1. Suppose f and $\bar{f}$ are analytic on some domain D. Show that f is constant.

   Let $f = u(x, y) + iv(x, y)$ and $\bar{f} = u(x.y) - iv(x, y)$ for some $u(x, y), v(x, y)$.

   Then by Cauchy-Riemann equations, all of these must hold:

$$\begin{cases} u_x = v_y \\ u_y = -v_x \\ u_x = -v_y \\ u_y = v_x \end{cases}$$

   Solve for u_x and v_x, we arrive at $u_x = v_x = 0$. So for any $z \in D$, $f'(z) = 0$, or f is constant.

2. Suppose f is analytic on domain D. Suppose $|f(z)|$ is constant. Show that f is constant.

   Let $|f(z)| = c$ for some constant c. If $c = 0$, f is constant.

   If $c \neq 0$, it follows that $f\bar{f} = c \neq 0$, so $f(z) \neq 0$ in D.

   Then $\bar{f} = \frac{1}{f}$, which is the quotient of two analytic functions, so $\bar{f}$ is analytic.

   Thus from Problem 1, f is constant.

3. Suppose f is analytic on domain D and f is real-valued. Show that f is constant.

   Since $f(z) = u(x, y) + iv(x, y)$ is real-valued, $v(x, y) = 0$. It follows that, from Cauchy-Riemann equations, $u_x = v_y = 0$ and $u_y = -v_x = 0$. Thus $f(z) = u(x, y) = c$ for some constant c. $\square$