2. Suppose there exists a sequence of function $\{g_n\}_{n=1}^{\infty}$ uniformly converging to 0 on A. Suppose we have a sequence of functions $\{f_n\}_{n=1}^{\infty}$ and a function f on A such that $|f_n(x) - g_n(x)| \leq g_n(x)$ for all $x \in A$. Show that $\{f_n\}$ converges uniformly to f on A.

Let $\epsilon > 0$.

Then there exists $N \in \mathbb{N}$ such that for all $n \geq N$, $|g_n(x) - 0| = |g_n(x)| \leq \epsilon$.

So, for all $n \geq N$ and $x \in A$, $|f_n(x) - f(x)| \leq g_n(x) \leq |g_n(x)| \leq \epsilon$.

Thus f_n converges uniformly to f in A. $\square$

3. Suppose $\{f_n\}$ and $\{g_n\}$ converge uniformly to f and g on some set A. Show that $\{f_n + g_n\}$ converges uniformly to $f + g$ on A.

Let $\epsilon > 0$.

Since f_n converges to f uniformly, there exists $N_1 \in \mathbb{N}$ such that for all $n \geq N_1$ and $x \in A$, $|f_n(x) - f(x)| \leq \frac{\epsilon}{2}$.

Since g_n converges to g uniformly, there exists $N_2 \in \mathbb{N}$ such that for all $n \geq N_2$ and $x \in A$, $|g_n(x) - g(x)| \leq \frac{\epsilon}{2}$.

So, with $N = max(N_1, N_2)$, then for all $n \geq N$ and $x \in A$, we have that $|f_n(x) + g_n(x) - f(x) - g(x)| \leq |f_n(x) - f(x)| + |g_n(x) - g(x)| \leq \epsilon$.

Thus $\{f_n + g_n\}$ converges uniformly to $f + g$. $\square$

4.