binary observations

Bayesian logistic regression

$$p(\beta) = N(\beta; b, B)$$

$$p(y_i \mid x_i, \beta) = \frac{e^{(x_i^T \beta) y_i}}{1 + e^{(x_i^T \beta)}}$$

$$\Downarrow \text{ augment } w_i \sim PG$$

$$p(w_i \mid \beta) = PG(w_i; 1, x_i^T \beta)$$

$$p(\beta) = N(\beta; b, B)$$

$$p(y_i \mid x_i, \beta, w_i) \propto \exp\left(K_i x_i^T \beta - \frac{w_i (x_i^T \beta)^2}{2} \right)$$

$$K_i = y_i - \frac{1}{2}$$

GP Classification

$$p(f) = GP(f; 0, k(\cdot, \cdot))$$

$$p(y_i \mid x_i, f) = \frac{e^{f(x_i) \cdot y_i}}{1 + e^{f(x_i)}}$$

$$\Downarrow \text{ augment } w_i \sim PG$$

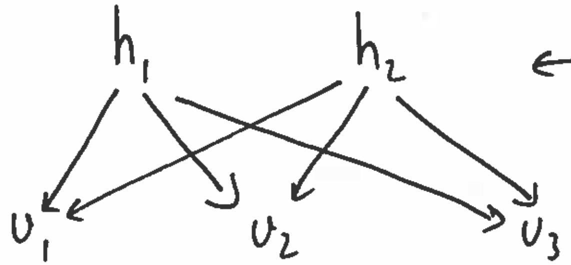
$$p(w_i \mid f) = PG(w_i; 1, f(x_i))$$

$$p(f) = GP(f; 0, k(\cdot, \cdot))$$

$$p(y_i \mid x_i, w_i, f) \propto \exp\left(K_i f(x_i) - \frac{w_i (f(x_i))^2}{2} \right)$$

$$K_i = y_i - \frac{1}{2}$$

$\boxed{\text{SBNs}}$



← binary hidden units/variables : $h_n \in \{0,1\}^Q$

← binary observations $v_n \in \{0,1\}^D$

$n = 1:N$, $W \in R^{D \times Q}$

$$p(h_{nq} = 1 \mid b_q) = \sigma(b_q)$$

$$p(v_{nd} = 1 \mid h_n, w_d, c_d) = \sigma(w_d^T \cdot h_n + c_d)$$

$$p(b_q) = N(b_q;\ 0,\ \theta_b)$$

$$p(c_d) = N(c_q;\ 0,\ \theta_c)$$

$$p(W_d) = N(w_d;\ 0,\ \mathrm{diag}(\theta_w))$$

Augment PG variables $\gamma_h \in \mathbb{R}^Q$ and $\gamma_v \in R^{N \times D}$

Similar to BayesLgReg :

$$p(\gamma_{h,q} \mid -) = PG(\gamma_{h,q} ; 1, b_q)$$

$$p(\gamma_{v,nd} \mid -) = PG(\gamma_{v,nd} ; 1, w_d^T h_n + c_d)$$

$$p(b_q) = N(b_q ; 0, \theta_b)$$

$$p(h_{nq} \mid b_q, \gamma_{hq}) \propto \exp\left(K_h \cdot b_q - \frac{\gamma_{hq} b_q^2}{2} \right)$$

$$\Rightarrow p(b_q \mid -) = N(b_q ; \dots)$$

$$p(w_d) = N(w_d ; 0, \operatorname{diag}(\theta_w))$$

$$p(v_{nd} \mid w_d, \gamma_{vnd}, -) \propto \exp\left(K_v \cdot w_d^T \cdot h_n - \gamma_{vnd} \cdot \frac{(w_d^T h_n)^2}{2} \right)$$

$$\Rightarrow p(w_d \mid -) = N(w_d ; \dots)$$