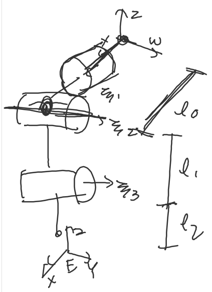
$$g_{AB} = \begin{bmatrix} R_{AB} & p_{AB} \\ \vec{0}^T & 1 \end{bmatrix}$$

$$p_{AB} = [\,l_0 \quad 0 \quad 0\,]^T$$

$$R_{AB} = R_x(\pi/2)$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$g_{AB} = \begin{bmatrix} 1 & 0 & 0 & l_0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$\xi_1 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix}$$
$$\omega = [\,1 \quad 0 \quad 0\,]^T$$
$$q = [\,0 \quad 0 \quad 0\,]^T$$
$$\xi_1 = [\,0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0\,]^T$$

$$\xi_2 = [\,-\omega \times q \quad \omega\,]^T$$
$$\omega = [\,0 \quad 1 \quad 0\,]^T$$
$$q = [\,l_0 \quad 0 \quad 0\,]^T$$
$$-\omega \times q = [\,0 \quad 0 \quad l_0\,]^T$$
$$\xi_2 = [\,0 \quad 0 \quad l_0 \quad 0 \quad 1 \quad 0\,]^T$$

$$\xi_3 = [\,-\omega \times q \quad \omega\,]$$
$$\omega = [\,0 \quad 1 \quad 0\,]^T$$
$$q = [\,l_0 \quad 0 \quad -l_1\,]^T$$
$$-\omega \times q = [\,l_1 \quad 0 \quad l_0\,]^T$$
$$\xi_3 = [\,l_1 \quad 0 \quad l_0 \quad 0 \quad 1 \quad 0\,]^T$$

$$g_{WE}(\vec{\theta}) = \underbrace{e^{\hat{\xi}_1\theta_1}}\,\underbrace{e^{\hat{\xi}_2\theta_2}}\,\underbrace{e^{\hat{\xi}_3\theta_3}}\,g_{WE}(0)$$

$$e^{\hat{\xi}\theta} = \begin{bmatrix} e^{\hat{\omega}\theta} & (1 - e^{\hat{\omega}\theta})(\omega \times v) + \omega\omega^T v\theta \\ \vec{0}^T & 1 \end{bmatrix}$$

$$e^{\hat{\xi}_1\theta_1} = \begin{bmatrix} R_x(\theta_1) & (1 - e^{\hat{\omega}\theta})(\omega_1 \times v_1) \\ \vec{0}^T & 1 \end{bmatrix}$$

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \qquad \omega_1 \times v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

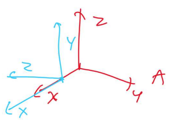
$$e^{\hat{\omega}_1\theta_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$q' = Rq + p \qquad R' = R^{-1} = R^T$$
$$q = R'q' + p'$$
$$q = R'(Rq + p) + p'$$

$= R' R q + R' p + p'$

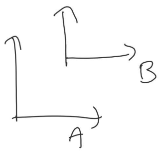
$p' + R' p = 0$

$p' = -R' p$



$$g_{AB} = \begin{bmatrix} R_{AB} & p_{AB} \\ \vec{0}^T & 1 \end{bmatrix}$$

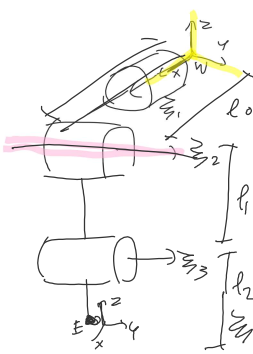
$p_{AB} = [\, l_0 \quad 0 \quad 0 \,]^T$



$R_{AB} = R_x(\pi/2)$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$g_{AB} = \begin{bmatrix} 1 & 0 & 0 & l_0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$\xi_1 = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix}$$

$\omega = [\, 1 \quad 0 \quad 0 \,]$

$v = -\omega \times q$

$= 0$

$\xi_1 = [\, 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \,]^T$

$\xi_2 = [\, -\omega \times q \quad \omega \,]^T$

$$\begin{bmatrix} 1 & 0 & 0 & l_0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_1 + l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\omega = [\, 0 \quad 1 \quad 0 \,]^T$

$q = [\, l_0 \quad 0 \quad 0 \,]^T$

$-\omega \times q = -\hat{\omega} q = [\, 0 \quad 0 \quad l_0 \,]^T$

$\xi_2 = [\, 0 \quad 0 \quad l_0 \quad 0 \quad 1 \quad 0 \,]^T$

ξ_3: $\omega = [\, 0 \quad 1 \quad 0 \,]^T$

$q = [\, l_0 \quad 0 \quad -l_1 \,]^T$

$-\omega \times q = [\, l_1 \quad 0 \quad l_0 \,]^T$

$\xi_3 = [\, l_1 \quad 0 \quad l_0 \quad 0 \quad 1 \quad 0 \,]^T$

$$g_{WE}(\theta_1, \theta_2, \theta_3) = e^{\hat{\xi}_1 \theta_1} e^{\hat{\xi}_2 \theta_2} e^{\hat{\xi}_3 \theta_3} g_{WE}(0)$$

given : $g = \begin{bmatrix} R & p \\ \vec{0}^T & 1 \end{bmatrix}$, what is g^{-1} ?

$$g^{-1} = \begin{bmatrix} R' & p' \\ \vec{0}^T & 1 \end{bmatrix}$$

$q' = \underline{Rq} + \underline{p}$

$q = R' q' + p'$

$q = R'(Rq + p) + p'$

$\underline{q} = \underline{R' R q} + R' p + p'$

$R' = R^{-1} = R^T$

$0 = R' p + p'$

$p' = -R' p$

$$g^{-1} = \begin{bmatrix} R^T & -R^T p \\ \vec{0}^T & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \end{bmatrix}$$

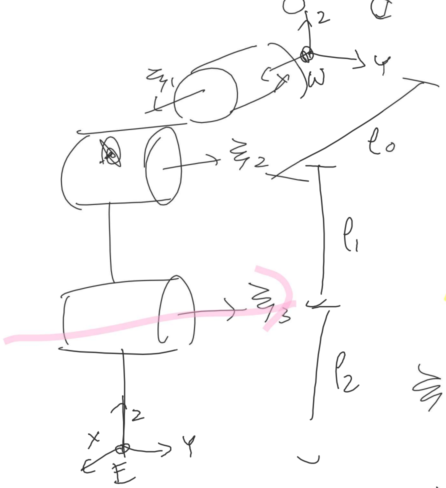
(a) $$g_{AB} = \begin{bmatrix} R_{AB} & p_{AB} \\ \vec{0}^T & 1 \end{bmatrix}$$

$$p_{AB} = [l_0 \quad 0 \quad 0]^T$$

$$R_{AB} = R_x(\pi/2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\pi/2 & -\sin\pi/2 \\ 0 & \sin\pi/2 & \cos\pi/2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$g_{AB} = \begin{bmatrix} 1 & 0 & 0 & l_0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$\xi_1 = \begin{bmatrix} -\omega_1 \times q_1 \\ \omega_1 \end{bmatrix}$$

$$\omega_1 = [1 \quad 0 \quad 0]^T$$

$$q_1 = [0 \quad 0 \quad 0]^T$$

$$\xi_1 = [0 \; 0 \; 0 \; 1 \; 0 \; 0]^T$$

ξ_2: $\omega_2 = [0 \quad 1 \quad 0]^T$

$$q_2 = [l_0 \quad 0 \quad 0]^T$$

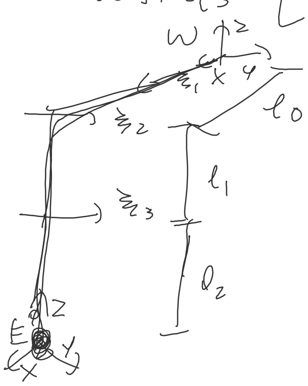
$$\xi_2 = \begin{bmatrix} 0 \\ 0 \\ l_0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

ξ_3: $\omega_3 = [0 \quad 1 \quad 0]^T$

$$q_3 = [l_0 \quad 0 \quad -l_1]^T$$

$$\xi_3 = [l_1 \; 0 \; l_0 \; 0 \; 1 \; 0]^T$$

$$-\omega_3 \times q_3 = [l_1 \quad 0 \quad l_0]^T$$



$$g_{WE}(\theta) = \underline{e^{\hat{\xi}_1\theta_1}}\;\underline{e^{\hat{\xi}_2\theta_2}}\;\underline{e^{\hat{\xi}_3\theta_3}}\; g_{WE}(0)$$

$$g_{WE}(0) = \begin{bmatrix} R_{WE} & p_{WE} \\ \vec{0}^T & 1 \end{bmatrix}$$

$$p_{WE} = [l_0 \quad 0 \quad -l_1 - l_2]$$

$$R_{WE} = I$$

$$g_{WE}(0) = \begin{bmatrix} 1 & 0 & 0 & l_0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -l_1-l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\xi = \begin{bmatrix} v \\ \omega \end{bmatrix}$$

$$e^{\hat{\xi}\theta} = \begin{bmatrix} e^{\hat{\omega}\theta} & (I - e^{\hat{\omega}\theta})(\omega \times v) + \omega\omega^T v \theta \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$e^{\hat{\omega}_1 \theta_1} = R_x(\pi/2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \qquad \omega \times v$$

$$\boxed{e^{\hat{\xi}_2 \theta_2} = I_4}$$

$$e^{\hat{\xi}_3 \theta_3} = \dots \qquad e^{\hat{\omega}_3 \theta_3} = R_y(\pi/2) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\omega \times v = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} l_1 \\ 0 \\ l_0 \end{bmatrix} = \begin{bmatrix} l_0 \\ 0 \\ -l_1 \end{bmatrix}$$

$$I - R_y(\pi/2) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_0 \\ 0 \\ -l_1 \end{bmatrix} = \begin{bmatrix} l_0 + l_1 \\ 0 \\ l_0 - l_1 \end{bmatrix}$$

$$\boxed{e^{\hat{\xi}_3 \theta_3} = \begin{bmatrix} 0 & 0 & 1 & l_0 + l_1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & l_0 - l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$