Name: ______________________ **PID:** ______________

TA: ________________ **Sec. No:** _____ **Sec. Time:** _____

Math 18
Exam 2 v. 0: Practice
March 2, 2018

Turn off and put away your cell phone and other electronic devices.
You may use one hand-written standard sheet of notes, but calculators, notes, and other assistance are prohibited during this exam.
Read each question carefully, and answer each question completely.
Show all of your work; no credit will be given for unsupported answers.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

1. Compute the determinant

$$\begin{vmatrix} -2 & -1 & -1 & 0 \\ 4 & 3 & 3 & 2 \\ -4 & -6 & -7 & 0 \\ -1 & -5 & -9 & 0 \end{vmatrix}$$

2. Let $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ -5 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} \right\}$.

(a) Explain why $\mathcal{B}$ is a basis of $\mathbb{R}^3$.

(b) Let $\mathbf{x} = \begin{pmatrix} -3 \\ -8 \\ -2 \end{pmatrix}$. Find the coordinate vector of $\mathbf{x}$ relative to $\mathcal{B}$.

3. The matrix

$$\mathbf{A} = \begin{pmatrix} | & | & & | \\ \mathbf{a}_1 & \mathbf{a}_2 & \dots & \mathbf{a}_6 \\ | & | & & | \end{pmatrix} = \begin{pmatrix} 1 & 0 & -2 & 4 & -2 & -4 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 5 & 0 & -1 \\ 2 & -2 & 3 & -6 & 2 & 6 \end{pmatrix}$$

is row equivalent to

$$\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & -2 \end{pmatrix}$$

Let H be the subspace of $\mathbb{R}^4$ spanned by $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ and K be the subspace of $\mathbb{R}^4$ spanned by $\mathbf{a}_4$, $\mathbf{a}_5$, $\mathbf{a}_6$. Find a basis for each of the following subspaces.

(a) $\operatorname{Col}\mathbf{A}$, the column space of $\mathbf{A}$.

(b) $\operatorname{Nul}\mathbf{A}$, the null space of $\mathbf{A}$.

(c) $H + K$, the sum of subspaces H and K.

(d) $H \cap K$, the intersection of H and K.

4. Let $\mathbf{A}$ be a 7×10 matrix. We also know that $\operatorname{Nul}\mathbf{A}$ can be written as the linear span of $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4$, but we do not know if $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \mathbf{b}_4$ are linearly independent or not.

   (a) Find all possible value(s) of $\dim \operatorname{Nul}\mathbf{A}$, the dimension of the null space of $\mathbf{A}$.

   (b) Find all possible value(s) of $\operatorname{rank}\mathbf{A}$, the rank of $\mathbf{A}$.