# HW04



DUE WEEK 7 AND 8

Reading: Chapter 3, *Foundations of Data Science* by Avrim Blum, John Hopcroft, and Ravindran Kannan

Solve Problems 3.5, 3.6, 3.10, 3.12, 3.13 and the following problems.

**Problem 1.** Let $M = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 2 & 0 \end{bmatrix}$.

(a) Find the SVD decomposition of $M$.

(b) Run the power method starting from $\boldsymbol{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ for $k = 3$ steps. Compare what you get with the first left singular vector $\boldsymbol{v}_1$ that you obtain from Part (a).

**Problem 3.**

(a) Let $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$ and its SVD is

$$U\Sigma V^T = \begin{bmatrix} \frac{2}{\sqrt{6}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \sqrt{3} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}.$$

What is the best rank-1 approximation matrix to $A$ (in Frobenius norm).

(b) Suppose that your image has a SVD of the form $M = \sigma_1 \boldsymbol{u}_1 \boldsymbol{v}_1^T + \sigma_2 \boldsymbol{u}_2 \boldsymbol{v}_2^T + \ldots + \sigma_{100} \boldsymbol{v}_{100} \boldsymbol{v}_{100}^T$, where $\sigma_1 = 10, \sigma_2 = 9, \sigma_3 = 8$, and $\sigma_4 = \ldots = \sigma_{100} = 0.01$. What is the matrix you want to use to save more memory but still preserve significant information of your image $M$?