VI

$$\begin{tikzcd} z \arrow[d, "p(x|z)"'] \\ x \arrow[u, bend right, "q_\phi(z|x)"'] \end{tikzcd}$$

$$L = \log p(x) = \log \int \frac{q(z|x)}{q(z|x)} \, p(x|z) \cdot p(z) \cdot dz$$

$$\geq -KL\left(q(z|x) \,\|\, p(z)\right) + \langle \log p(x|z) \rangle_{q(z|x)}$$

Need $\dfrac{dL}{d\phi} = -\dfrac{dKL}{d\phi} + \dfrac{dL_2}{d\phi}$; $\dfrac{dL_2}{d\phi} = \dfrac{d}{d\phi} \int q_\phi(z|x) \log p(x|z) \, dz$

1. Log derivative trick $\quad \dfrac{dL_2}{d\phi} = \int q_\phi(z|x) \cdot \dfrac{d}{d\phi} \log q_\phi(z|x) \log p(x|z) \, dz$

2. Reparameterisation trick $\quad q_\phi(z|x) = N(z; \mu, \sigma^2)$ or $z = \mu + \sigma\varepsilon$

$$\frac{dL_2}{d\phi} = \int N(\varepsilon; 0, 1) \, \nabla_\phi \log p(x | \mu + \sigma\varepsilon)$$

$q_\phi(z|x) = N(z; \mu, \Sigma) \xrightarrow{?}$ heavy tailed / multimodal

→ Mixture : does not work well

→ This paper : $z_0 \xrightarrow{f_1} z_1 \xrightarrow{f_2} z_2 \cdots \xrightarrow{f_K} z_K$ f_i : smooth + invertible

$$\therefore \log q_K(z_K) = \log q_0(z_0) - \sum_{k=1}^{K} \log \det \left| \frac{\partial f_k}{\partial z_k} \right|$$

Example: $f(z) = z + u h(w^T z + b)$

$$\det \left| \frac{\partial f}{\partial z} \right| = \left| 1 + u^T \psi(z) \right| \quad \text{where} \quad \psi(z) = h'(w^T z + b) \cdot w$$

and $$\log q_K(z_K) = \log q_0(z_0) - \sum_{k=1}^{K} \log \left| 1 + u_k^T + \psi_k(z_k) \right|$$

$$\text{ELBO} = E_{q_K(z_K)} \left[\log p(x, z_K) - \log q_K(z_K) \right]$$

$$= -E_{q_0(z_0)} \left[\log q_0(z_0) \right] + E_{q_0(z_0)} \left[\sum_{k=1}^{K} \log \left| 1 + u_k^T \psi_k(z_k) \right| \right] + E_{q_0(z_0)} \left[\log p(x, z_K) \right]$$