HW1 - ML2

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1. Biến đổi lại thuật toán PCA

Tập hợp đầu vào:

$$x = \{x_1, x_2, ..., x_3\} \ (x_i \in R^D)$$

$$x = \begin{bmatrix} {x_1}^T \\ {x_2}^T \\ ... \\ {x_3}^T \end{bmatrix} \in R^{NxD}, B = \begin{bmatrix} b_1 \\ b_2 \\ ... \\ b_N \end{bmatrix}$$

$$Z = x.B$$

$$\Leftrightarrow \begin{bmatrix} {z_1}^T \\ {z_2}^T \\ ... \\ {z_N}^T \end{bmatrix} = \begin{bmatrix} {x_1}^T \\ {x_2}^T \\ ... \\ {x_3}^T \end{bmatrix} . \begin{bmatrix} b_1 \\ b_2 \\ ... \\ b_N \end{bmatrix} = \begin{bmatrix} {x_1}^T b_1 \ {x_1}^T b_2 \ ... \ {x_1}^T b_M \\ {x_2}^T b_1 \ {x_2}^T b_2 \ ... \ {x_2}^T b_M \\ ... \\ {x_N}^T b_1 \ {x_N}^T b_2 \ ... \ {x_N}^T b_M \end{bmatrix}$$

Maximize variance:

$$V_1 = \frac{1}{N} \sum_{n=1}^{N} {z_{1n}}^2 \ where \ {z_{1n}}^2 = x_i^T b_1$$

$$V_1 = \frac{1}{N} \sum_{n=1}^{N} (x_i^T b_1)^2 = \frac{1}{N} \sum_{n=1}^{N} b_1^T x_i x_i^T b1 = b_1^T \left(\frac{1}{N} \sum_{n=1}^{N} x_i x_i^T \right) b_1 = b_1^T S \ b_1$$

$\rightarrow$ maximize $b_1^T S \ b_1$ subject to $||b_1||^2 = 1$

The Lagrangian:

$$L = b_1^T S \ b_1 + \alpha(1 - b_1^T b_1)$$

$$\frac{\partial L}{\partial b_1} = 0 \Leftrightarrow 2Sb_1 - 2\alpha b_1 = 0 \Leftrightarrow Sb1 = \alpha b_1$$

We can see that b_1, α is an eigenvector and eigenvalue of S

$$\frac{\partial L}{\partial \alpha} = 0 \Leftrightarrow b_1^T b_1 = 1$$

$$\rightarrow var = b_1^T S b_1 = b_1^T \alpha b_1 = \alpha b_1^T b_1 = \alpha$$