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Lambda Calculus

Question 1:

Not = \a.(a (False) (True))

= \a.(a (\x(\y.y)) (\x(\y.x)))

Question 2:

Lambda expession for ¬(¬p ∧ ¬q) when let a = p and b = q is:
NOT (NOT p AND NOT q)

Write this expression in prefix position, we have:
NOT (AND (NOT p NOT q))
= \a.(a (False) (True)) (\a.(\b.(ab)a) (\a.(a (False) (True)) p \a.(a (False) (True)) q))
= (a (False) (True)) [a := (\a.(\b.(ab)a) (\a.(a (False) (True)) p \a.(a (False) (True)) q))]
= (\a.(\b.(ab)a) (\a.(a (False) (True)) p \a.(a (False) (True)) q)) (False) (True)
= (\a.(\b.(ab)a) (\a.(a (False) (True)) p \a.(a (False) (True)) q)) (False) (True)
= (\a.(\b.(ab)a) ((a (False) (True))[a:= p] (a (False) (True)) [a:=q])) (False) (True)
= (\a.(\b.(ab)a) ((p (False) (True)) (q (False) (True)))) (False) (True)
= ((\b.(ab)a) [a = (p (False) (True))] (q (False) (True))) (False) (True)
= (\b.((p (False) (True))b)(p (False) (True)) (q (False) (True))) (False) (True)
= (((p (False) (True))b)(p (False) (True)) [b:= (q (False) (True))]) (False) (True)
= (((p(False)(True)) (q(False)(True))) (p(False)(True))) (False) (True)

Let this be (1)

We also have (2):

- True True True
  = (\x.(\y.x)) True True
  = x [x := True] [y := True]
  = True
- False False False
  = (\x.(\y.y)) False False
  = y [x := False] [y := False]
  = False
- False True False
  = (\x.(\y.y)) True False
  = y [x := True] [y := False]
  = False
- False False True
  = (\x.(\y.y)) False True
  = y [x := False] [y := True]
  = True

From (1) there are 4 cases of p and q which is simpler to solve by substitute (2) in:

Case 1: p = False and q = False
(((False(False)(True)) (False(False)(True))) (False(False)(True))) (False) (True)
= ((True) (True) (True)) (False) (True)
= False

Case 2: p = True and q = False
= (((True(False)(True)) (False(False)(True))) (True(False)(True))) (False) (True)
= ((False) (True) (False)) (False) (True)
= True

Case 3: p = False and q = True
= (((False(False)(True)) (True(False)(True))) (False(False)(True))) (False) (True)
= ((True) (False) (True)) (False) (True)

= True

Case 4: p = True and q = True
= (((True(False)(True)) (True(False)(True))) (True(False)(True))) (False) (True)
= ((False) (False) (False)) (False) (True)
= True

In summary, by grouping these cases and comparing them to OR ($p \lor q$) as the truth table below, we can make the conclusion that both of them are equivalent.

p	q	$\neg(\neg p \land \neg q)$	$(p \lor q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

Question 3:

= (\xy.x)(\abc.cab)z(\z.zz)
= (\xy.x)[x:=\abc.cab]z(\z.zz)
= (\y.(\abc.cab))z(\z.zz)
= ((\abc.cab))[y:=z](\z.zz)
= (\abc.cab)(\z.zz)
= (\bc.cab)[a:=\z.zz]
= \bc.(c(\z.zz)b)

Principle Types

(These questions are coded in file PrincipalType.hs)

Blockus (Question 10)

We can use The Principle of Mathematical Induction theorem to prove a $2^n \times 2^n$ Blockus board with northwest corner removed can be covered with V3 pieces (3 blocks).

Logically, by removing 1 block out of $2^n \times 2^n$ Blockus board we can have:

$$2^n \times 2^n - 1\ (blocks)$$

Moreover, to make sure that all V3 pieces can cover the whole board, we need the board is not contained a straight line of at least 2 blocks so that it can fit at least 1 V3 piece (fortunately, $2^n \times 2^n$ is always a square so that not necessary) and the number of remaining blocks is divisible by 3:

$$(2^n \times 2^n - 1)\ \vdots\ 3\ (blocks)$$

Let $P(n) \leftrightarrow (2^n \times 2^n - 1)\ \vdots\ 3$

Where base case is:

$$P(0) \leftrightarrow (2^0 \times 2^0 - 1)\ \vdots\ 3 \leftrightarrow 0\ \vdots\ 3 \leftrightarrow T.$$

Induction Hypothesis:

Assuming that $P(n) = T$ in $P(n) \to P(n+1)$ and showing $P(n+1)$ is a consequence of $P(n)$:

$$P(n+1) \leftrightarrow (2^{n+1} \times 2^{n+1} - 1) \vdots 3$$

Starting with the left-hand side of the expression:

$$((2^n \times 2^n) \times (2^1 \times 2^1) - 1)$$

$$= 4 \times (2^n \times 2^n) - 1\ (1)$$

Using the induction hypothesis, we already know that $(2^n \times 2^n - 1)$ is divisible by 3. Therefore, the expression can be written as:

$(2^n \times 2^n) - 1 = 3 \times m$, where m is an arbitrary integer.

$$\leftrightarrow (2^n \times 2^n) = 3 \times m + 1\ (2)$$

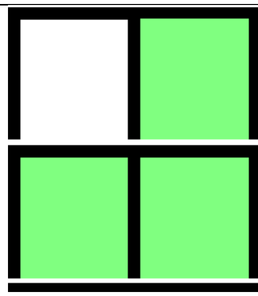
Substituting (2) into the equation (1) gives:

$$= 4 \times (3 \times m + 1) - 1$$

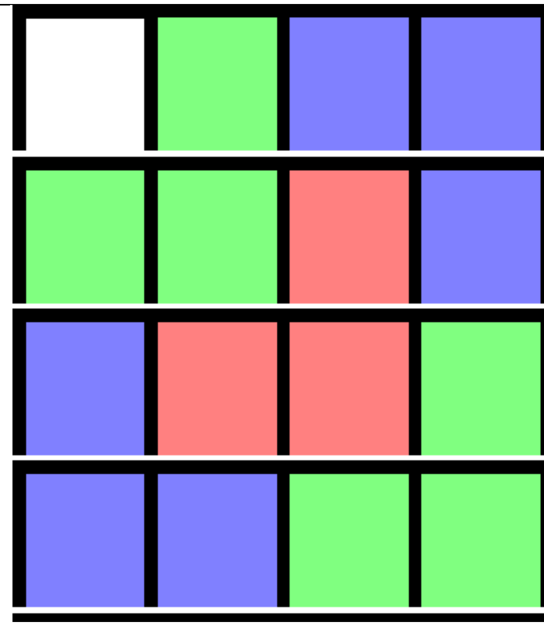
$$= 12 \times m + 3$$

Thus, $(2^{n+1} \times 2^{n+1} - 1)$ is equal to an integer m multiply by 3 which also divisible by 3.

For instance,



From n = 1



To n +1, which n = 2

By the principle of mathematical induction, we have proved that $(2^n \times 2^n - 1)$ is divisible by 3 for all positive integers n. Finally, there is always true that there is exist at least one order of V3 piece(s) that the square $2^n \times 2^n$ Blockus board with northwest corner removed can be covered with it.