Problem 1: $\vec{F}(x,y,z) = (3x^2z,\ z^2,\ x^3 + 2yz)$

$\vec{c}(t) = (\ln t/\ln 2,\ t^{3/2},\ t\cos(\pi t))$ for $1 \le t \le 2$

Sol. • First try: direct approach. $\Rightarrow$ hard, since we will get a complicated integral.

• $\Rightarrow$ Try different approach. Is $\vec{F}$ conservative? Yes!

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2z & z^2 & x^3+2yz \end{vmatrix} = (2z-2z, -(3x^2-3x^2), 0) = (0,0,0).$$

$\Rightarrow$ we need to find f such that $\nabla f = \vec{F}$.

$\boxed{\frac{\partial f}{\partial x} = 3x^2z}$

$\boxed{\frac{\partial f}{\partial y} = z^2}$

$\frac{\partial f}{\partial z} = x^3 + 2yz \Rightarrow f(x,y,z) = x^3z + yz^2 + h(x,y)$

$f = \int (x^3+2yz)\,dz$

$\therefore \frac{\partial f}{\partial y} = z^2 + \frac{\partial h}{\partial y} = z^2$

$h(x,y)$

$\Rightarrow \frac{\partial h}{\partial y} = 0 \Rightarrow h = g(x)$

$\Rightarrow f(x,y,z) = x^3z + yz^2 + g(x)$

$\frac{\partial f}{\partial x} = 3x^2z + 0 + g'(x) = 3x^2z$

$g'(x) = 0$

$g(x) = 0.$

$\Rightarrow f(x,y,z) = x^3z + yz^2$

By FTOLI:

$$\int_C \vec{F}\cdot d\vec{s} = f(\vec{c}(2)) - f(\vec{c}(1)) = f(1, 2^{3/2}, 2) - f(0,1,-1)$$
$$= 2 + 2^{3/2}(4) - 0 - 1$$
$$= 1 + 8\sqrt{2}$$