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The harmony of the world is made manifest in Form and Number, and the heart and soul and all the poetry of Natural Philosophy are embodied in the concept of mathematical beauty.

– D'Arcy Wentworth Thompson

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A FILTER-FUNCTION FORMALISM FOR QUANTUM OPERATIONS

Filter Functions

1

For a **given** quantum operation $\tilde{\mathcal{U}}$ resulting from the quantum system's evolution under the noise fully characterized by its one-sided power spectral density (PSD) $S(\omega)$, we define the filter function (FF) $\mathcal{F}(\omega; \tau)$ by

$$\tilde{\mathcal{U}}(\tau) = \exp \int \frac{d\omega}{2\pi} \mathcal{F}(\omega; \tau) S(\omega). \quad (1.1)$$

Now, suppose that

$$S_{\omega_i}(\omega) = \sigma_i^2 \delta(\omega - \omega_i), \quad (1.2)$$

that is, the PSD of a monochromatic sinusoid of frequency ω_i and root mean square (RMS) σ_i^2 . Then Equation 1.1 becomes

$$\begin{aligned} \tilde{\mathcal{U}}_{\omega_i}(\tau) &= \exp \left\{ \sigma_i^2 \int \frac{d\omega}{2\pi} \mathcal{F}(\omega; \tau) \delta(\omega - \omega_i) \right\} \\ &= \exp \left\{ \frac{\sigma_i^2}{2\pi} \mathcal{F}(\omega_i; \tau) \right\}, \end{aligned} \quad (1.3)$$

where $\tilde{\mathcal{U}}_{\omega_i}(\tau)$ is the noisy quantum operation generated by monochromatic noise with PSD $S_{\omega_i}(\omega)$ according to Equation 1.2. It is now easy to invert Equation 1.3, and we obtain

$$\mathcal{F}(\omega_i; \tau) = \frac{2\pi}{\sigma_i^2} \log \tilde{\mathcal{U}}_{\omega_i}(\tau). \quad (1.4)$$

Because we represent quantum operations as matrices in Liouville space, Equation 1.4 is easy to implement on a computer; we simply need to evaluate $\tilde{\mathcal{U}}_{\omega_i}(\tau)$ for a set of discrete frequencies $\{\omega_i\}_i$ using Monte Carlo (MC) and take the logarithm! [1].

Indeed, we can go a step further and split apart the coherent and incoherent contributions to the noisy evolution. Since (in-)coherent quantum operations are represented by (anti-)symmetric matrices in Liouville space, we may define the incoherent and coherent FFs by

$$\begin{aligned} \mathcal{F}_T(\omega; \tau) &= \frac{1}{2} (\mathcal{F}(\omega; \tau) + \mathcal{F}(\omega; \tau)^T) \\ &= \frac{\pi}{\sigma_i^2} (\log \tilde{\mathcal{U}}_{\omega_i}(\tau) + \log \tilde{\mathcal{U}}_{\omega_i}(\tau)^T), \end{aligned} \quad (1.5)$$

and

$$\begin{aligned} \mathcal{F}_\Delta(\omega; \tau) &= \frac{1}{2} (\mathcal{F}(\omega; \tau) - \mathcal{F}(\omega; \tau)^T) \\ &= \frac{\pi}{\sigma_i^2} (\log \tilde{\mathcal{U}}_{\omega_i}(\tau) - \log \tilde{\mathcal{U}}_{\omega_i}(\tau)^T), \end{aligned} \quad (1.6)$$

respectively. From a MC point of view, $\tilde{\mathcal{U}}$ is given by

$$\tilde{\mathcal{U}} \equiv \langle \tilde{\mathcal{U}}_{\omega_i} \rangle(\tau) = \mathcal{Q}^T \langle \mathcal{U}_{\omega_i}(\tau) \rangle, \quad (1.7)$$

where $\mathcal{U}_{\omega_i}(\tau)$ is the solution of the Schrödinger equation for a single realization of the noise and $\langle \cdot \rangle$ denotes the ensemble average. Solving Equa-

Equation 1.2 discretizes $S(\omega)$ by sampling it at points ω_i , i.e.,

$$S(\omega) = \lim_{n \rightarrow \infty} \sum_{i=1}^n S_{\omega_i}(\omega).$$

[1]: Geck (2021), *Scalable control electronics for a spin based quantum computer*

For N realizations of the stochastic process underlying $b(t)$, the ensemble average of a quantity $A(t)$ that is a function of $b(t)$ is given by

$$\langle A \rangle(t) = \frac{1}{N} \sum_{i=1}^N A_i(t)$$

where i enumerates the realizations of

tion 1.1 for $\mathcal{F}(\omega; \tau)$, we find

$$\begin{aligned}\mathcal{F}(\omega; \tau) &= \int \frac{d\omega'}{2\pi} \delta(\omega - \omega') \log \langle \tilde{\mathcal{U}}_{\omega_i}(\tau) \rangle \\ &= \mathcal{Q}^\top + \int \frac{d\omega'}{2\pi} \delta(\omega - \omega') \log \langle \mathcal{U}_{\omega_i}(\tau) \rangle.\end{aligned}\quad (1.8)$$

So what does a single noise realization of Equation 1.2 in the time domain look like? It's a sinusoid with amplitude $A_i \sim \mathcal{N}(0, \sigma_i^2)$, frequency ω_i , and phase $\phi \sim \mathcal{U}(0, 2\pi)$,

$$b(t) = A \sin(\omega_i t + \phi). \quad (1.9)$$

We can compute the expectation value of $\mathcal{U}_{\omega_i}(t)$ over A_i and ϕ by integrating over the probability distributions,

$$\mathbb{E}_{\sigma_i, \phi}[\mathcal{U}](t) = \int dx \rho_{\sigma_i}(x) \int_0^{2\pi} d\phi \mathcal{U}[x, \phi]. \quad (1.10)$$

In practice, Equation 1.10 will be evaluated numerically by dividing the integration intervals into sub-intervals of equal size and weighing the integrand with the probability density, approximated to be constant on the sub-interval.

We write the expectation value of an observable A with respect to a random variable X with the probability density function ρ_X as $\mathbb{E}_X[A]$.

The probability density function of the zero-mean normal distribution is

$$\rho_{\sigma_i}(x) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{x^2}{2\sigma_i^2}\right).$$

APPENDIX

Test

A

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Bibliography

Here are the references in citation order.

- [1] Lotte Geck. “Scalable control electronics for a spin based quantum computer.” ISBN: 9783958065406 Number: RWTH-2021-04930. PhD thesis. Jülich: Forschungszentrum Jülich GmbH, 2021. (Visited on 11/18/2024) (cited on page 2).