

Midterm 2vP

Math 18, Section A

May 19, 2017

Time Limit: 50 Minutes

Name (Print): \_\_\_\_\_

PID: \_\_\_\_\_

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- **DO NOT** begin working, or even open this packet, until instructed to do so.
- You should be in your assigned seat, unless instructed otherwise by Ed or one of the TAs.
- Enter all requested information on the top of this page, and put your name on the top of every page, in case the pages become separated.
- You may use a two-sided page of notes on this exam.
- You may **not** use your books, additional notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** Unless otherwise directed in the statement of the problem, a correct answer, unsupported by calculations, explanation, or algebraic work will receive little or no credit. An incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Question	Points
1	10
2	10
3	10
4	10
Total:	40

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1. (10 points) Let  $I$  be the  $3 \times 3$  identity matrix and let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

Find a matrix  $M$  satisfying  $MA = B + I$ .

2. (10 points) Let  $T : \mathbb{R}^{1903} \rightarrow \mathbb{R}^4$  be a linear transformation with range

$$\text{Range } T = \text{span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Find  $\dim \text{Nul } T$ .

3. (10 points) Let  $\mathbb{P}$  be the space of polynomials in  $x$  (with the usual addition and scalar multiplication) and let  $H = \text{span}\{x, x^2, x^5\} \subset \mathbb{P}$ . Then  $\mathcal{B} = \{x, x^2, x^5\}$  is a basis for  $H$ , and  $\mathcal{C} = \{x + x^5, x^2, x\}$  is also a basis for  $H$ . Find a matrix  $A$  satisfying the following condition:

- For any  $p(x) \in H$ ,

$$A[p(x)]_{\mathcal{B}} = [p(x)]_{\mathcal{C}}$$

4. (10 points) Suppose that  $S$  is a region of  $\mathbb{R}^3$  whose volume is 3. Let  $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 4 & 6 \\ 0 & 5 & 3 \end{bmatrix}$  and let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the linear transformation  $T(\vec{x}) = A^{2017}\vec{x}$ . Find the volume of  $T(S)$ .