1. Let z_1 and z_2 denote two complex numbers. Prove that $|z_1 z_2| = |z_1||z_2|$.

Proof:

Let $z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i$.

Then

$$\begin{aligned}
|z_1 z_2| &= |(a_1 + b_1 i)(a_2 + b_2 i)| \\
&= |a_1 a_2 - b_1 b_2 + (a_1 b_2 + a_2 b_1) i| \\
&= \sqrt{(a_1 a_2 - b_1 b_2)^2 + (a_1 b_2 + a_2 b_1)^2} \\
&= \sqrt{a_1^2 a_2^2 + b_1^2 b_2^2 + a_1^2 b_2^2 + a_2^2 b_1^2}
\end{aligned}$$

Also,

$$\begin{aligned}
|z_1||z_2| &= \sqrt{a_1^2 + b_1^2}\sqrt{a_2^2 + b_2^2} \\
&= \sqrt{a_1^2 a_2^2 + b_1^2 b_2^2 + a_1^2 b_2^2 + a_2^2 b_1^2}
\end{aligned}$$

So, $|z_1 z_2| = |z_1||z_2|$. $\square$

2. Let z be a complex number. For $n \in \mathbb{N}$, show that $|z^n| = |z|^n$.

Proof:

Base case: for $n = 1$, $|z| = |z|$.

Induction hypothesis: For $n \geq 1$, suppose that $|z^n| = |z|^n$.

Then $|z^{n+1}| = |z^n \times z| = |z^n| \times |z| = |z|^n \times |z| = |z|^{n+1}$.

So, for $n \in \mathbb{N}$, $|z^n| = |z|^n$. $\square$