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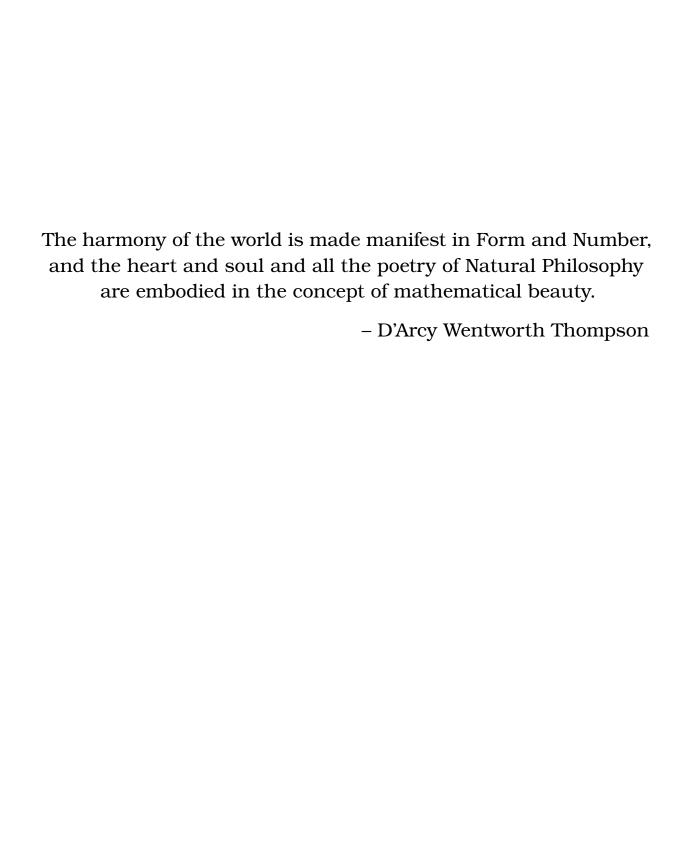
# Example and documentation of the kaobook class

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# CLASS OPTIONS, COMMANDS AND ENVIRONMENTS



### Filter Functions

For a given quantum operation  $\tilde{\mathcal{U}}$  resulting from the quantum system's evolution under the noise fully characterized by its one-sided power spectral density (PSD)  $S(\omega)$ , we define the filter function (FF)  $\mathscr{F}(\omega;\tau)$  by

$$\tilde{\mathcal{U}}(\tau) = \exp \int \frac{\mathrm{d}\omega}{2\pi} \mathcal{F}(\omega; \tau) S(\omega). \tag{1.1}$$

Now, suppose that

$$S_{\omega_i}(\omega) = \sigma_i^2 \delta(\omega - \omega_i),$$
 (1.2)

that is, the PSD of a monochromatic sinusoid of frequency  $\omega_i$  and root mean square (RMS)  $\sigma_i^{2*}$ . Then Equation 1.1 becomes

$$\begin{split} \tilde{\mathcal{U}}_{\omega_{i}}(\tau) &= \exp\left\{\sigma_{i}^{2} \int \frac{\mathrm{d}\omega}{2\pi} \mathcal{F}(\omega;\tau) \delta(\omega - \omega_{i})\right\} \\ &= \exp\left\{\frac{\sigma_{i}^{2}}{2\pi} \mathcal{F}(\omega_{i};\tau)\right\}, \end{split} \tag{1.4}$$

where  $\tilde{\mathcal{U}}_{\omega_i}(\tau)$  is the noisy quantum operation generated by monochromatic noise with PSD  $S_{\omega_i}(\omega)$  according to Equation 1.2. It is now easy to invert Equation 1.4, and we obtain

$$\mathcal{F}(\omega_i; \tau) = \frac{2\pi}{\sigma_i^2} \log \tilde{\mathcal{U}}_{\omega_i}(\tau). \tag{1.5}$$

Because we represent quantum operations as matrices in Liouville space, Equation 1.5 is easy to implement on a computer; we simply need to evaluate  $\tilde{\mathcal{U}}_{\omega_i}(\tau)$  for a set of discrete frequencies  $\{\omega_i\}_i$  using Monte Carlo (MC) and take the logarithm! [Geck2021].

Indeed, we can go a step further and split apart the coherent and incoherent contributions to the noisy evolution. Since (in-)coherent quantum operations are represented by (anti-)symmetric matrices in Liouville space, we may define the incoherent and coherent FFs by

$$\begin{split} \mathcal{F}_{\Gamma}(\omega;\tau) &= \frac{1}{2} \left( \mathcal{F}(\omega;\tau) + \mathcal{F}(\omega;\tau)^T \right) \\ &= \frac{\pi}{\sigma_i^2} \left( \log \tilde{\mathcal{U}}_{\omega}(\tau) + \log \tilde{\mathcal{U}}_{\omega}(\tau)^T \right), \end{split} \tag{1.6}$$

$$S(\omega) = \lim_{n \to \infty} \sum_{i=1}^{n} S_{\omega_i}(\omega). \tag{1.3}$$

<sup>\*</sup> Equation 1.2 discretizes  $S(\omega)$  by sampling it at points  $\omega_i$ , i.e.,

and

$$\begin{split} \mathcal{F}_{\Delta}(\omega;\tau) &= \frac{1}{2} \left( \mathcal{F}(\omega;\tau) - \mathcal{F}(\omega;\tau)^T \right) \\ &= \frac{\pi}{\sigma_i^2} \left( \log \tilde{\mathcal{U}}_{\omega}(\tau) - \log \tilde{\mathcal{U}}_{\omega}(\tau)^T \right), \end{split} \tag{1.7}$$

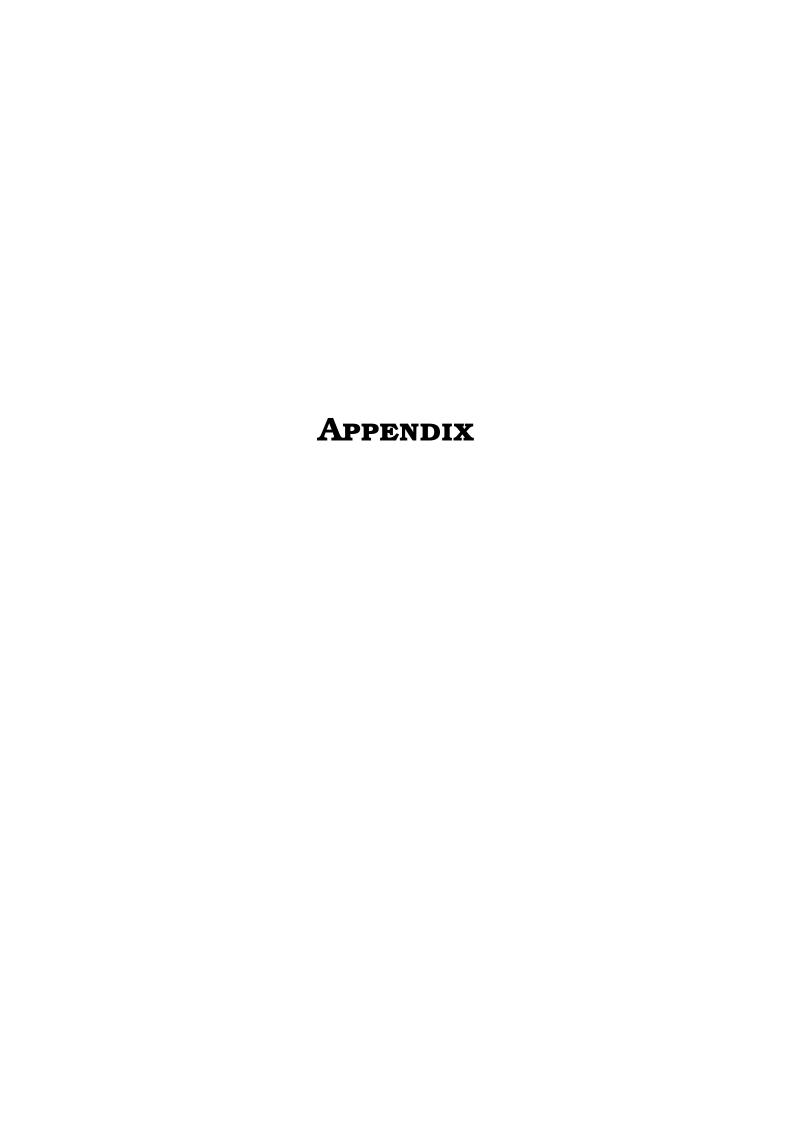
respectively.

From a MC point of view,  $\tilde{\mathcal{U}}$  is given by

$$\langle \tilde{\mathcal{U}} \rangle (\tau) = \mathcal{Q}^T \langle \mathcal{U}(\tau) \rangle,$$
 (1.8)

where  $\mathcal{U}(\tau)$  is the solution of the Schrödinger equation for a single realization of the noise. Solving Equation 1.1 for  $\mathcal{F}(\omega;\tau)$ , we find

$$\mathscr{F}(\omega;\tau) = \int \frac{d\omega'}{2\pi} \delta(\omega - \omega') \log \langle \tilde{\mathscr{U}}(\tau) \rangle$$
 (1.9)



# Test A

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