

COMP3400

Assignment 3 Written

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In addition to this written work there are *three* coding questions. The written work is worth 25 points and the coding questions are worth 60 points totalling 85 points.

Either

The **Functor and Applicative for the Either data-type** is as follows:

```
instance Functor (Either a) where
  fmap f (Right x) = Right (f x)
  fmap f (Left x)  = Left x

instance Applicative (Either e) where
  pure          = Right
  Left e  <*> _ = Left e
  Right f <*> r = fmap f r
```

Question 1. [4 MARKS]

Show Either satisfies the *second* functor law:

$$\text{fmap } (g \ . \ h) = \text{fmap } g \ . \ \text{fmap } h$$

Question 2. [6 MARKS]

Show Either satisfies the *third* applicative law:

$$x \ <*> \ \text{pure } y = \text{pure } (\backslash g \ -> \ g \ y) \ <*> \ x$$

ZipWith

Recall the alternate definition for a list applicative given in tutorial 8.

```
1 instance Functor [] where
2     fmap _ [] = []
3     fmap g (x:xs) = g x : (fmap g xs)
4
5 instance Applicative [] where
6     pure f = repeat f
7     [] <*> _ = []
8     _ <*> [] = []
9     (f:fs) <*> (x:xs) = (f x) : (fs <*> xs)
```

When writing your proofs use the line numbers given above when justifying your steps.

Question 3. [7 MARKS]

Show your Applicative satisfies the *second* applicative law:

$$\text{pure } (g \ x) = \text{pure } g \ \<*> \ \text{pure } x$$

Question 4. [8 MARKS]

Show your Applicative satisfies the *forth* applicative law:

$$x \ \<*> \ (y \ \<*> \ z) = (\text{pure } (.) \ \<*> \ x \ \<*> \ y) \ \<*> \ z$$