Homework 5

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October 2021

1 Biến đổi lại thuật toán logistic regression theo ma trận hệ số.

The model logistic regression is:

$$P(C_1|\phi) = y(\phi) = \sigma(w^T\phi)$$

$$P(C_2|\phi) = 1 - P(C_1|\phi)$$

For a data set ϕ_n, t_n where $t_n \in 0, 1$ and $\phi_n = \phi(x_n)$ with $n = 1, ..., N$, the likelihood function can be written :

$$p(t|w) = \prod_{n=1}^{N} y_n^{t_n}(1 - y_n)^{1-t_n}$$

where:

$$t = (t_1, ...t_N)^T \quad and \quad y_n = p(C_1|\phi_n)$$

We have:

$$L = -log\, p(t|w) = -\sum_{n=1}^{N}(\, t_n log y_n + (1-t_n) log(1-y_n)) = -\, t\, log y - (1-t)\, log(1-y)$$

where $y_n = \sigma(z)$ and $z = w^T\phi_n$
Apply Chain Rule:

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial y}.\frac{\partial y}{\partial z}.\frac{\partial z}{\partial w}$$

We have:

$$\frac{\partial L}{\partial y} = -(\frac{t}{y} - \frac{1-t}{1-y}) = -\frac{t(1-y) - y(1-t)}{y(1-y)} = \frac{y-t}{y(1-y)}$$

$$\frac{\partial y}{\partial z} = \frac{\partial \sigma(z)}{\partial z} = \sigma(z)(1 - \sigma(z)) = y(1-y)$$

$$\frac{\partial z}{\partial w} = \phi$$

$$\Rightarrow \frac{\partial L}{\partial w} = \phi(y-t) = \sum_{n=1}^{N} \phi^T(y_n - t_n)$$

2 Tìm hàm f(x), biết f'(x) = f(x)(1-f(x))

$$f'(x) = f(x)(1-f(x))$$

$\iff$

$$\frac{d(f(x))}{dx} = f(x)(1-f(x))$$

$\iff$

$$\frac{d(f(x))}{f(x)(1-f(x))} = dx$$

$\iff$

$$\int \frac{d(f(x))}{f(x)(1-f(x))} = \int dx$$

$\iff$

$$\int (\frac{1}{f(x)} dx + \frac{1}{1-f(x)} dx) = \int dx$$

$\iff$

$$\ln|f(x)| - \ln|1-f(x)| = x + C$$

$\iff$

$$\ln \frac{f(x)}{1-f(x)} = x + C$$

$\iff$

$$\frac{f(x)}{1-f(x)} = e^{x+C}$$

$\iff$

$$f(x) = e^{x+C}(1-f(x))$$

$\iff$

$$f(x) = e^{x+C} - f(x)e^{x+C}$$

$\iff$

$$f(x) = \frac{e^{x+C}}{1+e^{x+C}} = \sigma(x)$$