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Tobias Hangleiter\*

April 20, 2025

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The harmony of the world is made manifest in Form and Number, and the heart and soul and all the poetry of Natural Philosophy are embodied in the concept of mathematical beauty.

– D'Arcy Wentworth Thompson



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**Part I**

**A FLEXIBLE PYTHON TOOL FOR  
FOURIER-TRANSFORM NOISE  
SPECTROSCOPY**



# Introduction

# 1

**N**OISE is ubiquitous in condensed matter physics experiments, and in mesoscopic systems in particular it can easily drown out the sought-after signal. Hence, characterizing (and subsequently mitigating) noise is an essential task for the experimentalist. But noise comes in as many different forms as there are types of signal sources and detectors, whether it be a voltage source or a photodetector, and while some instruments have built-in solutions for noise analysis, they vary in functionality and capability. Moreover, the measured signal often does not directly correspond to the noisy physical quantity of interest, making it desirable to be able to manipulate the raw data before processing.



# Theory of spectral noise estimation

2

THERE exist various methods for estimating the properties of noise in a classical signal  $x(t)$ .<sup>1</sup> A simple metric quantifying the average noise amplitude of the signal observed from time  $t = 0$  to  $t = T$  is the root mean square (RMS) [3],

$$\text{RMS}_x = \sqrt{\frac{1}{T} \int_0^T dt x(t)^2}. \quad (2.1)$$

While this already tells us *something* about the noise, it is evident that a single number does not provide many clues if we were to attempt to mitigate the noise or say something qualitative about it beyond “small” and “large”. In cases such as these, physics has often turned to the *spectral* representation of the function of interest. Knowing the frequency content of a function gives access to a wealth of information about the underlying contributing processes. But how can we learn how a system behaves as a function of frequency?

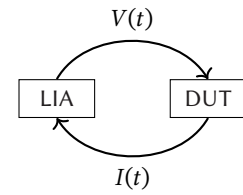
Consider an electrical black box (some DUT) with two leads connected to a LIA as shown in Figure 2.1. Assuming the DUT is conducting, we could simply measure the conductance  $G(t) = I(t)/V(t)$  through the device for some time  $T$  with a given lock-in modulation frequency  $f_i$ , subtract the constant offset<sup>2</sup> and calculate the RMS using Equation 2.1. Repeating this procedure for various different modulation frequencies  $\{f_i\}$ , we would collect a set of RMS-values that we could assign to the modulation frequencies at which they were measured and thus sample the noise amplitude spectrum of the conductance,  $S_G(f_i)$ .<sup>3</sup> This method has the advantage that we can choose the frequencies at which the spectrum should be sampled—in particular they do not have to be evenly spaced. However, it has two significant shortcomings. First, it is very inefficient and therefore time-consuming. For  $N$  frequency sample points, it takes a total time of  $NT$  to acquire all data, where  $T$  needs to be chosen such that the variance of  $\text{RMS}_G$  is sufficiently small. Second, and crucially, it is not always possible to excite the system at a certain frequency and measure its response. For example, it is generally considered hard<sup>4</sup> to – deliberately – excite vibrations of a specific frequency in a cryostat.

If the noisy process  $x(t)$  has Gaussian statistics, meaning that the value at a given point in time follows a normal distribution with some mean  $\mu$  and variance  $\sigma^2$  over multiple realizations of the process, it can be fully described by the power spectral density (PSD)  $S(\omega)$ .<sup>5</sup> For the purpose of noise estimation, the assumption of Gaussianity is a rather weak one as the noise typically arises from a large ensemble of individual fluctuators and is therefore well approximated by a Gaussian distribution by the central limit theorem [4].<sup>6</sup> Even if the process  $x(t)$  is not perfectly Gaussian, non-Gaussian contributions can be seen as higher-order contributions if viewed from the perspective of perturbation theory, and therefore the PSD still captures a significant part of the statistical properties. For this reason, the PSD is the central quantity of interest in noise spectroscopy and I will discuss some of its properties in the following.

For real signals  $x(t) \in \mathbb{R}$ ,  $S(\omega)$  is an even function and one therefore distinguishes the *two-sided* PSD  $S^{(2)}(\omega)$  defined over  $\mathbb{R}$  from the *one-sided* PSD  $S^{(1)}(\omega) = 2S^{(2)}(\omega)$  defined only over  $\mathbb{R}^+$ . Complex signals  $x(t) \in \mathbb{C}$

1: We discuss only classical noise here, meaning  $x(t)$  commutes with itself at all times. For descriptions of and spectroscopy protocols for quantum noise refer to References 1 and 2, for example.

lay out some others



**Figure 2.1:** Measuring the conductance through a device under test (DUT) using a lock-in amplifier (LIA).

2: The constant part  $G_0 = G(t) - \delta G(t)$  of course also holds some information about the system, for example about its bandwidth.

3: See Subsection 3.2.1 for a discussion on how the RMS at a certain frequency relates to other quantities discussed in this part.

4: And also unpopular with colleagues.

5: The term *power spectrum* is often used interchangeably. I will do so as well, but emphasize at this point that in digital signal processing in particular, the *spectrum* is a different quantity from the *spectral density*. See also sidenote 20 in Chapter 3.

maybe a classical signal processing ref?

6: As an example, consider electronic devices, where voltage noise is thought to arise from a large number of defects and other charge traps in oxides being populated and depopulated at certain rates  $\gamma$ . The ensemble average over these so-called two-level fluctuators (TLFs) then yields the well-known  $1/f$ -like noise spectra [5, 6] (at least for a large density [7]).

flesh out, organize

such as those generated by Lock-in amplifiers after demodulation in turn have asymmetric, two-sided PSDs.

## 2.1 Spectrum estimation from time series

7: For a wide-sense stationary (also called weakly stationary) process  $x(t)$ , the mean is constant and the auto-correlation function  $C(t, t') = \langle x(t)^* x(t') \rangle$  is given by  $\langle x(t)^* x(t + \tau) \rangle = \langle x(0)^* x(\tau) \rangle$  with  $\tau = t' - t$ . That is, it is a function of only the time lag  $\tau$  and not the absolute point in time. For Gaussian processes as discussed here, this also implies stationarity [8]. The property further implies that  $C(\tau)$  is an even function.

8: In this chapter we will always denote the Fourier transform of some quantity  $\xi$  using the same symbol with a hat,  $\hat{\xi}$ .

To see how the PSD may be estimated from time-series data, consider a continuous wide-sense stationary<sup>7</sup> signal in the time domain,  $x(t) \in \mathbb{C}$ , that is observed for some time  $T$ . We define the windowed Fourier transform of  $x(t)$  and its inverse by<sup>8</sup>

$$\hat{x}_T(\omega) = \int_0^T dt x(t) e^{-i\omega t} \quad (2.2)$$

$$\text{and } x(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \hat{x}_T(\omega) e^{i\omega t}, \quad (2.3)$$

i. e., we assume that outside of the window of observation  $x(t)$  is zero. The auto-correlation function of  $x(t)$  is given by

$$C(\tau) = \langle x(t)^* x(t + \tau) \rangle \quad (2.4)$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt x(t)^* x(t + \tau), \quad (2.5)$$

where  $\langle \cdot \rangle$  is the ensemble average over multiple realizations of the process and the last equality holds true for ergodic processes. Expressing  $x(t)$  in terms of its Fourier representation (Equation 2.2) and reordering the integrals, we get<sup>9</sup>

$$C(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \hat{x}_T(\omega)^* e^{-i\omega t} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \hat{x}_T(\omega') e^{i\omega'(t+\tau)} \quad (2.6)$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \hat{x}_T(\omega)^* \hat{x}_T(\omega') e^{i\omega'\tau} \int_0^T dt e^{it(\omega' - \omega)} \quad (2.7)$$

9: Mathematicians might at this point argue the integrability of  $x(t)$ , but as we deal with physical processes with finite bandwidth (and have no shame), we do not.

The innermost integral approaches a  $\delta$ -function for large  $T$ ,<sup>10</sup> allowing us to further simplify this under the limit as

$$C(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \hat{x}_T(\omega)^* \hat{x}_T(\omega') e^{i\omega'\tau} \delta(\omega' - \omega) \quad (2.8)$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} |\hat{x}_T(\omega)|^2 e^{i\omega\tau} \quad (2.9)$$

$$= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} S(\omega) e^{i\omega\tau} \quad (2.10)$$

with the PSD

$$S(\omega) = \lim_{T \rightarrow \infty} \frac{1}{T} |\hat{x}_T(\omega)|^2 \quad (2.11)$$

$$= \int_{-\infty}^{\infty} d\tau C(\tau) e^{-i\omega\tau} \quad (2.12)$$

10: Note that, because  $x(t)$  is wide-sense stationary, we may shift the limits of integration  $\int_0^T \rightarrow \int_{-T/2}^{+T/2}$ .

Equation 2.10 is the Wiener-Khinchin theorem that states that the auto-correlation function  $C(\tau)$  and the PSD  $S(\omega)$  are Fourier-transform pairs [8]. Furthermore, defining the latter through Equation 2.11 gives us an intu-

itive picture of the PSD if we recall Parseval's theorem,

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{T} |\hat{x}_T(\omega)|^2 = \frac{1}{T} \int_{-\infty}^{\infty} dt |x(t)|^2. \quad (2.13)$$

That is, the total power  $P$  contained in the signal  $x(t)$  is given by integrating over the PSD. Similarly, the power contained in a band of frequencies  $[\omega_1, \omega_2]$  is given by

$$P(\omega_1, \omega_2) = \text{RMS}_S(\omega_1, \omega_2)^2 \quad (2.14)$$

$$= \int_{\omega_1}^{\omega_2} \frac{d\omega}{2\pi} S(\omega) \quad (2.15)$$

where  $\text{RMS}_S(\omega_1, \omega_2)$  is the root mean square within this frequency band. These relations are helpful when analyzing noise PSDs to gauge the relative weight of contributions from different frequency bands to the total noise power.

To become familiar with the quantities  $C(\tau)$  and  $S(\omega)$ , consider the Ornstein-Uhlenbeck process [9], the only stationary Gaussian Markovian stochastic process [10]. The auto-correlation function of the Ornstein-Uhlenbeck process is given by

$$C(\tau) = \sigma^2 e^{-\tau/\tau_c}, \quad (2.16)$$

with  $\sigma$  the RMS and  $\tau_c$  the correlation time of the process, and the PSD is the Lorentzian function

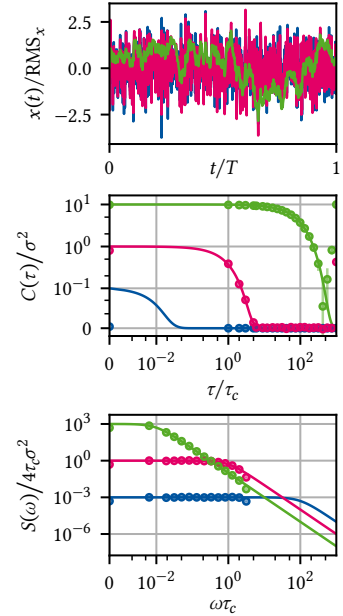
$$S(\omega) = \frac{2\sigma^2\tau_c}{1 + (\omega\tau_c)^2}. \quad (2.17)$$

For a given discretization time step  $\Delta t$  and hence bandwidth  $\omega \in [0, \pi f_s]$ , the Ornstein-Uhlenbeck process interpolates between perfectly uncorrelated, white noise ( $\Delta t/\tau_c \rightarrow 0, S(\omega) = \text{const.}$ ) and correlated, pink noise ( $\Delta t/\tau_c \rightarrow \infty, S(\omega) \propto \omega^{-2}$ ). Figure 2.2 depicts simulated data and its auto-correlation function and PSD for exemplary parameters: in the white noise limit ( $\tau_c = 10^{-2}\Delta t$ , blue), in the intermediate regime ( $\tau_c = 1\Delta t$ , magenta), and in the correlated limit ( $\tau_c = 10^2\Delta t$ , green). From the time series plot at the top it becomes clear that the RMS alone is insufficient to describe the properties of noisy signals as the curves differ significantly despite being normalized to their RMS. The auto-correlation functions averaged over  $10^3$  realizations of the noisy signals as well as their theoretical (continuous) value, Equation 2.16, are plotted in the middle panel, normalized to  $\tau_c = \Delta t$  and  $\sigma^2 = 1/4$ . For the white noise limit (blue), correlations are too short to be resolved with the given time discretization. The correlations decay to  $e^{-1}$  at  $\tau/\tau_c = 10^{-2}, 1, 10^2$ , respectively. Finally, the bottom panel shows the PSD, Equation 2.17, and its periodogram estimate, again averaged over  $10^3$  realizations of the signal and normalized to  $\tau_c = \Delta t$  and  $\sigma^2 = 1/4$ . The cross-over from white to pink PSD occurs at  $\omega = \tau_c$ . While the simulated data for  $\tau_c = 10^{-2}\Delta t$  appears perfectly white, that for  $\tau_c = 10^2\Delta t$  appears perfectly  $1/f$ -like.

Equation 2.11 represents the starting point for the experimental spectrum estimation procedure. Instead of a continuous signal  $x(t), t \in [0, T]$ , consider its discretized version<sup>11</sup>

$$x_n, \quad n \in \{0, 1, \dots, N-1\} \quad (2.18)$$

defined at times  $t_n = n\Delta t$  with  $T = N\Delta t$  and where  $\Delta t = f_s^{-1}$  is the



**Figure 2.2:** Ornstein-Uhlenbeck process. Simulated time traces (top), auto-correlation function (middle), PSD (bottom) of the Ornstein-Uhlenbeck process. Top: Simulated time traces using the algorithm presented in Chapter 11. The data are normalized to the computed RMS (equal to  $\sigma$  in the continuous case). Middle: Theoretical auto-correlation function (Equation 2.16, solid lines) and computed from the simulated data averaged over  $10^3$  traces (circles, subset of points). Error bars indicate the standard error of the mean and axes are scaled with respect to the parameters of the magenta data, and data are plotted on an asinh-scale. Bottom: Theoretical PSD (Equation 2.17, solid lines) and periodograms computed from the simulated data averaged over  $10^3$  traces using `scipy.signal.periodogram()`, cf. Equation 2.21 (circles, subset of points). Axes are again scaled with respect to the parameters of the magenta data and plotted on an asinh-scale. Parameters are  $\tau_c = \Delta t \times \{10^{-2}, 1, 10^2\}$  and  $\sigma^2 = \sqrt{\tau_c}/4$  for blue, magenta, and green data, respectively.

<sup>11</sup>: We only discuss the problem of

sampling interval (the inverse of the sampling frequency  $f_s$ ). Invoking the ergodic theorem, we can replace the long-term average in Equation 2.11 by the ensemble average over  $M$  realizations  $i$  of the noisy signal  $x_n^{(m)}$  and write

$$S_n = \frac{1}{M} \sum_{i=0}^{M-1} |\hat{x}_n^{(m)}|^2 \quad (2.19)$$

$$= \frac{1}{M} \sum_{i=0}^{M-1} S_n^{(m)} \quad (2.20)$$

where  $\hat{x}_n^{(m)}$  is the discrete Fourier transform of  $x_n^{(m)}$ , we defined the *periodogram* of  $x_n^{(m)}$  by

$$S_n^{(m)} = |\hat{x}_n^{(m)}|^2, \quad (2.21)$$

and  $S_n$  is an *estimate* of the true PSD sampled at the discrete frequencies  $\omega_n = 2\pi n/T \in 2\pi \times \{-f_s/2, \dots, f_s/2\}$ .<sup>12</sup> Equation 2.19 is known as Bartlett's method [13] for spectrum estimation.<sup>13</sup>

To better understand the properties of this estimate, let us take a look at the parameters  $\Delta t$ ,  $N$ , and  $M$ . The sampling interval  $\Delta t$  defines the largest resolvable frequency by the Nyquist sampling theorem,

$$f_{\max} = \frac{f_s}{2} = \frac{1}{2\Delta t}. \quad (2.22)$$

In turn, the number of samples  $N$  determines the frequency resolution  $\Delta f$ , or smallest resolvable frequency,

$$f_{\min} = \Delta f = \frac{1}{T} = \frac{1}{N\Delta t} = \frac{f_s}{N}. \quad (2.23)$$

Lastly,  $M$  determines the variance of the set of periodograms  $\{S_n^{(m)}\}_{i=0}^{M-1}$  and hence the accuracy of the estimate  $S_n$ .

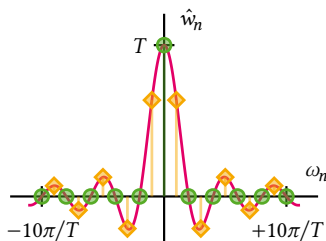
In practice, the ensemble realizations  $i$  are of course obtained sequentially, implying that one acquires a time series of data  $x_n, n \in \{0, 1, \dots, NM - 1\}$  and partitions these data into  $M$  sequences of length  $N$ . It becomes clear, then, that the Bartlett average (Equation 2.19) trades spectral resolution (larger  $N$ ) for estimation accuracy (larger  $M$ ) given the finite acquisition time  $T = NM\Delta t$ .

An improvement in data efficiency can be obtained using Welch's method [14]. To see how, we first need to discuss spectral windowing.

## 2.2 Window functions

Partitioning a signal  $x_n$  into  $M$  sections  $x_n^{(m)}$  of length  $N$  is mathematically equivalent to multiplying the signal with the rectangular *window function* given by<sup>14</sup>

$$w_n^{(m)} = \begin{cases} 1 & \text{if } (m-1)N \leq n < mN \\ 0 & \text{else} \end{cases} \quad (2.24)$$



**Figure 2.3:** The Fourier representation of the rectangular window in continuous time (solid line) and for discrete frequencies  $\omega_n = 2\pi n/T$  (circles). Introducing a phase shift, that is, shifting the window with respect to the signal in time, effec-

12: We blithely disregard integer algebra issues occurring here for conciseness and leave it as an exercise for the reader to figure out what the exact bounds of the set of  $\omega_n$  are.

13: By taking the limit  $M \rightarrow \infty$  one recovers the true PSD,

$$\lim_{M \rightarrow \infty} S_n = S(\omega_n).$$

The continuum limit is as always obtained by sending  $\Delta t \rightarrow 0, N \rightarrow \infty, N\Delta t = \text{const.}$



so that  $x_n^{(m)} = x_n w_n^{(m)}$ . Now recall that multiplication and convolution are duals under the Fourier transform, implying that

$$\hat{x}_n^{(m)} = \hat{x}_n * \hat{w}_n^{(m)}, \quad (2.25)$$

where the Fourier representation of the rectangular window<sup>15</sup>

$$\hat{w}_n^{(m)} = \hat{w}_n e^{-i(m-1/2)\omega_n T}, \quad (2.26)$$

$$\hat{w}_n = T \operatorname{sinc}\left(\frac{\omega_n T}{2}\right). \quad (2.27)$$

Figure 2.3 shows the unshifted rectangular window  $\hat{w}_n$  in Fourier space. We can hence understand the Fourier spectrum of  $x_n^{(m)}$  as sampling  $\hat{x}_n$  with the probe  $\hat{w}_n^{(m)}$ . However, while in the continuum limit (sidenote 13) Equation 2.27 tends towards  $\delta(\omega_n)$  and thus will produce a faithful reconstruction of the true spectrum, the finite frequency spacing  $\Delta f$  of discrete signals and finite observation length  $T$  introduce a finite bandwidth of the probe as well as *sidelobes*. These effects induce what is known as *spectral leakage* and *scalloping loss* [8, 15] and lead to artifacts and deviations of the spectrum estimator  $S_n$  from the true spectrum  $S(\omega_n)$ .

For this reason, a plethora of *window functions* have been introduced to mitigate the effects of spectral leakage. Key properties of a window are the spectral bandwidth (center lobe width) and sidelobe amplitude between which there typically is a tradeoff.<sup>16</sup> A window frequently used in spectral analysis is the Hann window [17],

$$w_n^{(m)} = \begin{cases} \cos^2\left(\frac{\pi n}{N}\right) & \text{if } (m-1)N \leq n < mN \text{ and} \\ 0 & \text{else} \end{cases} \quad (2.28)$$

with the Fourier representation of the unshifted window,

$$\hat{w}_n = T \operatorname{sinc}\left(\frac{\omega_n T}{2}\right) \times \frac{1}{2(1 - \omega_n T/2\pi)(1 + \omega_n T/2\pi)}, \quad (2.29)$$

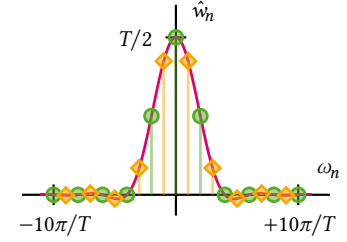
shown in Figure 2.4. The favorable properties of the Hann window are apparent when compared to the rectangular window in Equation 2.27 and Figure 2.3; the sidelobes are quadratically suppressed while the center lobe is only slightly broadened.

Another favorable property of the Hann window is that  $w_0^{(0)} = w_{N-1}^{(0)} = 0$ . This suppresses detrimental effects arising from a possible discontinuity ( $x_0^{(0)} \neq x_{N-1}^{(0)}$ ) at the edge of a data segment related to the discrete Fourier transform, which assumes periodic data.<sup>17</sup>

## 2.3 Welch's method

Contemplating Equation 2.28, one might come to the conclusion that using a window such as this is not very data efficient in the sense that a large fraction of samples located at the edge of the window is strongly suppressed and hence does not contribute significantly to the spectrum estimate. To alleviate this lack of efficiency, one can introduce an overlap between adjacent data windows. That is, instead of partitioning the data  $x_n$  into  $M$  non-overlapping sections of length  $N$ , one shifts the  $m$ th window forward by  $-mK$  with  $K > 0$  the overlap. Finally, the periodogram

15:  $\operatorname{sinc}(x) = \sin(x)/x$ .



**Figure 2.4:** The Fourier representation of the Hann window in continuous time (solid line) and for discrete frequencies  $\omega_n$  (circles). Diamonds indicate discrete sampling when the window completely out of phase with the signal (cf. Figure 2.3).

16: Wikipedia gives a good overview of existing window functions [16].

17: Although this can usually also be achieved approximately by detrending the data before performing the Fourier transform, which is a good idea in any case.

18: Again neglecting integer arithmetic issues.

**Figure 2.5:** Illustration of Welch's method for spectrum estimation. The data (pink) of length  $L$  is partitioned into  $K = 2L/N - 1$  segments of length  $N$ . Each segment is multiplied with a window function (gray) which reduces spectral leakage and other artifacts. A finite overlap  $K$  between adjacent windows (gray) ensures efficient sample use.

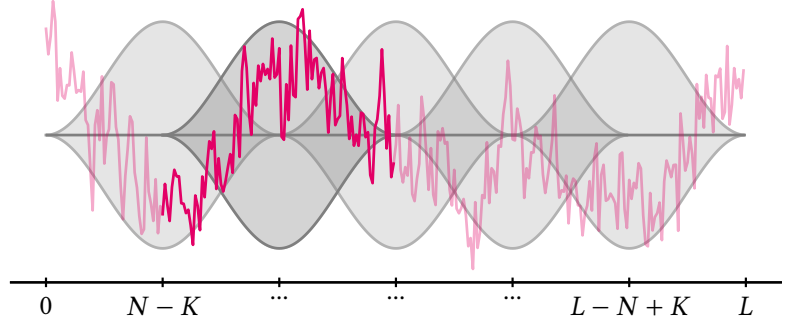


Figure 2.5 conceptually illustrates Welch's method for a trace of  $1/f$  noise with  $L = 300$  samples in total. Choosing the Hann window and an overlap of 50 % results in  $M = 5$  segments for a window length of  $N = 100$ . The data in the second window is highlighted.

### 2.3.1 Parameters

We are now in a position to discuss how the various parameters of a time series relate to both to physical parameters of the resulting spectrum estimate and to each other. To this end, we will go through the typical procedure of acquiring a spectrum estimate using Welch's method chronologically.

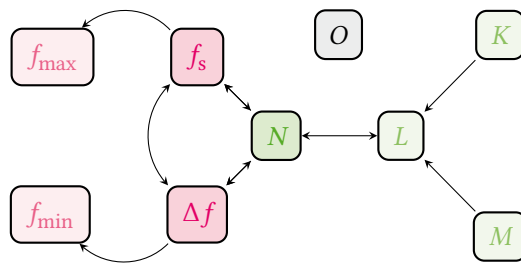
put discussion of properties here as well?

**Table 2.1:** Overview of spectrum estimation parameters. The parameters can be assigned into four groups: 1. DAQ parameters configuring the data acquisition device, 2. Welch parameters specifying the periodogram averaging, 3. Spectrum properties induced by the above, and 4. External parameters unrelated to the others.

1. DAQ parameters	
$L$	Total number of samples
$f_s$	Sample rate
2. Welch parameters	
$K$	Number of overlap samples
$N$	Number of segment samples
$M$	Number of Welch segments
3. Spectrum parameters	
$f_{\min}$	Smallest resolvable frequency
$f_{\max}$	Largest resolvable frequency
4. Miscellaneous parameters	
$O$	Number of outer averages

To acquire data using some form of (digital) data acquisition (DAQ), one usually needs to specify two parameters first: the total number of samples to be acquired,  $L$ , and the sample rate,  $f_s$ . This results in a measurement of duration  $T = L\Delta t$  where  $\Delta t = f_s^{-1}$  as previously mentioned. The choice of  $f_s$  already induces an upper bound on the first parameter characterizing the PSD estimate: the largest resolvable frequency  $f_{\max} \leq f_s/2$  (cf. Equation 2.22, but note that we allow  $f_{\max}$  to be smaller than half the sample rate in anticipation of hardware constraints). Next, we choose a number of Welch averages,  $M$ , i.e., data partitions, and their overlap,  $K$ . In doing so, one fixes the number of samples per partition  $N$  and thereby induces the lower bound on the second parameter characterizing the PSD estimate: the frequency spacing  $\Delta f = 1/N \leq f_{\min}$  (cf. Equation 2.23).<sup>19</sup> Finally, we can introduce a number of *outer* averages  $O$ , that is, the number of data batches that are acquired. While not directly related to Welch's method, choosing  $O > 1$  can, for instance, help achieve a certain variance if the number of samples per batch,  $L$ , is limited by the data acquisition hardware, or simply allow for updating the spectrum estimate as data is being acquired. Figure 2.6 shows the relationships of the various parameters among each other. In Subsection 3.1.1, I lay out how these inter-dependencies are implemented in software.

19: Technically, the smallest resolvable frequency in a fast Fourier transform (FFT) is zero, of course. But as data is typically detrended (a constant or linear trend subtracted) before computation of the periodogram, the smallest *meaningful* frequency is  $f_{\min} = 1/L$ .



**Figure 2.6:** Relationships of data acquisition parameters (cf. Tables 2.1 and 3.1, with arrows indicating dependencies.  $N$ ,  $f_s$ , and  $\Delta f$  are the central quantities defining the estimated spectrum's properties. From  $f_s$  and  $\Delta f$  follow (bounds for)  $f_{\max}$  and  $f_{\min}$ . From  $N$ , together with  $K$  and  $M$ , follows  $L$ , the total number of samples per data batch.



# The `python_spectrometer` software package

# 3

In this chapter, I introduce the `python_spectrometer` Python package<sup>1</sup> and lay out its design and functionality. This software package was developed to make it easier for experimentalists to transfer the mathematical machinery introduced in Chapter 2 to the lab. While in principle the entire process of spectrum estimation from a given array of time series data is already covered by the `welch()` routine in SciPy [18], obtaining the data array is not standardized. Different DAQ instruments have different capabilities, both on the hardware and the software level, and different driver interfaces to communicate with them. This implies custom data acquisition code is required for every instrument, introducing a significant entry barrier to spectral analysis. The `python_spectrometer` package implements a simple interface to different hardware instruments that allows for changing the hardware backend without having to adapt the user-facing code and also incorporates different hardware constraints.

What is more, noise spectroscopy tends to be a visual endeavor in practice; it is hard to compare different noise spectra based on quantitative reasoning alone. Data visualization is hence an integral part of noise spectroscopy, but plotting is not just plotting. Do we want the data to be shown on a log-log scale?<sup>2</sup> Do we want to show the relative magnitude of different data sets? Do we want to inspect the time traces as well? The `python_spectrometer` package addresses these questions by allowing users to interactively change features of the main plot window to adapt it to the form best suited to the situation at hand.

Moreover, when concerned with noise spectrum estimation, we are typically more interested in specifying parameters of the resulting PSD rather than parameters of the underlying time series data. The `python_spectrometer` package approaches data acquisition from the inverse direction: rather than inferring the spectrum properties from the time series data, users specify the properties they would like the resulting spectrum to have and the package chooses the correct parameters for data acquisition accordingly.

## 3.1 Package design and implementation

The `python_spectrometer` package provides a central class, `Spectrometer`, that users interact with to perform data acquisition, spectrum estimation, and plotting. It is instantiated with an instance of a child class of the DAQ base class that implements an interface to various DAQ hardware devices.<sup>3</sup> New spectra are obtained by calling the `Spectrometer.take()` method with all acquisition and metadata settings. In the following, I will go over the the design of these aspects of the package in more detail.

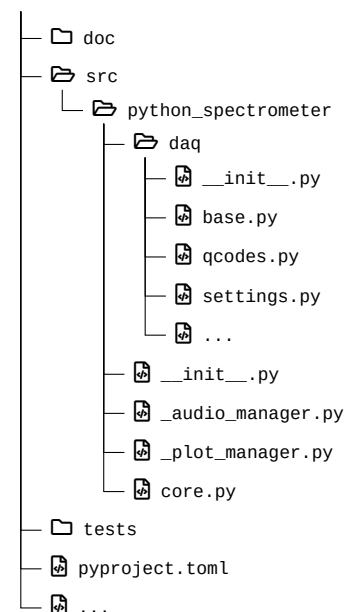
### 3.1.1 Data acquisition

Figure 3.1 shows the directory structure of the source code. The `daq` subpackage contains on the one hand the declaration of the DAQ abstract base class (`base.py`) and its child class implementations (`qcodes.py`, *etc.*), and on the other the `settings.py` module, which defines the `DAQSettings`

1: The package repository is hosted on [GitLab](#). Its documentation is automatically generated and hosted on [GitLab Pages](#). Releases are automatically published to [PyPI](#) and allow the package to be installed using `pip install python-spectrometer`.

[18]: (n.d.), *welch* — SciPy v1.15.2 Manual

2: The short answer is yes, but it comes with visual side-effects that demand other ways of plotting data at times. The long answer is therefore yes, *and ...*



**Figure 3.1:** Source tree structure of the `python_spectrometer` package. Driver wrappers are placed in the `daq` subpackage. `core.py` exports the `Spectrometer` class.

3: Actually *drivers* to be more precise.

**Table 3.1:** Variable names used in Chapter 2 and their corresponding parameter names as used in `python_spectrometer` and `scipy.signal.welch()` [18].

Variable	Parameter
$L$	<code>n_pts</code>
$N$	<code>nperseg</code>
$K$	<code>noverlap</code>
$M$	<code>n_seg</code>
$O$	<code>n_avg</code>
$f_s$	<code>fs</code>
$f_{\max}$	<code>f_max</code>
$f_{\min}$	<code>f_min</code>

4: Let's call her Alice.

5: Physicists generally are.

6: `DAQSettings` inherits from the builtin `dict` and as such can contain arbitrary other keys besides those listed in Table 3.1. However, automatic validation of parameter consistency is only performed for these special keys.

7: Since the graph spanned by the parameters is not acyclic, this only works most of the time.

8: Settings are automatically parsed when passed to the `take()` method of the `Spectrometer` class.

**Listing 3.1:** `DAQSettings` example showcasing automatic parameter resolution. `n_avg` determines the number of outer averages, i.e., the number of data buffers acquired and processed individually.

```
{'f_min': 14.30511474609375,
 'f_max': 72000.0,
 'fs': 234375.0,
 'df': 14.30511474609375,
 'nperseg': 16384,
 'noverlap': 0,
 'n_seg': 1,
 'n_pts': 16384,
 'n_avg': 1}
```

**Listing 3.2:** Resolved settings for the same input parameters as in Listing 3.1 but for the `ZurichInstrumentsMFLIScope` backend with hardware constraints on `n_pts` and `fs`.

9: [https://docs.zhinst.com/labone\\_api\\_user\\_manual/modules/scope/index.html](https://docs.zhinst.com/labone_api_user_manual/modules/scope/index.html)

10: And issued a warning to inform the user their requested settings could not be matched.

class. This class is used in the background to validate data acquisition settings both for consistency (see Subsection 2.3.1) and hardware constraints.

To better understand the necessity of this functionality, consider the typical scenario of a physicist<sup>4</sup> in the lab. Alice has wired up her experiment, performed a first measurement, and to her dismay discovered that the data is too noisy to see the sought-after effect. She sets up the `python_spectrometer` code to investigate the noise spectrum of her measurement setup. From her noisy data she could already estimate the frequency of the most harrowing noise, so she knows the frequency band  $[f_{\min}, f_{\max}]$  she is most interested in. But because she is lazy,<sup>5</sup> she does not want to do the mental gymnastics to convert  $f_{\min}$  to the parameter that her DAQ device understands,  $L$  (see Table 3.1), especially considering that  $L$  depends on the number of Welch averages and the overlap. Furthermore, while she could just about do the conversion from  $f_{\max}$  to the other relevant DAQ parameter,  $f_s$ , in her head, her device imposes hardware constraints on the allowed sample rates she can select! The `DAQSettings` class addresses these issues. It is instantiated with any subset of the parameters listed in Table 3.1<sup>6</sup> and attempts to resolve the parameter interdependencies lined out in Subsection 2.3.1 upon calling `DAQSettings.to_consistent_dict()`.<sup>7</sup> This either infers those parameters that were not given from those that were or, if not possible, uses a default value. Child classes of the DAQ class can subclass `DAQSettings` to implement hardware constraints such as a finite set of allowed sampling rates or a maximum number of samples per data buffer.

For instance, Alice might want to measure the noise spectrum in the frequency band [1.5 Hz, 72 kHz]. Although she would not have to do this explicitly,<sup>8</sup> she could inspect the parameters after resolution using the code shown in Listing 3.1.

```
>>> from python_spectrometer.daq import DAQSettings
>>> settings = DAQSettings(f_min=1.5, f_max=7.2e4)
>>> settings.to_consistent_dict()
{'f_min': 1.5,
 'f_max': 72000.0,
 'fs': 144000.0,
 'df': 1.5,
 'nperseg': 96000,
 'noverlap': 48000,
 'n_seg': 5,
 'n_pts': 288000,
 'n_avg': 1}
```

If the instrument she'd chosen for data acquisition had been a Zurich Instruments MFLI's Scope module,<sup>9</sup> the same requested settings would have resolved to those shown in Listing 3.2.<sup>10</sup> This is because the Scope module constrains  $L \in [2^{12}, 2^{14}]$  and  $f_s \in 60 \text{ MHz} \times 2^{[-16, 0]} \approx \{915.5 \text{ Hz}, \dots, 30 \text{ MHz}, 60 \text{ MHz}\}$ .

As already mentioned, the DAQ base class implements a common interface for different hardware backends, allowing the `Spectrometer` class to be hardware agnostic. That is, changing the instrument that is used to acquire the data does not necessitate adapting the code used to interact with the `Spectrometer`. To enable this, different instruments require small wrapper drivers that map the functionality of their actual driver onto the interface dictated by the DAQ class. This is achieved by subclassing `DAQ` and implementing the `DAQ.setup()` and `DAQ.acquire()` methods. Their

functionality is best illustrated by the internal workflow as representatively shown in Listing 3.3.

```
daq = MyDAQ(driver_handle)

parsed_settings = daq.setup(**user_settings)
acquisition_generator = daq.acquire(**parsed_settings)

for data_buffer in acquisition_generator:
    estimate_psd(data_buffer)
```

**Listing 3.3:** DAQ workflow pseudocode. A `MyDAQ` object (representing the instrument `My`) is instantiated with a driver object (for instance a `QCoDeS` Instrument). The instrument is configured with the given `user_settings`. Calling the generator function `daq.acquire()` with the actual device settings returns a generator, iterating over which yields one data buffer per iteration. The data buffers can then be passed to further processing functions (the PSD estimator in our example).

When acquiring a new spectrum, all settings supplied by the user are first fed into the `setup()` method where instrument configuration takes place. The method returns the actual device settings,<sup>11</sup> which are then forwarded to the `acquire()` generator function. Here, the instrument is armed (if necessary), and subsequently data is fetched from the device and yielded to the caller `n_avg` times, where `n_avg` is the number of outer averages.<sup>12</sup> An exemplary implementation of a DAQ subclass for a fictitious instrument is shown in Listing 3.4. In addition to the methods to configure the instrument and perform data acquisition, it is possible to override the `DAQSettings` property to implement instrument-specific hardware constraints such as, in this example, the number of samples per buffer being constrained to the discrete interval `[1, 2048]`. Leveraging the `util.domains` module, more complex constraints such as sample rates restricted to an internal clock rate divided by a power of two<sup>13</sup> can be specified.

11: Which might differ from the requested settings as outlined above.

12: *I.e.*, the number of time series data batches acquired, as opposed to the number of Welch averages `n_seg` within one batch.

13: See for example the implementation of the [AlazarTech ATS9440](#) digitizer card.

### 3.1.2 Data processing

Once time series data has been acquired using a given DAQ backend, it could in principle immediately be used to estimate the PSD following Equation 2.19. However, it is often desirable to transform, or process, the data in some fashion. This can include simple transformations such as accounting for the gain of a transimpedance amplifier (TIA) and convert the voltage back to a current,<sup>14</sup> or more complex ones such as applying calibrations. In particular, since the process of computing the PSD already involves Fourier transformation, the processing can also be performed in frequency space.

14: Although it is of course less than trivial to discriminate between current and voltage noise in a TIA.

In `python_spectrometer`, this can be done using a `procfn` (in the time domain) or `fourier_procfn` (in the Fourier domain). The former is specified as an argument directly to the `Spectrometer` constructor. It is a callable with signature `(x, **kwargs) -> xp`, that is, takes the time series data as its first (positional) argument and arbitrary settings that are passed through from the `take()` method as keyword arguments, and returns the processed data. Listing 3.5 shows a simple function that accounts for the gain of an amplifier. The latter is specified in the `psd_estimator` argument of the `Spectrometer` constructor. This argument allows the user to specify a custom estimator for the PSD, in which case a callable is expected. Otherwise, it should be a mapping containing parameters for the default PSD estimator, `scipy.signal.welch()` [18]. Here, the keyword `fourier_procfn` should be a callable with signature `(xf, f, **kwargs) -> (xfp, fp)`.<sup>15</sup> That is, it should take the frequency-space data, the corresponding frequencies, and arbitrary keyword arguments and return a tuple of the processed data and the corresponding frequencies.

[18]: (n.d.), *welch* — *SciPy v1.15.2 Manual*

15: *I.e.*, the `psd_estimator` argument would be `{"fourier_procfn": fn}`.



**Listing 3.4:** Exemplary code for a DAQ implementation of some instrument with given driver class `DeviceHandle` in the package `mydriver`. The `MyDAQ` class is instantiated with a `DeviceHandle` instance. Optionally, the `DAQSettings` property can be overridden to implement hardware constraints or default values for data acquisition parameters. For this, the `qutil.domains` module provides several classes that represent bounded domains and sets. The `setup()` method parses the given acquisition settings and configures the instrument through the external driver interface `handle.configure()`. The `acquire()` method arms the instrument (if necessary) and loops over the number of outer averages, `n_avg`. In the body of the loop, it can wait for external triggers (or send software triggers) before yielding a batch of data fetched from the external driver interface. Once acquisition is done, the method can return arbitrary metadata to the `Spectrometer` object to attach to the stored data.

```
# daq/mydaq.py
import dataclasses
from qutil.domains import DiscreteInterval
from .base import DAQ
from .settings import DAQSettings

@dataclasses.dataclass
class MyDAQ(DAQ):
    handle: mydriver.DeviceHandle

    @property
    def DAQSettings(self) -> type[DAQSettings]:
        class MyDAQSettings(DAQSettings):
            ALLOWED_NPERSEG = DiscreteInterval(1, 2048)
        return MyDAQSettings

    def setup(self, **settings) -> dict:
        settings = self.DAQSettings(settings)
        parsed_settings = settings.to_consistent_dict()
        self.handle.configure(parsed_settings)
        return parsed_settings

    def acquire(self, n_avg: int, *, **settings) -> Generator:
        self.handle.arm(n_avg)
        for _ in range(n_avg):
            self.handle.wait_for_trigger()
            yield self.handle.fetch()
        return self.handle.metadata
```

```
def comp_gain(x, gain=1.0, **_):
    return x / gain
```

**Listing 3.5:** A simple `procf`n, which converts amplified data back to the level before amplification. Note the token `**_` variable keyword argument that ensures no errors arise from other parameters being passed to the function. More complex processing chains can concisely be defined with `qutil.functools.FunctionChain` that pipes the output of one function into the input of the next.

16: One example is the `octave_band_rms()` function from the `qutil.signal_processing` subpackage [19] that decimates frequencies to fractions of octave bands. See Part II for a use-case. TODO: ref

```
def derivative(xf, f, n=0, **_):
    return xf / (2j * pi * f)**n
```

**Listing 3.6:** A simple `fourier_procf`n, which calculates the (anti-)derivative.

ref

blabla

The latter are required in case the function modifies the frequencies.<sup>16</sup> A simple example for a processing function in Fourier space is shown in Listing 3.6, which computes the (anti-)derivative of the data using the fact that

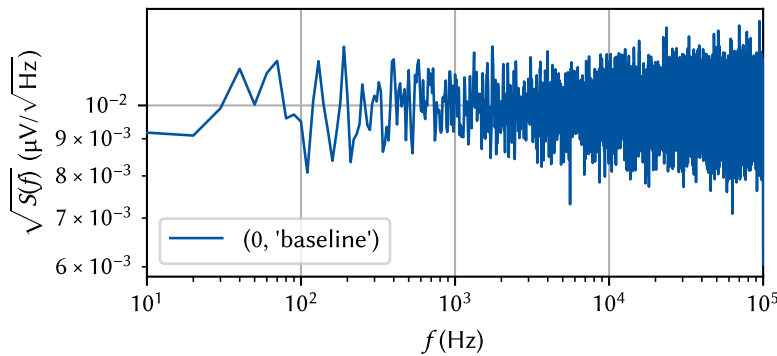
$$\frac{\partial^n}{\partial t^n} \xrightarrow{\text{F.T.}} (i\omega)^n \quad (3.1)$$

under the Fourier transform. In Part II, I discuss more complex use-cases of the processing functionality included in `python_spectrometer` in the context of vibration spectroscopy.

## 3.2 Feature overview

Now that we have a basic understanding of the design choices underlying `python_spectrometer`, let us discuss the typical workflow of using the package. Two modes of operation are to be distinguished: first, “serial” mode, in which users record new spectra manually, and second, “live” mode, in which new data is continuously being acquired. The former is well suited to a structured approach to noise spectroscopy, where data is retained persistently and discrete changes are made to the system between data acquisitions. The latter is aimed at a more fluent workflow in which data is not retained and data acquisition runs in the background.





**Figure 3.2:** The `python_spectrometer` plot after acquiring the (here: synthetic) baseline spectrum. By default, the amplitude spectral density (ASD) =  $\sqrt{\text{PSD}}$  is displayed in the main plot. Each spectrum is assigned a unique identifier key consisting of an incrementing integer and the user comment, and can be referred to by either (or both) when interacting with the object.

### 3.2.1 Serial spectrum acquisition

The default mode for spectrum acquisition using `python_spectrometer` revolves around the `take()` method. Key to this workflow is the idea that each acquired spectrum be assigned a comment that allows to easily identify it in the main plot. For instance, this comment could contain information about the particular settings that were active when the spectrum was recorded, or where a particular cable was placed.

Consider as an example the procedure of “noise hunting”, *i.e.*, debugging a noisy experimental setup. The experimentalist,<sup>17</sup> having discovered that his data is noisier than expected, sets up the `Spectrometer` class with an instance of the `DAQ` subclass for the `DAQ` instrument connected to his sample, a Zurich Instruments MFLI.<sup>18</sup> Choosing the frequency bounds, say  $f_{\min} = 10$  Hz and  $f_{\max} = 100$  kHz, and using the sensible defaults for the remaining spectrum parameters, Bob first grounds the input of his `DAQ` to record a *baseline* spectrum. Thus far, his code would hence look something like that shown in Listing 3.7, which produces the plot shown in Figure 3.2.

```
from python_spectrometer import Spectrometer, daq
from qutil.functools import scaled

mfli_daq = daq.ZurichInstrumentsMFLIDAQ(session, device)
spect = Spectrometer(mfli_daq, procfn=scaled(1e6),
                    processed_unit='uV')

spect.take('baseline', f_min=1e1, f_max=1e5, daq='grounded')
```

After acquiring the baseline, he next ungrounds the `DAQ` to obtain a representative spectrum of the noise in an actual measurement. He then proceeds by tweaking things on his setup, testing out different parameters, *etc.* Every time he changes something, he acquires another spectrum using `take()`, labeling each with a meaningful comment for identification.

```
settings = {'f_min': 1e1, 'f_max': 1e5, 'daq': 'connected'}
spect.take('connected', **settings)
spect.take('lifted cable', cable='lifted', **settings)
spect.take('jumped', **settings)
```

The code shown in Listing 3.8 would then leave him with the spectrometer plot as shown in Figure 3.3. While working, Bob realizes he’d like see

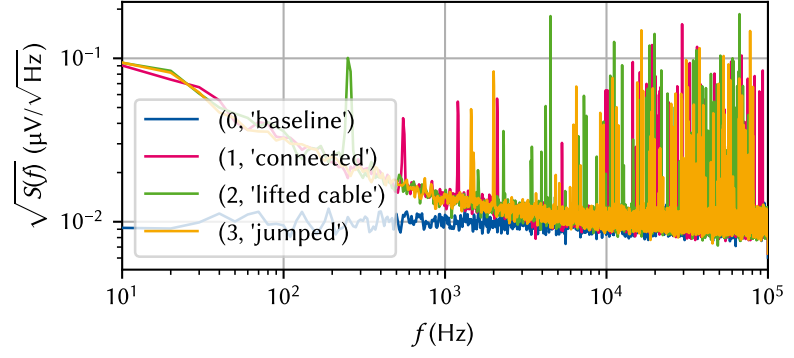
17: Let’s call him Bob.

18: For the MFLI, `DAQ` subclasses for both the `Scope` and the `DAQ` module are implemented. The former gives access to the signal before and the latter to the signal after demodulation.

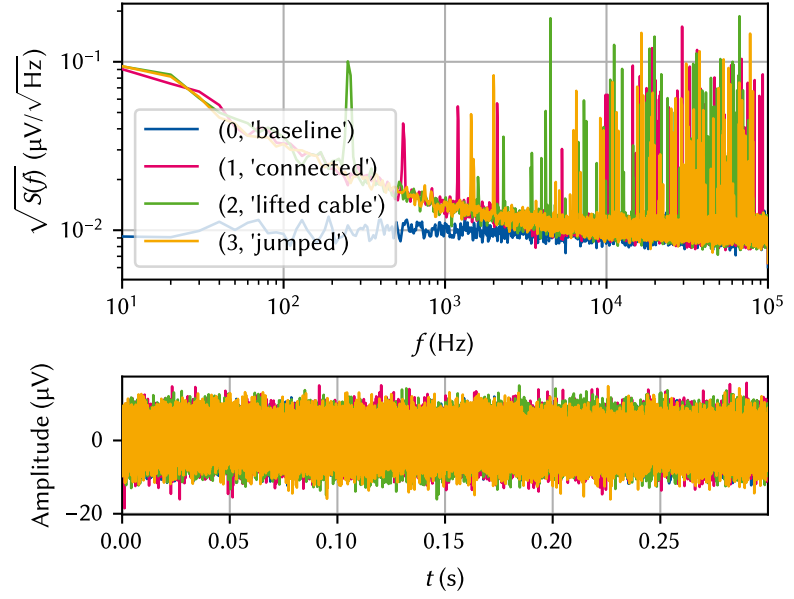
**Listing 3.7:** Setup and serial workflow using the `python_spectrometer` package. `session` and `device` are Application Programming Interface (API) objects of the `zhinst.toolkit` driver package. It is therefore possible to simply use the driver objects that are already in use in the measurement setup. The `procfn` and `processed_unit` arguments help converting raw data into a more human-friendly unit.

**Listing 3.8:** Code to acquire additional spectra. Arbitrary key-value pairs can be passed to the `take()` method, which are stored as metadata if they do not apply to any functions downstream in the data processing chain.

**Figure 3.3:** The `python_spectrometer` plot after acquiring additional (synthetic) spectra. Each spectrum is uniquely identified by a two-tuple of (index, comment).



**Figure 3.4:** The `python_spectrometer` plot shown in Figure 3.3 when setting `spect.plot_timetrace = True`. This adds a subplot that shows the time series data from which the PSD was computed akin to what an oscilloscope would show. Note that this is the entire time series, *i.e.*, the data of length  $L$ , which is (by default, using Welch's method) segmented for spectrum estimation.



the signal in the time-domain as well. He easily achieves this by setting `spect.plot_timetrace = True`, which adds an oscilloscope subplot to spectrometer figure as shown in Figure 3.4.

Bob now observes that the noise spectra he has recorded display many sharp peaks in particular at high frequencies while the  $1/f$  noise floor seems pretty consistent across different measurements. This makes it harder for him to evaluate whether any of his changes are actually an improvement or not. The `python_spectrometer` package allows addressing this by plotting the integrated spectra in another subplot. Bob's spectrometer figure after setting `spect.plot_cumulative = True` is shown in Figure 3.5. In the case that `spect.plot_amplitude == True`, this new subplot shows the RMS in the band  $[f_{\min}, f]$ ,

$$\text{RMS}_S(f) \equiv \text{RMS}_S(f_{\min}, f), \quad (3.2)$$

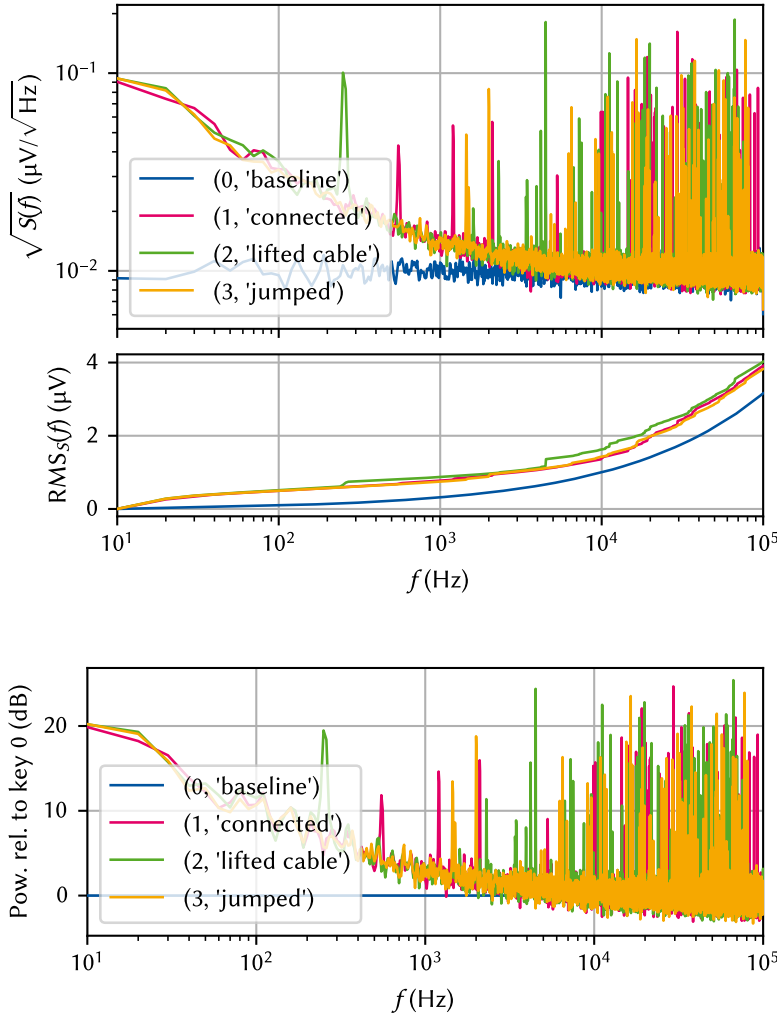
and the band power (Equation 2.14) otherwise.

The cumulative RMS plot already helps, but Bob would like a more quantitative comparison of relative spectral powers. Therefore, he rescales the spectra in terms of their relative powers expressed in dB<sup>19</sup> by applying the following settings, which produces the plot shown in Figure 3.6:

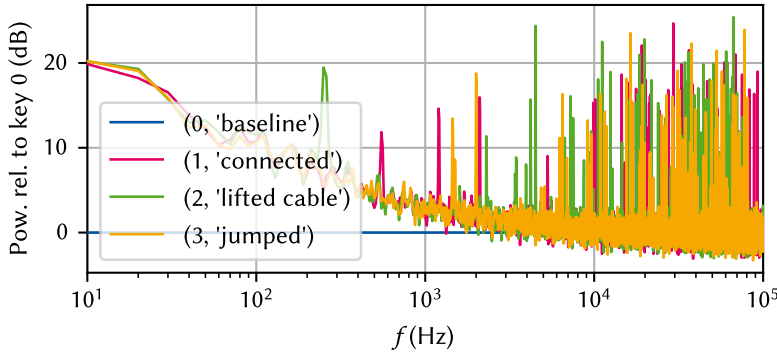
19: Recall that the decibel is defined by the ratio  $L_P$  of two powers  $P_1, P_2$  as [20]

$$L_P = 10 \log_{10} \left( \frac{P_1}{P_2} \right) \text{ dB}.$$

```
| spect.plot_dB_scale = True
| spect.plot_amplitude = False
```



**Figure 3.5:** The `python_spectrometer` plot shown in Figure 3.3 when setting `spect.plot_cumulative = True`. This adds a subplot that shows the RMS (see Equation 2.14) which can be helpful in evaluating the contribution of individual peaks in the spectrum to the total noise power. Both the oscilloscope subplot (Figure 3.4) and the RMS subplot can also be shown at the same time.



**Figure 3.6:** The `python_spectrometer` plot in relative mode. Starting from the state in Figure 3.3, we set `spect.plot_dB_scale = True` as well as `spect.plot_amplitude = False` and `spect.plot_density = False` to compare the relative noise powers with respect to the baseline.

```
| spect.plot_density = False
```

The attribute `plot_density` controls whether the *power spectral density* or the *power spectrum* is plotted.<sup>20</sup> Scaling the data to the power spectrum instead of the density, Bob can get an estimate of the RMS at a single frequency by reading off the peak height. Additionally displaying the data in dB then gives insight into relative noise powers of different spectra.

Bob carries on with his enterprise and continues to acquire spectra until, finally, he finds the source of his noise! Alas, his spectrometer plot is now overflowing with plotted data when really he just wants to compare the baseline, the original, noisy state, and the final, clean spectrum. He simply calls

```
| spect.hide(*range(2, 10))
```

to hide the eight spectra of unsuccessful debugging, leaving him with a plot as shown in Figure 3.7.

Finally, happy with the results, Bob serializes the state of the spectrometer to disk, allowing him to pick up where he left off at a later point in time:

```
| spect.serialize_to_disk('2032-12-24_noise_hunting')
```

The next week, Bob is asked by his team about his progress on debugging the noise in their setup. Even though he is working from home that day

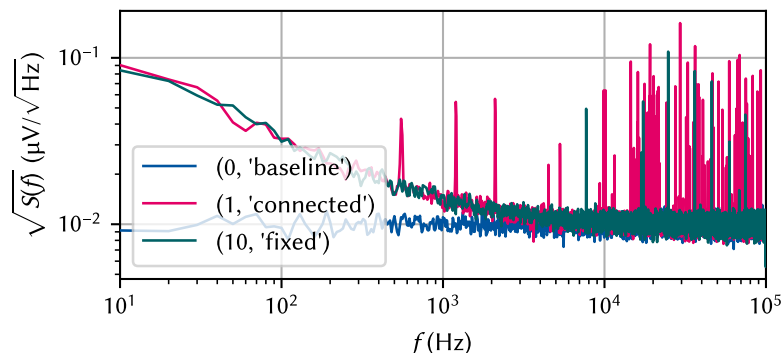
20: At this point, we *do* need to distinguish between the PSD and the power spectrum counter to sidenote 5 in Chapter 2. The PSD and power spectrum are related by the equivalent noise bandwidth (ENBW),

$$\text{Spectral density} \xrightarrow{\times \text{ENBW}} \text{Spectrum},$$

which is itself a function of the sampling rate and the properties of the spectral window [15],

$$\text{ENBW} = f_s \frac{\sum_n \hat{w}_n^2}{\left[\sum_n \hat{w}_n\right]^2}. \quad (3.3)$$

**Figure 3.7:** The `python_spectrometer` plot after multiple additional spectra were acquired and hidden. Hiding spectra that one is not interested in anymore is achieved through `spect.hide(*range(2, 10))`. This is reversed by `spect.show(*range(2, 10))`. Data can also be dropped (`spect.drop(key)`) or deleted (`spect.delete(key)`) from the internal cache and disk, respectively.



and does not have access to the lab computer, Bob simply uses his laptop computer and pulls up the Spectrometer session stored on the server, allowing them to interactively discuss the spectra:

```
file = '2032-12-24_noise_hunting'
# Read-only instance because no DAQ attached
spect = Spectrometer.recall_from_disk(savepath / file)
```

This opens up the plot shown in Figure 3.7 again. While they cannot acquire new spectra in this state,<sup>21</sup> they can still use all the plotting features like showing or hiding spectra, or changing plot types as discussed above.

21: They could of course always attach a DAQ instance to the spectrometer and continue as they were.

### 3.2.2 Live spectrum acquisition

Manually recording spectra in the workflow outlined in Subsection 3.2.1 becomes tedious at some point, and experimenters tend to become negligent with keeping metadata and comments up to date as they continue to change settings. Once a certain number of spectra has been obtained, the spectrometer plot also becomes crowded, and hiding old spectra manually is cumbersome. Moreover, each time a spectrum is captured, data is saved to disk, potentially accruing large amounts of disk space. For these reasons, the `python_spectrometer` package also offers a non-persistent live mode for displaying spectra continuously.

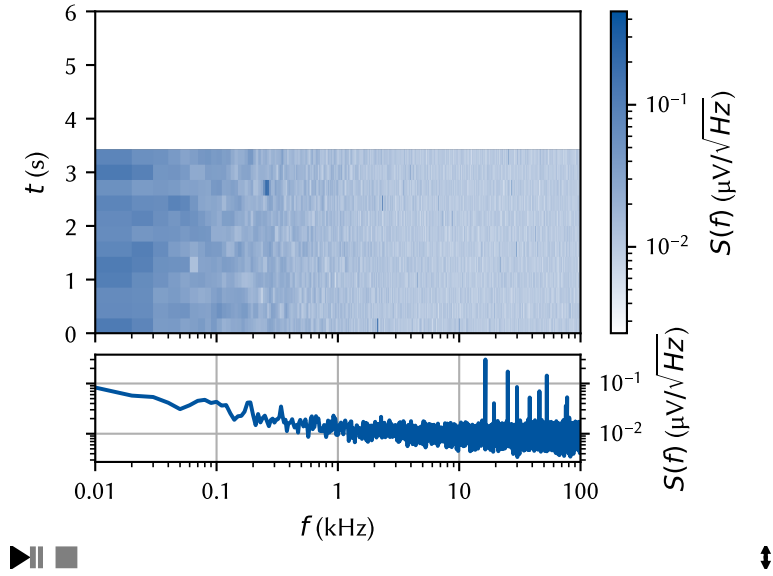
This mode is facilitated by the `qutil.plotting.live_view` module that provides asynchronous plotting functionality based on `matplotlib`. The `live_view` module supports both the multithreading and multiprocessing paradigms for concurrency in order to keep the interpreter responsive. In the former case, the window hosting the figure runs in the main thread and is kept responsive using `matplotlib`'s GUI event loop mechanisms. In the latter, the plotting takes place in a separate process, resulting in true parallelism.

The live mode is started with the `Spectrometer.live_view()` method. Data is continuously acquired<sup>22</sup> in a background thread using the same DAQ interface as the serial mode. Instead of saving the data on disk and managing plotting from `python_spectrometer`, however, the data is fed into a queue.<sup>23</sup> A `live_view.IncrementalLiveView2D` object then retrieves the data from the queue and handles plotting in a GUI event loop. To start a spectroscopy session with the same parameters as in Subsection 3.2.1 with a given `Spectrometer` object (see Listing 3.7), we would call

```
view = spect.live_view(f_min=1e1, f_max=1e5, in_process=True)
```

22: Technically, a very large `n_avg` is used.

23: A queue is a concurrency mechanism for exchanging data between multiple threads or processes.



**Figure 3.8:** Spectrometer live view. The bottom plot shows the most recently acquired spectrum, while the top plot shows a waterfall plot of the most recent ones. Data acquisition runs in a background thread, keeping the interpreter responsive and available to interact with instruments, for example. The icons in the bottom left and right corner allow interacting with the live view.

which would open a figure window such as that shown in Figure 3.8. Similar to `take()`, the data acquisition parameters are passed to `live_view()` as keyword arguments.<sup>24</sup> The `in_process` argument specifies if multiprocessing (`True`) or multithreading (`False`) is used. Dictionaries with customization parameters for the `live_view` object can further be passed to the method.

24: The difference is, of course, that we do not need to specify a comment since no data is retained.



# Conclusion and outlook

## 4

In this part, I presented the `python_spectrometer` Python package for interactive, backend-agnostic spectrum acquisition. I first introduced the theory behind spectral noise estimation in Chapter 2. There, I discussed

**Outlook.** There are several possible avenues for future development of the `python_spectrometer` package. An obvious case is adding support for more DAQ hardware instruments by implementing DAQ interfaces. Modular devices such as those offered by QBLOX<sup>1</sup> and Quantum Machines<sup>2</sup> are on track to become the new standard in quantum technology labs. Implementing drivers for these instruments would benefit the adoption of both the instruments and the `python_spectrometer` package. Another valuable addition would be to add support for generic instruments abstracted by QuMADA [21]. QuMADA is a QCoDeS-based measurement framework that provides a unified interface to instruments to, in a similar spirit to `python_spectrometer`'s DAQ interface, abstract away internals of individual instruments and provide users with a standardized way to interact with them. This approach should naturally lend itself to a single implementation of the DAQ class supporting various instruments through the unified QuMADA interface.

Next, incorporating noise spectroscopy into the standard measurement workflow of quantum device experiments would allow experimentalists to quickly gauge noise levels as they are performing measurements. If for some reason the noise changed<sup>3</sup> the experimentalist could quickly obtain insight into the noise by analyzing the spectrum. In a client-server architecture, which is inherently asynchronous, such as Zurich Instrument's LabOne<sup>4</sup> software, this is already possible using the web interface. But of course the strength of the `python_spectrometer` package stems from its capacity to be utilized in conjunction with any hardware instrument.

One way to implement such functionality would be to introduce a proxy DAQ subclass to be used together with the live mode presented in Subsection 3.2.2. This proxy class would serve as an interface to external measurement software and expose two attributes; first, a data queue, into which the external code could place arbitrary time series data that was obtained during some measurement, and second, a shared dictionary to hold acquisition parameters as these might change between measurements. Because the live view mode runs in the background, the external measurement framework could push data to the queue whenever new data was taken without obstructing the measurement workflow.

Listing 4.1 shows a template design for such a `TeeDAQ` class. The `setup()` method ignores the input parameters and instead obtains the current settings from the shared settings proxy. Similarly, instead of fetching data from an instrument itself, the `acquire()` method attempts to fetch data from the shared `data_queue` and blocks the thread if no data is present, thereby efficiently idling and consuming no resources unless triggered by the external caller. A measurement framework would then interact with the `TeeDAQ` object as exemplarized by the following code:

```
daq = TeeDAQ(...)
spect = Spectrometer(daq)
view = spect.live_view()
```

1: <https://www.qblox.com/research>  
2: <https://www.quantum-machines.co/>

3: As it happens often, unfortunately.

4: <https://www.zhinst.com/ch/en/instruments/labone/labone-instrument-control-software>

```
# daq/tee.py
import dataclasses
import threading
import multiprocessing as mp

from .base import DAQ

@dataclasses.dataclass
class TeeDAQ(DAQ):
    settings:
        ↪ mp.managers.DictProxy
    data_queue: mp.JoinableQueue
    stop_event: threading.Event

    def setup(self, **_):
        settings = self.
        ↪ DAQSettings(self.
        ↪ settings)
        return settings.
        ↪ to_consistent_dict()

    def acquire(self, **_):
        while not self.
        ↪ stop_event.is_set():
            yield self.
            ↪ data_queue.
            ↪ get(block=True)
```

**Listing 4.1:** Template design for a proxy DAQ implementation to stream noise spectra from an external measurement framework. The `settings` attribute is a dictionary proxy shared between processes and used to pass acquisition parameters from the measurement framework



```
...
data = measure(fs, n_pts)
daq.settings.update(fs=fs, n_pts=n_pts)
daq.data_queue.put(data)
```

Measurement frameworks integrating with this interface could thus provide experimentalists live feedback on current noise levels with negligible overhead and minimal code adaptation.

5: Again, we use the two terms interchangeably unless otherwise indicated, see sidenote 5 in Chapter 2.

Finally, it might be useful to not only allow estimating PSDs but also cross power spectral densities (CSDs) or *cross-spectra*. The cross-spectrum<sup>5</sup> is the Fourier transform of not the auto-correlation but the cross-correlation function  $C(\tau)$  (Equation 2.4) between two random processes. Take a set of processes  $\{x_1(t), x_2(t), \dots, x_n(t)\}$  that correspond to noise measured at different locations in a sample. The cross-correlation function between variables  $x_i$  and  $x_j$  is then given by<sup>6</sup>

$$C_{ij}(\tau) = \langle x_i(t)^* x_j(t + \tau) \rangle. \quad (4.1)$$

This function (and its Fourier pair the cross-spectrum  $S_{ij}(\omega)$ ) quantifies the degree of correlation between noise at site  $i$  and noise at site  $j$ . Unlike the *auto-spectrum* (or self-spectrum), the cross-spectrum is always a complex quantity, even for real  $x_i(t)$ . It is not hard to see that for quantum processors, for example, these kinds of correlations could have significant impact on operation, and on error correction in particular [22–24]. To incorporate cross-spectra in the `python_spectrometer` package, only small changes should be necessary.

7: If one sticks to single DAQ instances managing single instruments.

First, the data acquisition logic would need to be adapted. Two possible routes suggest themselves here; first, specialized DAQ classes could be implemented that, in place of yielding one batch of time series data, yield two batches each time they are queried. This approach first of all requires instruments with multiple channels,<sup>7</sup> which is not necessarily given. Furthermore, it would incur additional coding efforts by having to re-implement each DAQ class for cross-spectra. On the other hand, it would arguably make synchronization between channels easier to achieve.

8: Except possibly ensuring thread safety and timing synchronization if multiple DAQ objects communicate with the same physical instrument.

A less involved path would adapt the `Spectrometer` class to work with multiple DAQs. This would not involve additional driver work<sup>8</sup> and allow the `Spectrometer` object to ensure synchronization between the different DAQs. The internal workflow shown in Listing 3.3 would then need to be slightly adapted to the code shown in Listing 4.2. The downside of this approach is that synchronization of different instruments or channels would need to be taken care of externally.

**Listing 4.2:** Proposed DAQ workflow for estimating cross-spectra. Each hardware channel (same or different instruments) is assigned to a DAQ object. After instrument configuration, it is asserted that the parameters match. Finally, data is fetched from both channels and fed into a CSD estimator. Note that triggering would need to be implemented externally.

```
daq_1 = MyDAQ(driver_handle, channel=1)
# Or MyOtherDAQ(driver_handle_2) if another instrument
daq_2 = MyDAQ(driver_handle, channel=2)
daqs = (daq_1, daq_2)

parsed_settings = [daq.setup(**user_settings) for daq in daqs]
assert all_equal(parsed_settings), "DAQ settings do not match"

acquisition_generators = [daq.acquire(**parsed_settings[0])
                          for daq in daqs]
for data_buffers in zip(*acquisition_generators):
    estimate_csd(*data_buffers)
```

Further code adaptations would involve minor changes such as replacing the spectrum estimator with `scipy.signal.csd()` [25] for CSD estima-

[25]: (n.d.), *csd* — SciPy v1.15.2 Manual



tion<sup>9</sup> and make the plotting conform to complex data.<sup>10</sup>

9: In practice, working with the normalized CSD, or correlation coefficient [26, 27]

$$r_{ij}(\omega) = \frac{S_{ij}(\omega)}{\sqrt{S_i(\omega)S_j(\omega)}} \quad (4.2)$$

with  $S_i(\omega)$  the PSD of process  $x_i$  would likely be more favorable.

10: It could be worthwhile to add a subplot to display both the magnitude and phase of the complex quantity  $S_{ij}(\omega)$ .

improve this part.



## **Part II**

# **CHARACTERIZATION AND IMPROVEMENTS OF A MILLIKELVIN CONFOCAL MICROSCOPE**



### **Part III**

## **ELECTROSTATIC TRAPPING OF EXCITONS IN SEMICONDUCTOR MEMBRANES**



## **Part IV**

# **A FILTER-FUNCTION FORMALISM FOR UNITAL QUANTUM OPERATIONS**





# **APPENDIX**



# Bibliography

- [1] A. A. Clerk et al. “Introduction to Quantum Noise, Measurement, and Amplification.” In: *Rev. Mod. Phys.* 82.2 (Apr. 15, 2010), pp. 1155–1208. doi: [10.1103/RevModPhys.82.1155](https://doi.org/10.1103/RevModPhys.82.1155). (Visited on 01/19/2022) (cited on page 5).
- [2] Gerardo A. Paz-Silva, Leigh M. Norris, and Lorenza Viola. “Multiqubit Spectroscopy of Gaussian Quantum Noise.” In: *Physical Review A* 95.2 (Feb. 23, 2017), p. 022121. doi: [10.1103/PhysRevA.95.022121](https://doi.org/10.1103/PhysRevA.95.022121) (cited on page 5).
- [3] “Root-Mean-Square Value.” In: *A Dictionary of Physics*. Oxford University Press, 2009. (Visited on 04/17/2025) (cited on page 5).
- [4] Jan Krzywda and Łukasz Cywiński. “Adiabatic Electron Charge Transfer between Two Quantum Dots in Presence of  $1/f$  Noise.” In: *Phys. Rev. B* 101.3 (Jan. 15, 2020), p. 035303. doi: [10.1103/PhysRevB.101.035303](https://doi.org/10.1103/PhysRevB.101.035303). (Visited on 01/19/2022) (cited on page 5).
- [5] Josef Schrieffer et al. “Decoherence from Ensembles of Two-Level Fluctuators.” In: *New J. Phys.* 8 (Jan. 20, 2006), pp. 1–1. doi: [10.1088/1367-2630/8/1/001](https://doi.org/10.1088/1367-2630/8/1/001) (cited on page 5).
- [6] Félix Beaudoin and W. A. Coish. “Microscopic Models for Charge-Noise-Induced Dephasing of Solid-State Qubits.” In: *Physical Review B* 91.16 (Apr. 29, 2015), p. 165432. doi: [10.1103/PhysRevB.91.165432](https://doi.org/10.1103/PhysRevB.91.165432) (cited on page 5).
- [7] M. Mehmandoust and V. V. Dobrovitski. “Decoherence Induced by a Sparse Bath of Two-Level Fluctuators: Peculiar Features of  $1/f$  Noise in High-Quality Qubits.” In: *Phys. Rev. Res.* 6.3 (Aug. 15, 2024), p. 033175. doi: [10.1103/PhysRevResearch.6.033175](https://doi.org/10.1103/PhysRevResearch.6.033175). (Visited on 08/20/2024) (cited on page 5).
- [8] Lambert Herman Koopmans. *The Spectral Analysis of Time Series*. 2nd ed. Vol. 22. Probability and Mathematical Statistics. San Diego: Academic Press, 1995 (cited on pages 6, 9).
- [9] G. E. Uhlenbeck and L. S. Ornstein. “On the Theory of the Brownian Motion.” In: *Phys. Rev.* 36.5 (Sept. 1, 1930), pp. 823–841. doi: [10.1103/PhysRev.36.823](https://doi.org/10.1103/PhysRev.36.823). (Visited on 07/23/2024) (cited on page 7).
- [10] N. G Van Kampen. “Stochastic Differential Equations.” In: *Physics Reports* 24.3 (Mar. 1, 1976), pp. 171–228. doi: [10.1016/0370-1573\(76\)90029-6](https://doi.org/10.1016/0370-1573(76)90029-6). (Visited on 07/23/2024) (cited on page 7).
- [11] N. R. Lomb. “Least-Squares Frequency Analysis of Unequally Spaced Data.” In: *Astrophys Space Sci* 39.2 (Feb. 1, 1976), pp. 447–462. doi: [10.1007/BF00648343](https://doi.org/10.1007/BF00648343). (Visited on 04/10/2025) (cited on page 7).
- [12] J. D. Scargle. “Studies in Astronomical Time Series Analysis. II. Statistical Aspects of Spectral Analysis of Unevenly Spaced Data.” In: *The Astrophysical Journal* 263 (Dec. 1, 1982), pp. 835–853. doi: [10.1086/160554](https://doi.org/10.1086/160554). (Visited on 04/10/2025) (cited on page 7).
- [13] M. S. Bartlett. “Smoothing Periodograms from Time-Series with Continuous Spectra.” In: *Nature* 161.4096 (May 1948), pp. 686–687. doi: [10.1038/161686a0](https://doi.org/10.1038/161686a0). (Visited on 03/26/2025) (cited on page 8).
- [14] P. Welch. “The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging over Short, Modified Periodograms.” In: *IEEE Transactions on Audio and Electroacoustics* 15.2 (June 1967), pp. 70–73. doi: [10.1109/TAU.1967.1161901](https://doi.org/10.1109/TAU.1967.1161901) (cited on pages 8, 10).
- [15] F.J. Harris. “On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform.” In: *Proceedings of the IEEE* 66.1 (Jan. 1978), pp. 51–83. doi: [10.1109/PROC.1978.10837](https://doi.org/10.1109/PROC.1978.10837). (Visited on 03/27/2025) (cited on pages 9, 19).
- [16] *Window Function*. In: *Wikipedia*. Mar. 20, 2025. (Visited on 03/27/2025) (cited on page 9).
- [17] A. Nuttall. “Some Windows with Very Good Sidelobe Behavior.” In: *IEEE Transactions on Acoustics, Speech, and Signal Processing* 29.1 (Feb. 1981), pp. 84–91. doi: [10.1109/TASSP.1981.1163506](https://doi.org/10.1109/TASSP.1981.1163506). (Visited on 03/27/2025) (cited on page 9).
- [18] *welch* — *SciPy v1.15.2 Manual*. URL: <https://docs.scipy.org/doc/scipy/reference/generated/scipy.signal.welch.html> (visited on 03/31/2025) (cited on pages 13–15).
- [19] *octave\_band\_rms* — *Qutil 2025.3.1 Documentation*. URL: [https://qutech.pages.rwth-aachen.de/qutil/\\_autogen/qutil.signal\\_processing.fourier\\_space.octave\\_band\\_rms.html](https://qutech.pages.rwth-aachen.de/qutil/_autogen/qutil.signal_processing.fourier_space.octave_band_rms.html) (visited on 04/03/2025) (cited on page 16).

- [20] David M. Pozar. *Microwave Engineering*. Fourth edition. Hoboken, NJ: John Wiley & Sons, Inc, 2012. 1 p. (cited on page 18).
- [21] Till Huckemann. PhD thesis. Aachen: RWTH Aachen University, 2025 (cited on page 23).
- [22] Dorit Aharonov, Alexei Kitaev, and John Preskill. “Fault-Tolerant Quantum Computation with Long-Range Correlated Noise.” In: *Phys. Rev. Lett.* 96.5 (Feb. 7, 2006), p. 050504. DOI: [10.1103/PhysRevLett.96.050504](https://doi.org/10.1103/PhysRevLett.96.050504). (Visited on 01/19/2022) (cited on page 24).
- [23] Naomi H. Nickerson and Benjamin J. Brown. “Analysing Correlated Noise on the Surface Code Using Adaptive Decoding Algorithms.” In: *Quantum* 3 (Apr. 8, 2019), p. 131. DOI: [10.22331/q-2019-04-08-131](https://doi.org/10.22331/q-2019-04-08-131) (cited on page 24).
- [24] B. D. Clader et al. “Impact of Correlations and Heavy Tails on Quantum Error Correction.” In: *Phys. Rev. A* 103.5 (May 24, 2021), p. 052428. DOI: [10.1103/PhysRevA.103.052428](https://doi.org/10.1103/PhysRevA.103.052428). (Visited on 02/23/2022) (cited on page 24).
- [25] *csd — SciPy v1.15.2 Manual*. URL: <https://docs.scipy.org/doc/scipy/reference/generated/scipy.signal.csd.html> (visited on 04/15/2025) (cited on page 24).
- [26] J.S. Rojas-Arias et al. “Spatial Noise Correlations beyond Nearest Neighbors in  $^{28}\text{Si}/\text{Si-Ge}$  Spin Qubits.” In: *Phys. Rev. Appl.* 20.5 (Nov. 9, 2023), p. 054024. DOI: [10.1103/PhysRevApplied.20.054024](https://doi.org/10.1103/PhysRevApplied.20.054024). (Visited on 04/14/2025) (cited on page 25).
- [27] J. Yoneda et al. “Noise-Correlation Spectrum for a Pair of Spin Qubits in Silicon.” In: *Nat. Phys.* (Oct. 9, 2023). DOI: [10.1038/s41567-023-02238-6](https://doi.org/10.1038/s41567-023-02238-6). (Visited on 10/17/2023) (cited on page 25).

# Special Terms

## A

**API** Application Programming Interface. 17

**ASD** amplitude spectral density. 17

## C

**CSD** cross power spectral density. 24, 25

## D

**DAQ** data acquisition. 10, 13–15, 17, 23

**DUT** device under test. 5

## E

**ENBW** equivalent noise bandwidth. 19

## F

**FFT** fast Fourier transform. 10

## L

**LIA** lock-in amplifier. 5

## P

**PSD** power spectral density. 5–8, 10, 13, 15, 17–19, 24, 25

## R

**RMS** root mean square. 5, 7, 18, 19

## T

**TIA** transimpedance amplifier. 15

**TLF** two-level fluctuator. 5