

Final Exam vQ**Math 18, Section A****June 14, 2017****Time Limit: 180 Minutes****Name (Print):** _____**PID:** _____

- **DO NOT begin working, or even open this packet, until instructed to do so.**
- You should be in your assigned seat, unless instructed otherwise by Ed or one of the TAs.
- Enter all requested information on the top of this page, and put your name on the top of every page, in case the pages become separated.
- You may use a two-sided page of notes on this exam.
- You may **not** use your books, additional notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** Unless otherwise directed in the statement of the problem, a correct answer, unsupported by calculations, explanation, or algebraic work will receive little or no credit. An incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Question	Points
1	10
2	8
3	10
4	8
5	10
6	14
7	10
8	10
Total:	80

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1. (10 points) Let

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 3 \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Let W be the set of all vectors $\vec{w} \in \mathbb{R}^3$ such that $A\vec{w} \in \text{span}\{\vec{y}\}$. W is a subspace of \mathbb{R}^3 (you do not need to prove this). Find a basis for W .

2. (8 points) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}$ be the linear transformation

$$T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \det \begin{bmatrix} 2 & x_1 + x_2 & 0 \\ -1 & x_3 & 0 \\ 2 & 3 & 2 \end{bmatrix}$$

(You do not need to prove that T is linear). Find the matrix corresponding to T .

3. Let $A = \begin{bmatrix} 0 & 1 & 1 \\ 3 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix}$. Let $\mathcal{E} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ and let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \right\}$. \mathcal{E} is a

basis for \mathbb{R}^3 , and so is \mathcal{B} .

- (a) (4 points) Find a basis \mathcal{C} such that $A = P_{\mathcal{E} \leftarrow \mathcal{C}}$. (Recall that $P_{\mathcal{E} \leftarrow \mathcal{C}}$ is the change-of-coordinates matrix from \mathcal{C} to \mathcal{E}).

- (b) (6 points) Find a basis \mathcal{D} such that $A = P_{\mathcal{B} \leftarrow \mathcal{D}}$.

4. (8 points) Let V be the vector space of 2×2 matrices, and let $T : V \rightarrow \mathbb{R}^5$ be some linear transformation. Let k be the dimension of the range of T . Suppose that

$$T\left(\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad T\left(\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad T\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

This is not enough information to determine k . What is the biggest that k could be? And what is the smallest that k could be?

5. Let V be a vector space with two bases $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ and $\mathcal{C} = \{\vec{c}_1, \vec{c}_2, \vec{c}_3\}$. Let $T : V \rightarrow V$ be a linear transformation. Let M be the matrix of T relative to \mathcal{B} (also known as the \mathcal{B} -matrix of T). Suppose

$$M = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \\ 2 & 0 & -1 \end{bmatrix} \qquad {}_{\mathcal{C} \leftarrow \mathcal{B}} P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

(Recall that ${}_{\mathcal{C} \leftarrow \mathcal{B}} P$ is the change of coordinates matrix from \mathcal{B} to \mathcal{C} .)

- (a) (6 points) Find $T(\vec{c}_2)$.

- (b) (4 points) Find a matrix A with $A[\vec{v}]_{\mathcal{B}} = [T(\vec{v})]_{\mathcal{C}}$ for any $\vec{v} \in V$.

6. (a) (5 points) Find the eigenvalues of $\begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix}$.

(b) (5 points) Find the eigenspace of $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$ corresponding to the eigenvalue 2.

(c) (4 points) Let B be a 6×6 matrix with characteristic polynomial $(2 - \lambda)^2(5 - \lambda)^4$ and let I be the 6×6 identity matrix. If B is not similar to a diagonal matrix, what can you conclude about $\dim \text{Nul}(B - 2I)$ and $\dim \text{Nul}(B - 5I)$?

7. (10 points) Let $\vec{x} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ and let $W = \text{span}\{\vec{x}\} \subset \mathbb{R}^3$. Find an orthogonal basis for W^\perp .

8. (10 points) Let A be a matrix with the following QR-factorization:

$$A = \begin{bmatrix} 0 & \sqrt{1/3} \\ 1/\sqrt{2} & -\sqrt{1/3} \\ 1/\sqrt{2} & \sqrt{1/3} \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$$

Find the distance from $(3, 1, 1)$ to $\text{Col } A$.