The kaobook class

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Example and documentation of the kaobook class

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The harmony of the world is made manifest in Form and Number, and the heart and soul and all the poetry of Natural Philosophy are embodied in the concept of mathematical beauty.

– D'Arcy Wentworth Thompson

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For a given quantum operation $\tilde{\mathcal{U}}$ resulting from the quantum system's evolution under the noise fully characterized by its one-sided power spectral density (PSD) $S(\omega)$, we define the filter function (FF) $\mathcal{F}(\omega;\tau)$ by

$$\tilde{\mathcal{U}}(\tau) = \exp\int\frac{\mathrm{d}\omega}{2\pi}\mathcal{F}(\omega;\tau)S(\omega). \tag{1.1}$$

Now, suppose that

$$S_{\omega_i}(\omega) = \sigma_i^2\delta(\omega-\omega_i), \tag{1.2}$$

that is, the PSD of a monochromatic sinusoid of frequency ω_i and root mean square (RMS) σ_i^2. Then Equation 1.1 becomes

Equation 1.2 discretizes $S(\omega)$ by sampling it at points ω_i, *i. e.*,

$$S(\omega) = \lim_{n\to\infty}\sum_{i=1}^{n} S_{\omega_i}(\omega).$$

$$\begin{aligned}\tilde{\mathcal{U}}_{\omega_i}(\tau) &= \exp\left\{\sigma_i^2\int\frac{\mathrm{d}\omega}{2\pi}\mathcal{F}(\omega;\tau)\delta(\omega-\omega_i)\right\}\\ &= \exp\left\{\frac{\sigma_i^2}{2\pi}\mathcal{F}(\omega_i;\tau)\right\},\end{aligned} \tag{1.3}$$

where $\tilde{\mathcal{U}}_{\omega_i}(\tau)$ is the noisy quantum operation generated by monochromatic noise with PSD $S_{\omega_i}(\omega)$ according to Equation 1.2. It is now easy to invert Equation 1.3, and we obtain

$$\mathcal{F}(\omega_i;\tau) = \frac{2\pi}{\sigma_i^2}\log\tilde{\mathcal{U}}_{\omega_i}(\tau). \tag{1.4}$$

Because we represent quantum operations as matrices in Liouville space, Equation 1.4 is easy to implement on a computer; we simply need to evaluate $\tilde{\mathcal{U}}_{\omega_i}(\tau)$ for a set of discrete frequencies $\{\omega_i\}_i$ using Monte Carlo (MC) and take the logarithm! [**Geck2021**].

Indeed, we can go a step further and split apart the coherent and incoherent contributions to the noisy evolution. Since (in-)coherent quantum operations are represented by (anti-)symmetric matrices in Liouville space, we may define the incoherent and coherent FFs by

$$\begin{aligned}\mathcal{F}_{\Gamma}(\omega;\tau) =& \frac{1}{2}\left(\mathcal{F}(\omega;\tau)+\mathcal{F}(\omega;\tau)^{\mathsf{T}}\right)\\ =& \frac{\pi}{\sigma_i^2}\left(\log\tilde{\mathcal{U}}_{\omega_i}(\tau)+\log\tilde{\mathcal{U}}_{\omega_i}(\tau)^{\mathsf{T}}\right),\end{aligned} \tag{1.5}$$

and

$$\begin{aligned}\mathcal{F}_{\Delta}(\omega;\tau) =& \frac{1}{2}\left(\mathcal{F}(\omega;\tau)-\mathcal{F}(\omega;\tau)^{\mathsf{T}}\right)\\ =& \frac{\pi}{\sigma_i^2}\left(\log\tilde{\mathcal{U}}_{\omega_i}(\tau)-\log\tilde{\mathcal{U}}_{\omega_i}(\tau)^{\mathsf{T}}\right),\end{aligned} \tag{1.6}$$

respectively. From a MC point of view, $\tilde{\mathcal{U}}$ is given by

$$\tilde{\mathcal{U}} \equiv \left\langle\tilde{\mathcal{U}}_{\omega_i}\right\rangle(\tau) = \mathcal{Q}^{\mathsf{T}}\left\langle\mathcal{U}_{\omega_i}(\tau)\right\rangle, \tag{1.7}$$

where $\mathcal{U}_{\omega_i}(\tau)$ is the solution of the Schrödinger equation for a single realization of the noise and $\langle\cdot\rangle$ denotes the ensemble average. Solving Equa-

For N realizations of the stochastic process underlying $b(t)$, the ensemble average of a quantity $A(t)$ that is a function of $b(t)$ is given by

$$\langle A\rangle(t) = \frac{1}{N}\sum_{i=1}^{N}A_i(t)$$

where i enumerates the realizations of tion 1.1 for $\mathscr{F}(\omega;\tau)$, we find

$$\begin{aligned}\mathscr{F}(\omega;\tau) =& \int \frac{\mathrm{d}\omega'}{2\pi}\delta(\omega-\omega')\log\left\langle \tilde{\mathscr{U}}_{\omega_i}(\tau)\right\rangle \\ =&\mathscr{Q}^\mathsf{T} + \int \frac{\mathrm{d}\omega'}{2\pi}\delta(\omega-\omega')\log\left\langle \mathscr{U}_{\omega_i}(\tau)\right\rangle.\end{aligned} \tag{1.8}$$

So what does a single noise realization of Equation 1.2 in the time domain look like? It's a sinusoid with amplitude $A_i \sim \mathcal{N}(0,\sigma_i^2)$, frequency ω_i, and phase $\phi \sim \mathscr{U}(0,2\pi)$,

$$b(t) = A\sin(\omega_i t + \phi). \tag{1.9}$$

We can compute the expectation value of $\mathscr{U}_{\omega_i}(t)$ over A_i and ϕ by integrating over the probability distributions,

$$\mathbb{E}_{\sigma_i,\phi}[\mathscr{U}](t) = \int \mathrm{d}x\,\rho_{\sigma_i}(x)\int_0^{2\pi}\mathrm{d}\phi\,\mathscr{U}[x,\phi]. \tag{1.10}$$

We write the expectation value of an observable A with respect to a random variable X with the probability density function ρ_X as $\mathbb{E}_X[A]$.

In practice, Equation 1.10 will be evaluated numerically by dividing the integration intervals into sub-intervals of equal size and weighing the integrand with the probability density, approximated to be constant on the sub-interval.

The probability density function of the zero-mean normal distribution is

$$\rho_{\sigma_i}(x) = \frac{1}{\sqrt{2\pi\sigma_i^2}}\exp\left(\frac{x^2}{2\sigma_i^2}\right).$$

Appendix

Test A

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.