# HW3

Thanh Binh

October 2022

1. Biến đổi lại linear regression trên lớp ra latex, từ t = y(x,w) + $\varepsilon \rightarrow$ w = $(X^TX)^{-1}X^Tt$

$$L = \frac{1}{N}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2, \hat{y}_i = w_0 + w_1 x_i$$

$$w = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix},\ x = \begin{bmatrix} 1\ x_1 \\ 1\ x_2 \\ ... \\ 1\ x_n \end{bmatrix},\ y = \begin{bmatrix} y_1 \\ y_2 \\ ... \\ y_n \end{bmatrix}$$

$$\hat{y} = \begin{bmatrix} \hat{y_1} \\ \hat{y_2} \\ ... \\ \hat{y_n} \end{bmatrix} = \begin{bmatrix} w_0 + w_1 x_1 \\ w_0 + w_1 x_2 \\ ... \\ w_0 + w_1 x_n \end{bmatrix} = x_w$$

$$L\ = \parallel \hat{y} - y \parallel_2^2 = \parallel X_w - y \parallel_2^2 = (X_w - y)^T(X_w - y)$$

$$\rightarrow \frac{L'}{w'} = 2X^T(X_w - y) = 0$$

$$\Leftrightarrow X^TX_w = X^Ty$$

$$\Leftrightarrow w = (X^TX)^{-1}X^Ty$$

4. Chứng minh $X^TX$ invertible khi X full rank.

$$\Rightarrow \vec{v}^T X^T X \vec{v} = \vec{v}^T \vec{0} = 0 \Rightarrow (X\vec{v})^T X\vec{v} = 0 \Rightarrow (X\vec{v}) \cdot (X\vec{v}) = 0 \Rightarrow X\vec{v} = \vec{0}$$

So we have: if $\vec{v} \in N\left(X^TX\right) \Rightarrow \vec{v} \in N(X)$

$\Rightarrow \vec{v}$ can be $\vec{0}$
$\Rightarrow N\left(X^TX\right) = N(X) = \{\vec{0}\}$

$\Rightarrow$ $X^TX$ is linearly independent; and $X^TX$ is a square matrix $\Rightarrow$ $X^TX$ is invertible