Things you should know:

1) After mid 2 material:

- Eigenvalues & Eigenvectors of matrices.
- If $A\vec{v}_i = \lambda_i \vec{v}_i$ and $\lambda_1, \dots, \lambda_n$ are distinct.
  $\{\vec{v}_1, \dots, \vec{v}_n\}$ are linearly independent.
- Characteristic polynomial of $n \times n$ matrix A:

$$p_A(\lambda) = \det(A - \lambda I).$$

  $\Rightarrow$ know how to compute $p_A(\lambda)$
  how to find roots of $p_A(\lambda)$
  A is diagonalizable if all roots of $p_A(\lambda)$ are distinct.
  If there are repeated roots, A may or may not be diagonalizable.
- Know how to find eigenvector / eigenbasis of an eigenvalue, i.e. find a basis for $\text{Nul}(A - \lambda I)$.
- Definition of a diagonalizable $n \times n$ matrix A.
  A is diagonalizable $\Leftrightarrow$ algebraic multiplicity of λ_i
  $=$ geometric multiplicity of λ_i.
  for all λ_i

If A is diagonalizable and $A\vec{v}_i = \lambda_i \vec{v}_i$.

$$P = [\vec{v}_1 \dots \vec{v}_n].$$

$$D = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix}$$

$$A = PDP^{-1}.$$

$$\Rightarrow A^k = PD^kP^{-1} \quad , \text{where } D^k = \begin{bmatrix} \lambda_1^k & & & \\ & \lambda_2^k & & \\ & & \ddots & \\ & & & \lambda_n^k \end{bmatrix}.$$

- Know definition of dot/inner product; norm or length of a vector. Know the distance between two vectors.

- ~~Def~~ Meaning of orthogonal sets, orthonormal sets, ~~org~~ orthogonal bases, orthonormal bases.
- Orthogonal projections.

  If $B = \{\vec{u}_1, \dots, \vec{u}_n\}$ orthogonal basis for subspace $W \subset \mathbb{R}^m$.

$$\mathrm{proj}_W \vec{v} = \sum_{i=1}^{n} \frac{\vec{v}\cdot\vec{u}_i}{\|u_i\|^2} \vec{u}_i$$

- If $U = [\vec{u}_1 \dots \vec{u}_n]$ is an $n \times n$ matrix, the following are equivalent:

  i) U is an orthogonal matrix

  $$U^T U = I = U U^T.$$

  ii) the columns of U form an orthonormal basis for $\mathbb{R}^n$.

- Gram-Schmidt process.
- QR Decomposition.

2) Midterm 2 review material.

- Vector spaces?
- Subspaces and important subspaces (e.g. $\mathbb{P}_n$ = polynomials of degree at most n.)
- How to determine if a set $H \subset V$ a subspace or not.
- linear independence, span of vectors.
  basis, dimension, and coordinates relative to a basis.
- Can check if vectors in $\mathbb{R}^m$ are a basis or not by putting the vectors in the columns of a matrix and row reducing.
  - if $n > m$ the number of vectors, they are linearly dependent.
  - if $n < m$, they ~~dee~~ do not span $\mathbb{R}^m$.

- Matrix operations.
- Notion of linear transformation. $T: V \to W$.
  - $Nul(T) = \{\vec{v} \in V : T(\vec{v}) = \vec{0}\}$.
  - T is one-to-one iff $Nul(T) = \{0\}$.
  - if $T(x) = Ax$, be able to find a basis for $Nul(A)$
  - $Ran(T) = \{T(\vec{v}) \in W : \vec{v} \in V\}$.
  - Know how to find a basis for $Col(A)$, $Row(A)$.
    $\dim Row(A) = \dim Col(A) = rank(A)$.
- Understand the rank-nullity theorem
  Theorem 14, p. 233.

- Thm: if A not square, A is not invertible.
- Thm: if A $n \times n$, the following are equivalent:
  - a) A invertible.
  - b) $Col(A) = \mathbb{R}^n$ $\Leftrightarrow$ $\dim Row\, A = n$.
  - c) $Row(A) = \mathbb{R}^n$.
  - d) $Nul(A) = \{0\}$.
  - e) $\det(A) \neq 0$
  - f) $RREF(A) = I$.

- Can find A^{-1} using $[A|I] \rightsquigarrow [I|A^{-1}]$.
- Know how to compute determinants of matrices. know the determinants behavior under row ~~and~~ operations.

3) Midterm 1 Review material.

$A\vec{x} = \vec{b}$

- Thm: if $A\vec{p} = \vec{b}$, then general solution to $A\vec{x} = \vec{b}$ is $\vec{x} = \vec{p} + \vec{v}_h$, where $\vec{v}_h$ is the generic solution to $A\vec{v}_h = \vec{0}$.

- Linear independence/ dependence.
- Know how to get REF and RREF.
- Can write down the general solution to $A\vec{x} = \vec{b}$ and ~~condi~~ find out the condition of $\vec{b}$ such that $A\vec{x} = \vec{b}$ consistent / inconsistent.

Thm: Let A $m \times n$ and $U = RREF(A)$. Then.

a) $A\vec{x} = \vec{b}$ consistent iff $RREF([A \mid \vec{b}])$ does not have a pivot in the last column.

b) $A\vec{x} = \vec{b}$ consistent
iff $U = RREF(A)$ has a pivot in every row
i.e. U does not contain a row of zeros.

~~c) $m \geq n$~~

c) $A\vec{x} = \vec{b}$ has at most 1 solution
iff $Nul(A) = \{0\}$.
iff $\{\vec{a}_1, \dots, \vec{a}_n\}$ linearly independent.
iff $RREF(A)$ has no free variable