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November 27, 2024

* A LaTeX lover/hater

The harmony of the world is made manifest in Form and Number, and the heart and soul and all the poetry of Natural Philosophy are embodied in the concept of mathematical beauty.

– D’Arcy Wentworth Thompson

Contents

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Publications

This might come in handy for PhD theses: some ideas and figures have appeared previously in the following publications:

[1] Yaiza Aragonés-Soria et al. "Minimising Statistical Errors in Calibration of Quantum-Gate Sets." June 7, 2022. DOI: 10.48550/arXiv.2206.03417. (Visited on 06/08/2022). Pre-published.

[2] Pascal Cerfontaine, Tobias Hangleiter, and Hendrik Bluhm. "Filter Functions for Quantum Processes under Correlated Noise." In: *Physical Review Letters* 127.17 (Oct. 18, 2021), p. 170403. DOI: 10.1103/PhysRevLett.127.170403.

[3] Thomas Descamps et al. "Semiconductor Membranes for Electrostatic Exciton Trapping in Optically Addressable Quantum Transport Devices." In: *Phys. Rev. Appl.* 19.4 (Apr. 28, 2023), p. 044095. DOI: 10.1103/PhysRevApplied.19.044095. (Visited on 04/28/2023).

[4] Thomas Descamps et al. "Millikelvin Confocal Microscope with Free-Space Access and High-Frequency Electrical Control." In: *Review of Scientific Instruments* 95.8 (Aug. 9, 2024), p. 083706. DOI: 10.1063/5.0200889. (Visited on 08/12/2024).

[5] Sarah Fleitmann et al. "Noise Reduction Methods for Charge Stability Diagrams of Double Quantum Dots." In: *IEEE Transactions on Quantum Engineering* 3 (2022), pp. 1–19. DOI: 10.1109/TQE.2022.3165968.

[6] Fabian Hader et al. "On Noise-Sensitive Automatic Tuning of Gate-Defined Sensor Dots." In: *IEEE Transactions on Quantum Engineering* 4 (2023), pp. 1–18. DOI: 10.1109/TQE.2023.3255743.

[7] Tobias Hangleiter, Pascal Cerfontaine, and Hendrik Bluhm. "Filter-Function Formalism and Software Package to Compute Quantum Processes of Gate Sequences for Classical Non-Markovian Noise." In: *Phys. Rev. Research* 3.4 (Oct. 18, 2021), p. 043047. DOI: 10.1103/PhysRevResearch.3.043047. (Visited on 01/19/2022).

[8] Tobias Hangleiter, Pascal Cerfontaine, and Hendrik Bluhm. "Erratum: Filter-function Formalism and Software Package to Compute Quantum Processes of Gate Sequences for Classical Non-Markovian Noise [Phys. Rev. Research 3, 043047 (2021)]." In: *Phys. Rev. Res.* 6.4 (Oct. 16, 2024), p. 049001. DOI: 10.1103/PhysRevResearch.6.049001. (Visited on 10/16/2024).

[9] Isabel Nha Minh Le et al. "Analytic Filter-Function Derivatives for Quantum Optimal Control." In: *Phys. Rev. Applied* 17.2 (Feb. 2, 2022), p. 024006. DOI: 10.1103/PhysRevApplied.17.024006. (Visited on 02/03/2022).

[10] Kui Wu et al. "Modeling an Efficient Singlet-Triplet-Spin-Qubit-to-Photon Interface Assisted by a Photonic Crystal Cavity." In: *Phys. Rev. Appl.* 21.5 (May 24, 2024), p. 054052. DOI: 10.1103/PhysRevApplied.21.054052. (Visited on 08/21/2024).

Attention: This requires a separate run of `bibtex` for your `refsection`, *e.g.*, `ClassicThesis1-blx` for this file. You might also use `biber` as the backend for `biblatex`. See also http://tex.stackexchange.com/questions/128196/problem-with-refsection.

Software

The following open-source software packages were developed (at least partially) during the work on this thesis.

[1] Tobias Hangleiter, Isabel Nha Minh Le, and Julian D. Teske. *Filter_functions*. Version v1.1.3. Zenodo, May 14, 2024. DOI: 10.5281/ZENODO.4575000. (Visited on 09/26/2024).

[2] Tobias Hangleiter et al. *Python-Spectrometer*. Version 2024.11.1. Zenodo, Nov. 21, 2024. DOI: 10.5281/zenodo.14198682. (Visited on 11/21/2024).

[3] Tobias Hangleiter et al. *Qutil*. Version 2024.11.1. Zenodo, Nov. 21, 2024. DOI: 10.5281/zenodo.14200666. (Visited on 11/21/2024).

A Filter-Function Formalism For Quantum Operations

Monte Carlo and Lindblad equation simulations 1

1.1 Validation of QFT fidelities

In this section, we perform Lindblad master equation and Monte Carlo simulations to verify the fidelities predicted for the QFT circuit in the main text. We focus on noise exclusively on the third qubit, entering through the noise operator $B_\alpha \equiv \sigma_y^{(3)}$.

To validate the fidelity for white noise, we use a Lindblad master equation [1, 2] in superoperator form. We represent linear maps $\mathscr{A} : \rho \to \mathscr{A}(\rho)$ by matrices in the Pauli transfer matrix representation as (see Ref. 3 for more details)

$$\mathscr{A}_{ij} \coloneqq \mathrm{tr}\big(\sigma_i \mathscr{A}(\sigma_j)\big) \tag{1.1}$$

and operators as column vectors (*i.e.* generalized Bloch vectors) as

$$\rho_i \coloneqq \mathrm{tr}(\sigma_i \rho), \tag{1.2}$$

allowing us to write the Lindblad equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = -\mathrm{i}[H(t), \rho(t)] + \sum_\alpha \gamma_\alpha \left(L_\alpha \rho(t) L_\alpha^\dagger - \frac{1}{2}\Big\{L_\alpha^\dagger L_\alpha, \rho(t)\Big\}\right) \tag{1.3}$$

as a linear differential equation in matrix form,

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_i(t) = \sum_j \left(-\mathrm{i}\mathscr{H}_{ij}(t) + \sum_\alpha \gamma_\alpha \mathscr{D}_{\alpha,ij} \right) \rho_j(t). \tag{1.4}$$

Here, $\mathscr{H}_{ij}(t) = \mathrm{tr}\big(\sigma_i\big[H(t), \sigma_j\big]\big)$ and $\mathscr{D}_{\alpha,ij} = \mathrm{tr}\Big(\sigma_i L_\alpha \sigma_j L_\alpha^\dagger - \frac{1}{2}\sigma_i\Big\{L_\alpha^\dagger L_\alpha, \sigma_j\Big\}\Big)$. By setting $L_\alpha \equiv \sigma_y^{(3)}$ as well as $\gamma_\alpha \equiv S_0/2$ with S_0 the amplitude of the one-sided noise power spectral density (PSD) so that $S(\omega) = S_0$, we can compare the fidelity obtained from the filter functions to that from the explicit simulation of Equation 1.4. The latter is computed as $F_{\mathrm{avg}} = \mathrm{tr}\big(\mathscr{Q}^\dagger \mathscr{U}\big)\big/ d^2$, where $\mathscr{Q}$ is the superpropagator due to the Hamiltonian evolution alone (*i.e.* the ideal evolution without noise).

For the Monte Carlo simulation, we explicitly generate time traces of $b_Q(t)$ (c.f. **??**) by drawing pseudo-random variables from a distribution whose PSD is $S(f) = S_0/f$. To do this, we draw complex, normally distributed samples in frequency space (*i.e.* white noise), scale it with the square root of the PSD, and finally perform the inverse Fourier transform. We choose an oversampling factor of 16, so that the time discretization of the simulation is $\Delta t_{\mathrm{MC}} = \Delta t/16 = 62.5\,\mathrm{ps}$ ($\Delta t = 1\,\mathrm{ns}$ is the time step of the pulses used in the FF simulation), leading to a highest resolvable frequency of $f_{\max} = 16\,\mathrm{GHz}$. Conversely, we increase the frequency resolution by sampling a time trace longer by a given factor, giving frequencies below $f_{\min}$ (16 kHz for pink, 0 Hz for white noise) weight zero, and truncating it to the number of time steps in the algorithm times the oversampling factor. This yields a time trace with frequencies $f \in [f_{\min}, f_{\max}]$ and a given resolution (we choose $\Delta f = 160\,\mathrm{Hz}$).

We then proceed to diagonalize the Hamiltonian $H(t) = H_c(t) + H_n(t)$ and compute the propagator for one noise realization as

$$U(t) = \prod_g V^{(g)} \exp\{-\mathrm{i}\Omega^{(g)} \Delta t_{\mathrm{MC}}\} V^{(g)\dagger}, \tag{1.5}$$

where $V^{(g)}$ is the unitary matrix of eigenvectors of $H(t)$ during time segment g and $\Omega^{(g)}$ the diagonal matrix of eigenvalues. Finally, we obtain an estimate for the average gate fidelity F_{avg} from the entanglement fidelity F_{e} as

$$\left\langle F_{\mathrm{avg}} \right\rangle = \left\langle \frac{dF_{\mathrm{e}} + 1}{d + 1} \right\rangle = \left\langle \frac{\left|\mathrm{tr}\left(Q^\dagger U(\tau)\right)\right|^2 + d}{d(d + 1)} \right\rangle. \tag{1.6}$$

Here, $Q \equiv U_{\mathrm{c}}(t = \tau)$ is the noise-free propagator at time τ of completion of the circuit and $\langle \cdot \rangle$ denotes the ensemble average over N Monte Carlo realizations of Equation 1.5, *i. e.* $\langle A \rangle = \frac{1}{N} \sum_{i=1}^{N} A_i$. The standard error of the mean can be obtained as $\sigma_{\langle F_{\mathrm{avg}} \rangle} = \sigma_{F_{\mathrm{avg}}} / \sqrt{N}$ with $\sigma_{F_{\mathrm{avg}}}$ the standard deviation over the Monte Carlo traces.

Filter Functions 2

For a given quantum operation $\tilde{\mathscr{U}}$ resulting from the quantum system's evolution under the noise fully characterized by its one-sided power spectral density (PSD) $S(\omega)$, we define the filter function (FF) $\mathscr{F}(\omega;\tau)$ by

$$\tilde{\mathscr{U}}(\tau) = \exp \int \frac{\mathrm{d}\omega}{2\pi} \mathscr{F}(\omega;\tau) S(\omega). \tag{2.1}$$

Now, suppose that

$$S_{\omega_i}(\omega) = \sigma_i^2 \delta(\omega - \omega_i), \tag{2.2}$$

that is, the PSD of a monochromatic sinusoid of frequency ω_i and root mean square (RMS) σ_i^2. Then Equation 2.1 becomes

Equation 2.2 discretizes $S(\omega)$ by sampling it at points ω_i, *i. e.*,

$$S(\omega) = \lim_{n\to\infty} \sum_{i=1}^{n} S_{\omega_i}(\omega).$$

$$\begin{aligned} \tilde{\mathscr{U}}_{\omega_i}(\tau) &= \exp\left\{\sigma_i^2 \int \frac{\mathrm{d}\omega}{2\pi} \mathscr{F}(\omega;\tau)\delta(\omega-\omega_i)\right\} \\ &= \exp\left\{\frac{\sigma_i^2}{2\pi} \mathscr{F}(\omega_i;\tau)\right\}, \end{aligned} \tag{2.3}$$

where $\tilde{\mathscr{U}}_{\omega_i}(\tau)$ is the noisy quantum operation generated by monochromatic noise with PSD $S_{\omega_i}(\omega)$ according to Equation 2.2. It is now easy to invert Equation 2.3, and we obtain

$$\mathscr{F}(\omega_i;\tau) = \frac{2\pi}{\sigma_i^2} \log \tilde{\mathscr{U}}_{\omega_i}(\tau). \tag{2.4}$$

Because we represent quantum operations as matrices in Liouville space, Equation 2.4 is easy to implement on a computer; we simply need to evaluate $\tilde{\mathscr{U}}_{\omega_i}(\tau)$ for a set of discrete frequencies $\{\omega_i\}_i$ using Monte Carlo (MC) and take the logarithm [4]!

[4]: Geck (2021), *Scalable Control Electronics for a Spin Based Quantum Computer*

Indeed, we can go a step further and split apart the coherent and incoherent contributions to the noisy evolution. Since (in-)coherent quantum operations are represented by (anti-)symmetric matrices in Liouville space, we may define the incoherent and coherent FFs by

$$\begin{aligned} \mathscr{F}_\Gamma(\omega;\tau) &= \frac{1}{2}\left(\mathscr{F}(\omega;\tau) + \mathscr{F}(\omega;\tau)^\mathsf{T}\right) \\ &= \frac{\pi}{\sigma_i^2}\left(\log \tilde{\mathscr{U}}_{\omega_i}(\tau) + \log \tilde{\mathscr{U}}_{\omega_i}(\tau)^\mathsf{T}\right), \end{aligned} \tag{2.5}$$

and

$$\begin{aligned} \mathscr{F}_\Delta(\omega;\tau) &= \frac{1}{2}\left(\mathscr{F}(\omega;\tau) - \mathscr{F}(\omega;\tau)^\mathsf{T}\right) \\ &= \frac{\pi}{\sigma_i^2}\left(\log \tilde{\mathscr{U}}_{\omega_i}(\tau) - \log \tilde{\mathscr{U}}_{\omega_i}(\tau)^\mathsf{T}\right), \end{aligned} \tag{2.6}$$

respectively.

From a MC point of view, $\tilde{\mathscr{U}}$ is given by

$$\tilde{\mathscr{U}} \equiv \left\langle \tilde{\mathscr{U}}_{\omega_i} \right\rangle(\tau) = \mathscr{Q}^\mathsf{T} \left\langle \mathscr{U}_{\omega_i}(\tau) \right\rangle, \tag{2.7}$$

where $\mathcal{U}_{\omega_i}(\tau)$ is the solution of the Schrödinger equation for a single realization of the noise and $\langle \cdot \rangle$ denotes the ensemble average. Solving Equation 2.1 for $\mathcal{F}(\omega;\tau)$, we find

For N realizations of the stochastic process underlying $b(t)$, the ensemble average of a quantity $A(t)$ that is a function of $b(t)$ is given by

$$\langle A \rangle (t) = \frac{1}{N} \sum_{i=1}^{N} A_i(t)$$

where i enumerates the realizations of the stochastic process.

$$\begin{aligned} \mathcal{F}(\omega;\tau) &= \int \frac{\mathrm{d}\omega'}{2\pi} \delta(\omega-\omega') \log \left\langle \tilde{\mathcal{U}}_{\omega_i}(\tau) \right\rangle \\ &= \mathcal{Q}^\mathsf{T} + \int \frac{\mathrm{d}\omega'}{2\pi} \delta(\omega-\omega') \log \left\langle \mathcal{U}_{\omega_i}(\tau) \right\rangle . \end{aligned} \tag{2.8}$$

So what does a single noise realization of Equation 2.2 in the time domain look like? It's a sinusoid with amplitude $A_i \sim \mathcal{N}(0, \sigma_i^2)$, frequency ω_i, and phase $\phi \sim \mathcal{U}(0, 2\pi)$,

$$b(t) = A \sin(\omega_i t + \phi). \tag{2.9}$$

We can compute the expectation value of $\mathcal{U}_{\omega_i}(t)$ over A_i and ϕ by integrating over the probability distributions,

We write the expectation value of an observable A with respect to a random variable X with the probability density function ρ_X as $\mathbb{E}_X[A]$.

$$\mathbb{E}_{\sigma_i,\phi}[\mathcal{U}](t) = \int \mathrm{d}x\, \rho_{\sigma_i}(x) \int_0^{2\pi} \mathrm{d}\phi\, \mathcal{U}[x, \phi]. \tag{2.10}$$

In practice, Equation 2.10 will be evaluated numerically by dividing the integration intervals into sub-intervals of equal size and weighing the integrand with the probability density, approximated to be constant on the sub-interval.

The probability density function of the zero-mean normal distribution is

$$\rho_{\sigma_i}(x) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(\frac{x^2}{2\sigma_i^2}\right).$$

Appendix

Test A

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Bibliography

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[2] Vittorio Gorini, Andrzej Kossakowski, and E. C. G. Sudarshan. “Completely Positive Dynamical Semigroups of N-level Systems.” In: *J. Math. Phys.* 17.5 (1976), p. 821. DOI: 10.1063/1.522979. (Visited on 01/19/2022) (cited on page 2).

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