Adaptive Tracking Control of Surface Vessel Using Optimized Backstepping Technique

Guoxing Wen, Shuzhi Sam Ge, *Fellow, IEEE*, C. L. Philip Chen, *Fellow, IEEE*, Fangwen Tu, and Shengnan Wang

***Abstract*—In this paper, a tracking control approach for surface vessel is developed based on the new control technique named optimized backstepping (OB), which considers optimization as a backstepping design principle. Since surface vessel systems are modeled by second-order dynamic in strict feedback form, backstepping is an ideal technique for finishing the tracking task. In the backstepping control of surface vessel, the virtual and actual controls are designed to be the optimized solutions of corresponding subsystems, therefore the overall control is optimized. In general, optimization control is designed based on the solution of Hamilton–Jacobi–Bellman equation. However, solving the equation is very difficult or even impossible due to the inherent nonlinearity and complexity. In order to overcome the difficulty, the reinforcement learning (RL) strategy of actor-critic architecture is usually considered, of which the critic and actor are utilized for evaluating the control performance and executing the control behavior, respectively. By employing the actor-critic RL algorithm for both virtual and actual controls of the vessel, it is proven that the desired optimizing and tracking performances can be arrived. Simulation results further demonstrate effectiveness of the proposed surface vessel control.**

***Index Terms*—Actor-critic architecture, Lyapunov stability, optimized backstepping (OB), reinforcement learning (RL), surface vessel.**

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I. Introduction

With the rapid development of deep-sea exploitation and ocean transportation, trajectory tracking and path following of surface vessel had become an active research topic and attracted considerable attentions [1]. However, due to the complexity and unpredictability of deep ocean environments, the research topic always keeps the big challenge. It is well known that backstepping is the most popular and fundamental technique for the tracking control of high order systems [2]–[4]. Its basic idea is that the control behaviors are carried out by a recursive process, which considers many state variables as "virtual controls" and designs the control laws for them. Recently, backstepping technique has been applied to the tracking control of surface vessel, and many excellent research results have been published, such as [5]–[9].

In control community, optimization had become a fundamental design principle, many pioneering works based on adaptive fuzzy backstepping have been proposed for nonlinear systems with unmeasured states [10]–[12]. Since sea voyage for deep-sea exploitation or ocean transportation is supported by massive energy consumption, it is very necessary to consider optimization in surface vessel control. Optimal control means to minimize a structured cost index, which describes the balance between desired performance and available control resources. However, up to now, few of the optimal control methods are reported for surface vessel. Usually, the optimal control is designed by using the solution of Hamilton–Jacobi–Bellman (HJB) equation [13], which becomes Riccati equation for the case of linear systems, but the HJB equation of vessel dynamic is solved difficultly owing to its inherent nonlinearity and intractability. Therefore, these exiting optimal schemes, such as [14]–[16], cannot be directly applied to surface vessel, especially for tracking control.

Although most tracking control methods of high order systems are designed based on backstepping technique [17]–[19], integrating optimization into backstepping is still very challenging due to the technical and mathematical complexities. Recently, a new technique named optimized backstepping (OB) is proposed in [20], which fuses optimization control into backstepping design. The basic idea is that all virtual controls and the actual control are designed to be the optimized solution of corresponding subsystems, therefore the control of overall system is optimized. Since backstepping control has been well developed to surface vessel control [2]–[4], OB technique can be performed for optimizing the tracking control of surface

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vessel. However, due to surface vessels modeled in multidimensional form and its velocity states described by body-fixed frame, applying OB technique to surface vessel control is still a challenging work.

Since neural networks (NNs) and fuzzy logical systems (FLSs) had been proven to have the excellent approximation and learning abilities, they had become the powerful and popular tools in the nonlinear system modeling and controlling. In the past decades, a great number of NN or FLS-based nonlinear control methods are published [21]–[29], in which [21]–[25] are based on NNs and [26]–[29] are based on FLSs. In [30]–[33], the NN or FLS-based reinforcement learning (RL) is successfully applied to adaptive optimization control and has become a popular means in recent. The basic idea of RL is to obtain the appropriate actions by evaluating the feedback from the environment. One of effective means for implementing RL is actor-critic architecture, of which the actor and critic are utilized for performing control actions and evaluating the actions, respectively, [34].

Motivated by the above discussion, in this paper, an optimized control scheme is developed for surface vessel systems based on the OB control idea. Since the vessel model is depicted in multidimensional form, it is a challenging work to implement RL algorithm and stability analysis. In order to design the optimized control, the vessel model in body-fixed frame is transformed to earth-fixed frame. Therefore, the OB control is first obtained in earth-fixed frame, then the control for original system can be got by coordinate transformation.

II. Problem Description

Consider the following surface ship modeled in 3-degrees of freedom, which are surge, sway, and yaw shown in Fig. 1:

$$\begin{aligned}\dot{\eta}(t) &= J(\eta)v(t)\\ M\dot{v}(t) &= -C(v)v(t) - D(v)v(t) - g(\eta) + \tau\end{aligned} \tag{1}$$

where $\eta(t) = [\eta_x(t), \eta_y(t), \eta_z(t)] \in R^3$ are the position and heading states in the earth-fixed frame, respectively; $v(t) = [v_x(t), v_y(t), v_z(t)] \in R^3$ are the surge, sway, and yaw velocities in the body-fixed frame, respectively.

$$J(\eta) = \begin{bmatrix}\cos(\eta_z) & -\sin(\eta_z) & 0\\ \sin(\eta_z) & \cos(\eta_z) & 0\\ 0 & 0 & 1\end{bmatrix} \in SO(3),$$

that means $J^{-1} = J^T$, is the Jacobian transformation matrix for the coordinate transforming between the body-fixed and earth-fixed frames; $M = M^T$ is the inertia matrix, which is assumed a positive definite constant matrix; $C(v) = -C^T(v)$ is the Coliolis centripetal matrix; $D(v)$ is the damping matrix; $g(\eta) \in R^3$ is the restoring force vector in the presence of gravity and buoyancy; and $\tau \in R^3$ is the control input.

Let $\nu(t) = J(\eta)v(t)$, then the vessel model (1) can be rewritten as

$$\begin{aligned}\dot{\eta}(t) &= \nu(t)\\ \dot{\nu}(t) &= C^o(\eta,\nu)\nu(t) + D^o(\eta,\nu)\nu(t) + g^o(\eta) + u\end{aligned} \tag{2}$$

where $C^o(\eta,\nu) = -JM^{-1}C(J^{-1}\nu(t))J^{-1}$, $D^o(\eta,\nu) = \dot{J}J^{-1} - JM^{-1}D(J^{-1}\nu(t))J^{-1}$, $g^o(\eta) = -JM^{-1}g(\eta)$ and $u = JM^{-1}\tau$.

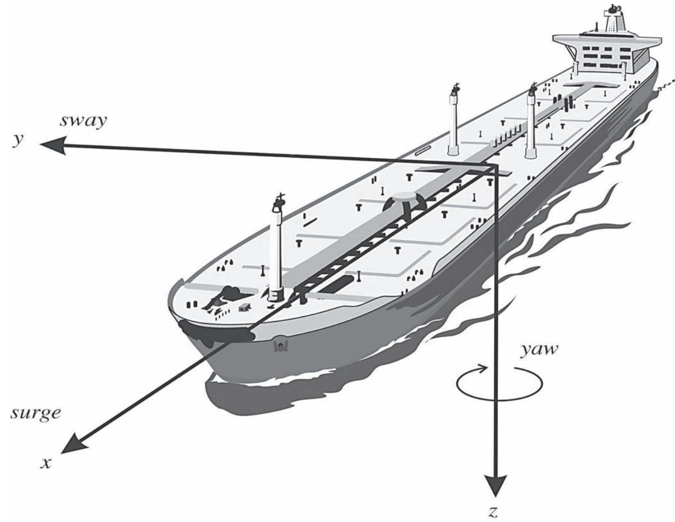

Fig. 1. Model ship.

Let $f(\chi) = C^o(\eta,\nu)\nu(t) + D^o(\eta,\nu)\nu(t) + g^o(\eta)$, where $\chi(t) = [\eta^T(t), \nu^T(t)]^T \in R^6$, then the dynamic equation (2) can be rewritten as

$$\dot{\eta}(t) = \nu(t), \quad \dot{\nu}(t) = f(\chi) + u. \tag{3}$$

Remark 1: It should be mentioned that the control law is designed for the dynamic model (2), therefore the control is got in earth-fixed frame first. Then the control for dynamic model (1) can be obtained by left multiplying the term $MJ^{-1}(\eta)$.

III. Optimized Backstepping Control Design

In this section, optimized tracking control is designed for the surface vessel by the 2-step OB [20]. In order to archive optimization, the actor-critic RL algorithm is carried out in every backstepping step, where the actor is utilized to perform the control policy and the critic is utilized to evaluate the optimization performance.

Control Objective: Based on the OB technique, design an optimized tracking control scheme for surface vessel (1) such that: 1) all error signals are semi-globally uniformly ultimately bounded (SGUUB) and 2) the vessel follows the desired trajectory $\eta_d = [\eta_{dx}, \eta_{dy}, \eta_{dz}]^T$, where η_d is sufficiently smooth and η_d with its derivative is bounded, to desired accuracy.

Definition 1 (SGUUB [20]): Consider the nonlinear system

$$\dot{x}(t) = g(x,t)$$

where $x(t) \in R^n$ is the state vector. Its solution is said to be SGUUB if, for $x(0) \in \Omega_x$ where $\Omega_x \in R^n$ is a compact set, there exist two constants σ and $T(\sigma, x(0))$ such that $\|x(t)\| \le \sigma$ is held for all $t > t_0 + T(\sigma, x(0))$.

Step 1: Define the tracking error vector as $z_\eta(t) = \eta(t) - \eta_d(t)$, and its time derivative along (3) is

$$\dot{z}_\eta(t) = \nu(t) - \dot{\eta}_d(t) \tag{4}$$

where $\nu(t)$ is viewed as intermediate control of the backstepping.

Let $\alpha(z_\eta) \in R^3$ denote virtual control of the z_η-subsystem (4), the infinite horizon value function is defined as

$$V_\eta(z_\eta) = \int_t^\infty r_\eta(z_\eta(s), \alpha(z_\eta))ds \tag{5}$$

where $r_\eta(z,\alpha) = z_\eta^T z_\eta + \alpha^T\alpha$ is the immediate or local cost function.

Remark 2: The optimal problem for a dynamic system is to find an admissible control policy [20] such that the control objective is realized by expending the minimal cost. For example, for the z_η-subsystem (4), the optimal virtual control is designed to guarantee that the infinite horizon value function (5) is minimized.

View $\nu(t)$ as the optimal virtual control $\alpha^*(z_\eta)$, i.e., $\nu(t) \triangleq \alpha^*$, the optimal value function is yielded as

$$\begin{aligned} V_\eta^*(z_\eta) &= \min_{\alpha\in\Psi(\Omega_\eta)} \int_t^\infty r_\eta(z_\eta, \alpha)ds \\ &= \int_t^\infty r_\eta(z_\eta, \alpha^*)ds \end{aligned} \tag{6}$$

where $\Psi(\Omega_\eta)$ denotes the set of admissible control policies over Ω_η, $\Omega_\eta \subset R^3$ is a compact set.

The Hamiltonian function associating with the infinite horizon value function (6) is

$$H_\eta\left(z_\eta, \alpha, \frac{\partial V_\eta}{\partial z_\eta}\right) = r_\eta(z_\eta, \alpha) + \left(\frac{\partial V_\eta}{\partial z_\eta}\right)^T \dot{z}_\eta(t) \tag{7}$$

where $\partial V_\eta/\partial z_\eta$ denotes the gradient of V_η with respect to z_η.

According to both (6) and (7), there is the following HJB equation:

$$\begin{aligned} H_\eta\left(z_\eta, \alpha^*, \frac{\partial V_\eta^*}{\partial z_\eta}\right) &= z_\eta^T z_\eta + \alpha^{*T}\alpha^* \\ &\quad + \left(\frac{\partial V_\eta^*}{\partial z_\eta}\right)^T (\alpha^* - \dot{\eta}_d) = 0. \end{aligned} \tag{8}$$

Assuming the solution of (8) is existent and unique, the optimal virtual control α^* can be obtained by solving $\partial H\left(z_\eta, \alpha^*, \partial V_\eta^*/\partial z_\eta\right)/\partial\alpha^* = 0$

$$\alpha^*(t) = -\frac{1}{2}\frac{\partial V_\eta^*}{\partial z_\eta}. \tag{9}$$

Substituting (9) into (8), the following result yields:

$$\|z_\eta(t)\|^2 - \left(\frac{\partial V_\eta^*}{\partial z_\eta}\right)^T \dot{\eta}_d(t) - \frac{1}{4}\left(\frac{\partial V_\eta^*}{\partial z_\eta}\right)^T\left(\frac{\partial V_\eta^*}{\partial z_\eta}\right) = 0. \tag{10}$$

By substituting solution of (10) into (9), the optimal virtual control can be obtained. However, solving the equation is very difficult or impossible because of its strong nonlinearities. In order to realize the control scheme, the online RL of actor-critic architecture is performed by employing adaptive NN approximation.

Rewrite the optimal value function (6) as

$$V_\eta^*(z_\eta) = \beta_\eta\|z_\eta(t)\|^2 + V_\eta^o(z_\eta) \tag{11}$$

where β_η is a positive design constant, $V_\eta^o(z_\eta) = -\beta_\eta\|z_\eta(t)\|^2 + V_\eta^*(z_\eta)$.

Inserting (11) into (9), the optimal virtual control can be rewritten as

$$\alpha^* = -\frac{1}{2}\frac{\partial V_\eta^*}{\partial z_\eta} = -\beta_\eta z_\eta(t) - \frac{1}{2}\frac{\partial V_\eta^o}{\partial z_\eta}. \tag{12}$$

It is well known that NNs have the excellent adaptive learning and function approximating abilities, it can approximate any continuous function to desired accuracy. Since the scalar value function V_η^o is continuous for $z_\eta \in \Omega_\eta$, it can be approximated by NNs in the following form:

$$V_\eta^o(z_\eta) = W_\eta^{*T} S_\eta(z_\eta) + \varepsilon_\eta(z_\eta) \tag{13}$$

where $W_\eta^* \in R^{n_\eta}$ is the ideal NN weight, n_η is the neuron number; $S_\eta(z_\eta) \in R^{n_\eta}$ is basis function vector; $\varepsilon_\eta(z_\eta) \in R$ is the NN approximation error, which is required that it and its derivative are bounded (more details see [20]).

Based on the ideal approximation (13), $V_\eta^*(z_\eta)$ and α^* can be re-expressed as

$$\begin{aligned} V_\eta^*(z_\eta(t)) &= \beta_\eta\|z_\eta(t)\|^2 + W_\eta^{*T} S_\eta(z_\eta) + \varepsilon_\eta(z_\eta) \\ \alpha^* &= -\beta_\eta z_\eta(t) - \frac{1}{2}\frac{\partial^T S_\eta}{\partial z_\eta} W_\eta^* - \frac{1}{2}\frac{\partial\varepsilon_\eta}{\partial z_\eta} \end{aligned} \tag{14}$$

where $(\partial^T S_\eta/\partial z_\eta) \in R^{3\times n_\eta}$ and $(\partial\varepsilon_\eta/\partial z_\eta) \in R^3$ are the gradients with respect to z_η.

Using the NN approximation (14), HJB equation (8) can be rewritten as

$$\begin{aligned} H_\eta(z_\eta, \alpha^*, W_\eta^*) &= -\left(\beta_\eta^2 - 1\right)\|z_\eta\|^2 - 2\beta_\eta z_\eta^T \dot{\eta}_d \\ &\quad - W_\eta^{*T}\frac{\partial S_\eta}{\partial z_\eta^T}(\beta_\eta z_\eta + \dot{\eta}_d) \\ &\quad - \frac{1}{4}\left\|\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^*\right\|^2 + \rho_\eta(t) = 0 \end{aligned} \tag{15}$$

where $\rho_\eta(t) = (\partial\varepsilon_\eta/\partial z_\eta^T)\alpha^*(t) + (1/4)\|(\partial\varepsilon_\eta/\partial z_\eta)\|^2 - (\partial\varepsilon_\eta/\partial z_\eta^T)\dot{\eta}_d(t)$, which is a bounded term by a positive constant ψ_η, i.e., $|\rho_\eta(t)| \le \psi_\eta$.

The optimal virtual control (14) is unavailable because the ideal weight matrix W_η^* is unknown. In order to achieve the control scheme, the RL algorithm is performed by constructing the following both critic and actor NNs, which are utilized to evaluate the controlling performance and execute the virtual control, respectively:

$$\hat{V}_\eta^*(z_\eta) = \beta_\eta\|z_\eta(t)\|^2 + \hat{W}_{\eta c}^T(t)S_\eta(z_\eta) \tag{16}$$

$$\hat{\alpha}(z_\eta) = -\beta_\eta z_\eta(t) - \frac{1}{2}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t) \tag{17}$$

where $\hat{V}_\eta^*$ denote the estimations of V_η^*; $\hat{W}_{\eta c}^T \in R^{n_\eta}$ and $\hat{W}_{\eta a} \in R^{n_\eta}$ are the critic and actor NN weights, respectively.

Adding (16) and (17) into (8), the approximated HJB equation can be yielded as

$$\begin{aligned} H_\eta\left(z_\eta, \hat{\alpha}, \hat{W}_\eta\right) &= \|z_\eta\|^2 + \left\|\beta_\eta z_\eta + \frac{1}{2}\frac{\partial^T S_\eta(z_\eta)}{\partial z_\eta}\hat{W}_{\eta a}(t)\right\|^2 \\ &\quad - \left(2\beta_\eta z_\eta(t) + \frac{\partial^T S_\eta(z_\eta)}{\partial z_\eta}\hat{W}_{\eta c}(t)\right)^T \\ &\quad \times \left(\beta_\eta z_\eta(t) + \frac{1}{2}\frac{\partial^T S_\eta(z_\eta)}{\partial z_\eta}\hat{W}_{\eta a}(t) + \dot{\eta}_d(t)\right). \end{aligned} \tag{18}$$

From (15) and (18), the Bellman residual error is derived as

$$\begin{aligned} e_\eta(t) &= H_\eta\left(z_\eta, \hat{\alpha}, \hat{W}_\eta\right) - H_\eta\left(z_\eta, \alpha^*, W_\eta^*\right) \\ &= H_\eta\left(z_\eta, \hat{\alpha}, \hat{W}_\eta\right). \end{aligned} \tag{19}$$

By applying gradient descent algorithm to the positive definite function

$$E_\eta(t) = \frac{1}{2}e_\eta^2(t) \tag{20}$$

the following critic NN updating law is yielded so that the Bellman residual error $e(t)$ is minimized:

$$\begin{aligned} &\dot{\hat{W}}_{\eta c}(t) \\ &= -\frac{\gamma_{\eta c}}{1+\|\sigma_\eta(t)\|^2} e_\eta(t)\frac{\partial e_\eta(t)}{\partial \hat{W}_{\eta c}(t)} \\ &= -\frac{\gamma_{\eta c}}{1+\|\sigma_\eta\|^2}\sigma_\eta\left(\sigma_{\eta\hat{W}_{\eta c}}^T(t) - \left(\beta_\eta^2 - 1\right)\|z_\eta(t)\|^2 - 2\beta_\eta z_\eta^T(t)\dot{\eta}_d(t)\right. \\ &\qquad \left. + \frac{1}{4}\left\|\frac{\partial^T S_\eta(z_\eta)}{\partial z_\eta}\hat{W}_{\eta a}(t)\right\|^2\right) \end{aligned} \tag{21}$$

where $\gamma_{c1} > 0$ is the learning rate; $\sigma_\eta(t) = -([\partial S_\eta(z_\eta)]/\partial z_\eta^T)(\beta_\eta z_\eta(t) + (1/2)(\partial^T S_\eta/\partial z_\eta)\hat{W}_{\eta a}(t) + \dot{\eta}_d(t))$.

The actor NN updating law is designed in the following:

$$\begin{aligned} \dot{\hat{W}}_{\eta a}(t) &= \frac{1}{2}\frac{\partial S_\eta(z_\eta)}{\partial z_\eta^T}z_\eta(t) - \gamma_{\eta a}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t) \\ &\quad + \frac{\gamma_{\eta c}}{4\left(1+\|\sigma_\eta\|^2\right)}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t)\sigma_{\eta\hat{W}_{\eta c}}^T(t) \end{aligned} \tag{22}$$

where $\gamma_{\eta a} > 0$ is the learning rate.

Assumption 1 ([35] Persistence of Excitation (PE)): The signal of $\sigma_\eta(t)\sigma_\eta^T(t)$ is required persistent excitation over the interval $[t, t+t_\eta]$, i.e., there exist constants $\underline{k}_\eta > 0$, $\overline{k}_\eta > 0$, $t_\eta > 0$ for all t to satisfy

$$\underline{k}_\eta I_3 \le \sigma_\eta(t)\sigma_\eta^T(t) \le \overline{k}_\eta I_3 \tag{23}$$

where $I_3 \in R^{3\times 3}$ is identity matrix.

Remark 3: The PE assumption is also carried out in next backstepping step. The signal of $\sigma_\nu(t)\sigma_\nu^T(t)$, which is defined in next backstepping step, is required to meet the PE condition over the interval $[t, t+t_\nu]$, $t_\nu > 0$, i.e., there exist constants $\underline{k}_\nu > 0$, $\overline{k}_\nu > 0$ for all t to satisfy $\underline{k}_\nu I_3 \le \sigma_\nu(t)\sigma_\nu^T(t) \le \overline{k}_\nu I_3$.

By introducing the error variable $z_\nu(t) = \nu(t) - \hat{\alpha}(z_\eta)$, the error dynamic (4) can be rewritten as

$$\dot{z}_\eta(t) = z_\nu(t) + \hat{\alpha}\left(z_\eta\right) - \dot{\eta}_d(t). \tag{24}$$

For the z_η-subsystem, Lyapunov function candidate is designed as

$$L_\eta(t) = \frac{1}{2}\|z_\eta(t)\|^2 + \frac{1}{2}\tilde{W}_{\eta a}^T(t)\tilde{W}_{\eta a}(t) + \frac{1}{2}\tilde{W}_{\eta c}^T(t)\tilde{W}_{\eta c}(t) \tag{25}$$

where $\tilde{W}_{\eta c}(t) = \hat{W}_{\eta c}(t) - W_\eta^*$, and $\tilde{W}_{\eta a}(t) = \hat{W}_{\eta a}(t) - W_\eta^*$. Its time derivative along (21), (22), and (24) is

$$\begin{aligned} \dot{L}_\eta(t) &= z_\eta^T(t)\left(z_\nu(t) + \hat{\alpha}(z_\eta) - \dot{\eta}_d(t)\right) + \tilde{W}_{\eta a}^T(t) \\ &\quad \times \left(\frac{1}{2}\frac{\partial S_\eta(z_\eta)}{\partial z_\eta^T}z_\eta(t) - \gamma_{\eta a}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t)\right. \\ &\qquad + \frac{\gamma_{\eta c}}{4\left(1+\|\sigma_\eta\|^2\right)}\frac{\partial S_\eta(z_\eta)}{\partial z_\eta^T}\frac{\partial^T S_\eta(z_\eta)}{\partial z_\eta}\hat{W}_{\eta a}(t) \\ &\qquad \left. \times\ \sigma_\eta^T(t)\hat{W}_{\eta c}(t)\right) - \frac{\gamma_{\eta c}}{1+\|\sigma_\eta(t)\|^2} \\ &\quad \times \left(\tilde{W}_{\eta c}^T(t)\sigma_\eta \times \left(\sigma_{\eta\hat{W}_{\eta c}}^T(t) - \left(\beta_\eta^2 - 1\right)\|z_\eta\|^2\right.\right. \\ &\qquad \left.\left. - 2\beta_\eta z_\eta^T(t)\dot{\eta}_d(t) + \frac{1}{4}\left\|\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t)\right\|^2\right)\right). \end{aligned} \tag{26}$$

Substituting (17) into (26) yields

$$\begin{aligned} \dot{L}_\eta(t) &= -\beta_\eta\|z_\eta\|^2 + z_\eta^T z_\nu - z_\eta^T\dot{\eta}_d - \frac{1}{2}z_\eta^T\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t) \\ &\quad + \frac{1}{2}\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta(z_\eta)}{\partial z_\eta^T}z_\eta(t) - \gamma_{\eta a}\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta} \\ &\quad \times \hat{W}_{\eta a}(t) + \frac{\gamma_{\eta c}}{4\left(1+\|\sigma_\eta\|^2\right)}\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta(z_\eta)}{\partial z_\eta^T}\frac{\partial^T S_\eta(z_\eta)}{\partial z_\eta} \\ &\quad \times \hat{W}_{\eta a}(t)\sigma_{\eta\hat{W}_{\eta c}}^T(t) - \frac{\gamma_{\eta c}}{1+\|\sigma_\eta\|^2} \\ &\quad \times \left(\tilde{W}_{\eta c}^T(t)\sigma_\eta\left(\sigma_{\eta\hat{W}_{\eta c}}^T(t) - \left(\beta_\eta^2 - 1\right)\|z_\eta\|^2 - 2\beta_\eta z_\eta^T\dot{\eta}_d\right.\right. \\ &\qquad \left.\left. + \frac{1}{4}\left\|\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t)\right\|^2\right)\right). \end{aligned} \tag{27}$$

Using $\tilde{W}_{\eta a}(t) = \hat{W}_{\eta a}(t) - W_\eta^*$, there are the following results:

$$\begin{aligned} &-\frac{1}{2}z_\eta^T\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a} + \frac{1}{2}z_\eta^T\frac{\partial^T S_\eta}{\partial z_\eta}\tilde{W}_{\eta a} \\ &\quad = -\frac{1}{2}z_\eta^T\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^* - \gamma_{\eta a}\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t) \\ &\quad = -\frac{\gamma_{\eta a}}{2}\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T} \times \frac{\partial^T S_\eta}{\partial z_\eta}\tilde{W}_{\eta a}(t) \\ &\qquad - \frac{\gamma_{\eta a}}{2}\hat{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t) + \frac{\gamma_{\eta a}}{2} \\ &\qquad \times W_\eta^{*T}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^*. \end{aligned}$$

Adding the above results to (27) has

$$\begin{aligned}\dot{L}_\eta(t) &= -\beta_\eta\|z_\eta\|^2 + z_\eta^T z_v - z_\eta^T\dot{\eta}_d - \frac{1}{2}z_\eta^T\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^* \\ &- \frac{\gamma_{\eta a}}{2}\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\tilde{W}_{\eta a}(t) - \frac{\gamma_{\eta a}}{2}\hat{W}_{\eta a}^T(t) \\ &\times \frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t) + \frac{\gamma_{\eta a}}{2}W_\eta^{*T}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^* \\ &+ \frac{\gamma_{\eta c}}{4\left(1+\|\sigma_\eta\|^2\right)}\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta(z_\eta)}{\partial z_\eta^T}\frac{\partial^T S_\eta(z_\eta)}{\partial z_\eta}\hat{W}_{\eta a}(t) \\ &\times \sigma_{\eta\hat{W}_{\eta c}}^T(t) - \frac{\gamma_{\eta c}}{1+\|\sigma_\eta\|^2} \\ &\times \left(\tilde{W}_{\eta c}^T(t)\sigma_\eta\left(\sigma_{\eta\hat{W}_{\eta c}}^T(t) - \left(\beta_\eta^2-1\right)\|z_\eta\|^2 - 2\beta_\eta z_\eta^T\dot{\eta}_d \right.\right. \\ &\left.\left. + \frac{1}{4}\left\|\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t)\right\|^2\right)\right). \end{aligned} \tag{28}$$

Using Cauchy inequality that $(\sum_{k=1}^n a_k b_k)^2 \le \sum_{k=1}^n a_k^2 \sum_{k=1}^n b_k^2$ and Young's inequality that $ab \le (a^2/2) + (b^2/2)$, there are the following facts:

$$\begin{aligned} z_\eta^T(t)z_v(t) &\le \frac{1}{2}\|z_\eta(t)\|^2 + \frac{1}{2}\|z_v(t)\|^2 \\ -z_\eta^T(t)\dot{\eta}_d(t) &\le \frac{1}{2}\|z_\eta(t)\|^2 + \frac{1}{2}\|\dot{\eta}_d(t)\|^2 \\ -\frac{1}{2}z_\eta^T(t)\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^* &\le \|z_\eta(t)\|^2 + W_\eta^{*T}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^*. \end{aligned}$$

Applying the above inequalities to (28) has

$$\begin{aligned}\dot{L}_\eta(t) &\le \frac{1}{2}\|z_v(t)\|^2 - (\beta_\eta-2)\|z_\eta(t)\|^2 - \frac{\gamma_{\eta a}}{2}\tilde{W}_{\eta a}^T(t) \\ &\times \frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\tilde{W}_{\eta a}(t) - \frac{\gamma_{\eta a}}{2}\hat{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta} \\ &\times \hat{W}_{\eta a}(t) + \frac{\gamma_{\eta c}}{4\left(1+\|\sigma_\eta\|^2\right)}\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta} \\ &\times \hat{W}_{\eta a}(t)\sigma_{\eta\hat{W}_{\eta c}(t)}^T - \frac{\gamma_{\eta c}}{1+\|\sigma_\eta(t)\|^2} \\ &\times \left(\tilde{W}_{\eta c}^T(t)\sigma_\eta \times \left(\sigma_{\eta\hat{W}_{\eta c}}^T(t) - \left(\beta_\eta^2-1\right)\|z_\eta(t)\|^2\right.\right. \\ &\left.\left. - 2\beta_\eta z_\eta^T(t)\dot{\eta}_d + \frac{1}{4}\left\|\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}\right\|^2\right)\right) \\ &+ \frac{1}{2}\|\dot{\eta}_d\|^2 + \left(1+\frac{\gamma_{\eta a}}{2}\right)\times W_\eta^{*T}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^*. \end{aligned} \tag{29}$$

Based on (15), there is the following one:

$$\begin{aligned} -&\left(\beta_\eta^2-1\right)\|z_\eta(t)\|^2 - 2\beta_\eta z_\eta^T(t)\dot{\eta}_d(t) \\ &= -\sigma^T(t)W_\eta^* - \frac{1}{2}\hat{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^* \\ &+ \frac{1}{4}W_\eta^{*T}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^* - \rho_\eta(t). \end{aligned} \tag{30}$$

Substituting (30) into (29) yields

$$\begin{aligned}\dot{L}_\eta(t) &\le \frac{1}{2}\|z_v(t)\|^2 - (\beta_\eta-2)\|z_\eta\|^2 - \frac{\gamma_{\eta a}}{2}\tilde{W}_{\eta a}^T(t) \\ &\times \frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\tilde{W}_{\eta a}(t) - \frac{\gamma_{\eta a}}{2}\hat{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta} \\ &\times \hat{W}_{\eta a}(t) + \frac{\gamma_{\eta c}}{4\left(1+\|\sigma_\eta\|^2\right)}\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta} \\ &\times \hat{W}_{\eta a}(t)\sigma_{\eta\hat{W}_{\eta c}(t)}^T - \frac{\gamma_{\eta c}}{1+\|\sigma_\eta\|^2} \\ &\times \left(\tilde{W}_{\eta c}^T(t)\sigma_\eta \times \left(\sigma_{\eta\tilde{W}_{\eta c}}^T(t) - \frac{1}{2}\hat{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^*\right.\right. \\ &+ \frac{1}{4}W_\eta^{*T}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^* + \frac{1}{4}\hat{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta} \\ &\left.\left.\times \hat{W}_{\eta a}(t) - \rho_\eta(t)\right)\right) + \frac{1}{2}\|\dot{\eta}_d\|^2 + \left(1+\frac{\gamma_{\eta a}}{2}\right) \\ &\times W_\eta^{*T}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^*. \end{aligned} \tag{31}$$

Using the following facts:

$$\begin{aligned} &-\frac{1}{2}\hat{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^* + \frac{1}{4}W_\eta^{*T}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^* \\ &+ \frac{1}{4}\hat{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t) = \frac{1}{4}\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta} \\ &\times \hat{W}_{\eta a}(t) - \frac{1}{4}W_\eta^{*T}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\tilde{W}_{\eta a}(t) \end{aligned} \tag{32}$$

$$\begin{aligned} \frac{\gamma_{\eta c}}{1+\|\sigma_\eta(t)\|^2}\tilde{W}_{\eta c}^T(t)\sigma_\eta(t)\rho_\eta(t) &\le \frac{\gamma_{\eta c}}{2\left(1+\|\sigma_\eta\|^2\right)}\rho_\eta^2(t) \\ &+ \frac{\gamma_{\eta c}}{2\left(1+\|\sigma_\eta\|^2\right)}\tilde{W}_{\eta c}^T(t)\sigma_\eta\sigma_\eta^T\tilde{W}_{\eta c}(t) \end{aligned} \tag{33}$$

the inequality (31) can be rewritten as

$$\begin{aligned}\dot{L}_\eta(t) &\le \frac{1}{2}\|z_v(t)\|^2 - (\beta_\eta-2)\|z_\eta(t)\|^2 - \frac{\gamma_{\eta a}}{2}\tilde{W}_{\eta a}^T(t) \\ &\times \frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\tilde{W}_{\eta a}(t) - \frac{\gamma_{\eta c}}{2\left(1+\|\sigma_\eta(t)\|^2\right)}\tilde{W}_{\eta c}^T(t) \\ &\times \sigma_\eta\sigma_{\eta\hat{W}_{\eta c}}^T(t) - \frac{\gamma_{\eta a}}{2}\hat{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t) \\ &+ \frac{\gamma_{\eta c}}{4\left(1+\|\sigma_\eta\|^2\right)}\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t)\sigma_{\eta\hat{W}_{\eta c}(t)}^T \\ &- \frac{\gamma_{\eta c}}{4\left(1+\|\sigma_\eta\|^2\right)}\tilde{W}_{\eta c}^T(t)\sigma_{\eta\tilde{W}_{\eta a}^T}(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t) \\ &+ \frac{\gamma_{\eta c}}{4\left(1+\|\sigma_\eta\|^2\right)}\tilde{W}_{\eta c}^T(t)\sigma_{\eta W_\eta^{*T}}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t) \\ &+ \left(1+\frac{\gamma_{\eta a}}{2}\right)W_\eta^{*T}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^* + \frac{\gamma_{\eta c}}{2\left(1+\|\sigma_\eta\|^2\right)}\rho_\eta^2 \\ &+ \frac{1}{2}\|\dot{\eta}_d\|^2. \end{aligned} \tag{34}$$

Based on the following condition:

$$\frac{\gamma_{c1}}{4\left(1+\|\sigma_\eta\|^2\right)}\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t)\sigma_{\eta\hat{W}_{\eta c}}^T(t)$$
$$-\frac{\gamma_{\eta c}}{4\left(1+\|\sigma_\eta\|^2\right)}\tilde{W}_{\eta c}^T(t)\sigma_{\eta\tilde{W}_{\eta a}^T(t)}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}^T(t)$$
$$=\frac{\gamma_{\eta c}}{4\left(1+\|\sigma_\eta\|^2\right)}\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}W_\eta^{*T}\sigma_\eta\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t).$$

Equation (33) can become the following equation:

$$\begin{aligned}\dot{L}_\eta(t)\le{}&\frac{1}{2}\|z_\nu(t)\|^2-(\beta_\eta-2)\|z_\eta(t)\|^2-\frac{\gamma_{\eta a}}{2}\tilde{W}_{\eta a}^T(t)\\&\times\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\tilde{W}_{\eta a}(t)-\frac{\gamma_{\eta c}}{2\left(1+\|\sigma_\eta\|^2\right)}\tilde{W}_{\eta c}^T(t)\\&\times\sigma_\eta\sigma_{\eta\tilde{W}_{\eta c}}^T(t)-\frac{\gamma_{\eta a}}{2}\hat{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t)\\&+\frac{\gamma_{\eta c}}{4\left(1+\|\sigma_\eta\|^2\right)}\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}W_\eta^{*T}\sigma_\eta\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t)\\&+\frac{\gamma_{\eta c}}{4\left(1+\|\sigma_\eta\|^2\right)}\tilde{W}_{\eta c}^T(t)\sigma_{\eta W_\eta^{*T}}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\tilde{W}_{\eta a}(t)\\&+\left(1+\frac{\gamma_{\eta a}}{2}\right)W_\eta^{*T}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^*+\frac{\gamma_{\eta c}}{2\left(1+\|\sigma_\eta\|^2\right)}\rho_\eta^2\\&+\frac{1}{2}\|\dot{\eta}_d\|^2.\end{aligned}\tag{35}$$

According to Young's inequality and Cauchy inequality, there are the following results:

$$\begin{aligned}&\frac{\gamma_{\eta c}}{4\left(1+\|\sigma_\eta\|^2\right)}\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}W_\eta^{*T}\sigma_\eta\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t)\\&\quad\le\frac{1}{32}\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}W_\eta^{*T}\sigma_\eta\sigma_{\eta W_\eta^*}^T\frac{\partial^T S_\eta}{\partial z_\eta}\tilde{W}_{\eta a}(t)\\&\qquad+\frac{\gamma_{\eta c}^2}{2}\hat{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t)\\&\frac{\gamma_{\eta c}}{4\left(1+\|\sigma_\eta\|^2\right)}\tilde{W}_{\eta c}^T(t)\sigma_{\eta W_\eta^{*T}}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\tilde{W}_{\eta a}(t)\\&\quad\le\frac{\gamma_{\eta c}^2}{2}\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\tilde{W}_{\eta a}(t)+\frac{1}{32\left(1+\|\sigma_\eta\|^2\right)}\\&\qquad\times\tilde{W}_{\eta c}^T(t)\sigma_{\eta W_\eta^{*T}}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^*\sigma_\eta^T\tilde{W}_{\eta c}(t).\end{aligned}$$

Substituting the above inequalities into (35) has

$$\begin{aligned}\dot{L}_\eta(t)\le{}&\frac{1}{2}\|z_\nu(t)\|^2-(\beta_\eta-2)\|z_\eta(t)\|^2\\&-\left(\frac{\gamma_{\eta a}}{2}-\frac{\gamma_{\eta c}^2}{2}-\frac{1}{32}W_\eta^{*T}\sigma_\eta\sigma_{\eta W_\eta^*}^T\right)\tilde{W}_{\eta a}^T(t)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\tilde{W}_{\eta a}(t)\\&-\frac{1}{1+\|\sigma_\eta\|^2}\left(\frac{\gamma_{\eta c}}{2}-\frac{1}{32}W_\eta^{*T}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^*\right)\\&\times\tilde{W}_{\eta c}^T(t)\sigma_\eta\sigma_{\eta\tilde{W}_{\eta c}}^T(t)-\left(\frac{\gamma_{\eta a}}{2}-\frac{\gamma_{\eta c}^2}{2}\right)\hat{W}_{\eta a}^T(t)\\&\times\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t)+\left(1+\frac{\gamma_{\eta a}}{2}\right)W_\eta^{*T}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\\&\times W_\eta^*+\frac{\gamma_{\eta c}}{2\left(1+\|\sigma_\eta\|^2\right)}\rho_\eta^2+\frac{1}{2}\|\dot{\eta}_d\|^2.\end{aligned}\tag{36}$$

Then the above inequality can be rewritten to compact form as

$$\begin{aligned}\dot{L}_\eta(t)\le{}&-\xi_\eta^T(t)A_\eta(t)\xi_\eta(t)+C_\eta(t)+\frac{1}{2}\|z_\nu(t)\|^2\\&-\left(\frac{\gamma_{\eta a}}{2}-\frac{\gamma_{\eta c}^2}{2}\right)\hat{W}_{\eta a}^T\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}\end{aligned}\tag{37}$$

where

$$\xi_\eta(t)=[z_\eta^T(t),\quad\tilde{W}_{\eta a}^T(t),\quad\tilde{W}_{\eta c}^T(t)]^T$$

$$A_\eta(t)=\begin{bmatrix}\beta_\eta-2&0&0\\0&\left(\frac{\gamma_{\eta a}}{2}-\frac{\gamma_{\eta c}^2}{2}-\frac{1}{32}W_\eta^{*T}\sigma_\eta\sigma_{\eta W_\eta^*}^T\right)\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}&0\\0&0&\frac{1}{1+\|\sigma_\eta\|^2}\left(\frac{\gamma_{\eta c}}{2}-\frac{1}{32}W_\eta^{*T}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^*\right)\sigma_\eta\sigma_\eta^T\end{bmatrix}$$

$$C_\eta(t)=(1+\frac{\gamma_{\eta a}}{2})W_\eta^{*T}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^*+\frac{\gamma_{\eta c}}{2(1+\|\sigma_\eta\|^2)}\rho_\eta^2+\frac{1}{2}\|\dot{\eta}_d\|^2.$$

Based on Assumption 1, the matrix $A_\eta(t)$ can be made positive definite via choosing the design parameters β_η, $\gamma_{\eta a}$, $\gamma_{\eta c}$ satisfying the following conditions:

$$\begin{aligned}&\beta_\eta>2,\gamma_{\eta a}>\gamma_{\eta c}^2+\frac{\bar{k}_\eta}{16}W_\eta^{*T}W_\eta^*\\&\gamma_{\eta c}>\frac{1}{16}\sup_{t\ge 0}\left\{W_\eta^{*T}\frac{\partial S_\eta}{\partial z_\eta^T}\frac{\partial^T S_\eta}{\partial z_\eta}W_\eta^*\right\}.\end{aligned}\tag{38}$$

Then (37) can become the following one:

$$\dot{L}_\eta(t)\le\frac{1}{2}\|z_\nu(t)\|^2-a_\eta\|\xi_\eta(t)\|^2+c_\eta\tag{39}$$

where $a_\eta=\inf_{t\ge 0}\{\lambda_{\min}\{A_\eta(t)\}\}$, $c_\eta=\sup_{t\ge 0}\{C_\eta(t)\}$.

Step 2: The actual control u is obtained in the step. From the dynamic equation (3), the time derivative of error variable $z_\nu(t)=\nu(t)-\hat{\alpha}$ is

$$\dot{z}_\nu(t)=f(\chi)-\dot{\hat{\alpha}}+u.\tag{40}$$

Define the optimal cost function as

$$\begin{aligned}V_\nu^*(z_\nu)&=\min_{u\in\Psi(\Omega_\nu)}\int_t^\infty r_\nu(z_\nu,u)ds\\&=\int_t^\infty r_\nu\left(z_\nu,u^*\right)ds\end{aligned}\tag{41}$$

where $r_\nu(z_\nu,u)=z_\nu^Tz_\nu+u^Tu$, Ω_ν is a compact set, u^* is the optimal control. Then the HJB equation for z_ν-subsystem is derived as

$$H_\nu\left(z_\nu, u^*, \frac{\partial V_\nu^*}{\partial z_\nu}\right) = z_\nu^T(t) z_\nu(t) + u^{*T} u^* + \frac{\partial V_\nu^*}{\partial z_\nu^T}\left(f(\chi) - \dot{\hat{\alpha}} + u^*\right) = 0. \quad (42)$$

By solving $(\partial H_\nu / \partial u^*) = 0$, the optimal control u^* is obtained as

$$u^* = -\frac{1}{2}\frac{\partial V_\nu^*}{\partial z_\nu}. \quad (43)$$

Rewrite the optimal cost function as

$$V_\nu^*(z_\nu) = \beta_\nu \|z_\nu(t)\|^2 + V_\nu^o(z_\nu) \quad (44)$$

where β_ν is a positive design constant, $V_\nu^o(z_\nu) = -\beta_\nu \|z_\nu(t)\|^2 + V_\nu^*(z_\nu)$. Substituting (44) into (42), the optimal control can be rewritten as

$$u^* = -\beta_\nu z_\nu(t) - \frac{1}{2}\frac{\partial V_\nu^o(z_\nu)}{\partial z_\nu} \quad (45)$$

Since the uncertaintied term $[(\partial V_\nu^o(z_\nu))/(\partial z_\nu)]$ is continuous and well defined in the compact set Ω_ν, it can be approximated by NNs as

$$V_\nu^o(z_\nu) = W_\nu^{*T} S_\nu(z_\nu) + \varepsilon_\nu(z_\nu) \quad (46)$$

where $W_\nu^{*T} \in R^{n_\nu}$ is the ideal weight; $S_\nu(z_\nu) \in R^{n_\nu}$ is the basis function vector; $\varepsilon_\nu(z_\nu) \in R$ is the approximation error.

Substituting (46) into (45) has

$$u^* = -\beta_\nu z_\nu(t) - \frac{1}{2}\frac{\partial^T S_\nu(z_\nu)}{\partial z_\nu} W_\nu^* - \frac{1}{2}\frac{\partial \varepsilon_\nu}{\partial z_\nu} \quad (47)$$

where $(\partial \varepsilon_\nu / \partial z_\nu)$ is bounded by a constant δ_ν, i.e., $\|(\partial \varepsilon_\nu / \partial z_\nu)\| \le \delta_\nu$.

Inserting (46) and (47) into (42) yields

$$\begin{aligned} H_\nu\left(z_\nu, u^*, W_\nu^*\right) = & -(\beta_\nu^2 - 1)\|z_\nu(t)\|^2 + 2\beta_\nu z_\nu^T(t) \\ & \times \left(f(\chi) - \dot{\hat{\alpha}}\right) + W_\nu^{*T}\frac{\partial S_\nu(z_\nu)}{\partial z_\nu^T} \\ & \times \left(f(\chi) - \dot{\hat{\alpha}} - \beta_\nu z_\nu\right) \\ & - \frac{1}{4}\left\|\frac{\partial^T S_\nu(z_\nu)}{\partial z_\nu} W_\nu^*\right\|^2 + \rho_\nu(t) = 0 \end{aligned} \quad (48)$$

where $\rho_\nu(t) = (\partial \varepsilon_\nu / \partial z_\nu^T) u^* + (\partial \varepsilon_\nu / \partial z_\nu^T)(f(x) - \dot{\hat{\alpha}}) + (1/4)\|(\partial \varepsilon_\nu / \partial z_\nu)\|^2$. Since all terms of $\rho_\nu(t)$ are bounded, it can be bounded by a constant, i.e., $|\rho_\nu(t)| \le \psi_\nu$.

Because the ideal constant matrix W_ν^* is unknown, the optimal control (47) is unavailable. For getting the available control, the online critic-actor RL is employed to implement the optimizing scheme

$$\hat{V}_\nu^*(z_\nu) = \beta_\nu \|z_\nu(t)\|^2 + \hat{W}_{\nu c}^T(t) S_\nu(z_\nu) \quad (49)$$

$$u = -\beta_\nu z_\nu(t) - \frac{1}{2}\frac{\partial^T S_\nu(z_\nu)}{\partial z_\nu}\hat{W}_{\nu a} \quad (50)$$

where $\hat{V}_\nu^*(z_\nu)$ are the approximation of $V_\nu^*(z_\nu)$; $\hat{W}_{\nu c}^T(t) \in R^{n_\nu}$ and $\hat{W}_{\nu a}^T(t) \in R^{n_\nu}$ are the critic and actor NN weights, respectively.

The optimized control u for the vessel dynamic (3) is depicted in earth-fixed frame, the control for the dynamic system (1) can be obtained by the following equation:

$$\tau = MJ^{-1}(\eta) u(t). \quad (51)$$

Substituting (49) and (50) into (42), the approximated HJB equation can be obtained as

$$\begin{aligned} H_\nu\left(z_\nu, u, \hat{W}_\nu\right) = & \|z_\nu\|^2 + \left\|\beta_\nu z_\nu + \frac{1}{2}\frac{\partial^T S_\nu(z_\nu)}{\partial z_\nu}\hat{W}_{\nu a}(t)\right\|^2 \\ & + \left(2\beta_\nu z_\nu(t) + \frac{\partial^T S_\nu(z_\nu)}{\partial z_\nu}\hat{W}_{\nu c}(t)\right)^T \\ & \times \left(f(\chi) - \dot{\hat{\alpha}} - \beta_\nu z_\nu(t) - \frac{1}{2}\frac{\partial^T S_\nu}{\partial z_\nu}\hat{W}_{\nu a}(t)\right). \end{aligned} \quad (52)$$

Similar with step 1, the critic NN weight updating law is constructed by minimizing Bellman error $e_\nu(t) = H_\nu(z_\nu, u, \hat{W}_{\nu c})$. Define a positive definite function as $E_\nu(t) = (1/2)e_\nu^2(t)$, then the critic NN weight updating law is derived based on the gradient descent algorithm

$$\begin{aligned} \dot{\hat{W}}_{\nu c}(t) = & -\frac{\gamma_{\nu c}}{1 + \|\sigma_\nu\|^2} e_\nu(t) \frac{\partial e_\nu(t)}{\partial \hat{W}_{\nu c}(t)} \\ = & -\frac{\gamma_{\nu c}}{1 + \|\sigma_\nu\|^2}\sigma_\nu\left(\sigma_\nu^T \hat{W}_{\nu c}(t) - (\beta_\nu^2 - 1) z_\nu^2(t)\right. \\ & + 2\beta_\nu z_\nu^T(t)\left(f(\chi) - \dot{\hat{\alpha}}\right) \\ & \left. + \frac{1}{4}\hat{W}_{\nu a}^T \frac{\partial S_\nu}{\partial z_\nu^T}\frac{\partial^T S_\nu}{\partial z_\nu}\hat{W}_{\nu a}\right) \end{aligned} \quad (53)$$

where $\gamma_{\nu c} > 0$ is the learning rate, $\sigma_\nu = (\partial S_\nu / \partial z_\nu^T)(f(\chi) - \dot{\hat{\alpha}} - \beta_\nu z_\nu(t) - (1/2)(\partial^T S_\nu / \partial z_\nu)\hat{W}_{\nu a}(t))$.

The weight updating law of actor NN is

$$\begin{aligned} \dot{\hat{W}}_{\nu a}(t) = & \frac{1}{2}\frac{\partial S_\nu}{\partial z_\nu^T} z_\nu(t) - \gamma_{\nu a}\frac{\partial S_\nu}{\partial z_\nu^T}\frac{\partial^T S_\nu}{\partial z_\nu}\hat{W}_{\nu a}(t) \\ & + \frac{\gamma_{\nu c}}{4\left(1 + \|\sigma_\nu\|^2\right)}\frac{\partial S_\nu}{\partial z_\nu^T}\frac{\partial^T S_\nu}{\partial z_\nu}\hat{W}_{\nu a}(t)\sigma_\nu^T \hat{W}_{\nu c}(t) \end{aligned} \quad (54)$$

where $\gamma_{\nu a} > 0$ are the learning rate.

Consider the overall Lyapunov function candidate as follows:

$$L(t) = L_\eta(t) + \frac{1}{2} z_\nu^T(t) z_\nu(t) + \frac{1}{2}\tilde{W}_{\nu a}^T(t)\tilde{W}_{\nu a}(t) + \frac{1}{2}\tilde{W}_{\nu c}^T(t)\tilde{W}_{\nu c}(t) \quad (55)$$

where $\tilde{W}_{\nu c}(t) = \hat{W}_{\nu c}(t) - W_\nu^*$ and $\tilde{W}_{\nu a}(t) = \hat{W}_{\nu a}(t) - W_\nu^*$ are the critic and actor NN approximation errors, respectively.

The time derivative of $L(t)$ along (40), (53), and (54) is

$$\begin{aligned} \dot{L}(t) = & \dot{L}_\eta(t) + z_\nu^T(t)\left(f(\chi) - \dot{\hat{\alpha}} + u\right) + \tilde{W}_{\nu a}^T(t) \\ & \times \left(\frac{1}{2}\frac{\partial S_\nu}{\partial z_\nu^T} z_\nu(t) - \gamma_{\nu a}\frac{\partial S_\nu}{\partial z_\nu^T}\frac{\partial^T S_\nu}{\partial z_\nu}\hat{W}_{\nu a}(t)\right. \\ & \left. + \frac{\gamma_{\nu c}}{4\left(1 + \|\sigma_\nu\|^2\right)}\frac{\partial S_\nu}{\partial z_\nu^T}\frac{\partial^T S_\nu}{\partial z_\nu}\hat{W}_{\nu c}(t)\sigma_\nu^T \hat{W}_{\nu c}(t)\right) \end{aligned}$$

$$-\frac{\gamma_{vc}}{1+\|\sigma_v\|^2}\tilde{W}_{vc}^T(t)\sigma_v \times \left(\sigma_v^T\hat{W}_{vc}(t) - (\beta_v^2-1)z_v^2(t) + 2\beta_v z_v^T\left(f(\chi)-\dot{\hat{\alpha}}\right) + \frac{1}{4}\hat{W}_{va}^T(t)\frac{\partial S_v}{\partial z_v^T}\frac{\partial^T S_v}{\partial z_v}\hat{W}_{va}(t)\right). \tag{56}$$

Applying the control (50)–(56), similar with the first step, there is the following one:

$$\begin{aligned}\dot{L}(t) \le{}& \dot{L}_\eta(t) - (\beta_v - 3)\|z_v(t)\|^2 - \frac{\gamma_{va}}{2}\tilde{W}_{va}^T(t)\frac{\partial S_v}{\partial z_v^T}\\ &\times\frac{\partial^T S_v}{\partial z_v}\tilde{W}_{va}(t) - \frac{\gamma_{va}}{2}\hat{W}_{va}^T(t)\frac{\partial S_v}{\partial z_v^T}\frac{\partial^T S_v}{\partial z_v}\hat{W}_{va}(t)\\ &-\frac{\gamma_{vc}}{4\left(1+\|\sigma_v\|^2\right)}\tilde{W}_{va}^T(t)\frac{\partial S_v}{\partial z_v^T}\frac{\partial^T S_v}{\partial z_v}\hat{W}_{vc}\sigma_v^T\hat{W}_{vc}(t)\\ &-\frac{\gamma_{vc}}{1+\|\sigma_v\|^2}\tilde{W}_{vc}^T(t)\sigma_v\\ &\times\left(\sigma_v^T\hat{W}_{vc}(t) - (\beta_v^2-1)z_v^2(t) + 2\beta_v z_v^T\left(f(\chi)-\dot{\hat{\alpha}}\right)\right.\\ &\left.+\frac{1}{4}\hat{W}_{va}^T(t)\frac{\partial S_v}{\partial z_v^T}\frac{\partial^T S_v}{\partial z_v}\hat{W}_{va}(t)\right)\\ &+\left(1+\frac{\gamma_{va}}{2}\right)W_v^{*T}\frac{\partial S_v}{\partial z_v^T}\frac{\partial^T S_v}{\partial z_v}W_v^*\\ &+\frac{1}{2}f^2(\chi)+\frac{1}{2}\|\dot{\alpha}\|^2.\end{aligned} \tag{57}$$

Rewrite (48) to the following one:

$$\begin{aligned}&-(\beta_v^2-1)\|z_v(t)\|^2 + 2\beta_v z_v^T(t)\left(f(\chi)-\dot{\hat{\alpha}}\right)\\ &\quad= -\sigma_v^T W_v^* - \hat{W}_{va}^T(t)\frac{\partial S_v(z_v)}{\partial z_v^T}\frac{\partial^T S_v(z_v)}{\partial z_v}W_v^*\\ &\quad\quad+\frac{1}{4}\left\|\frac{\partial^T S_v(z_v)}{\partial z_v}W_v^*\right\|^2 - \rho_v(t).\end{aligned} \tag{58}$$

Similar to the first step, applying (58) to (57) yields

$$\begin{aligned}\dot{L}(t) \le{}& \dot{L}_\eta(t) - (\beta_v-3)\|z_v(t)\|^2\\ &-\left(\frac{\gamma_{va}}{2}-\frac{\gamma_{vc}^2}{2}-\frac{1}{32}W_v^{*T}\sigma_v\sigma_v^T W_v^*\right)\tilde{W}_{va}^T(t)\frac{\partial S_v}{\partial z_v^T}\frac{\partial^T S_v}{\partial z_v}\tilde{W}_{va}(t)\\ &-\frac{\gamma_{vc}}{1+\|\sigma_v\|^2}\left(\frac{\gamma_{vc}}{2}-\frac{1}{32}\frac{\partial S_v}{\partial z_v^T}\frac{\partial^T S_v}{\partial z_v}\right)\tilde{W}_{vc}^T(t)\sigma_v\sigma_v^T\tilde{W}_{vc}(t)\\ &-\left(\frac{\gamma_{va}}{2}-\frac{\gamma_{vc}^2}{2}\right)\hat{W}_{va}^T\frac{\partial S_v}{\partial z_v^T}\frac{\partial^T S_v}{\partial z_v}\hat{W}_{va}\\ &+\left(1+\frac{\gamma_{va}}{2}\right)W_v^{*T}\frac{\partial S_v}{\partial z_v^T}\frac{\partial^T S_v}{\partial z_v}W_v^* + \frac{1}{2}f^2(\chi)\\ &+\frac{1}{2}\|\dot{\alpha}\|^2 + \frac{\gamma_{vc}}{2}\rho_v^2.\end{aligned} \tag{59}$$

Using the previous results, (59) is rewritten to compact form as

$$\begin{aligned}\dot{L}(t) \le{}& -a_\eta\left\|\xi_\eta(t)\right\|^2 + c_\eta - \xi_v^T(t)A_v(t)\xi_v(t) + C_v\\ &-\left(\frac{\gamma_{va}}{2}-\frac{\gamma_{vc}^2}{2}\right)\hat{W}_{va}^T\frac{\partial S_v}{\partial z_v^T}\frac{\partial^T S_v}{\partial z_v}\hat{W}_{va}\end{aligned} \tag{60}$$

where

$$\xi_v(t) = [z_v^T(t), \tilde{W}_{vc}^T(t), \tilde{W}_{va}^T(t)]^T$$

$$A_v(t) = \begin{bmatrix}\beta_v - 3\frac{1}{2} & 0 & 0\\ 0 & \frac{\gamma_{va}}{2}-\frac{\gamma_{vc}^2}{2}-\frac{1}{32}W_v^{*T}\sigma_v\sigma_v^T W_v^* & 0\\ 0 & 0 & \frac{\gamma_{vc}}{1+\|\sigma_v\|^2}\left(\frac{\gamma_{vc}}{2}-\frac{1}{32}W_v^{*T}\frac{\partial S_v}{\partial z_v^T}\frac{\partial^T S_v}{\partial z_v}W_v^*\right)\sigma_v\sigma_v^T\end{bmatrix}$$

$$C_v(t) = (1+\frac{\gamma_{va}}{2})W_v^{*T}\frac{\partial S_v}{\partial z_v^T}\frac{\partial^T S_v}{\partial z_v}W_v^* + \frac{\gamma_{vc}}{2}f^2(\chi) + \frac{\gamma_{vc}}{2}\rho_v^2 + \frac{1}{2}\|\dot{\alpha}\|^2.$$

Based on PE assumption, $A_v(t)$ can be made positive definite by designing the parameters β_v, γ_{va}, and γ_{vc} to satisfy the following conditions:

$$\begin{aligned}&\beta_v > 4,\ \gamma_{va} > \gamma_{vc}^2 + \frac{\bar{k}_v}{16}W_v^{*T}W_v^*\\ &\gamma_{vc} > \frac{1}{16}\sup_{t\ge 0}\left\{W_v^{*T}\frac{\partial S_v}{\partial z_v^T}\frac{\partial^T S_v}{\partial z_v}W_v^*\right\}\end{aligned} \tag{61}$$

then (60) can become the following one:

$$\dot{L}(t) < -a_\eta\left\|\xi_\eta(t)\right\|^2 - a_v\|\xi_v(t)\|^2 + c_\eta + c_v \tag{62}$$

where $a_v = \inf_{t\ge 0}\{\lambda_{\min}\{A_v(t)\}\}$, $c_v = \sup_{t\ge 0}\{\|C_v(t)\|\}$.

The main results are concluded in the following theorem.

Theorem 1: Consider the surface vessel (1) with bounded initial condition and reference signals. If the OB control utilizes the weight updating laws (21), (22) for the virtual control (17), and (53), (54) for the actual control (50), and the design parameters satisfy (38), (61), and PE conditions (Assumptions 1) are satisfied, then:

1) all error signals of the optimized control are SGUUB;
2) the surface vessel can track the reference trajectory to desired accuracy.

Proof: See the Appendix. ■

IV. Simulation Examples

The simulation is carried out by a mode ship of 1:75 scale-down replica. The mass of the model ship is $m = 21$ kg, its length and width are 1.2 and 0.3 m, respectively. The inertia, Coliolis centripetal, damping matrices are

$$M = \begin{bmatrix}20 & 0 & 0\\ 0 & 19 & 0.72\\ 0 & 0.72 & 2.7\end{bmatrix}$$

$$C = \begin{bmatrix}0 & 0 & -19v_y - 0.72v_z\\ 0 & 0 & 20v_x\\ 19v_y + 0.72v_z & -20v_x & 0\end{bmatrix}$$

$$D = \begin{bmatrix}0.72 + 1.3|v_x| + 5.8v_x^2 & 0 & 0\\ 0 & 0.86 + 36|v_y| + 3|v_z| & -0.1 - 2|v_y| + 2|v_z|\\ 0 & -0.1 - 5|v_y| + 3|v_z| & 6 + 4|v_y| + 4|v_z|\end{bmatrix}.$$

For simplicity, the restoring force vector $g(\eta)$ is assumed to be 0. The initial states of position and velocity are $\eta(0) = [0.3,\quad 0.1,\quad 0.2]^T$ and $v(0) = [0.1,\quad 0.2,\quad 0.3]^T$.

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

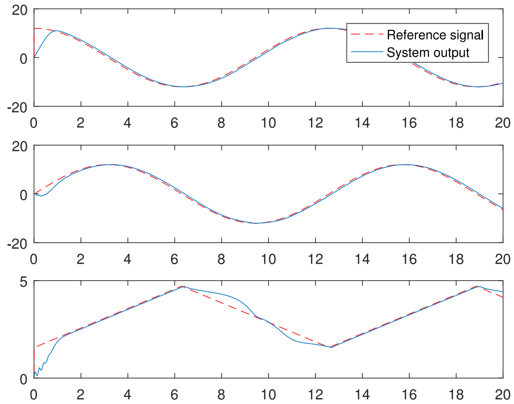

Fig. 2. Tracking performance of position and head states.

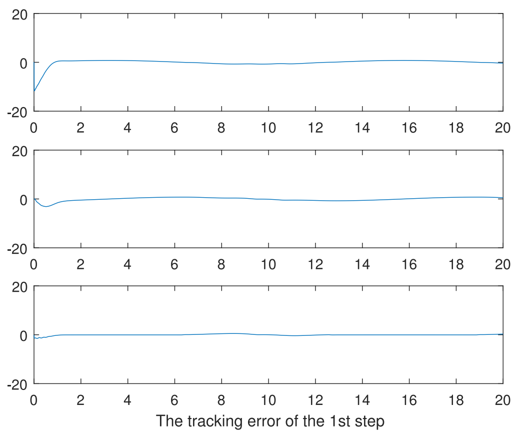

Fig. 3. Positive tracking error z_η.

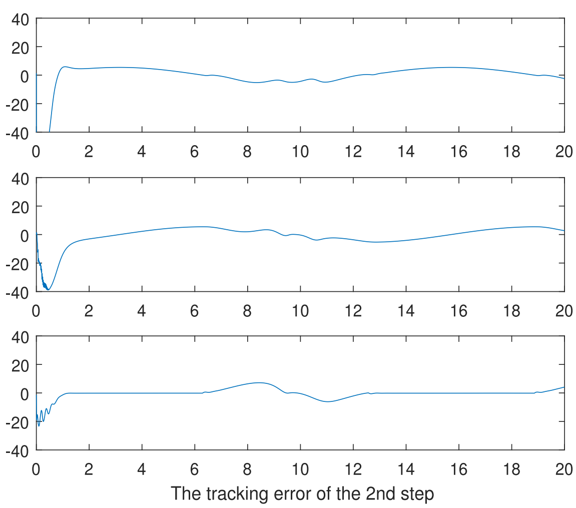

Fig. 4. Velocity tracking error z_ν.

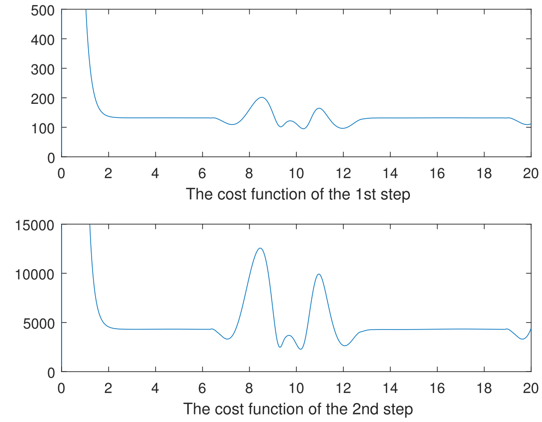

Fig. 5. Cost functions of two backstepping steps.

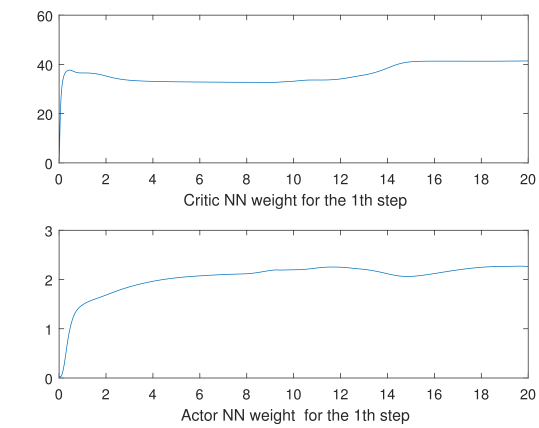

Fig. 6. Critic and actor weights of the first step.

The desired reference signal is $\eta_d(t) = [12\sin(0.2t + (\pi/2)), 12\sin(0.2t), \arcsin(\sin(0.2t)) + (\pi/2)]^T$ with the initial value $\eta_d(0) = [0.2, \quad 0.1, \quad 0.1]^T$.

For the two backstepping steps, the NNs are designed to contain 12 nodes, i.e., $n_\eta = n_\nu = 12$. The basis function vector are $S_\eta(z_\eta) = [s_1(z_\eta), \ldots, .s_{12}(z_\eta)]^T$ and $S_\nu(z_\nu) = [s_1(z_\nu), \ldots, .s_{12}(z_\nu)]^T$, where $s_i(z_\eta) = \exp[-(z_\eta - \mu_i)^T(z_\eta - \mu_i)/\phi_i^2]$, $s_i(z_\nu) = \exp[-(z_\nu - \mu_i)^T(z_\nu - \mu_i)/\phi_i^2]$, $i = 1, 2, \ldots, 12$ and the centers μ_i evenly spaced in the range of $[-6, 6] \times [-6, 6] \times [-3, 3]$, the widths are $\phi_i = 2$ for all.

The updating laws are designed based on (21), (53) for critic NNs and (22), (54) for actor NNs, respectively, of which the learning rates are $\gamma_{c1} = 0.1$, $\gamma_{c2} = 0.01$, $\gamma_{a1} = 0.3$, $\gamma_{a2} = 0.4$, and the initial conditions are $W_{c1}(0) = W_{c2}(0) = [0.01, \ldots, 0.02]^T \in R^{72\times 1}$, $W_{a1}(0) = W_{a2}(0) = [0.01, \ldots, 0.01]^T \in R^{72\times 1}$. Then the virtual control and actual control are obtained based on (17) and (50), of which the control parameters are $\beta_\eta = 10$ and $\beta_\nu = 14$.

The simulation results are shown in Fig. 2–7. Figs. 2 shows the tracking performance for the position and heading states. The tracking error vectors, $z_\eta(t)$ and $z_\nu(t)$, are presented by Figs. 3 and 4, which converges to zero. The cost terms, $r_\eta(z_\eta, \alpha_\eta)$ for the first step and $r_\nu(z_\nu, u)$ for the second step, are presented by Fig. 5. The bounded critic and actor weight vectors are shown by Fig. 6 for the first step and Fig. 7 for the second step. Figs. 2–7 further demonstrate the proposed optimizing scheme can guarantee that the control objective is achieved.

In order to display the good optimizing performance of the method, a comparison with the vessel control method of literature [7] is implemented. The simulation results are shown in Figs. 8 and 9. Fig. 8 shows that the similar performance

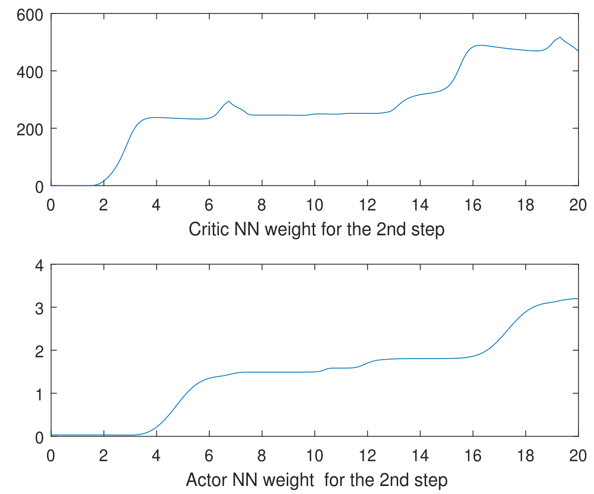

Fig. 7. Critic and actor weights of the second step.

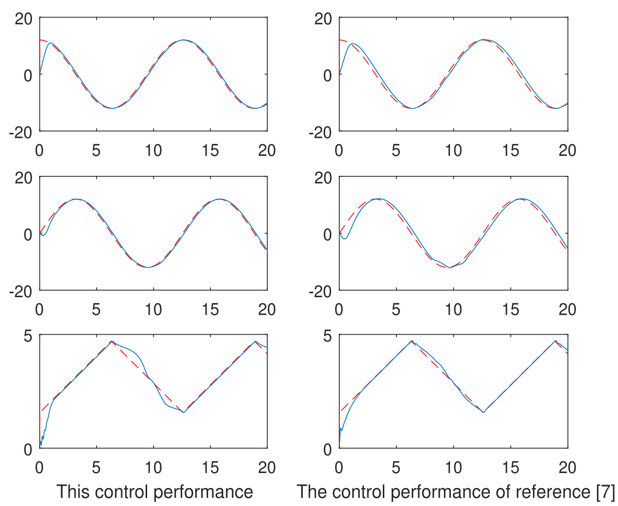

Fig. 8. Similar control performance.

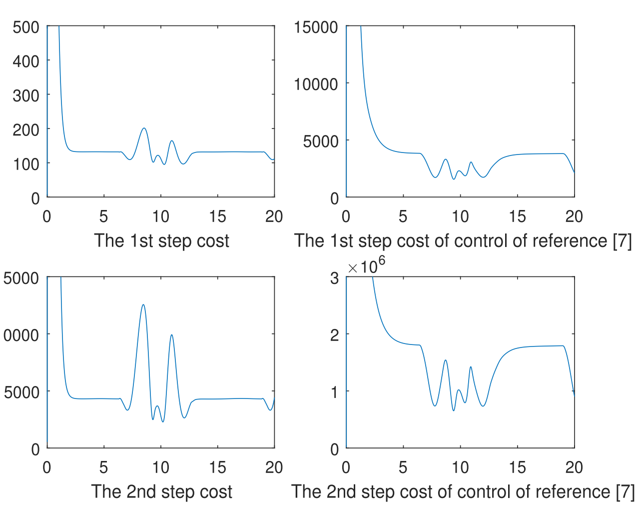

Fig. 9. Cost functions of two control methods.

is archived, and Fig. 9 shows the control costs of two control methods. Obviously, the proposed control method is lower-cost under the same control performances.

V. Conclusion

Based on the new optimizing technique OB [20], an optimized tracking control for surface vessel is developed. In the optimized control, the NN-based RL strategy of actor-critic architecture is employed, where the critic NN is used to evaluate the control performance and the actor NN is used to carry out the control behavior. The overall control for the vessel system is optimized by designing both virtual and actual controls to be the optimized solutions of corresponding subsystems. Based on the Lyapunov analysis, it is proven that the proposed optimal algorithm can achieve the control objective. The effectiveness is further demonstrated by simulation results.

Appendix
Proof of Theorem 1

The following lemma is used in the proof.

Lemma 1 [36]: Let $G(t) \in R$ be a continuous positive function with bounded initial value $G(0)$. If $\dot{G}(t) \leq -aG(t) + c$ is held, where a and c are two constants, then there is following inequality:

$$G(t) \leq e^{-at}G(0) + \frac{c}{a}\left(1 - e^{-at}\right). \tag{63}$$

Proof of Theorem 1:

1) Taking $a = \min\{a_\eta, a_\nu\}$ and $c = \max\{c_\eta, c_\nu\}$, then (62) can be rewritten as

$$\dot{L}(t) < -aL(t) + c. \tag{64}$$

According to Lemma 1, the following one can be obtained:

$$L(t) < e^{-at}L(0) + \frac{c}{a}\left(1 - e^{-at}\right). \tag{65}$$

The above inequality implies that all error signals, $z_\eta(t)$, $z_\nu(t)$, $\tilde{W}_{\eta a}(t)$, $\tilde{W}_{\eta c}(t)$, $\tilde{W}_{\nu a}(t)$, $\tilde{W}_{\nu c}(t)$, are SGUUB.

2) Let $L_z(t) = (1/2)z_\eta^T(t)z_\eta(t) + (1/2)z_\nu^T(t)z_\nu(t)$, its time derivative along (24) and (40) is

$$\dot{L}_z(t) = z_\eta^T(t)\left(z_\nu(t) + \hat{\alpha} - \dot{\eta}_d(t)\right) + z_\nu^T(t)\Big(f(\chi) - \dot{\hat{\alpha}} + u\Big). \tag{66}$$

Substituting (17) and (50) into (66) has

$$\begin{aligned}\dot{L}_z(t) = &-\beta_\eta\left\|z_\eta(t)\right\|^2 - \frac{1}{2}z_\eta^T(t)\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t) + z_\eta^T(t)z_\nu(t)\\ &- z_\eta^T(t)\dot{\eta}_d(t) - \beta_\nu\|z_\nu(t)\|^2 - \frac{1}{2}z_\nu^T(t)\frac{\partial^T S_\nu}{\partial z_\nu}\hat{W}_{\nu a}(t)\\ &+ z_\nu^T(t)f(\chi(t)) - z_\nu^T(t)\dot{\hat{\alpha}}.\end{aligned} \tag{67}$$

Based on the following result facts:

$$\begin{aligned}z_\eta^T(t)z_\nu(t) &\leq \frac{1}{2}\left\|z_\eta(t)\right\|^2 + \frac{1}{2}\|z_\nu(t)\|^2\\ -z_\eta^T(t)\dot{\eta}_d(t) &\leq \frac{1}{2}\left\|z_\eta(t)\right\|^2 + \frac{1}{2}\|\dot{\eta}_d(t)\|^2\\ -\frac{1}{2}z_\eta^T(t)\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t) &\leq \left\|z_\eta(t)\right\|^2 + \left\|\frac{\partial^T S_\eta}{\partial z_\eta}\hat{W}_{\eta a}(t)\right\|^2\\ z_\nu^T(t)f(\chi(t)) &\leq \frac{1}{2}\|z_\nu(t)\|^2 + \frac{1}{2}\|f(\chi)\|^2\\ -z_\nu^T(t)\dot{\hat{\alpha}}(t) &\leq \frac{1}{2}\|z_\nu(t)\|^2 + \frac{1}{2}\left\|\dot{\hat{\alpha}}(t)\right\|^2\\ -\frac{1}{2}z_\nu^T(t)\frac{\partial^T S_\nu}{\partial z_\nu}\hat{W}_{\nu a}(t) &\leq \|z_\nu(t)\|^2 + \left\|\frac{\partial^T S_\nu}{\partial z_\nu}\hat{W}_{\nu a}(t)\right\|^2.\end{aligned}$$

Equation (67) can be rewritten as

$$\dot{L}_z(t) \le -(\beta_\eta - 2)\left\|z_\eta(t)\right\|^2 - (\beta_\nu - 3)\|z_\nu(t)\|^2 + P(t) \quad (68)$$

where $P(t) = (1/2)\|\dot{\eta}_d(t)\|^2 + (1/2)\|\dot{\hat{\alpha}}\|^2 + (1/2)\|f(\chi)\|^2 + \|(\partial^T S_\eta/\partial z_\eta)\hat{W}_{\eta a}(t)\|^2 + \|(\partial^T S_\nu/\partial z_\nu)\hat{W}_{\nu a}(t)\|^2$. Because $\tilde{W}_{\eta a}(t)$ and $\tilde{W}_{\nu a}(t)$ are SGUUB, which are proven by part 1, it is concluded that $P(t)$ are bounded by a constant ϵ, i.e., $P(t) < \epsilon$. Further, the following fact holds:

$$\dot{L}_z(t) < -\beta L_z(t) + \epsilon$$

where $\beta = \min\{\beta_\eta - 2, \beta_\nu - 3\}$. Applying Lemma 1 has

$$L_z(t) < e^{-\beta t} L_z(0) + \frac{\epsilon}{\beta}(1 - e^{-\beta t})$$

it implies that the tracking errors can arrive to the desired accuracy by making β large enough, as a result that the surface vessel can track the predefined trajectory to desired accuracy.

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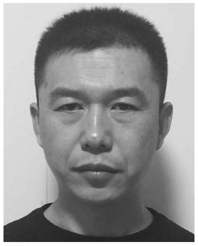

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