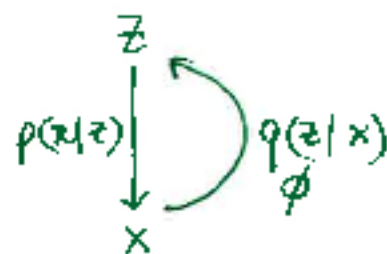


VI

①



$$L = \log p(x) = \log \int \frac{q(z|x)}{q(z|x)} p(x|z) \cdot p(z) \cdot dz$$

$$\geq -KL(q(z|x) || p(z)) + \langle \log p(x|z) \rangle_{q(z|x)}$$

Need  $\frac{dL}{d\phi} = - \frac{dKL}{d\phi} + \frac{dL_2}{d\phi}$  ;  $\frac{dL_2}{d\phi} = \frac{d}{d\phi} \int q_{\phi}(z|x) \log p(x|z) dz$

① Log derivative trick  $\frac{dL_2}{d\phi} = \int q_{\phi}(z|x) \cdot \frac{d}{d\phi} \log q_{\phi}(z|x) \log p(x|z) \cdot dz$

② Reparameterization trick  $q_{\phi}(z|x) = N(z; \mu, \sigma^2)$  or  $z = \mu + \sigma \epsilon$

$$\frac{dL_2}{d\phi} = \int N(\epsilon; 0, 1) \nabla_{\phi} \log p(x | \mu + \sigma \epsilon)$$

$$q_\phi(z|x) = N(z; \mu, \Sigma) \xrightarrow{?} \text{heavy tailed / multimodal} \quad (2)$$

→ Mixture : does not work well

→ This paper :

$$z_0 \xrightarrow{f_1} z_1 \xrightarrow{f_2} z_2 \dots \xrightarrow{f_k} z_k \quad f_i : \text{smooth + invertible}$$

$$\therefore \log q_k(z_k) = \log q_0(z_0) - \sum_{h=1}^k \log \det \left| \frac{\partial f_h}{\partial z_h} \right|$$

Example:  $f(z) = z + u h(w^T z + b)$

$$\det \left| \frac{\partial f}{\partial z} \right| = |1 + u^T \psi(z)| \quad \text{where } \psi(z) = h'(w^T z + b) \cdot w$$

$$\text{and } \log q_k(z_k) = \log q_0(z_0) - \sum_{h=1}^k \log |1 + u_h^T \psi_h(z_h)|$$

$$\begin{aligned} \text{ELBO} &= E_{q_k(z_k)} [\log p(x, z_k) - \log q_k(z_k)] \\ &= -E_{q_0(z_0)} [\log q_0(z_0)] + E_{q_0(z_0)} \left[ \sum_{h=1}^k \log |1 + u_h^T \psi_h(z_h)| \right] + E_{q_0(z_0)} [\log p(x, z)] \end{aligned}$$