Math 18 **Final Examination v. A** **Name:** ____________________

March 22, 2017 **(Total Points: *50*)** **PID:** ____________________

Instructions

1. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
2. **You may use one handwritten page of notes, but no books or other assistance during this exam.**
3. **Read each question carefully and answer each question completely.**
4. **Show all of your work. No credit will be given for unsupported answers, even if correct.**
5. **Write your Name at the top of each page.**

(2 points) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Consider the following matrix equation $A\mathbf{x} = \mathbf{b}$:

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & -3 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

(a) Determine the solution set of the matrix equation $A\mathbf{x} = \mathbf{b}$ and, if appropriate, write it in parametric form.

(b) Determine the solution set of the corresponding homogeneous matrix equation $A\mathbf{x} = \mathbf{0}$ and, if appropriate, write it in parametric form.

(6 points) 2. Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Find A^{-1}, the inverse of A.

(b) Find the matrix X such that $A\,X = A^T$, the transpose of A.

 Name: ____________________

(6 points) 3. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & -1 \end{bmatrix}$.

(a) Compute AB. Is AB an invertible matrix? Justify your answer.

(b) Compute BA. Is BA an invertible matrix? Justify your answer.

(6 points) 4. The matrices $A = \begin{bmatrix} 3 & -1 & 1 & -6 \\ 2 & 1 & 9 & 1 \\ -3 & 2 & 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ are row equivalent.

(a) Find a basis for $\mathrm{Col}(A)$, the column space of A.

(b) Find a basis for $\mathrm{Nul}(A)$, the null space of A.

(c) Find a basis for $\mathrm{Col}\left(A^T\right)^{\perp}$, the orthogonal complement of the column space of A^T. Be sure to explain how you know that it is a basis for $\mathrm{Col}\left(A^T\right)^{\perp}$.

(6 points) 5. Let W be a subspace of $\mathbb{R}^n$ with an orthogonal basis $\mathcal{B}_W = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_p\}$, and let $\mathcal{B}_{W^\perp} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_q\}$ be an orthogonal basis for $W^\perp$, the orthogonal complement of W.

(a) Explain why $\mathcal{S} = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_p, \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_q\}$ is an orthogonal set.

(b) Explain why the set $\mathcal{S}$ spans $\mathbb{R}^n$.

(c) Explain why $\mathcal{S}$ is linearly independent.

(d) Explain why $\dim(W) + \dim\left(W^\perp\right) = n$.

(6 points) 6. The set $\mathcal{B} = \{1,\ 1+2t,\ 1+2t+4t^2\}$ is a basis for $\mathbb{P}_2$, the vector space of polynomials of degree at most two. The polynomial $\mathbf{p} = 1 + 4t^2$. Find $[\mathbf{p}]_{\mathcal{B}}$, the coordinate vector for $\mathbf{p}$ with respect to the basis $\mathcal{B}$.

(6 points) 7. Consider the matrix $A = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 4 & 0 \\ 2 & 0 & 2 \end{bmatrix}$.

(a) Determine the eigenvalues of A. (Note: One of the eigenvalues of A is 0.)

(b) Find a matrix P that diagonalizes A. That is, find P so that $P^{-1}A\,P = D$, where D is a diagonal matrix.

(6 points) 8. Let $A = \begin{bmatrix} -1 & 2 & 10 \\ 2 & 1 & 10 \\ 1 & 2 & 0 \\ 2 & -1 & 0 \end{bmatrix}$.

(a) Find an orthonormal basis for $\mathrm{Col}(A)$, the column space of A.

(b) Find an orthogonal matrix Q and an upper triangular matrix R such that $Q\,R = A$.