIME SERIES APPLIED TO HOURLY LOAD FORECASTING.

Method	MAPE	IQR
Weighted	3.4546	3.2890
Exponential	3.0827	3.1297
Transformation	1.0135	0.8564
High-order	3.0951	3.3393
Interval	2.1301	1.1494

TABLE II
AVERAGE MAPE AND IQR OF FTS APPROACHES APPLIED TO DAILY LOAD FORECASTING.

Method	MAPE	IQR
Weighted	7.0030	6.1449
Exponential	6.5257	5.3246
Transformation	6.3924	5.0735
High-order	4.6974	4.8980
Interval	5.3653	4.5187

The performance of FTS models are measured through mean absolute percentage errors (MAPE) and their interquartile ranges (IQR). Errors generated by the models are compared in pairs considering the Wilcoxon rank sum test with 5% significance level in order to confirm if each pair of models are statistically different.

Some parameters are described as follows: in Exponential FTS, c is set to 1.2, High-order FTS uses information from three instants, and Interval FTS uses mean value to determine a single point forecasting.

B. Results

Some comparisons between fuzzy forecast methods applied to short-term load forecasting are presented according three different case studies previously described.

1) Hourly forecast: The mean average percent error and interquartile range for FTS methods applied to hourly load forecast are presented in Table I. All FTS approaches have presented error rate of less than 4%. Noteworthy, the Transformation FTS method has presented error rate closer to 1% in these tests.

2) Daily forecast: The mean average percent for FTS methods applied to daily load forecast are presented in Figure 4. As presented, FTS approaches have presented error rate of less than 10%, except for Transformation FTS, which presented a worse result than in hourly forecast. High-order and Interval FTS have presented better results in this case. The performance of methods was better for night hours forecasting. Considering the forecasted period, the performance of such methods may be different.

The average values of MAPE and IQR were also calculated in order to measure the overall performance of FTS methods. In Table II, the average MAPE and IQR are presented.

3) Weekly forecast: The mean average percent for FTS methods applied to weekly load forecast are presented in Figure 5. In this case, FTS approaches have presented error rate of less than 7%, with better results for Transformation FTS in this case. These algorithms have performed better for weekends forecasting. The results for Interval FTS were omitted in the figure because of its poor performance in the analysis, as presented in Table III, where the average MAPE and IQR are presented.

4) Statistical Relevance and Discussion: For all approaches, the Wilcoxon rank sum test with 5% significance level test was applied. In hourly tests, results of these tests presented statistical differences between models, except in the case of Weighted and Exponential FTS models, which presented p-value of 0.618, with no rejection of null hypothesis. In daily and weekly analysis, however, tests pointed that FTS results were statistically different, with null hypothesis

TABLE III
AVERAGE MAPE AND IQR OF FTS APPROACHES APPLIED TO WEEKLY LOAD FORECASTING.

Method	MAPE	IQR
Weighted	5.3694	5.4577
Exponential	4.6480	4.6977
Transformation	2.1187	1.8886
High-order	4.2228	4.3423
Interval	9.1721	9.5512

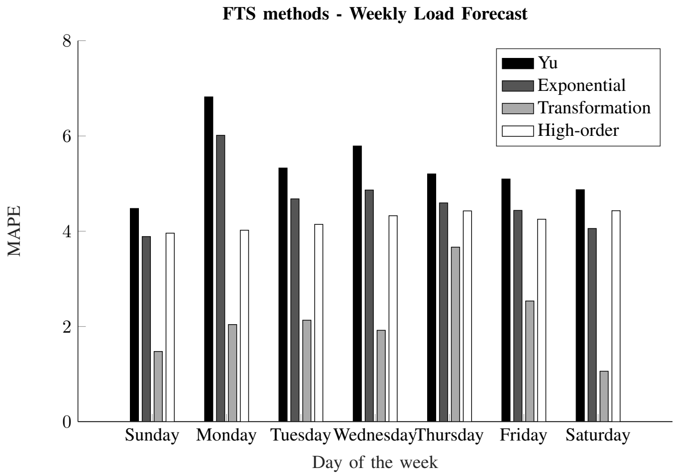

Fig. 5. FTS applied to weekly load forecasting, according to the weekday.

rejection in all cases.

In general, results can be considered as promising. However, these algorithms were used without any additional procedure beside the ones presented in this paper, and no optimization was performed for partitions and intervals definitions as well. This may explain the performance of FTS methods being different compared to methods presented in [6], for example. An investigation of these points can be performed in order to evaluate how to improve these results in this sense.

V. Conclusion

In this work, we applied some Advanced FTS algorithms to Short Term Load Forecasting problems according to different periods of time. The presented results, it is possible to verify that Transformation FTS algorithm offered best results to fit demand forecasting for the hourly and weekly case. The use of FTS algorithms to predict load forecasting has the following points to be taken into account:

- the mathematical and statistics background in FTS is much more simple and easy to understand and implement than other forecasting methods;
- load forecasting may present high accuracy in FTS for very short term analysis, in daily analysis, results were very different, possibly due to seasonal factors related to hourly events, such as peak demands and unusual demand in some hours;
- even considering these results, the analysis performed in the Polish Electric System Data was considerably different, since most methods evaluated have pattern-similarity features.

For future improvements of this work, some other features can be considered, such as the optimal intervals and partitions evaluation, integration with the pattern similarity-based model, multiple points forecasting, multivariate models, and the incorporation of adaptive features in FTS methods. In order to provide better performances, these methods can also be combined to evolutionary algorithms, such as Genetic Algorithms, (GA), Differential Evolution (DE) or Particle Swarm Optimization (PSO) in some extensions: finding the best sequence of former states or finding the optimum length of the intervals.

Acknowledgment

The authors acknowledge the support of CNPq, FAPEMIG and CAPES in this study.

References

[1] Q. Song and B. S. Chissom, *Fuzzy time series and its models*, Fuzzy Sets and Systems vol. 54, no. 3 (1993), pp. 269–277. North-Holland. Elsevier.
[2] G. Gross, F. D. Galiana, *Short-term load forecasting*, Proceedings of the IEEE , vol.75, no.12 (1987), pp.1558–1573.
[3] A. K. Srivastava, A. S. Pandey, D. Singh, *Short-term load forecasting methods: A review*, 2016 International Conference on Emerging Trends in Electrical Electronics & Sustainable Energy Systems (ICETEESES), Sultanpur, 2016, pp. 130-138.
[4] M. Q. Raza, A. Khosravi, *A review on artificial intelligence based load demand forecasting techniques for smart grid and buildings*, Renewable and Sustainable Energy Reviews, vol. 50 (2015), pp. 1352–1372.
[5] G. Dudek, *Pattern similarity-based methods for short-term load forecasting - Part 1: Principles*, Appl. Soft Comput., vol. 37 (2015), Pages 277–287.
[6] G. Dudek, *Pattern similarity-based methods for short-term load forecasting - Part 2: Models*, Appl. Soft Comput., vol. 36 (2015), Pages 422–441.
[7] G. Dudek, *Neural networks for pattern-based short-term load forecasting: A comparative study*, Neurocomputing, vol. 205 (2016), pp. 64–74.
[8] G. Dudek, *Pattern-based local linear regression models for short-term load forecasting*, Electric Power Systems Research, vol. 130 (2016), Pages 139-147.
[9] S. Li, L. Goel, P. Wang, *An ensemble approach for short-term load forecasting by extreme learning machine*, Appl. Energy, Vol. 170 (2016), pp. 22-29.
[10] C. Antunes, A. L. Oliveira, *Temporal data mining: an overview*, KDD Workshop on Temporal Data Mining (2001), pp. 113.
[11] T.-C. Fu, *A review on time series data mining*, Eng. Appl. of Artif. Intel., vol. 24, no. 1 (2011), pp. 164-181.
[12] Singh, P. *A brief review of modeling approaches based on fuzzy time series*, Int. J. Mach. Learn. & Cyber. vol. 8, no. 2 (2017), pp. 397–420.
[13] Z. Hu, Y. Bao, T. Xiong, *Electricity Load Forecasting Using Support Vector Regression with Memetic Algorithms*, The Scientific World Journal (2013), Article ID 292575, 10 pages.
[14] S.-M. Chen, *Forecasting enrollments based on fuzzy time series*, Fuzzy Sets and Systems, vol. 81, no. 3, 1996, pp. 311–319.
[15] H.–K. Yu, *Weighted fuzzy time series models for TAIEX forecasting*, Physica A: Statistical Mechanics and its Applications, vol. 349, no. 3, 2005, pp. 609–624.
[16] K. Huarng, *Effective lengths of intervals to improve forecasting in fuzzy time series*, Fuzzy Sets and Systems, vol. 123, no. 3, pp. 387–394, 2001.
[17] R. Efendi, Z. Ismail, M. M. Deris, *Improved Weight Fuzzy Time Series As Used In The Exchange Rates Forecasting Of Us Dollar To Ringgit Malaysia*, International Journal of Computational Intelligence and Applications vol. 12, no. 1, article ID: 1350005, 19 pages, 2013.
[18] F. M. Talarposhti, H. J. Sadaei, R. Enayatifar, F. G. Guimarães, M. Mahmud, T. Eslami, *Stock market forecasting by using a hybrid model of exponential fuzzy time series*, International Journal of Approximate Reasoning, vol. 70, 2016, pp. 79-98.
[19] M. H. Lee, H. J. Sadaei, Suhartono, *Improving TAIEX forecasting using fuzzy time series with Box–Cox power transformation*, Journal of Applied Statistics vol. 40 , no. 11, 16 pages, 2013.
[20] S.-M. Chen, *Forecasting Enrollments Based On High-Order Fuzzy Time Series*, Cybernetics and Systems vol. 33 , no. 1, pp. 1-16, 2002.
[21] P. C. L. Silva, H. J. Sadaei, F. G. Guimarães, *Interval forecasting with Fuzzy Time Series*, 2016 IEEE Symposium Series on Computational Intelligence (SSCI), Athens, 2016, pp. 1-8.