Statistical Modeling 2
Exercise 2

January 31, 2017

1 A simple Gaussian location model

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The joint prior over the mean parameter θ and precision parameter ω is:

$$p(\theta, \omega) \approx \omega^{(d+1)/2-1} \exp\left\{-\omega \frac{\kappa(\theta-\mu)^2}{2}\right\} \exp\left\{-\omega \frac{\eta}{2}\right\}$$

To get the marginal prior, we integrate out the parameter ω:

$$\begin{aligned}
p(\theta) &\propto \int_0^\infty \omega^{(d+1)/2-1} \exp\left\{-\omega \frac{\kappa(\theta-\mu)^2+\eta}{2}\right\} \\
&\propto \left(\frac{\kappa(\theta-\mu)^2+\eta}{2}\right)^{-(d+1)/2} \\
&= \left(\frac{\eta}{2} + \frac{\kappa(\theta-\mu)^2}{2}\right)^{-(d+1)/2} \\
&= \left(1 + \frac{\kappa(\theta-\mu)^2}{\eta}\right)^{-(d+1)/2} \left(\frac{\eta}{2}\right)^{-(d+1)/2} \\
&\propto \left(1 + \frac{\kappa(\theta-\mu)^2}{\eta}\right)^{-(d+1)/2} \\
&= \left(1 + \frac{1}{d}\frac{\kappa(\theta-\mu)^2}{\eta}\right)^{-(d+1)/2}
\end{aligned}$$

Let $\nu = d, m = \mu$ and $s = \sqrt{\eta/\kappa}$, we have a Student t distribution with ν degrees of freedom and scale s:

$$p(\theta) \propto \left(1 + \frac{1}{\nu}\frac{(\theta-m)^2}{s^2}\right)^{-(\nu+1)/2}$$