1. Show that a composition of natural transformations is a natural transformation.

Let $\mathcal{C}, \mathcal{D}, \mathcal{E}$ be categories.

Let $F, G : \mathcal{C} \rightrightarrows \mathcal{D}$ and $K, L : \mathcal{D} \rightrightarrows \mathcal{E}$ be functors.

Let $\alpha : F \Rightarrow G$ and $\beta : K \Rightarrow G$ be natural transformations.

Since the composition of functors is functor, we can define the functor $FK, GL : \mathcal{C} \rightrightarrows \mathcal{E}$. We WTS the composition of natural transformations of these functors, $\alpha\beta : FK \Rightarrow GL$, is a natural transformation.

Let c, d be objects and a morphism $f : c \to d$ in $\mathcal{C}$. Define the component of $\alpha\beta$ at c as $\alpha\beta_c \coloneqq \beta_{cF}$. Since $\beta : K \Rightarrow G$ is a natural transformation, its component at cF is $\beta_{cF} : cFK \to cGL$, which are the same image of c under FK and GL.

Moreover, the morphism f, under FK and GL, becomes f^{FK} and f^{GL}, each together with components of $\alpha\beta$ as defined above, commutes since β and α are natural transformations.

Thus, $\alpha\beta$ is a natural transformation. $\square$

2.