

1. Prove that for any set  $S$ ,  $z \in \mathbb{C}$  is an interior point to  $S$  if and only if  $z$  is an exterior point to  $S^c$ .

For any point  $z \in \mathbb{C}$ ,  $z$  is an interior point to  $S$  if and only if there exists  $\epsilon > 0$  such that the  $\epsilon$ -neighborhood is contained within  $S$  which happens if and only if that neighborhood does not intersect  $S^c$ . So  $z$  is an exterior point to  $S^c$ .  $\square$

2. Prove that for any set  $S$ ,  $\partial S = \partial S^c$ .

$\partial S$  is the set of all boundary points of  $S$ , or points with every neighborhoods intersect with both  $S$  and  $S^c$ .

$\partial S^c$  is the set of all boundary points of  $S^c$ , or points with every neighborhoods intersect with both  $S^c$  and  $(S^c)^c = S$ .

So  $\partial S = \partial S^c$ .  $\square$

3. Prove that a set  $S$  is open if and only if  $S^c$  is closed.

A set  $S$  is open if and only if it does not contain any of its boundary points. So it will also not contain any boundary points of its complement, since  $\partial S = \partial S^c$ , or  $\partial S^c \subset S^c$ .

Thus  $S^c$  is closed.  $\square$