

1. Liouville's Theorem: If a function f is entire and bounded in the complex plane, then $f(z)$ is constant throughout the plane.

Let M be an upper bound of f . Since f is entire, for any $r \in \mathbb{R}$, by Cauchy's Inequality, for any $z_0 \in \mathbb{C}$, $0 \leq |f'(z_0)| \leq \frac{M}{r}$.

Since $\lim_{r \rightarrow \infty} \frac{M}{r} = 0$, $f'(z_0) = 0$. So $f'(z) = 0$ for all $z \in \mathbb{C}$, so $f(z)$ is constant. \square