[Cauchy Condensation Test] Let $\{a_k\}$ be a sequence such that:

1. $a_1 \geq a_2 \geq a_3 \geq \cdots \geq a_k \geq a_{k+1} \geq \cdots$

2. $a_k \geq 0$ for all $k \in \mathbb{N}$

Then the series $\sum_{k=1}^{\infty} a_k$ converges if and only if $\sum_{k=0}^{\infty} 2^k a_{2^k}$ converges.

Corollary: The series $\sum_{k=1}^{\infty} \frac{1}{k^p}$ converges if and only if $p > 1$.

Proof:

For any $k \in \mathbb{N}$, let $a_k \coloneqq \frac{1}{k^p} > 0$. It is easy to see that $a_k > a_{k+1}$.

Note the series:

$$\sum_{k=1}^{2^n} \frac{1}{k^p}$$
$$= \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{2^{np}}$$
$$\leq \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{2^p} + \cdots + \frac{1}{2^{n-1}p}$$
$$\leq \frac{1}{1^p} + \frac{2}{2^p} + \cdots + \frac{2^{n-1}}{2^{(n-1)p}}$$
$$\leq \sum_{k=0}^{n-1} 2^k a_{2^k} = \sum_{k=0}^{n-1} \frac{1}{2^{k(p-1)}}$$

Consider the series $\sum\limits_{k=0}^{\infty} 2^k a_{2^k} = \sum\limits_{k=0}^{\infty} \frac{2^k}{2^{kp}} = \sum\limits_{k=0}^{\infty} \frac{1}{2^{k(p-1)}}$.

If $p \leq 1$, $\frac{1}{2^{k(p-1)}} \geq 1$, then by Comparison Test, $\sum\limits_{k=0}^{\infty} \frac{1}{2^{k(p-1)}}$ diverges.

So, by Cauchy Condensation Test, the series $\sum\limits_{k=1}^{\infty} \frac{1}{k^p}$ converges if and only if $p > 1$. $\square$