P1.3

$x_{\mathsf{new}} = (1-\alpha)x_{\mathsf{old}} + \alpha x_{\mathsf{update}}$

Rewrite the update equation:

$x_{\mathsf{n}} = (1-\alpha)x_{\mathsf{n-1}} + \alpha x_{\mathsf{update}}$

The same equation can be applied to $x_{\mathsf{n-1}}$

$x_{\mathsf{n-1}} = (1-\alpha)x_{\mathsf{n-2}} + \alpha x_{\mathsf{update}}$

...

$x_1 = (1-\alpha)x_0 + \alpha x_{\mathsf{update}}$

$\Rightarrow x_{\mathsf{k}}$ consists of an element A scaled by α and an element B scaled by $1-\alpha$.

However, formula of A contains element scaled by α and an element scaled by $1-\alpha$.

$\Rightarrow x_{\mathsf{n}}$ contains element scaled by $\alpha^k(1-\alpha)^{n-k}$ $\forall k < n, k \in \mathrm{N}$

Hence $x_{\mathsf{new}} = (1-\alpha)x_{\mathsf{old}} + \alpha x_{\mathsf{update}}$ is exponentially weighted

P1.4

1. Addition of 2 vectors in R^n

$[a_{\mathsf{n}}, a_{\mathsf{n-1}}, ..., a_1] + [b_{\mathsf{n}}, b_{\mathsf{n-1}}, ..., b_1] = [a_{\mathsf{n}} + b_{\mathsf{n}}, a_{\mathsf{n-1}} + b_{\mathsf{n-1}}, ..., a_1 + b_1]$

And scalar multiplication of a vector in R^n

$\alpha[a_{\mathsf{n}}, a_{\mathsf{n-1}}, ..., a_1] = [\alpha a_{\mathsf{n}}, \alpha a_{\mathsf{n-1}}, ..., \alpha a_1]$

result in a vector living in R^n ,hence R^n is closed under addition and scalar multiplication .

Therefore, R^n is a vector space.

2.

Addition of 2 tensors in $R^{m*n*...*q}$ is element-wise addition, thus the output tensors will retain the old dimesion,thus be an element of $R^{m*n*...*q}$

The same to scalar multiplication ,which is element-wise multiplication of every elements in the tensor with a scalar.

Hence $R^{m*n*...*q}$ is closed under addition and scalar multiplication. Therefore, $R^{m*n*...*q}$ is a vector space.

3.

Given 2 functions $f(x)$ and $g(x)$ in $C[a,b]$,we have $f(x) + g(x) = (f+g)(x)$, resulting in a continuous function with domain $D_f \bigcap D_g = [a,b]$

Similarly, $f(x)$ * $g(x) = (f*g)(x)$, resulting in a continuous function with domain $D_f \bigcap D_g = [a,b]$

Hence $C[a,b]$ is a vector space.

4.

Obviously sum of 2 polynomials with degree at most n will result in another polynomial with degree at most n.

And also multiplying a polynomial by a scalar will not increase nor decrease the degree, thus the output still stay in $P_n(R) := \{\sum_{i=0}^{n} a_i x^i\}$

Hence $P_n(R) := \{\sum_{i=0}^{n} a_i x^i\}$ is closed under addition and scalar multiplication, thus is a vector space.

P1.5

1.The space of positive real axis:

Multiplying an element of the positive real axis by a negative scalar will result in an element not belong to the positive real axis.

2.Unit vectors

Addition of 2 unit vectors will result in a vector which is not a unit vector, since unit vectors are linearly independant.

3.Latitude and longitude

$(90^o N, 0^o S)$ + $(50^o N, 10^o S)$ = $(140^o N, 10^o S)$, which does not belong to Earth's coordinate anymore, so latitude and longitude space is not closed under addition.

4.Monomials $\{x^k\}$

$x^k + x^{k-1}$ is a sum of 2 monomials but obviously not a monomial.

So the space of monomials is not closed under addition.

P1.6

tu is vector u stretched or shrinked by a factor t, so tu $\forall$ $t \in R$ will be a line crossing the point u and the origin.

$+v$ means translating the line in a way that move the origin to the point .

Hence $L_1 = \{w \in V : w = v + tu,\ \forall t \in R,\ u, v \in V\}$ is a line crossing v in direction of u.

P1.7

$w = (1-t)v + tu = t(u-v) + v$ $u - v$ is a vector that have a direction from point v to point u

$\Rightarrow t(u-v)$ is a line crossing the origin that have a direction from point v to point u

$+v$ is a translation that translate the origin to point v. Since the line has a v-to-u direction, the translated line will also cross the point u.

P1.8:

Given 2 random points u and v in a vector space V. The formula of the line D crossing points u and v is:

$L_2 = \{w \in V : w = (1-t)v + tu,\ \forall t \in R,\ u, v \in V\}$

Thus every point on D and between 2 points u and v is also in vector space V

Recall the formula of line crossing a point: $L_1 = \{w \in V : w = v + tu,\ \forall t \in R,\ u, v \in V\}$,with v being the point on D , u being any point in the same space as v, which is V

Since a line only consists of linear combinations of u and v,which are also elements of V ,it must be contained in V.

Hence V is flat.

P1.9

$(u-\vec{0})$ is a segment with direction from origin to point u, as $(u-\vec{0})=u$ so scalar t $\forall t \in R$ allow the segment to stretch freely, hence, become a line , as t can reach $+-\infty$

P1.10

w = (1-t) v + tu = t(u-v) + v

It is similar to the form seen on P1.7, however, t here is restricted from 0 to 1, so the length of t(u-v) is now limited to the distance between u and v

So $L3$ will be only a segment connecting point u and v.

P1.11:

Since we allow v to move freely on another segment, L_3 will become a set of points inside a triangle with vertices u, u_2, v_2 or only a line if u_2, v_2, u are aligned.

Repeating the same process, we will consecutively add a point in space and L_3 will be a set of points inside a polyhedron with vertices being the points $u, u_2, u_3, ..$ introduced in the formula of L_3

P1.12

Basis vectors for R_{d}

- [1,0,0,0,0,...0]
- [0,1,0,0,0,...0]

...

- [0,0,0,0,0,...1]

associated coordinate vectors: $[a_1, a_2, ..., a_d]$

Basis vectors for $R_{\mathsf{m*n}}$

$[1, 0, 0, 0, 0, ...0$
$0, 0, 0, 0, 0, ...0$
...
$0, 0, 0, 0, 0, ...0]$
$[0, 1, 0, 0, 0, ...0$
$0, 0, 0, 0, 0, ...0$
...
$0, 0, 0, 0, 0, ...0]$
...
$[0, 0, 0, 0, 0, ...0$
$0, 0, 0, 0, 0, ...0$
...
$0, 0, 0, 0, 0, ...1]$

associated coordinate vectors: $[a_1, a_2, ..., a_{m*n}]$

Basis vectors for $P_{\mathsf{n}}(R)$

- [x^n,0,0,0,0,...0]
- [0,x^{n-1},0,0,0,...0]

...
- [0,0,0,0,0,...x^0]
associated coordinate vectors: $[a_1, a_2, ..., a_d]$

P1.13

"linear" in linear combinations means however the input is distorted, the output will be distorted in the same way.

P1.14
Denote (x,y,z) is the coordinate in the standard basis of R^3.
the coordinates in this basis B is:

$$(\frac{x+y}{\sqrt{2}}, \frac{y-x}{\sqrt{2}}, z)$$