1.

a. Doesn't imply anything about median

If add info abt right-skew / left-skew → can we conclude?

E.g. Company A (5 employee) : 30, 32, 35, 38, 100

Company B (5 employee) : 34, 35, 36, 37, 38

Company C (5 employee) : 33, 34, 34, 35, 36

$\Sigma_A > \Sigma_B$: $Med_A < Med_B$

$\Sigma_A > \Sigma_C$. $Med_A > Med_B$

b.

$\bar{x}_A > \bar{x}_B$ (since $\Sigma_A > \Sigma_B$ & $\#_A < \#_B$)

3.

$$\bar{x}_j = \frac{1}{j} \sum_{i=1}^{j} x_i \quad , \quad j = 1 \dots n$$

$$s_j^2 = \frac{\sum_{i=1}^{j} (x_i - \bar{x})^2}{j-1} \quad , \quad j = 2 \dots n$$

$$s_1^2 = 0$$

Add new data point $j+1$,

no need to cal sum from scratch

$$\bar{x}_{j+1} = \bar{x}_j + \left(\frac{x_{j+1} - \bar{x}_j}{j+1} \right)$$

since $\bar{x}_j - \frac{\bar{x}_j}{j+1} = \frac{\bar{x}_j \cdot j}{j+1}$

weight ave of old value + new value

$$s_{j+1}^2 = \left(1 - \frac{1}{j}\right) s_j^2 + (j+1)(\bar{x}_{j+1} - x_j)^2$$

a. 3, 4, 7, 2, 9, 6

$\bar{x}_1 = 3$

$\bar{x}_2 = 3 + \frac{4-3}{2} = 3.5$

$\bar{x}_3 = 3.5 + \frac{7-3.5}{3} = 4.67$

$\bar{x}_4 = 4.67 + \frac{2-4.67}{4} = 4$

$\bar{x}_5 = 4 + \frac{9-4}{5} = 5$

$\bar{x}_6 = 5 + \frac{6-5}{6} = 5.167$

$s_1^2 = 0$

$s_2^2 = \left(1 - \frac{1}{1}\right) \cdot 0 + 2(3.5-3)^2 = 0.5$

$s_3^2 = \left(1 - \frac{1}{2}\right) \cdot (0.5) + 3.(4.67 - 3.5)^2 = 4.336$

$s_4^2 = \left(1 - \frac{1}{3}\right) \cdot (4.336) + 4.(4 - 4.67)^2 = 4.686$

$s_5^2 = \left(1 - \frac{1}{4}\right) \cdot (4.686) + 5.(5-4)^2 = 8.515$

$s_6^2 = \left(1 - \frac{1}{5}\right) \cdot (8.515) + 6.(5.167 - 5)^2 = 6.98$

4.

$S = \{a, b, c\}$ $P(a) = P(b) = 2P(c)$

$P(a) + P(b) + P(c) = 1$

$\rightarrow$ $5P(c) = 1 \rightarrow P(c) = 0.2$

$P(a) = P(b) = 0.4$

5.

a. Show $P(AB^c) = P(A) - P(AB)$

$A = AB \cup AB^c$, $AB \cap AB^c = \emptyset$

$\Rightarrow P(A) = P(AB) + P(AB^c)$

$\rightarrow P(AB^c) = P(A) - P(AB)$

b. Show $A \subseteq B \Rightarrow P(A) \leq P(B)$

$B = A \cup A^cB$, $A \cap A^cB = \emptyset$

$\rightarrow P(B) = P(A) + P(A^cB)$

$\downarrow$

$0 \leq P(A^cB) \leq 1$

$\rightarrow P(B) \geq P(A)$

6.

PDF

$$f(x) = \begin{cases} 0, & 0 < x < 30 \\ \frac{c}{x^2}, & x > 30 \end{cases}$$

- Determine c

$$\int_{30}^{\infty} c \cdot x^{-2} dx = 1 \rightarrow -c \cdot x^{-1} \Big|_{30}^{\infty} = 1$$

$$\rightarrow -c \cdot \left[0 - \frac{1}{30} \right] = 1$$

$$\Leftrightarrow \frac{1}{30} c = 1 \rightarrow c = 30$$

$$F(x) = \begin{cases} 0, & x < 30 \\ -30x^{-1}, & x > 30 \end{cases}$$

Prob 2 games within 2 hours:

$$Pr(30 \leq X \leq 120) = \int_{30}^{120} \frac{30}{x^2} dx = 0.75$$

$\downarrow$ can use

30 $\times$ FC(120) - FC(30)

$\Rightarrow Pr(X > 120) = 1 - Pr(X \leq 120) = 0.25$

$\Rightarrow$ 2 out of 6 games $= \binom{6}{2} (2^{0.75})(4^{0.25})$