

Let m be the Lebesgue measure as described in class. Let \mathcal{C} be the Cantor set as described in class. Compute $m(\mathcal{C})$.

Proof:

Let \mathcal{C} be the Cantor Set.

Then, with deMorgan's law and Assessment 17 part 3,

$$\begin{aligned}\mu(\mathcal{C}) &= \mu\left(\bigcap_i^\infty C_i\right) \\ &= 1 - \mu\left(\bigcup_i^\infty C_i^c\right) \\ &= 1 - \lim_{n \rightarrow \infty} \mu(\mathcal{C}_n^c) \\ &= 1 - \lim_{n \rightarrow \infty} \sum_{i=0}^n \frac{1}{3} \left(\frac{2}{3}\right)^i \\ &= 1 - \frac{1}{3} \times \frac{1}{1 - \frac{2}{3}} \\ &= 1 - 1 = 0.\end{aligned}$$

Since μ is a measurement, $\mu(\mathcal{C}) = 0$. \square