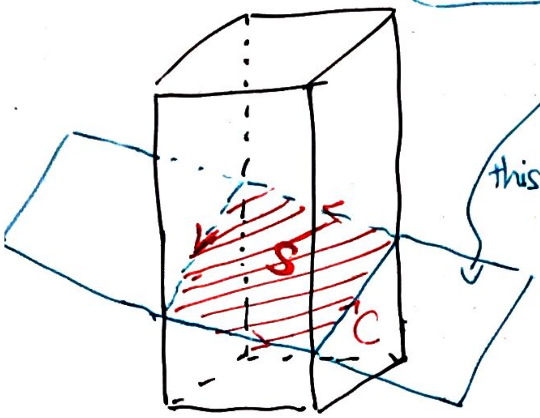
Problem 4.

$\underbrace{2x + y + z = 5}$ $\qquad \vec{F} = \left(-y, \tfrac{1}{2}z^2, 2\right).$

this plane.



Find $\displaystyle\int_{\vec{C}} \vec{F}\cdot d\vec{s}$

Solving directly may be difficult as it's hard to parametrize C.

$\Rightarrow$ use Stokes' theorem.

$$\int_C \vec{F}\cdot d\vec{s} = \iint_S (\nabla\times\vec{F})\cdot d\vec{S} = \iint_S (-z, 0, 1)\cdot d\vec{S}$$

$$\nabla\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & \frac{1}{2}z^2 & 2 \end{vmatrix} = (-z, 0, 1)$$

We can think of S as a graph of a function

$$(x, y, 5-2x-y). \quad 0 \le x \le 1, \; 0 \le y \le 1$$

$$T_x \times T_y = (-g_x, -g_y, 1) = (+2, +1, 1)$$

$$= \int_0^1\int_0^1 (-z, 0, 1)\cdot(2, 1, 1)\, dx\, dy$$

$$= \int_0^1\int_0^1 (-2z + 1)\, dx\, dy \qquad z = 5 - 2x - y$$

$$= \int_0^1\int_0^1 -2(5-2x-y) + 1 \; dx\, dy$$

$$= \ldots = -6$$