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The harmony of the world is made manifest in Form and Number, and the heart and soul and all the poetry of Natural Philosophy are embodied in the concept of mathematical beauty.

– D’Arcy Wentworth Thompson

Contents

A flexible Python tool for Fourier-transform noise spectroscopy

Introduction 1

Noise is ubiquitous in condensed matter physics experiments, and in mesoscopic systems in particular it can easily drown out the sought-after signal. Hence, characterizing (and subsequently mitigating) noise is an essential task for the experimentalist. But noise comes in as many different forms as there are types of signal sources and detectors, whether it be a voltage source or a photodetector, and while some instruments have built-in solutions for noise analysis, they vary in functionality and capability. Moreover, the measured signal often does not directly correspond to the noisy physical quantity of interest, making it desirable to be able to manipulate the raw data before processing.

Theory of noise estimation | 2

There exists a multitude of methods for estimating noise properties.

lay out some others

If the noisy process $x(t)$[1] has Gaussian statistics, meaning that the amplitude at a given frequency follows a normal distribution with some mean μ and variance σ^2, it can be fully described by the power spectral density (PSD) $S(\omega)$.[2] Given the windowed Fourier transform of the process $x(t)$,[3]

$$\hat{x}(\omega) = \frac{1}{\sqrt{T}} \int_{-T/2}^{+T/2} \mathrm{d}t\, \mathrm{e}^{\mathrm{i}\omega t} x(t) \tag{2.1}$$

one defines the PSD as the ensemble average [1]

$$S(\omega) = \lim_{T\to\infty} \langle \hat{x}(\omega)^* \hat{x}(\omega) \rangle \,. \tag{2.2}$$

Using Equation 2.1, the argument of the limit in Equation 2.2 becomes

$$\begin{aligned} S(\omega) &= \lim_{T\to\infty} \frac{1}{T} \int_0^T \mathrm{d}t \int_0^T \mathrm{d}t'\, \mathrm{e}^{\mathrm{i}\omega(t'-t)} \langle x(t)^* x(t') \rangle & (2.3)\\ &= \lim_{T\to\infty} \frac{1}{T} \int_0^T \mathrm{d}t \int_{-t}^{T-t} \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\omega\tau} \langle x(t)^* x(t+\tau) \rangle & (2.4)\\ &= \lim_{T\to\infty} \frac{1}{T} \int_0^T \mathrm{d}t \int_{-t}^{T-t} \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\omega\tau} \langle x(t-\tau/2)^* x(t+\tau/2) \rangle \,. & (2.5) \end{aligned}$$

If the observation time T is much longer than the characteristic correlation time τ_c of $x(t)$ over which the auto-correlation function $\langle x(0)x(\tau)\rangle$ decays,

For the purpose of noise estimation, the assumption of Gaussianity is a rather weak one as the noise typically arises from a large ensemble of individual fluctuators and is therefore well approximated by a Gaussian distribution by the central limit theorem.[4] Even if the process $x(t)$ is not perfectly Gaussian, non-Gaussian contributions can be seed as higher-order contributions if viewed from the perspective of perturbation theory, and therefore the PSD still captures a large part of the statistical properties. For this reason, the PSD is the central quantity of interest in noise spectroscopy and I will discuss some of its properties in the following.

For real signals $x(t) \in \mathbb{R}$, $S(\omega)$ is an even function and one therefore distinguishes the *two-sided* PSD $S^{(2)}(\omega)$ from the *one-sided* PSD $S^{(1)}(\omega) = 2S^{(2)}(\omega)$ defined only over $\mathbb{R}^+$. Complex signals $x(t) \in \mathbb{C}$ such as those generated by Lock-in amplifiers after demodulation in turn have an even real and an odd imaginary part.

for,for,for

1: We discuss only classical noise here, meaning $x(t)$ commutes with itself at all times. For descriptions of and spectroscopy protocols for quantum noise, refer to Refs. 1 and 2, for example.

2: As we will see below, a further necessary condition for this is that the process $x(t)$ is *wide-sense stationary*, which means that the autocorrelation function $\langle x(t)^* x(t')\rangle = \langle x(t)^* x(t+\tau)\rangle$ with $\tau = t' - t$. That is, it is a function of only the time lag τ and not the absolute point in time. For simplicity, we therefore set $t \equiv 0$ below.

maybe a classical signal processing ref?

3: The windowing arises naturally when observing the process $x(t)$ for a finite amount of time T as one would do in an experiment. Mathematicians might at this point argue the integrability of $x(t)$, but as we deal with physical processes and have no shame, we do not.

4: As an example, consider electronic devices, where voltage noise arises from a large number of defects and other charge traps in oxides being populated and depopulated at certain rates γ. The ensemble average over these so-called two-level fluctuators (TLFs) then yields the well-known $1/f$-like noise spectra.

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A Filter-Function Formalism For Quantum Operations

Appendix

Bibliography

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Special Terms

P

PSD power spectral density. 3

T

TLF two-level fluctuator. 3