Networks: Friends, Epidemics & Terrorists

Homework #4

Centrality [★]

Assume that $\mathcal{N}_x(y)$ denote the number of nodes that are y distance away from x. Let's define a new centrality measure $\mathcal{X}$ as follows

$$\mathcal{X}_i = \sum_{j=0}^{d} \alpha^j \mathcal{N}_i(j)$$

where $\alpha < 1$ is a constant. Essentially, this measure assigns 1 to the source node itself; α to each node that are one distance away; α^2 to nodes that are two distances away and so on and sums them up.

Describe and analyze in terms of complexity an algorithm for calculating this centrality measure.

Road Trip [★★]

You are planning to drive from Hamilton to Saratoga Springs. You are given a road map of the state and you know the average amount of gas spent in each road segment.

Choose an algorithm for each of the following scenarios, describe the time complexity of the algorithm of your choice in terms of the appropriate variables.

1) Is it even possible to drive to Saratoga Springs? Find all of the towns you can go to and those you can't.
2) The average price of gas in June 2022 was $4.94 a gallon. Pick the best path if you're strapped for cash.
3) If you want to see the sights along the way, how would you know if it's possible to have a completely unique set of scenery to and fro? If it's possible, find those paths.

Efficiency [★]

You are given a program that takes in any `networkx` graph and prints out the space consumed if it is stored as an adjacency matrix and as an adjacency list alongside the time taken for each of the common operations.

Experiment with this program on graphs of different sizes (10 to 10,000 nodes) and sparsity and analyze how these metrics vary.