Force and Displacement Measurement

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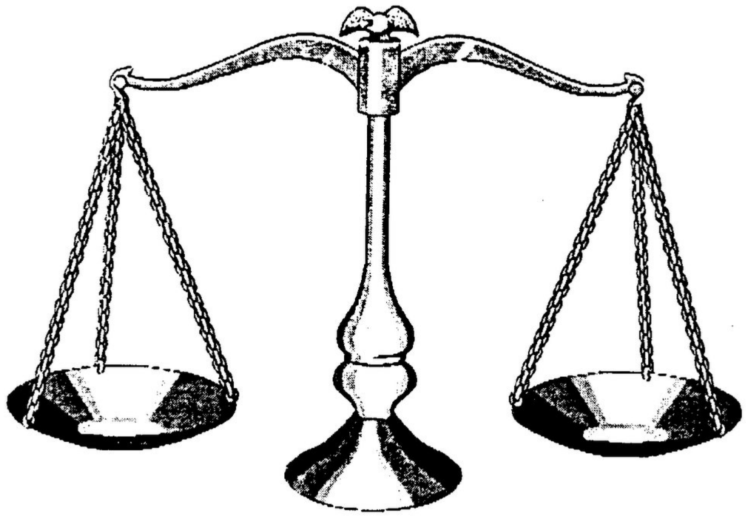
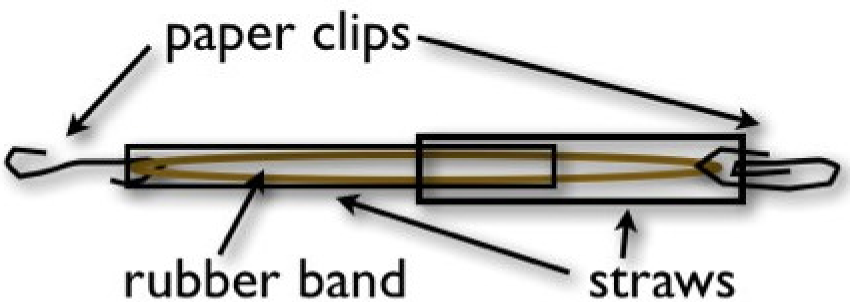
Updated Fall 2011

Simple ways to measure a force

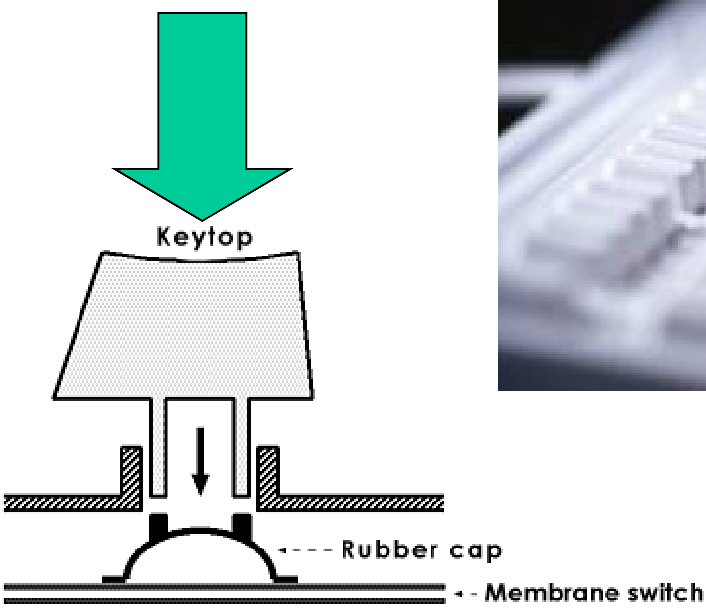


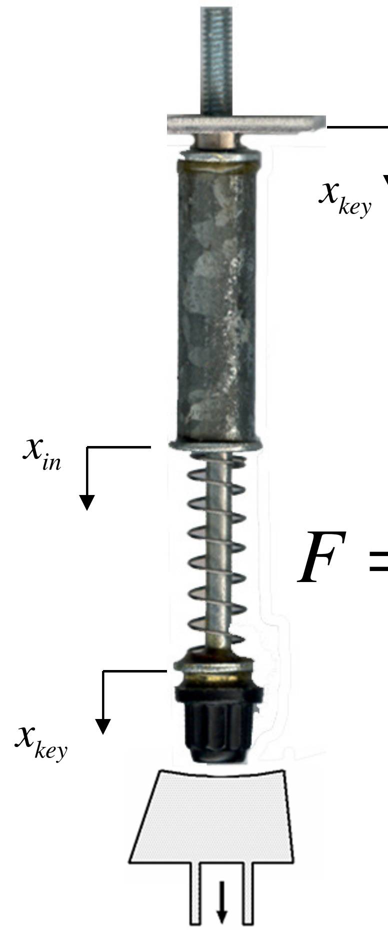


http://scienceblogs.com/dotphysics/2010/02/diy_force_probe.php

Example: Key Force/Deflection

measure low-level forces required to trigger key-switch on a computer keyboard





$$F = k \cdot \Delta x$$

Force and torque sensors

A force or torque sensor provides an output in the form of an **electrical signal** (voltage, current).

Force → Force Sensing Device → Voltage

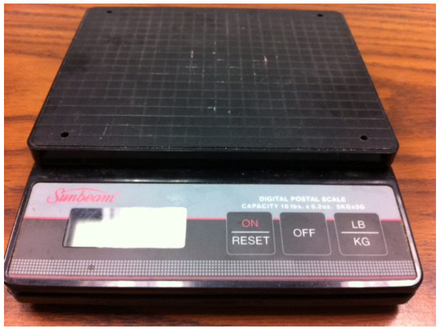
Sensitivity = voltage/force

Examples:

Strain-gauge force sensors

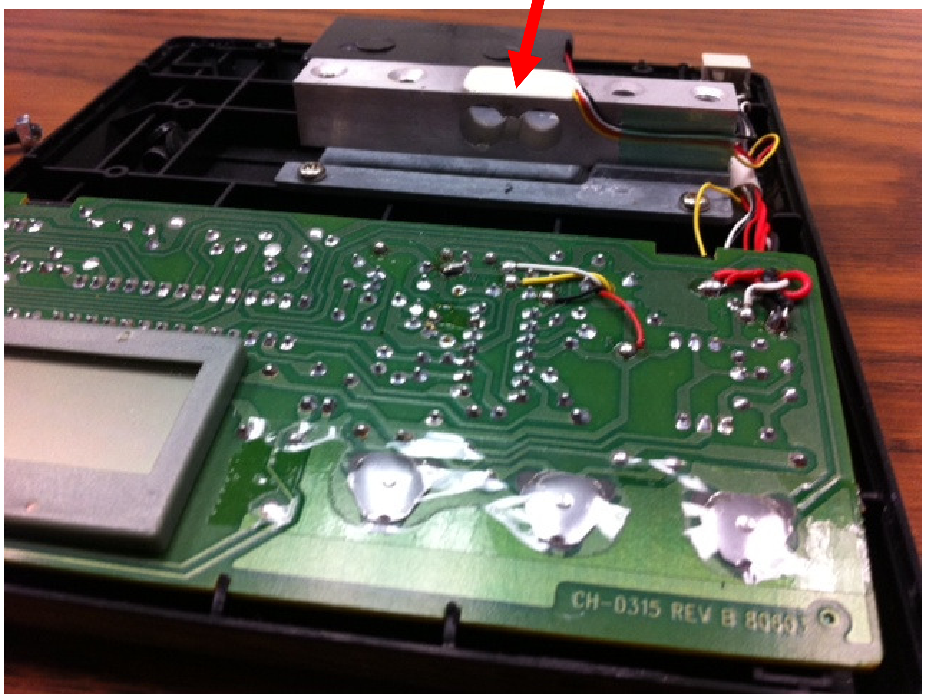
Optical in-line torque sensor

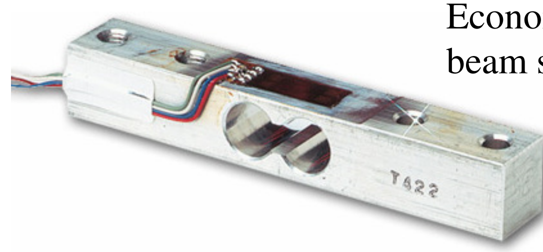
Force sensing in products



Postal scale, ~$30

Strain gauge bending beam sensor





Economical bending beam scale, ~$165

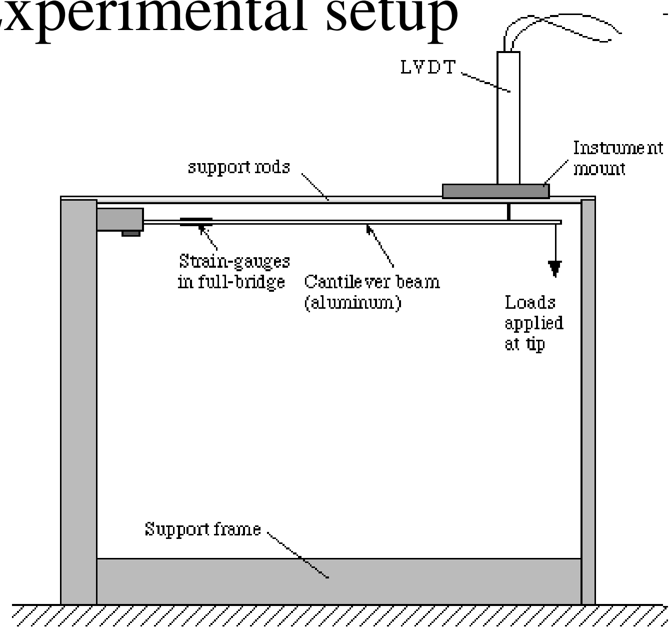
Omega Engineering

Sensing Mechanism

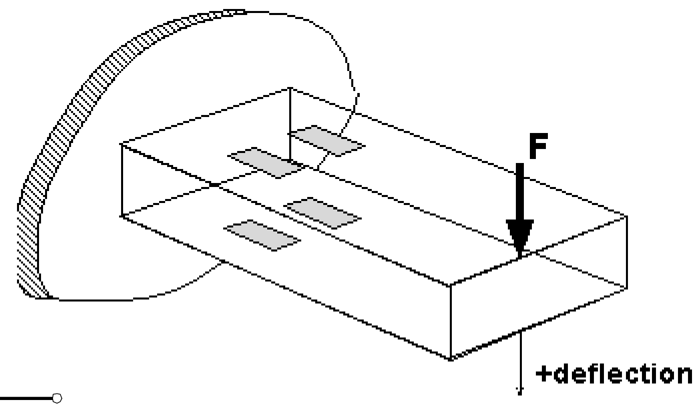
- To measure **force (or torque)**, it is usually necessary to design a **compliant** **mechanical structure**. This structure may itself be a **sensing material**.
- Force will induce stress, leading to strain which can be detected in various ways, for example:
  - using strain gauges (piezoresistive effect)
  - using crystals or ceramics (piezoelectric effect)
  - optically
- Sometimes force can be measured using other types of displacement sensing devices.

Lab study: beam sensor

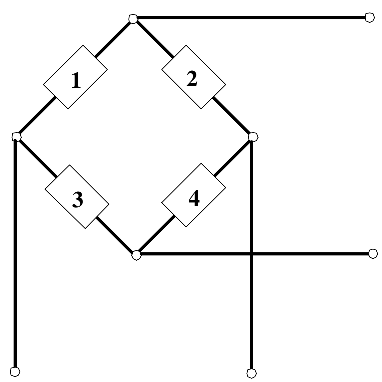
Experimental setup



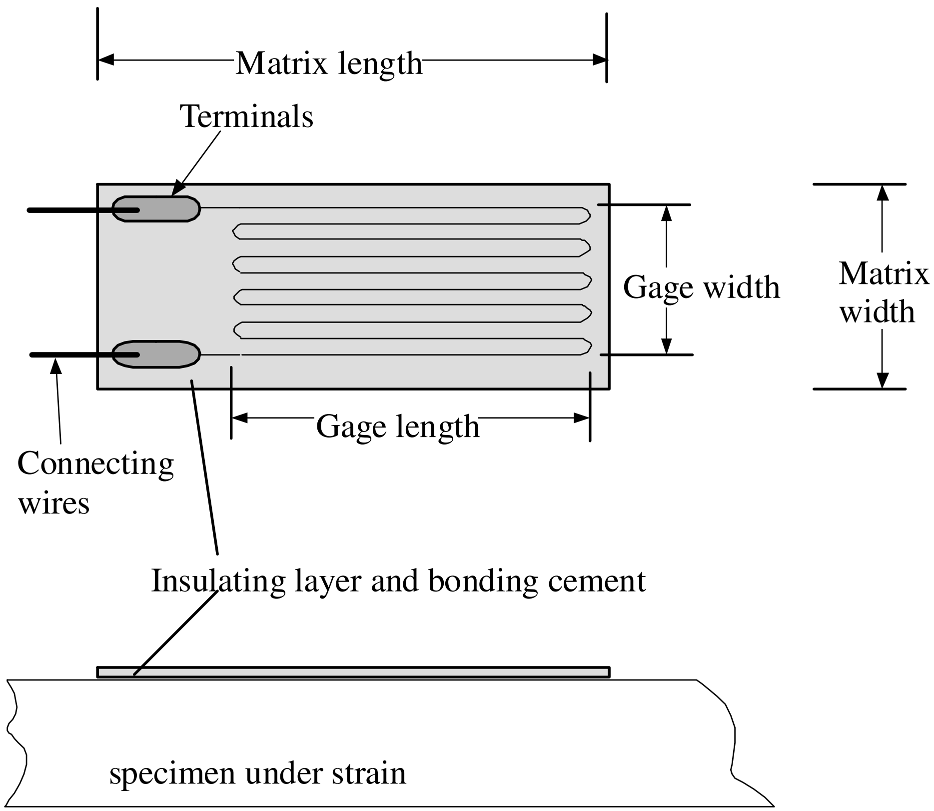
Beam configuration with a 'full-bridge' strain gauge configuration



Wheatstone bridge



Strain gauge concepts



Strain gauges exhibit **piezoresistive** behavior, and are one of the most common ways to measure strain.

<u>Types</u>

- **unbonded wire** - basically a wire under strain (c. 1940s)
- **foil** - type shown to left (c. 1950s) are most common
- **semiconductor** (c. 1960s)

Strain gauge Sensitivity
The gauge Factor, *G*

A measure of the "sensitivity" of a strain gauge is given by the gauge factor, which is defined as,

$$G = \frac{\text{fractional change in resistance}}{\text{fractional change in strain}}$$

Using the derivation in Appendix B,

$$G = \frac{1}{\varepsilon}\frac{dR}{R} = (1+2\nu) + \boxed{\frac{1}{\varepsilon}\frac{d\rho}{\rho}}$$

"Piezoresistive effect"

Typical values:
80% Ni, 20% Cr, G = 2
45% Ni, 55% Cu, G = 2
Platinum, G = 4.8
95% Pt, 5% Ir, G = 5.1
Semiconductor, G =70 to 135

More on gauge Types

- Strain gauges come in many specialized forms and typically include a calibrated gauge factor, G.
- Semiconductor strain gauges have the highest values of G. These strain gauges can have G values of 70 to 135, and they are typically very small. However, there are some disadvantages which include:
  - output is not linear with strain,
  - very temperature dependent,
  - usually have a much lower strain limit than metallic type,
  - more expensive than metallic type.

Strain Detection

Order of Magnitude Calculation

Consider a situation where the strain is on the order of 1 microstrain.

For a metallic foil strain gauge with G = 2, R = 120 ohm,

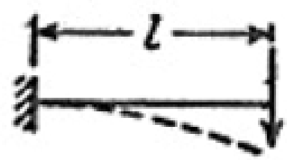
$$\Delta R = G \cdot \varepsilon \cdot R = 2 \cdot 1\times 10^{-6} \cdot 120 = 0.0024\Omega$$

You need to measure a 0.002% change in R!

How would you detect such a change?

Beam Sensors

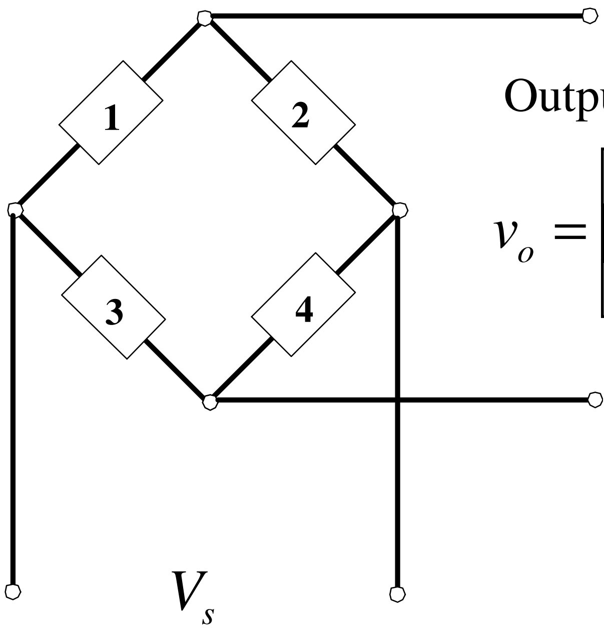
- The beam structure/geometry is used extensively in designing many types of force and torque sensors.
- A beam offers certain advantages:
  - easy geometry for basic analysis and design
  - strain gauges can be mounted easily and configured in several different ways to achieve different objectives



Cantilever $k = \frac{3EI}{l^3}$

See beam configurations in Appendix C.

Wheatstone Bridge Configuration



Output DC voltage

$$v_o = \left[\frac{R_1 R_4 - R_2 R_3}{\left(R_1 + R_2\right)\left(R_3 + R_4\right)} \right] \cdot V_s$$

Null condition is satisfied when: $\dfrac{R_1}{R_2} = \dfrac{R_3}{R_4}$

If all the gauges have the same resistance, you can show:

$$\frac{dv_o}{V_s} = \frac{dR_1 - dR_2 - dR_3 + dR_1}{R} = \frac{G}{4}\left[\varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4\right]$$

This equation can be used to guide placement of gauges on a specimen.

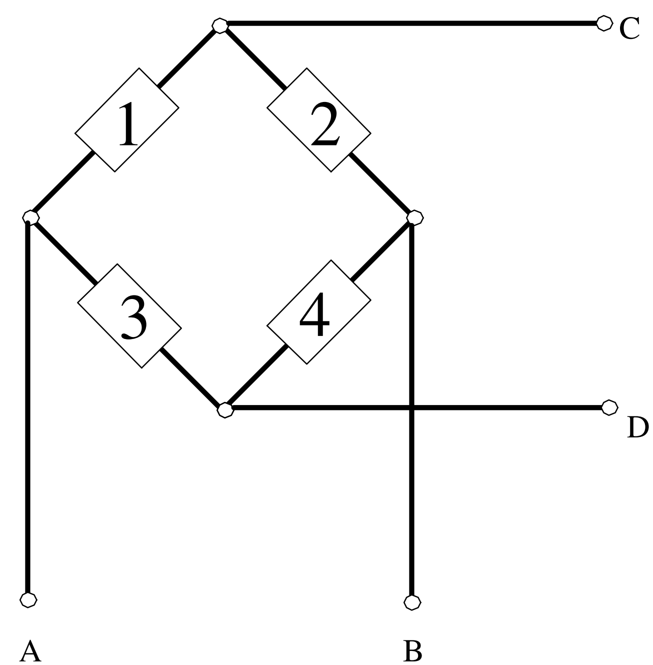
Balanced Bridge

The bridge is balanced when the ratio of resistances of any two adjacent arms is equal to the ratio of resistances of the remaining two arms (taken in the same sense).

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

or,

$$\frac{R_1}{R_3} = \frac{R_2}{R_4}$$



Signal Conditioning: Impedance Bridges

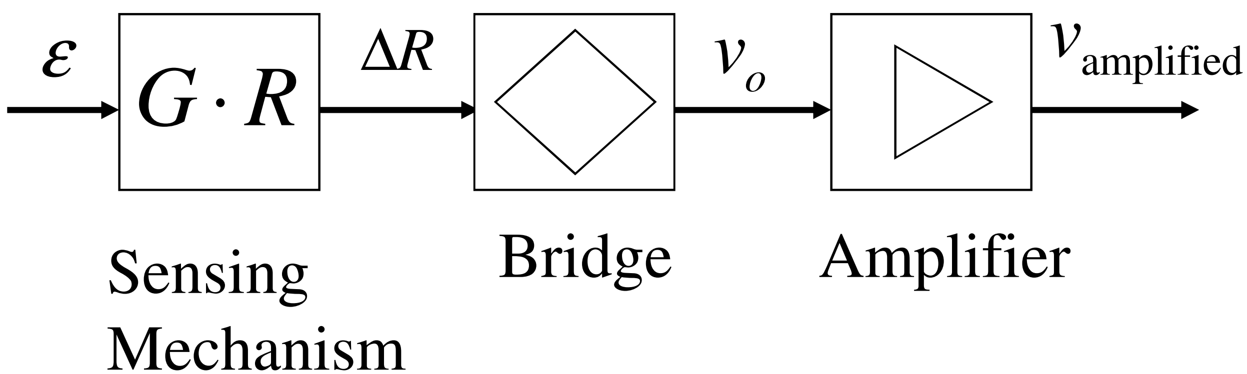
Impedance bridges are used to convert the output of resistive, capacitive, or inductive sensors into a voltage signal

IMPEDANCE $\Rightarrow$ VOLTAGE

Many types of impedance bridges exist; see examples in Appendix D.

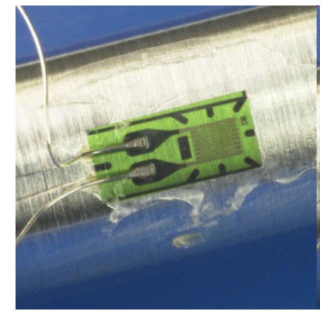
Strain Gauge Measurement System

The strain gauge is part of a multi-stage process that generates a voltage signal proportional to the strain.

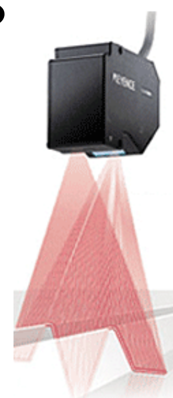


The amplifier used in our lab experiments is described in Appendix E.

Displacement measurements



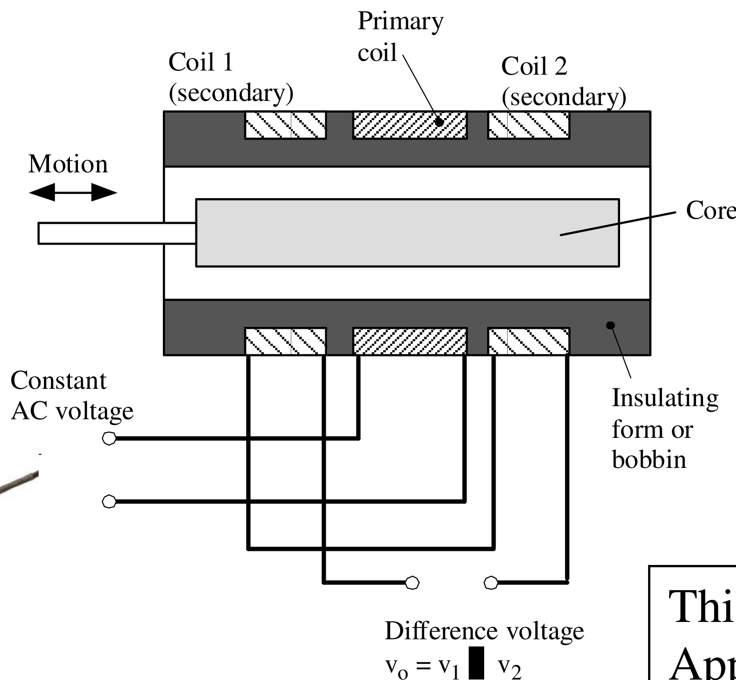
- Mechanical (gage blocks, rulers, etc.)
- Strain gauges measure deflection
- Contact sensors
  - LVDT
  - Inertial sensors (accelerometers, seismometers)
- Non-contact sensors
  - Optical



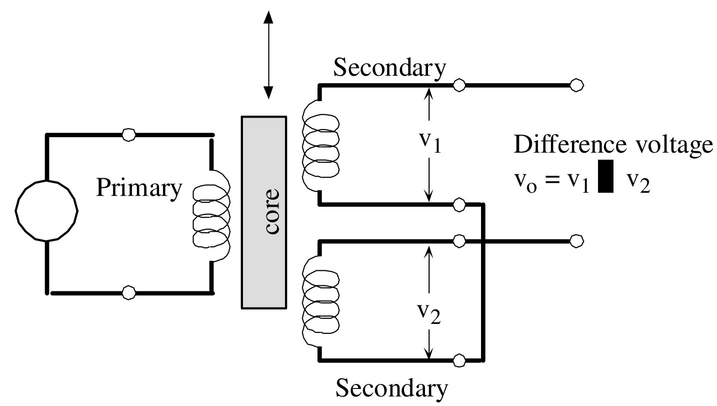
There are MANY ways to measure displacement…

LVDT – displacement sensor

Linear variable differential transformer (LVDT) monitors displacement of a core which modulates the mutual inductance between two coils.



(a)



This sensor type is described briefly in Appendix G.

(b)

Schaevitz Sensors
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FINAL TEST DC-EC-1000

30810100-000

RANGE +/-1 Inches

S/N J0677

INDEPENDENT LINEARITY DATA

LEAST SQUARES LINE

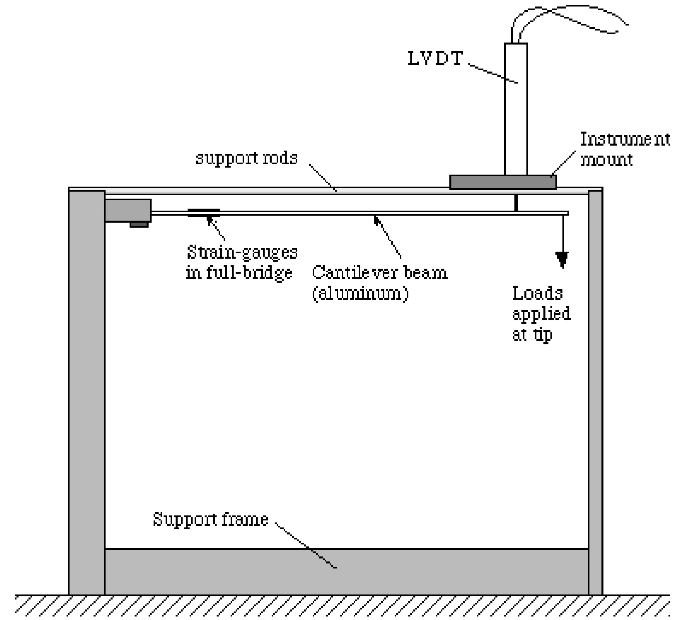
07-09-2001

MEASURED Inches	MEASURED Volts dc	CALC. Volts dc	CALC. DEVIATION
-1.0000	-10.0812	-10.0936	+0.0124
-0.8000	-8.0700	-8.0741	+0.0041
-0.6000	-6.0694	-6.0546	-0.0148
-0.4000	-4.0505	-4.0351	-0.0154
-0.2000	-2.0177	-2.0156	-0.0020
+0.2000	+2.0256	+2.0234	+0.0022
+0.4000	+4.0532	+4.0429	+0.0103
+0.6000	+6.0788	+6.0624	+0.0164
+0.8000	+8.0865	+8.0819	+0.0046
+1.0000	+10.0836	+10.1014	-0.0178

NULL (Actual) = -0.0005 Volts dc
Linearity = .09%
Scalefactor = 10.0975 Volts dc / Inches

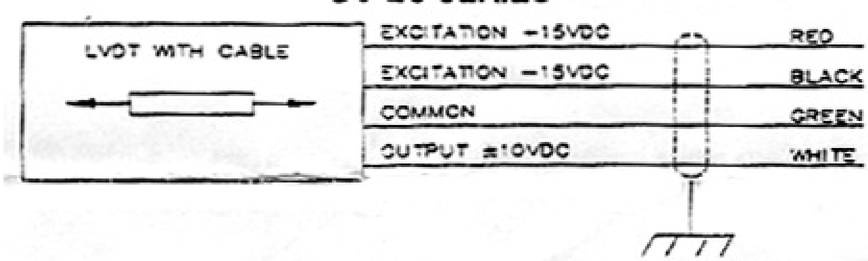
Tested by

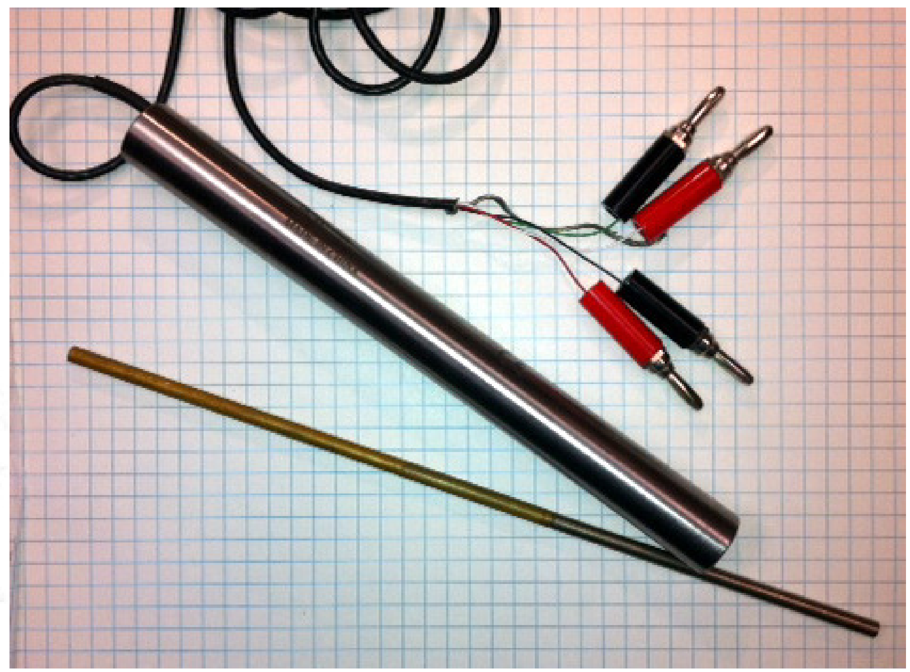
Inspected by



EXCITATION ±15VDC OUTPUT ±10VDC

DC-EC SERIES





Summary

- Force measurement takes advantage of the relationship between force, displacement and stiffness.
- **Strain gauges** are a common basis for sensors that can measure force or torque
- We ‘wrap’ the sensor with signal conditioning to get a measurable signal (voltage or current)
- Strain gauges are core knowledge for mechanical engineers

Appendix A: Example force and torque sensors
Appendix B: Piezoresistivity and gauge factor
Appendix C: Beams as basis for strain-gauge sensors
Appendix D: Summary of impedance bridges
Appendix E: DMD-465W – strain-gauge amplifier
Appendix F: Examples of ‘home-made’ force sensors
Appendix G: Inductive-type sensors

Appendix A:

Example force sensors

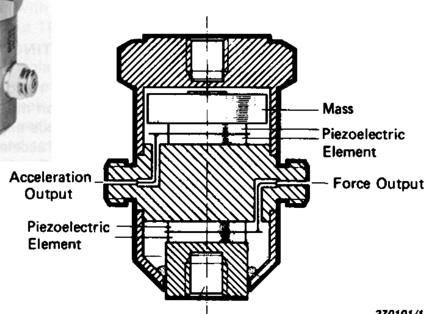
- Strain-gauge force sensors

Omega

Omega

Sensotec

- Piezoelectric



Force and acceleration
Bruel and Kjaer

XYZ force sensor from PCB Piezotronics

General purpose PCB Piezotronics

Appendix A:

Example torque sensors

Reaction torque sensor

In-line torque sensor

Appendix B:

Piezoresistivity (1)

We know that for a conductor of uniform area, the resistance is given by,

$$R \equiv \frac{\rho l}{A}$$

where ρ is the **resistivity** (cm ohm)., l is the length, and A is the cross-sectional area.

Under strain, the change in R is,

$$dR = \frac{\partial R}{\partial l} dl + \frac{\partial R}{\partial A} dA + \frac{\partial R}{\partial \rho} d\rho$$

which for uniform A is,

$$dR = \frac{\rho}{A} dl - \frac{\rho l}{A^2} dA + \frac{l}{A} d\rho$$

For typical conductors, the resistivity values in units of ohm mm^2/m are: Aluminum 0.0278, Pure Iron 0.1, Constantan 0.48, Copper 0.0172, Gold 0.0222, Tungsten 0.059, Manganese 0.423, Nickel 0.087.

Appendix B:

Piezoresistivity (2)

The fractional change of R is of more interest, so we find,

$$\frac{dR}{R} = \frac{dl}{l} - \frac{dA}{A} + \frac{d\rho}{\rho}$$

$$\frac{dl}{l} = \text{fractional change in length}$$

$$\frac{dA}{A} = \text{fractional change in area}$$

$$\frac{d\rho}{\rho} = \text{fractional change in resistivity}$$

Appendix B:

Piezoresistivity (3)

For a linearly elastic body, $\sigma_{xx} = F / A_o = E \cdot \varepsilon_x = E \cdot \frac{dl}{l}$

where E is the Young's modulus. Recall $\varepsilon_x = \frac{dl}{l}, \quad \varepsilon_y = -\nu \frac{dl}{l}, \quad \varepsilon_z = -\nu \frac{dl}{l}$

And for an area $A = w\, t$, the fractional change is, $\frac{dA}{A} = \frac{dw}{w} + \frac{dt}{t} = -2\nu\varepsilon_x$

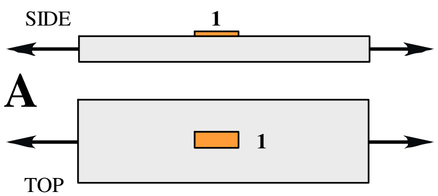
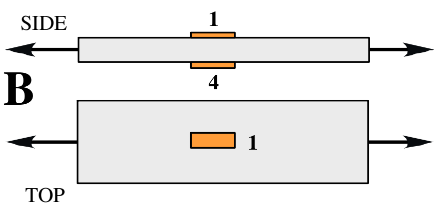
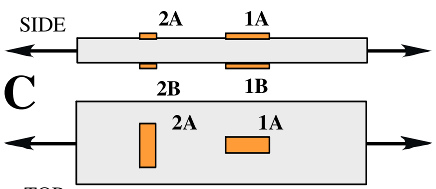
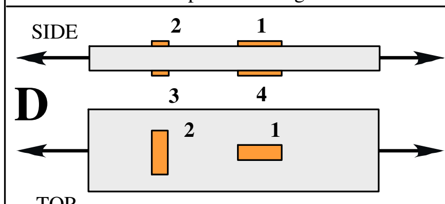
Recall that n is Poisson's ratio. Now the fractional change in R is,

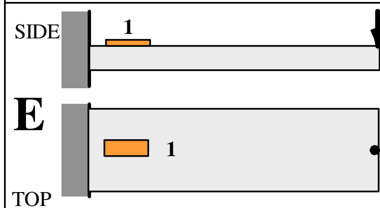
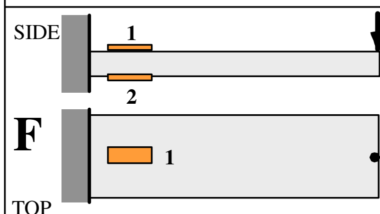
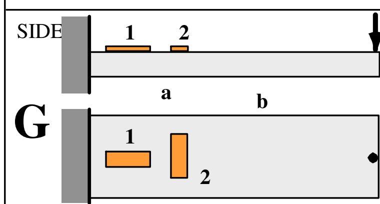
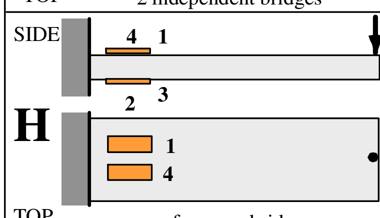
Input a strain → $$\boxed{\frac{dR}{R} = \boxed{(1+2\nu)} \cdot \varepsilon_x + \boxed{\frac{d\rho}{\rho}}}$$ → Output a resistance change

Geometric

Material

Appendix C:
Strain gauge orientations for axially-loaded and cantilevered Beams

Axially-Loaded Beam	K	Bending	Temperature
A 	1	Sensitive	Compensated with dummy gage in arm 2 or arm 3
B 	2	Compensated	Compensated (with four-arm bridge and dummy gages in arms 2 and 3)
C  2 independent bridges	1+ν	Compensated	Compensated with two-arm bridges.
D  one four arm bridge	2(1+ν)	Compensated	Compensated through four-arm bridge.

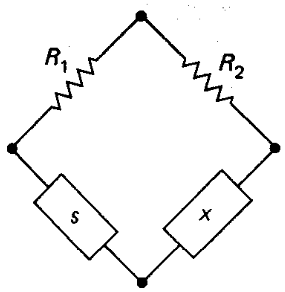
Cantilevered Beam	K	Axial/ Torsion	Temperature
E 	1	Sensitive to both axial and torsion loads	Compensated with dummy gage in arm 2 or arm 3
F 	2	Compensated for both axial and torsion loads	Compensated
G  2 independent bridges	1+bν/a	Compensated for both axial and torsion loads	Compensated
H  one four arm bridge	4	Compensated for both axial and torsion loads	Compensated

NOTES: All axially-loaded beams sensitive to torsion. Requirement for null: R1/R2 = R3/R4
K = Bridge constant = (output of bridge)/(output of primary gage)

Appendix D:

Impedance Bridges

From Beckwith, Buck, and Marangoni, "Mechanical Measurements", Addison-Wesley, 3rd ed, 1982.



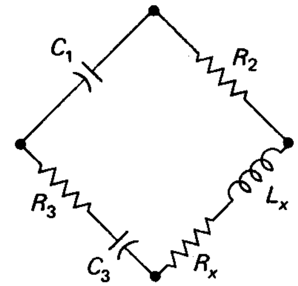
Comparison with series constants

Measures L or C
Balance equations:

$$R_x = R_s \frac{R_2}{R_1}$$

If inductive, $L_x = L_s \frac{R_2}{R_1}$.

If capacitive, $C_x = C_s \frac{R_1}{R_2}$.

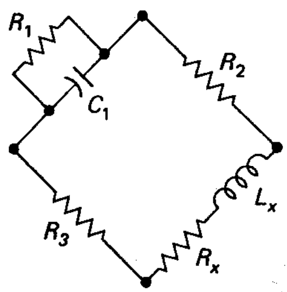


Owen circuit

Measures L
Balance equations:

$$L_x = R_2 R_3 C_1$$

$$R_x = R_2 \frac{C_1}{C_3}$$

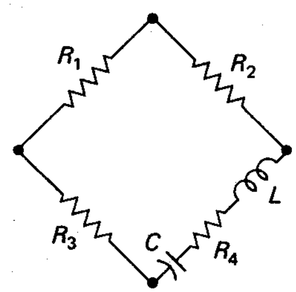


Maxwell circuit

Measures L
Balance equations:

$$L_x = R_2 R_3 C_1$$

$$R_x = \frac{R_2 R_3}{R_1}$$

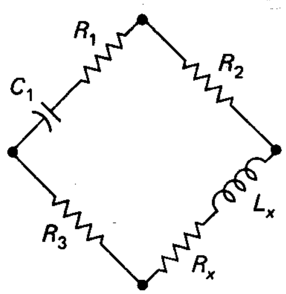


Resonance circuit

Measures L or C (f known), f (L and C known)
Balance equations:

$$X_L = X_C \text{ or } L \cdot C = \frac{1}{\omega^2}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

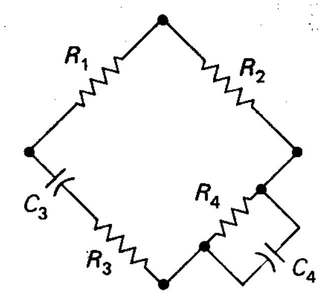


Hay circuit

Measures L
Balance equations:

$$L_x = \frac{R_2 R_3 C_1}{1 + \omega^2 C_1^2 R_1^2}$$

$$R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1 + \omega^2 C_1^2 R_1^2}$$



Wien, or R-C frequency bridge

Measures f
Balance equations:

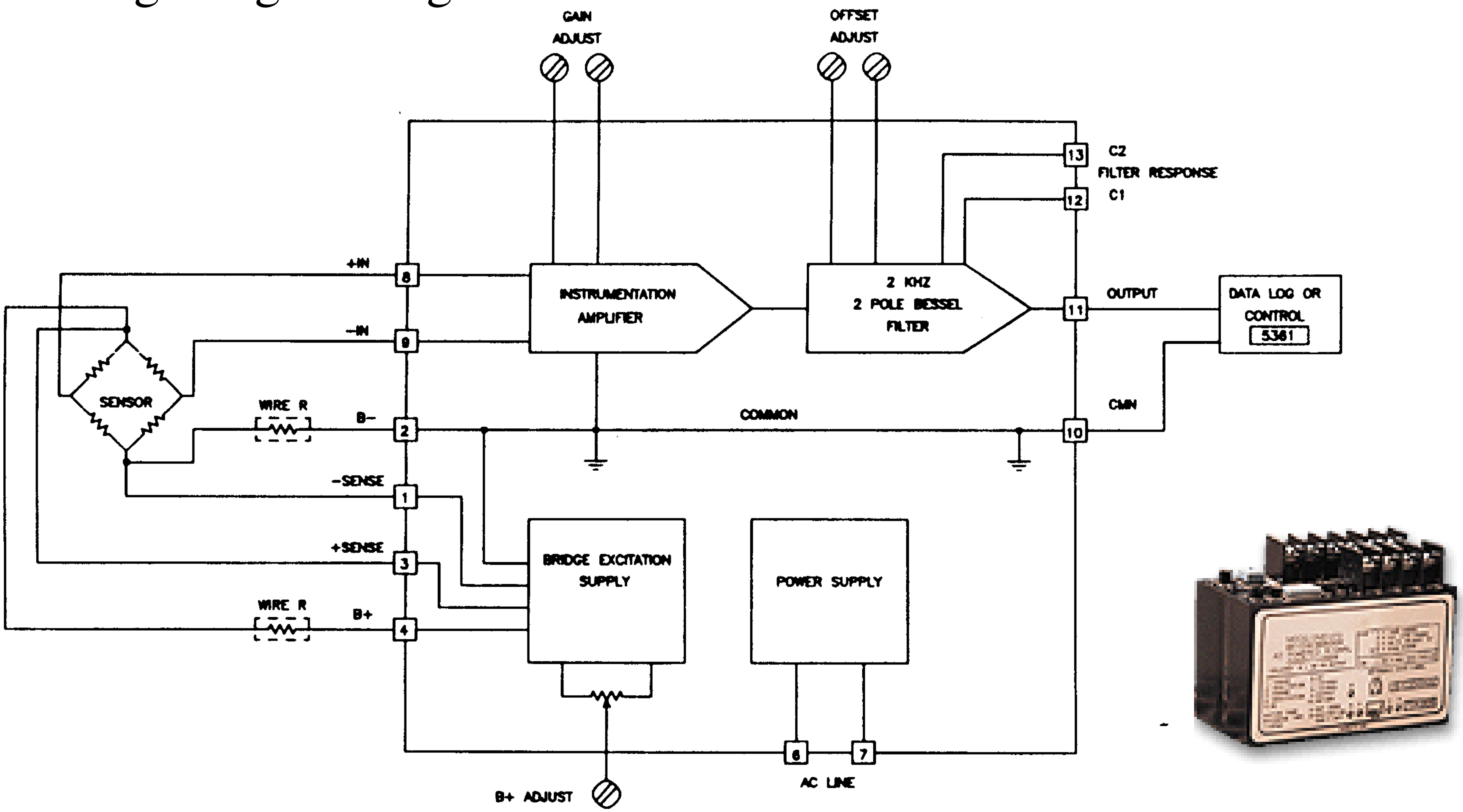
$$f = \frac{1}{2\pi\sqrt{R_3 R_4 C_3 C_4}}$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} + \frac{C_4}{C_3}$$

Appendix E:

DMD 465WB

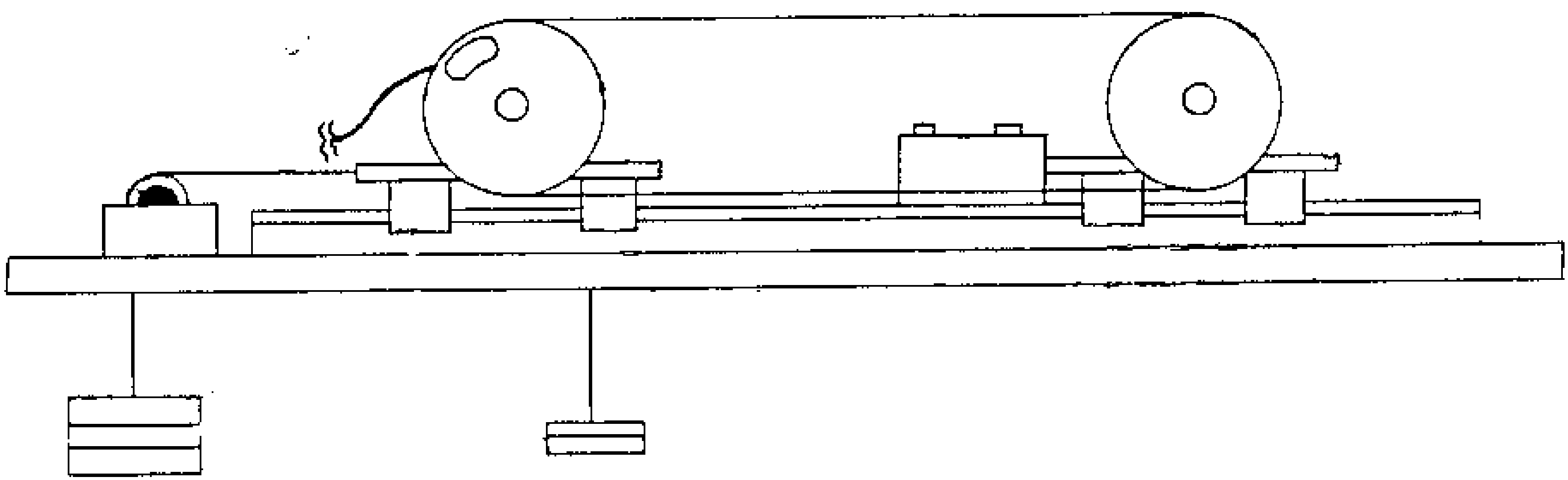
Omega Engineering



Appendix F:

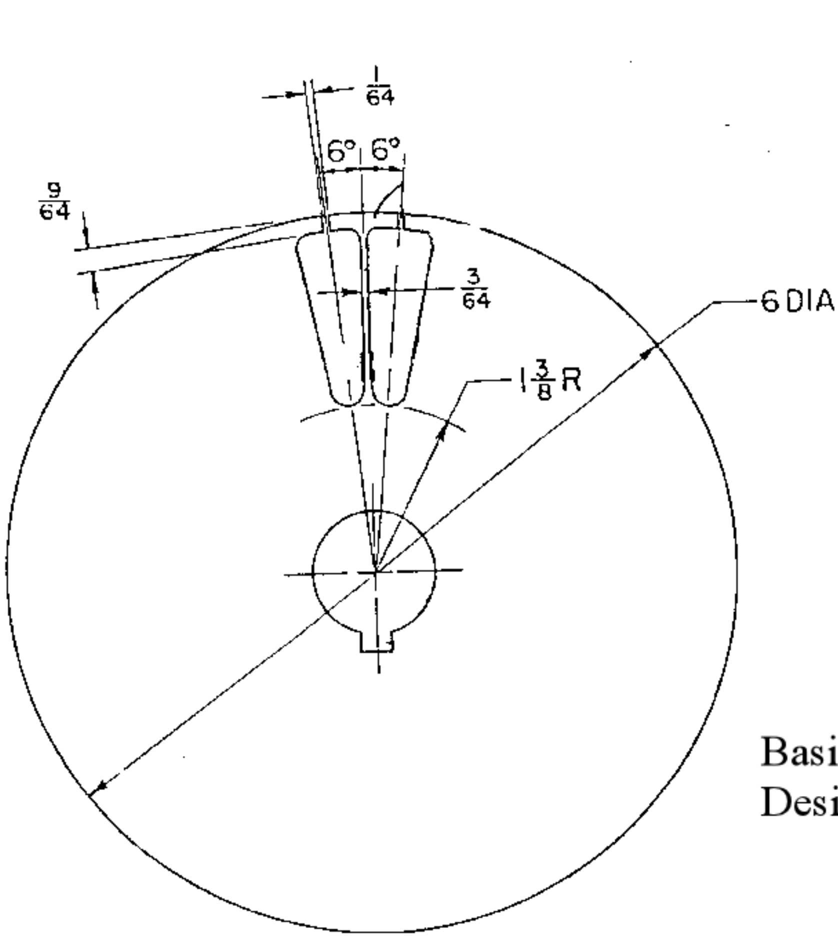
‘Home-made’ force sensors

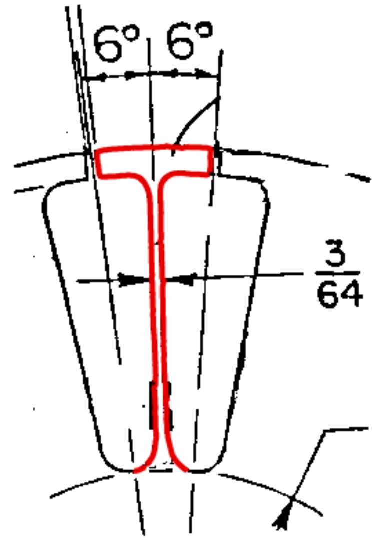
Belt force measurement (Kim and Marshek, c. 1981)
This apparatus used two different types of ‘home-made’ force sensors to measure the forces induced by a grinding belt on the disk.

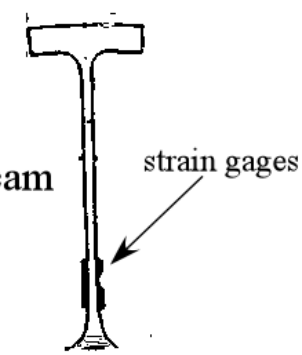


Appendix F:

Measuring tangential belt forces





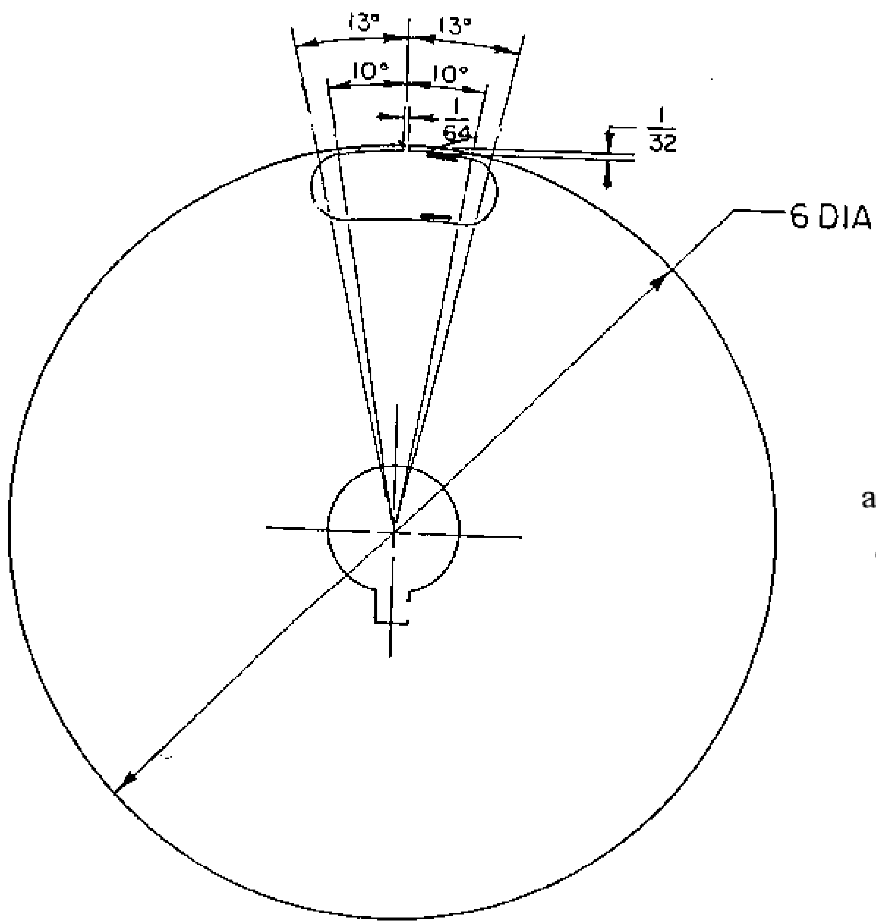


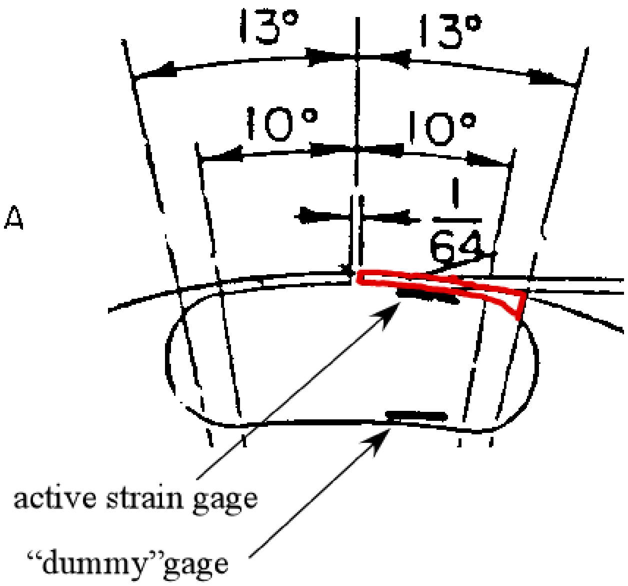
Basic Beam Design

This beam has a full-bridge of strain gauges.

Appendix F:

Measuring normal belt forces

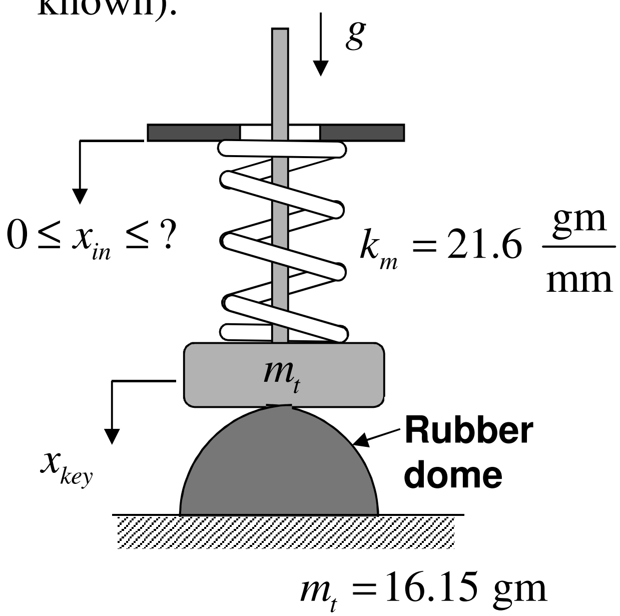


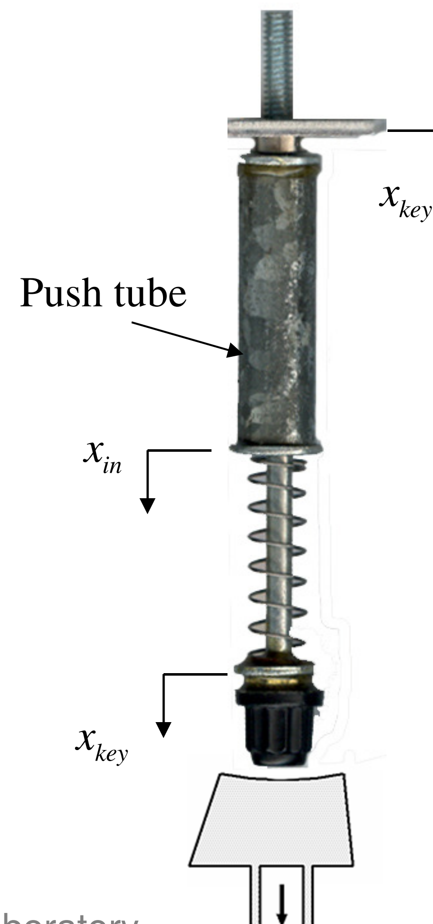


Appendix F:

Using displacement to measure low-range forces

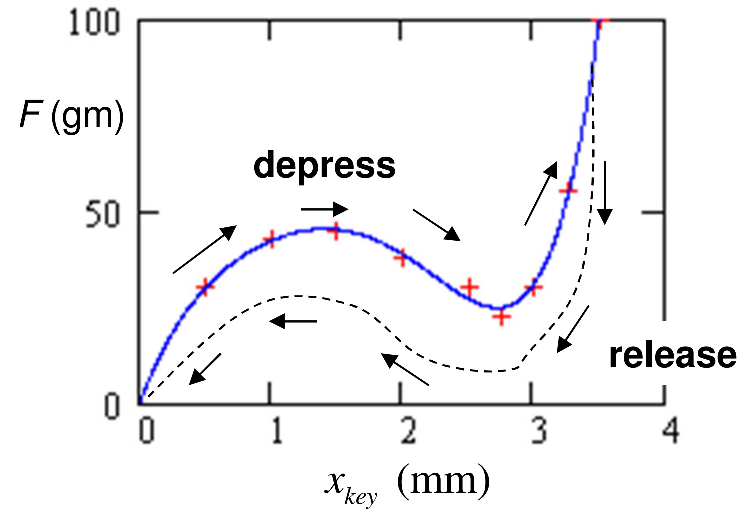
Measuring the difference in displacement between the key and the push tube allows force measurement (assuming stiffness is known).





data :=

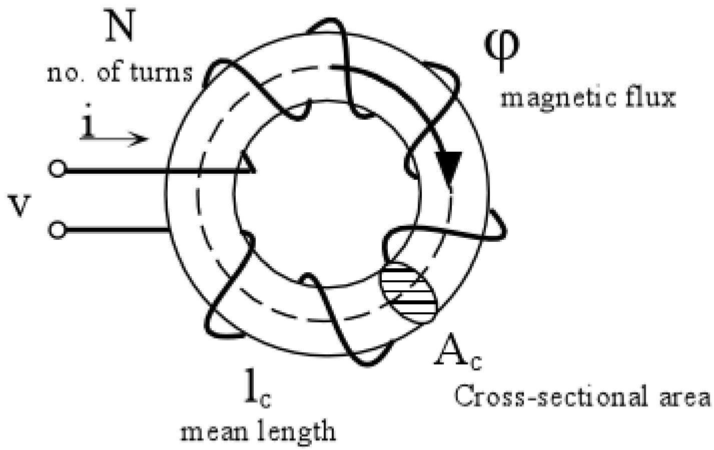
(mm)	(gm)
0	0
0.5	30
1	42.5
1.5	45
2	38
2.5	30
2.75	22.5
3	30
3.25	55
3.5	100



Appendix G:

Inductors store magnetic energy

- Inductors store magnetic energy



$$\xrightarrow[i]{v=\dot{\lambda}} \mathbf{I}$$

$$\xrightarrow[i]{v=\dot{\lambda}} \overset{N}{\mathbf{G}} \xrightarrow[\dot{\varphi}]{M} \mathbf{C}$$

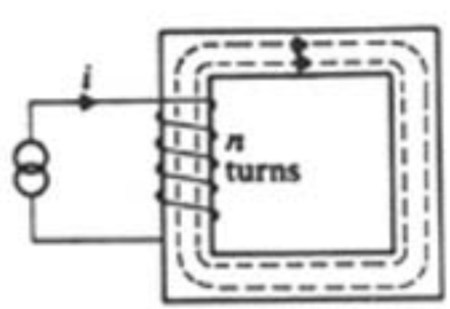
$$v = \dot{\lambda} = N\,\dot{\varphi}$$

- An inductive (or variable-reluctance) sensor design takes advantage of how magnetic flux passes through the circuit, and this can be detected as a change in inductance.

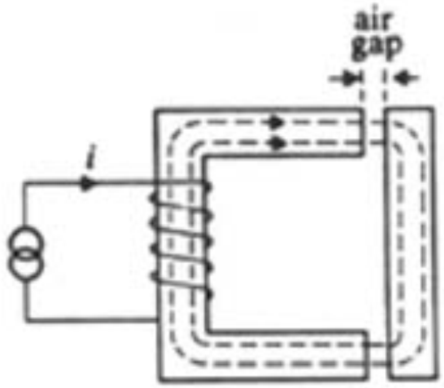
Appendix G:

‘Variable-reluctance’ sensors

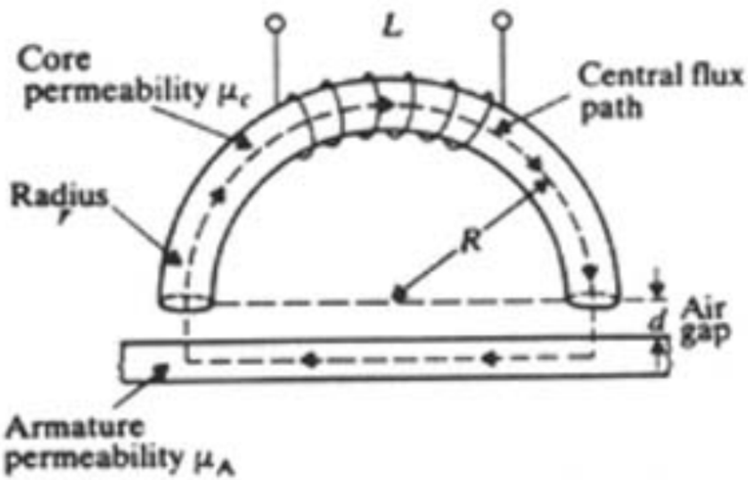
Reluctance is inversely related to inductance

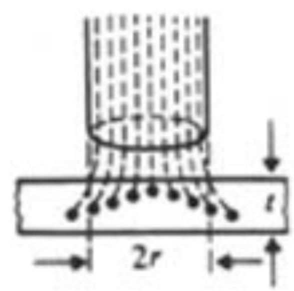


(a)



In all of these devices, the inductance, L, is changing based on **geometry**.





You can also have **material** change to affect reluctance.