Homework 1

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1 Exercise 1

P(Hansen) = 0.05 $\Rightarrow P(noHansen) = 0.95$

$$P(+) = P(+|Hansen) * P(Hansen) + P(+|noHansen) * P(noHansen)$$

=

$$0.98 * 0.05 + 0.03 * 0.95$$

=

$$0.0775$$

$$P(Hansen|+) = \frac{P(+|Hansen).P(Hansen)}{P(+)}$$

=

$$\frac{0.98 * 0.05}{0.0775}$$

$$\approx\ 0.6323$$

2 Exercise 2

2.1 Proof the distributions are normalized

The format of univariate normal distribution is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

To proof the distribution is normalized, we need show that:

$$\int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx = 1$$

Let

$$Z = \frac{x-\mu}{\sigma}$$

So

$$\sigma Z = x - \mu$$

$$\sigma dZ = dx$$

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(Z)^2}{2}} \sigma dZ$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-(Z)^2}{2}} dZ$$

... We have:

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$or \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-(Z)^2}{2}} dZ = 1$$

2.2 Mean

$$E(x) = xf(x)$$

So

$$E(x) = \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx$$

Let

$$Z = x - \mu$$

$$dZ = dx$$

$$E(x) = \int_{-\infty}^{\infty} (Z + \mu) \frac{1}{\sqrt{2\pi}} e^{\frac{-(Z)^2}{2}} dZ$$

$$E(x) = \int_{-\infty}^{\infty} Z \frac{1}{\sqrt{2\pi}} e^{\frac{-(Z)^2}{2}} dZ + \mu \frac{1}{\sqrt{2\pi}} e^{\frac{-(Z)^2}{2}} dZ$$

2.3 Standard Deviation