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# **Millikelvin Confocal Microscopy of Semiconductor Membranes and Filter Functions for Unital Quantum Operations**

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The harmony of the world is made manifest in Form and Number, and the heart and soul and all the poetry of Natural Philosophy are embodied in the concept of mathematical beauty.

– D'Arcy Wentworth Thompson

# Contents

Contents	iii
<b>I A FLEXIBLE PYTHON TOOL FOR FOURIER-TRANSFORM NOISE SPECTROSCOPY</b>	<b>1</b>
<b>II CHARACTERIZATION AND IMPROVEMENTS OF A MILLIKELVIN CONFOCAL MICROSCOPE</b>	<b>2</b>
<b>III OPTICAL MEASUREMENTS OF ELECTROSTATIC EXCITON TRAPS IN SEMICONDUCTOR MEMBRANES</b>	<b>3</b>
1 Introduction	4
2 Observations	5
2.1 Transfer-matrix method simulations of the membrane structure . . . . .	5
3 Conclusion & outlook	8
<b>IV A FILTER-FUNCTION FORMALISM FOR UNITAL QUANTUM OPERATIONS</b>	<b>9</b>
<b>APPENDIX</b>	<b>10</b>
A Additional TMM simulations	11
A.1 Dependence on epoxy thickness . . . . .	11
A.2 Optimization of the barrier thickness . . . . .	11
Bibliography	12
List of Terms	13

# List of Figures

2.1	Generated by <code>img/py/experiment/tmm.py</code> . . . . .	7
2.2	Generated by <code>img/py/experiment/tmm.py</code> . . . . .	7
2.3	Generated by <code>img/py/experiment/tmm.py</code> . . . . .	7
2.4	Generated by <code>img/py/experiment/tmm.py</code> . . . . .	7
A.1	Generated by <code>img/py/experiment/tmm.py</code> . . . . .	11
A.2	Generated by <code>img/py/experiment/tmm.py</code> . . . . .	11

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# Software

The following open-source software packages were developed (at least partially) during the work on this thesis.

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**Part I**

**A FLEXIBLE PYTHON TOOL FOR  
FOURIER-TRANSFORM NOISE  
SPECTROSCOPY**



**Part II**

**CHARACTERIZATION AND  
IMPROVEMENTS OF A MILLIKELVIN  
CONFOCAL MICROSCOPE**

**Part III**

**OPTICAL MEASUREMENTS OF  
ELECTROSTATIC EXCITON TRAPS IN  
SEMICONDUCTOR MEMBRANES**

# Introduction

1



## 2.1 Transfer-matrix method simulations of the membrane structure

The transfer-matrix method (TMM) is a computationally efficient method of obtaining the electric field in layered structures. In this section, I perform simulations of the heterostructure membranes investigated in this part of the present thesis using the PyMoosh package [1] to elucidate the observed quenching of photoluminescence (PL) when illuminating gate electrodes as well as the overall optical efficiency.<sup>1</sup> I will first briefly recap the simulation method following Reference 1. For more details, refer to *ibid.* and references therein.

Consider a layered structure along  $z$  with interfaces at  $z_i, i \in \{0, 1, \dots, N+1\}$  that is translationally invariant along  $x$  and  $y$ . Each layer  $i$  may consist of a different dielectric material characterized by a (complex) relative permittivity  $\epsilon_{r,i}$ .<sup>2</sup> The electric field component along  $y$  of a transverse electric (TE) mode originating in some far away point satisfies the Helmholtz equation

$$\frac{\partial^2 E_y}{\partial z^2} + \gamma_i^2 E_y = 0, \quad (2.1)$$

where  $\gamma_i = \sqrt{\epsilon_{r,i}k_0^2 - k_x^2}$  with  $k_0 = \omega/c$  the wave vector in vacuum and  $k_x$  the component along  $x$ . In layer  $i$  of the structure, the solution to Equation 2.1 may be written as a superposition of plane waves incident and reflected on the lower and upper interfaces,

$$\begin{cases} E_{y,i}(z) = A_i^+ \exp\{i\gamma_i[z - z_i]\} + B_i^+ \exp\{-i\gamma_i[z - z_i]\}, \\ E_{y,i}(z) = A_i^- \exp\{i\gamma_i[z - z_{i+1}]\} + B_i^- \exp\{-i\gamma_i[z - z_{i+1}]\}, \end{cases} \quad (2.2)$$

where the coefficients with superscript  $+$  ( $-$ ) are referenced to the phase at the upper (lower) interface, respectively. Matching these solutions at  $z = z_i$  for all  $i$  to satisfy the interface conditions imposed by Maxwell's equations gives rise to a linear system of equations, the solution to which can be obtained through several different methods.

A particularly simple method is the transfer-matrix method ( $T$ -matrix formalism), which corresponds to writing the interface conditions at  $z = z_i$  as the matrix equation

$$\begin{pmatrix} A_{i+1}^+ \\ B_{i+1}^+ \end{pmatrix} = T_{i,i+1} \begin{pmatrix} A_i^- \\ B_i^- \end{pmatrix} \quad (2.3)$$

with

$$T_{i,i+1} = \frac{1}{2\gamma_{i+1}} \begin{pmatrix} \gamma_i + \gamma_{i+1} & \gamma_i - \gamma_{i+1} \\ \gamma_i - \gamma_{i+1} & \gamma_i + \gamma_{i+1} \end{pmatrix} \quad (2.4)$$

1: Strictly speaking, the term TMM only refers to one of the several formalisms implemented in the PyMoosh package. While fast, it is not the most numerically stable, and other methods may be preferred if wall time is not a limiting issue.

2: We disregard magnetic materials with relative permeability  $\mu_r \neq 1$  for simplicity.

the transfer matrix for interface  $i$ . Connecting the coefficients for adjacent interfaces within a layer of height  $h_i = z_{i+1} - z_i$  requires propagating the phase,

$$\begin{pmatrix} A_i^- \\ B_i^- \end{pmatrix} = C_i \begin{pmatrix} A_i^+ \\ B_i^+ \end{pmatrix}, \quad (2.5)$$

with

$$C_i = \exp \{ \text{diag}(-i\gamma_i h_i, i\gamma_i h_i) \}. \quad (2.6)$$

Iterating Equations 2.4 and 2.6, the total transfer matrix  $T = T_{0,N+1}$  then reduces to the matrix product

$$T = T_{N,N+1} \prod_{i=0}^{N-1} T_{i,i+1} C_i. \quad (2.7)$$

From  $T$ , the reflection and transmission coefficients can be obtained as  $r = A_0^- = -T_{01}/T_{00}$  and  $t = B_{N+1}^+ = rT_{10} + T_{11}$ . Taking the absolute value square of reflection and transmission coefficients then yields the reflectance  $\mathcal{R}$  and the transmittance  $\mathcal{T}$ , which correspond to the fraction of total incident power being reflected and transmitted, respectively. To obtain the absorptance  $\mathcal{A}$ , the fraction of power being absorbed, in layer  $i$ , one can compute the difference of the  $z$ -components of the Poynting vectors (*cf.* ??) at the top of layers  $i$  and  $i+1$ . In the TE case considered here, ?? reduces to

$$S_i = \text{Re} \left[ \frac{\gamma_i^*}{\gamma_0} (A_i^+ - B_i^+) (A_i^+ + B_i^+) \right] \quad (2.8)$$

and is hence straightforward to extract from the calculation of either the  $S$  or  $T$  matrices.

Equation 2.7 is simple to evaluate on a computer, making this method attractive for numerical applications. However, the opposite signs in the argument of the exponentials in Equation 2.6 can lead to instabilities for evanescent waves ( $\gamma_i \in \mathbb{C}$ ) due to finite-precision floating point arithmetic [2]. Rewriting Equation 2.4 to have incoming and outgoing fields on opposite sides of the equality alleviates this issue while sacrificing the simple matrix-multiplication composition rule in what is known as the scattering matrix ( $S$ -matrix) formalism.

Beyond the calculation of the aforementioned coefficients, the TMM formalism also allows to compute the full spatial dependence of the fields. Two cases are implemented in PyMoosh: irradiation of the layered structured with a Gaussian beam rather than plane waves of infinite extent, and a current line source inside the structure. In the first case, the previously assumed translational invariance along  $x$  leading to a plane-wave spatial dependence is replaced by a superposition of plane waves weighted with a normally distributed amplitude,<sup>3</sup>

$$E_{y,i}(x) = \exp(ik_x x) \rightarrow \int \frac{dk_x}{2\pi} \mathcal{E}_0(k_x) \exp(ik_x x), \quad (2.9)$$

with (*cf.* ??)

$$\mathcal{E}_0(k_x) = \frac{w_0}{2\sqrt{\pi}} \exp \left\{ -ik_x x_0 - \left[ \frac{w_0 k_x}{2} \right]^2 \right\}, \quad (2.10)$$

3: *I.e.*, the inverse Fourier transform of  $\mathcal{E}_0(k_x)$ .

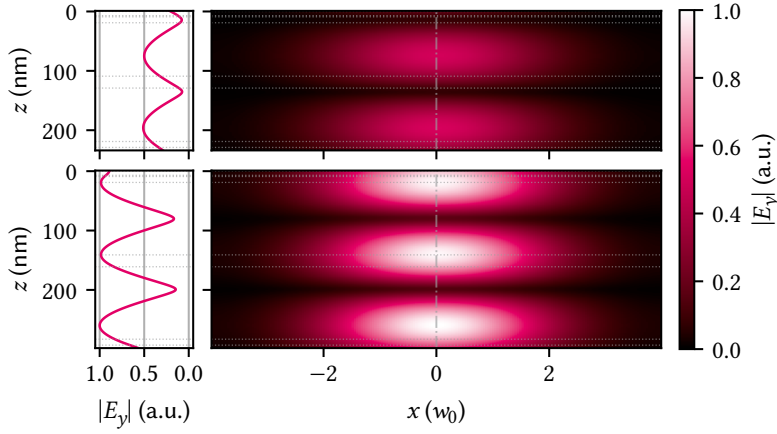


Figure 2.1

so that

$$E_{y,i}(x, z) = E_{y,i}(z)E_{y,i}(x) \quad (2.11)$$

with

$$E_{y,i}(z) = A_i^- \exp\{i\gamma_i[z - z_{i+1}]\} + B_i^+ \exp\{-i\gamma_i[z - z_i]\} \quad (2.12)$$

and where we considered only normal incidence for simplicity. In the second case, Langevin et al. [1] consider an AC current  $I$  flowing through a translationally invariant, one-dimensional wire along  $y$  at  $x = x_s$ . This introduces a source term into the Helmholtz equation Equation 2.1 which, upon Fourier transforming in  $x$  direction, leads to

$$\frac{\partial^2 \hat{E}_y}{\partial z^2} + \gamma_i^2 \hat{E}_y = -i\omega\mu_0 I \delta(z) \exp(ik_x x_s). \quad (2.13)$$

The electric field  $\hat{E}_{y,i}(k_x, z)$  is thus proportional to the Green's function of Equation 2.13 and can be obtained using a similar procedure as in the case of a distant source incident on the structure by matching the interface conditions. Performing the inverse Fourier transform by means of Equation 2.9 with constant weights,  $\mathcal{E}_0(k_x) \equiv 1$ , then yields the two-dimensional spatial distribution of the electric field,  $E_{y,i}(x, z)$ .

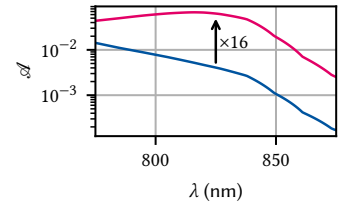


Figure 2.2

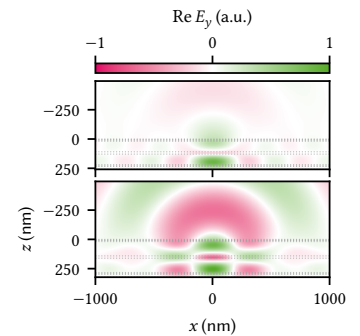


Figure 2.3

## Conclusion & outlook

3



## **Part IV**

# **A FILTER-FUNCTION FORMALISM FOR UNITAL QUANTUM OPERATIONS**



# **APPENDIX**

# Additional TMM simulations



## A.1 Dependence on epoxy thickness

## A.2 Optimization of the barrier thickness

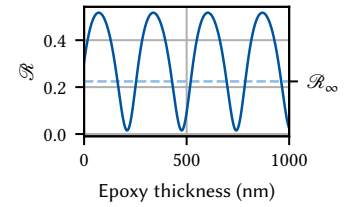


Figure A.1

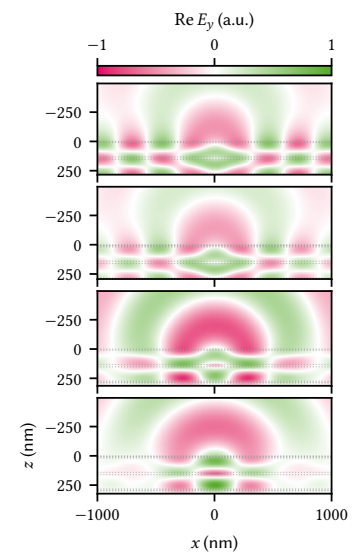


Figure A.2

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# Special Terms

## P

**PL** photoluminescence. 5

## T

**TE** transverse electric. 5, 6

**TMM** transfer-matrix method. 5, 6, 11