Modelling, Control, Parameter Identification & State Estimation

Introduction to Robotics: Discussion 9
Robert Peter Matthew
20151027

EXAMPLE V: PENDULUM

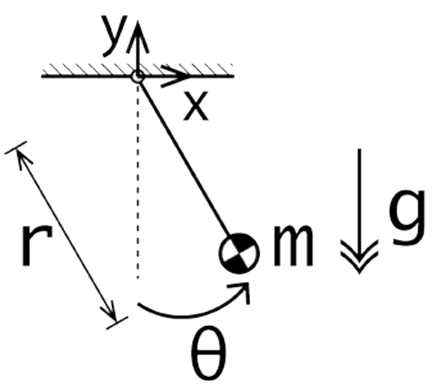
$$L = T - V$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \gamma_i$$

Kinetic Energy

Potential Energy

Lagrangian



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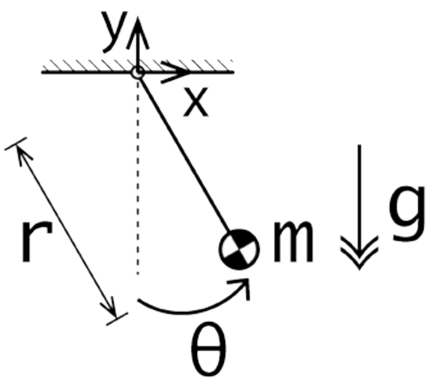

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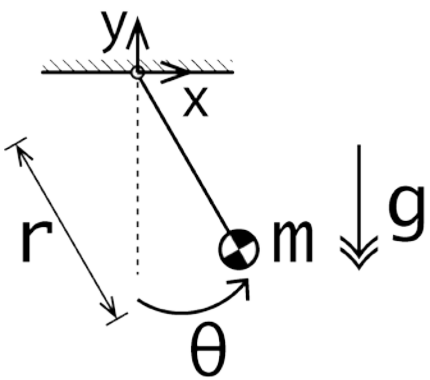

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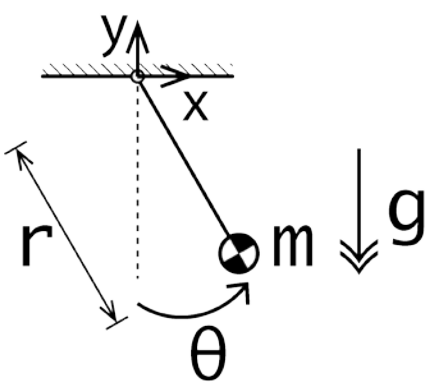

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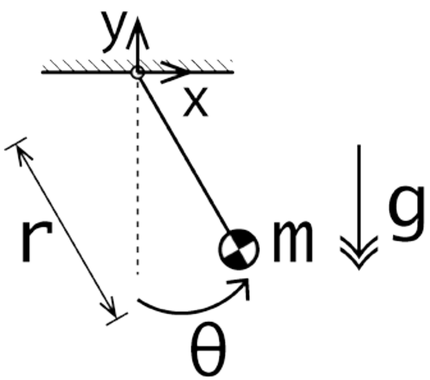

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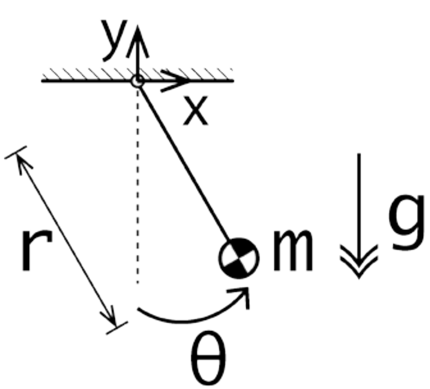

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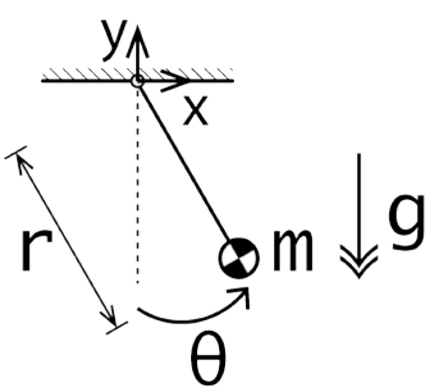

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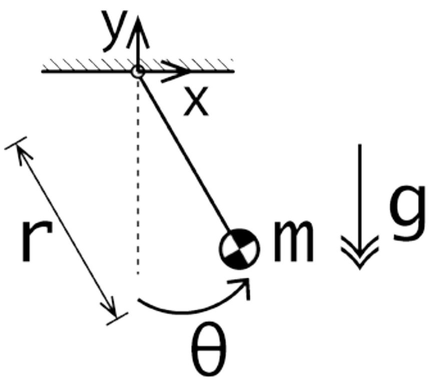
MODELLING

Given the differential equations of a system, simulate the system evolution over time.



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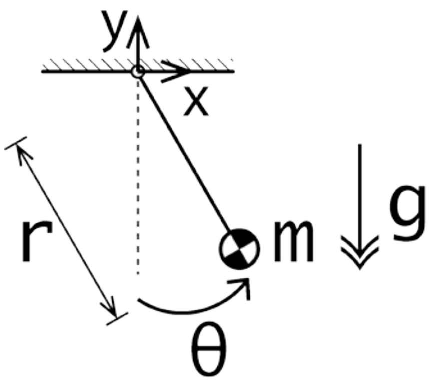

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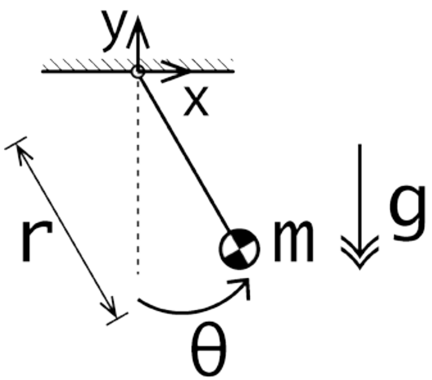

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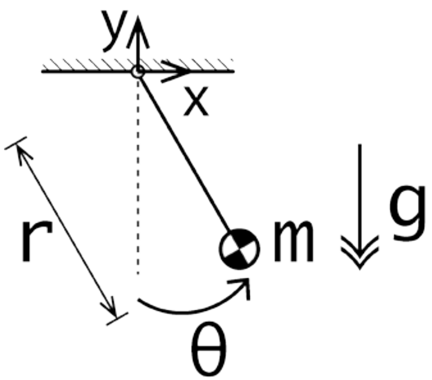

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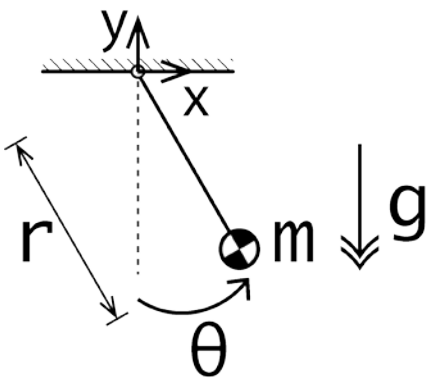

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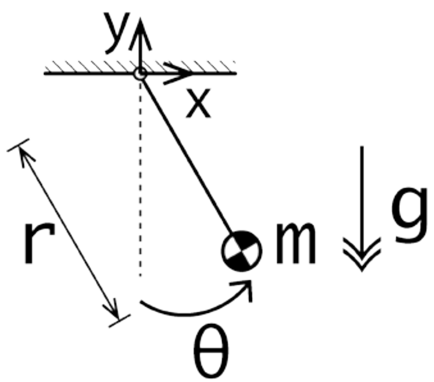

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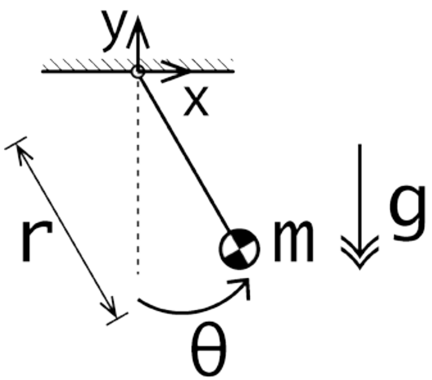

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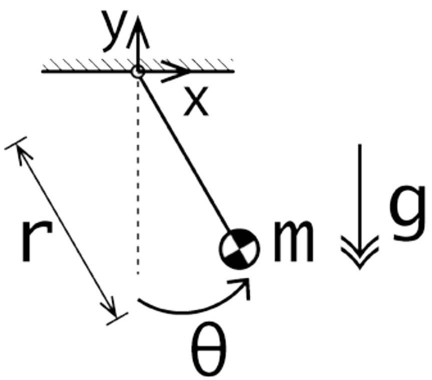

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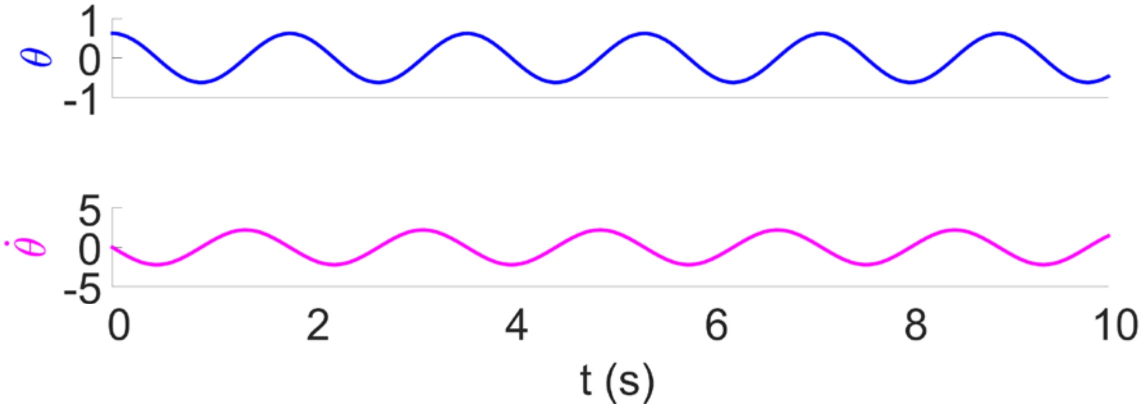

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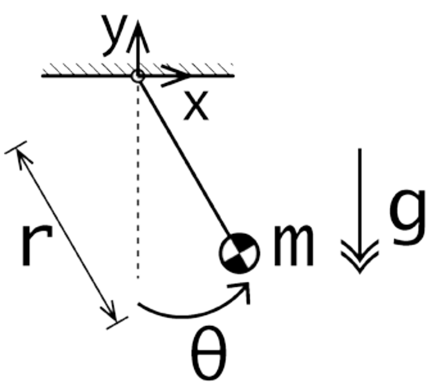

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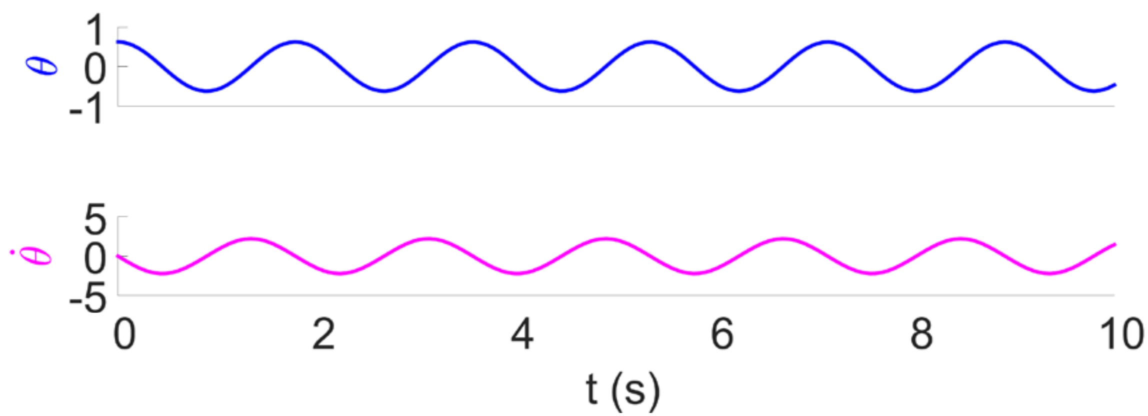

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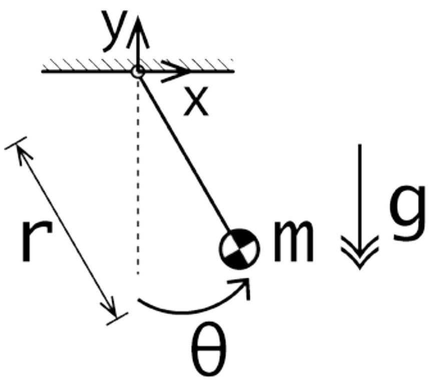

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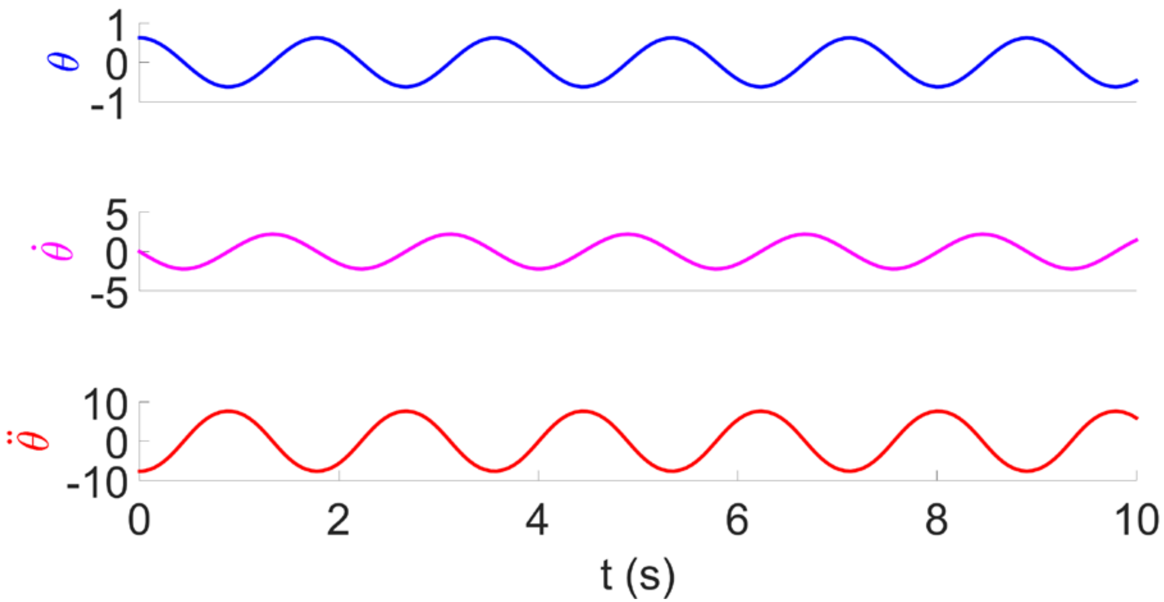

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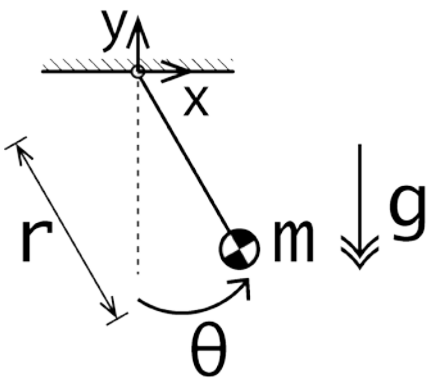

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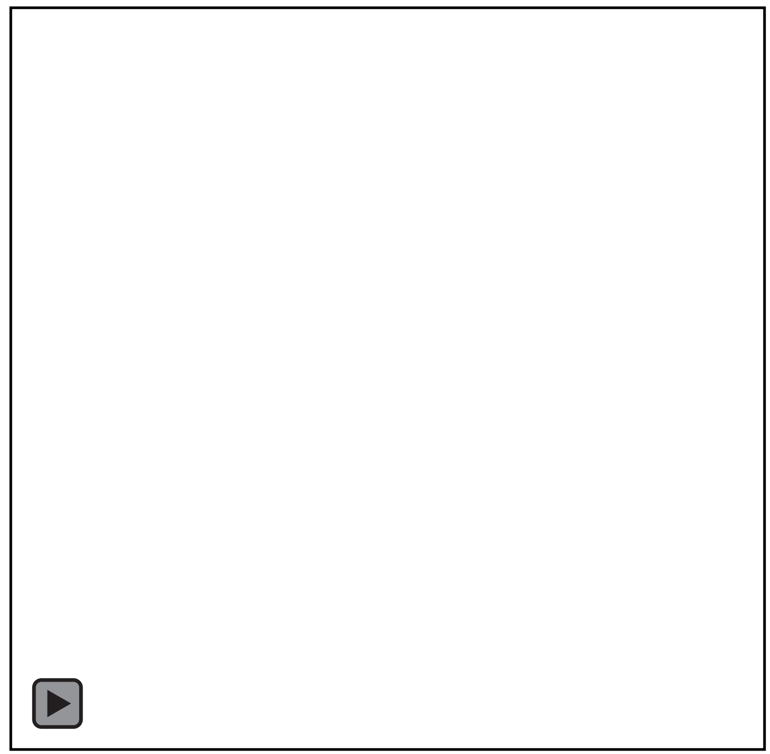

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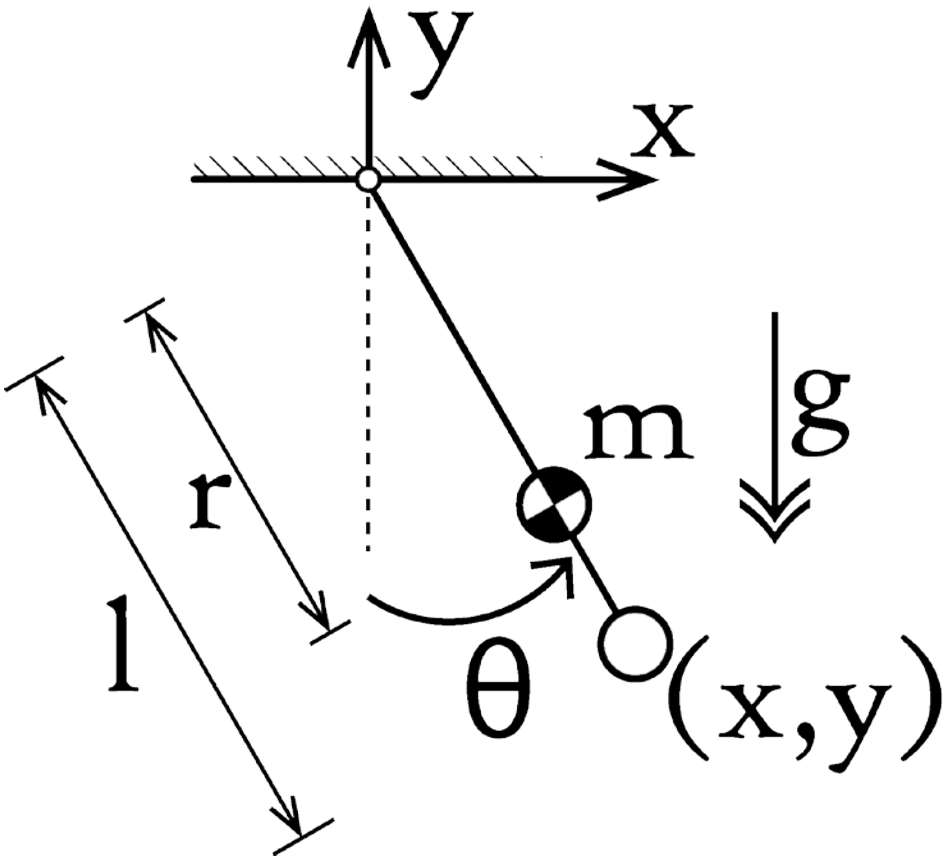

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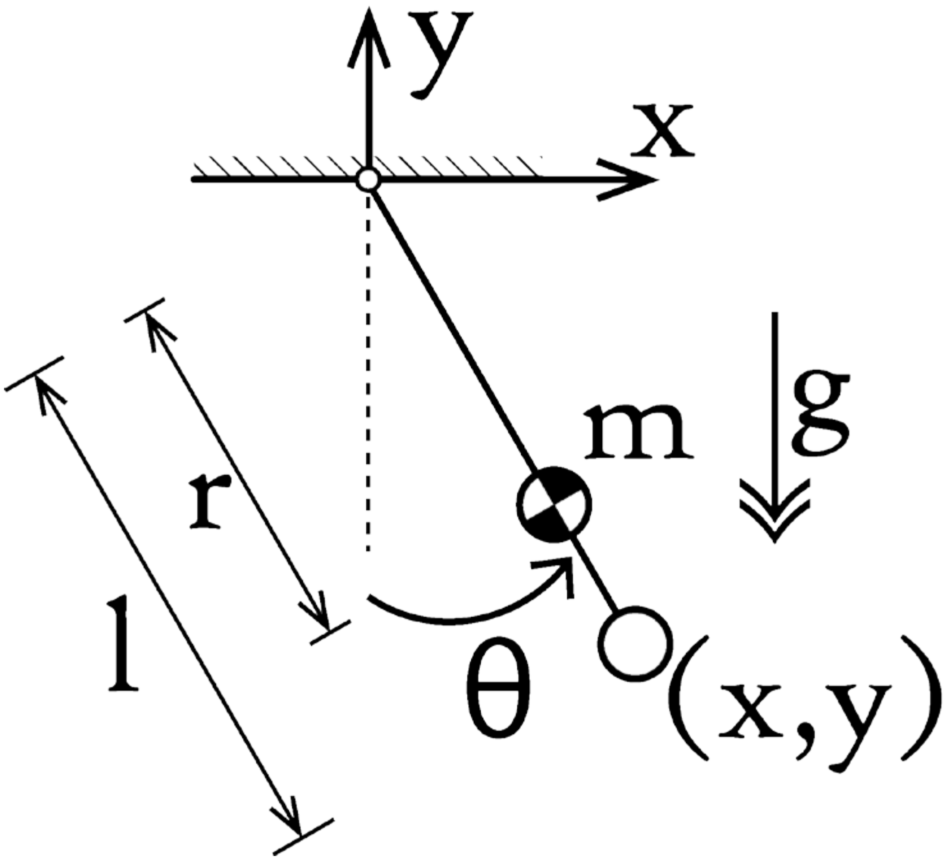

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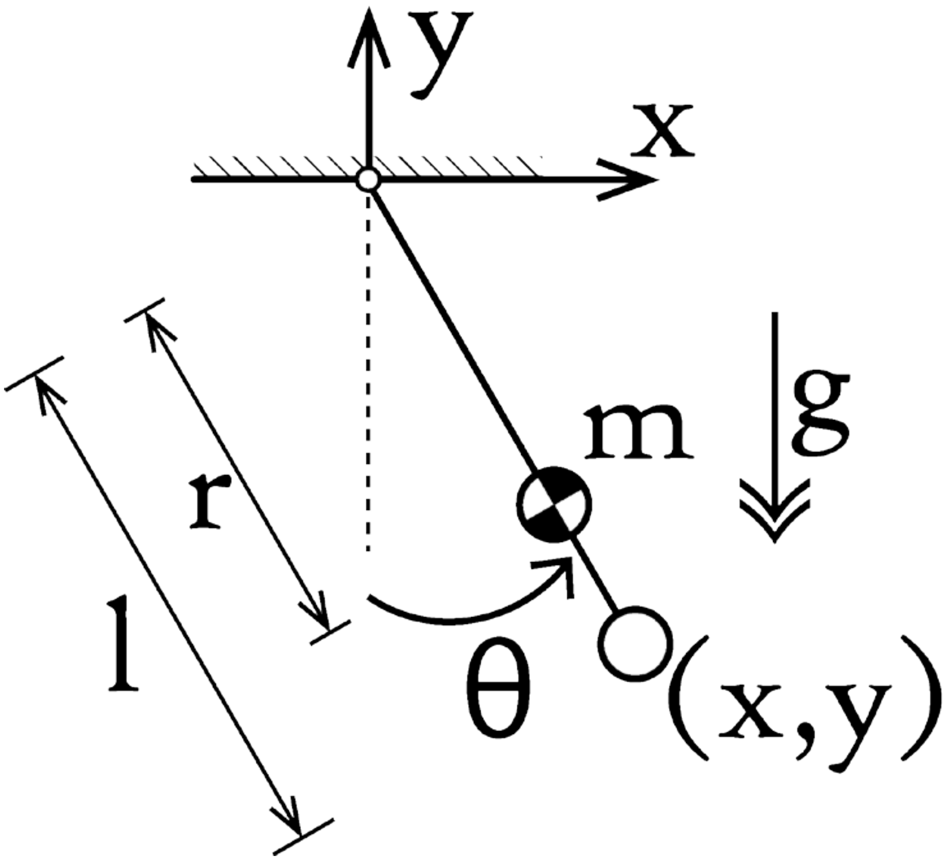

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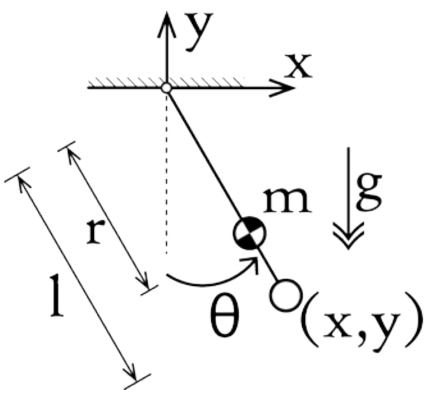

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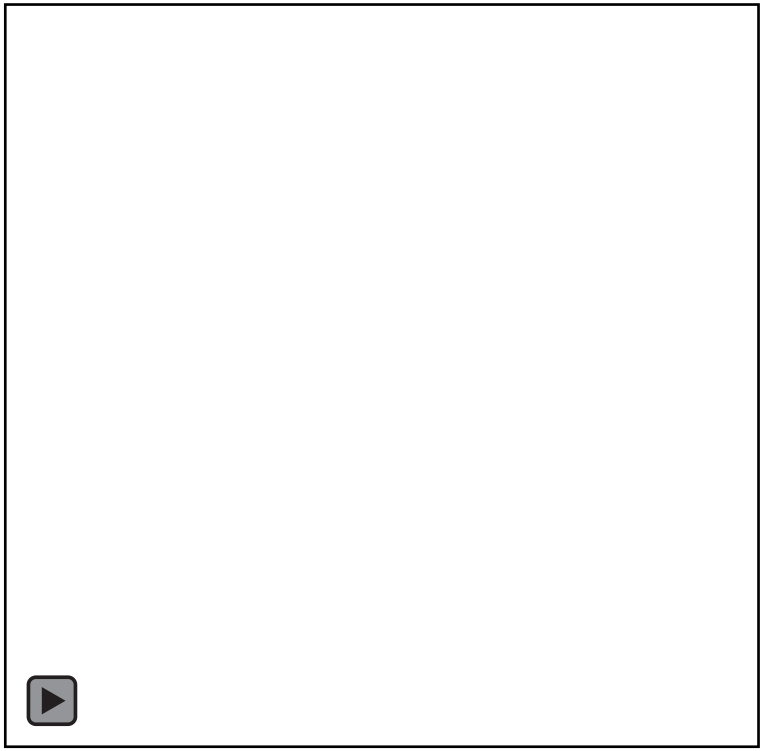

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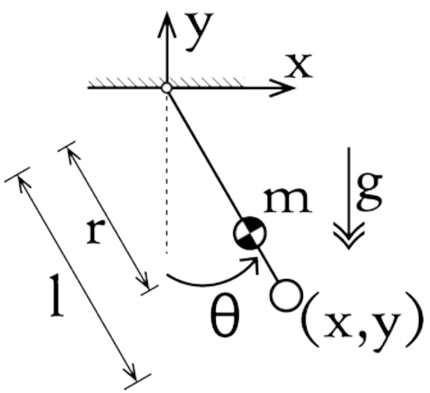

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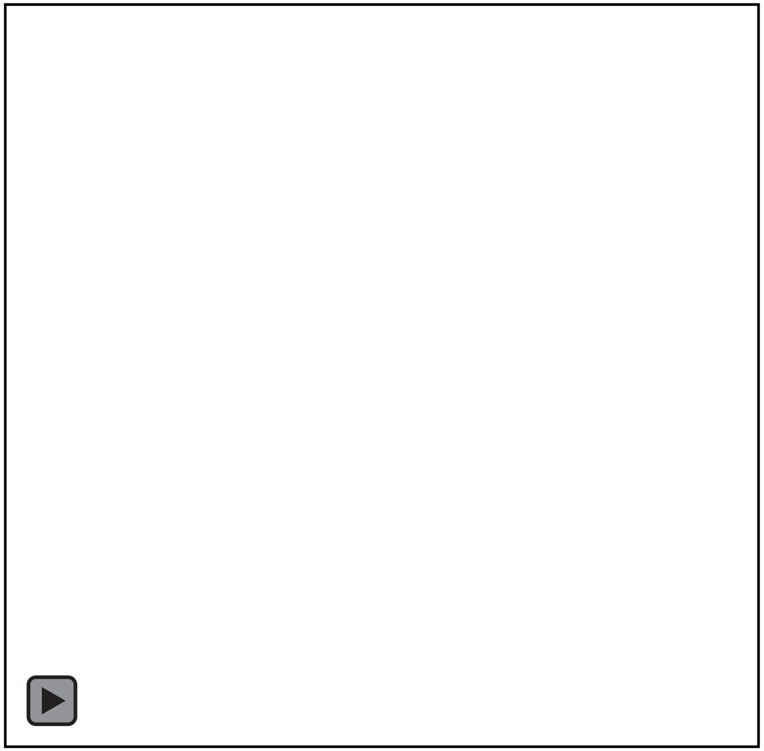

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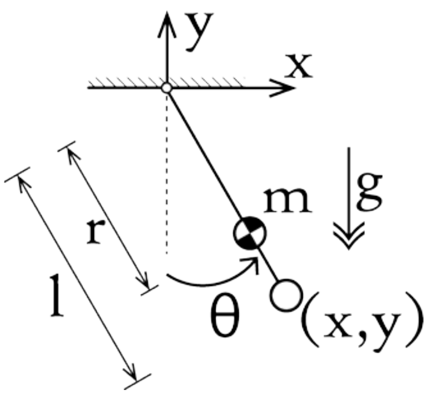

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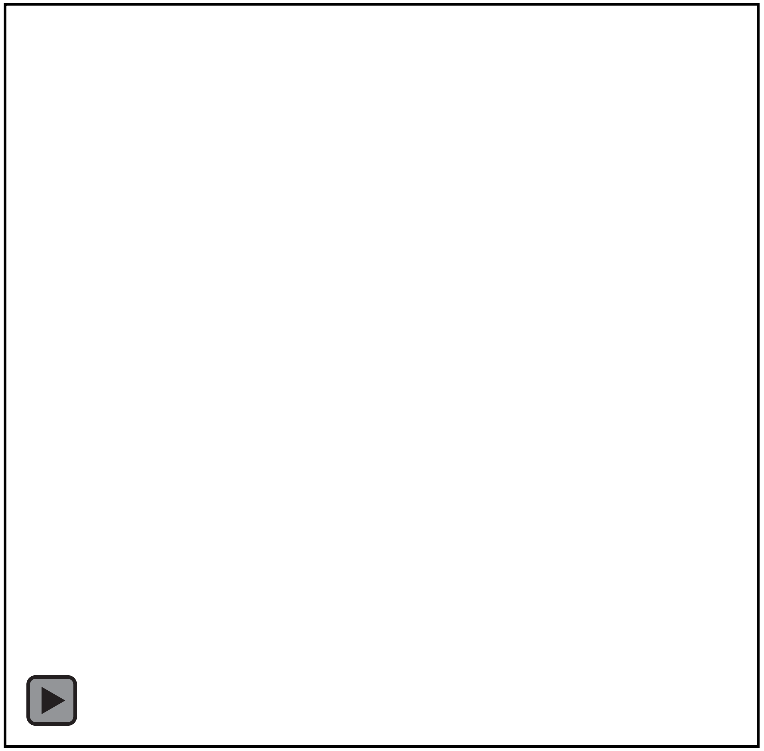

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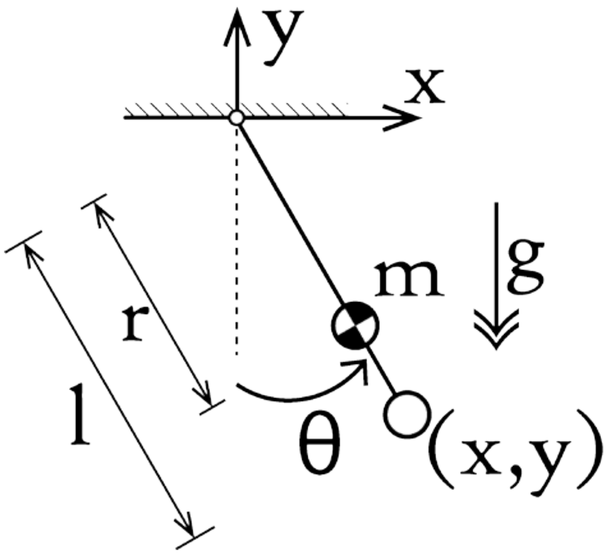

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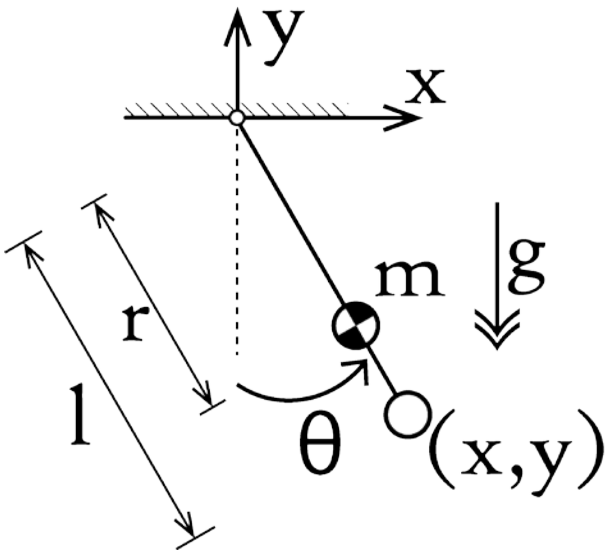

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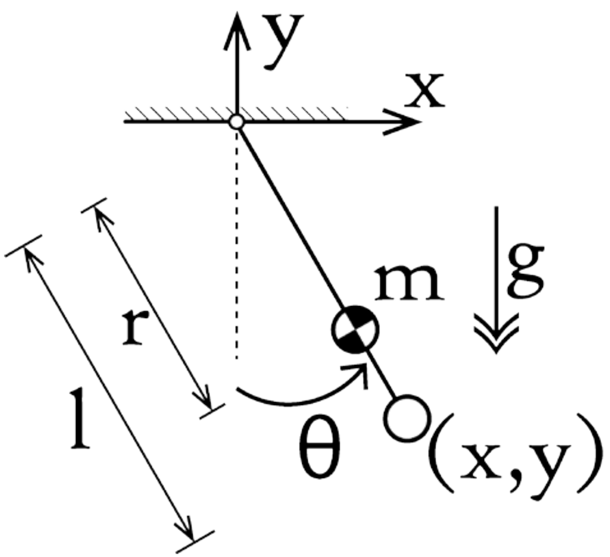
State Estimation:
Given some **observable** data, estimate the **time varying states** of the system

PI: FREE CASE



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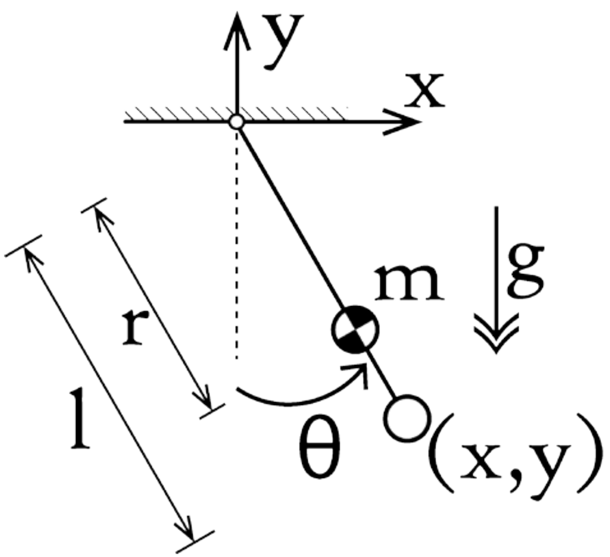

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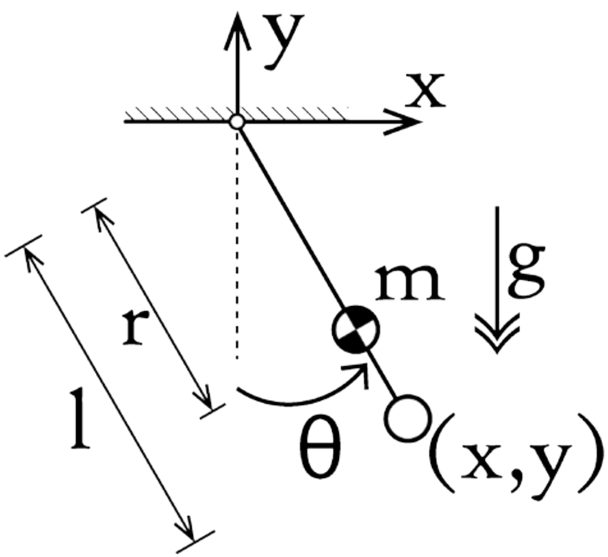
Given one measurement of (x, y)

- Two equations: x, y
- One unknown state: θ
- Three unknown parameters: T_x, T_y, l

PI: FREE CASE



$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c_\theta & -s_\theta & T_x \\ s_\theta & c_\theta & T_y \\ & \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -l \\ 1 \end{bmatrix}$$

$$\ddot{\theta} = \frac{1}{mr^2}\tau - \frac{g}{r} s_\theta$$

Parameter Identification:

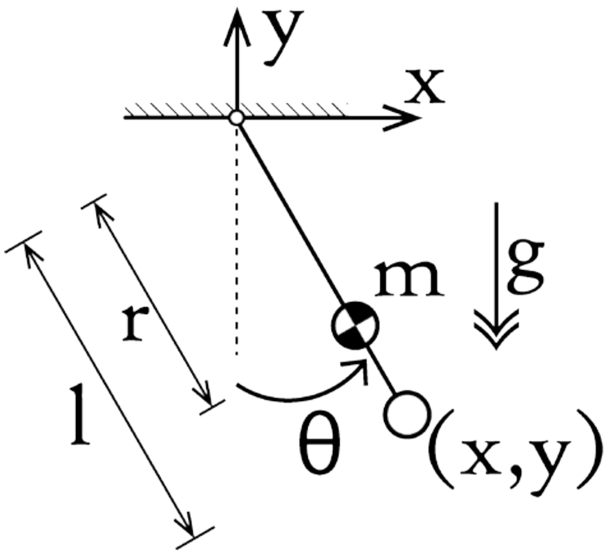
Given some **observable** data, estimate the **fixed parameters** of the system

Given one measurement of (x, y)

- Two equations: x, y
- One unknown state: θ
- Three unknown parameters: T_x, T_y, l

⇒ Two equations, four unknowns

PI: FREE CASE



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⇒ Unsolvable

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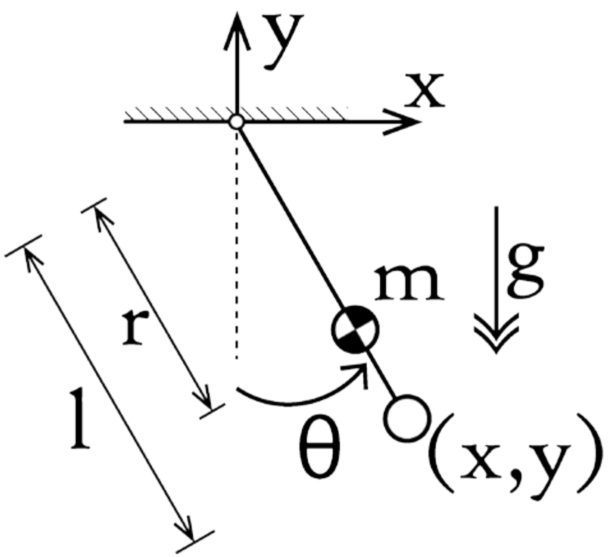

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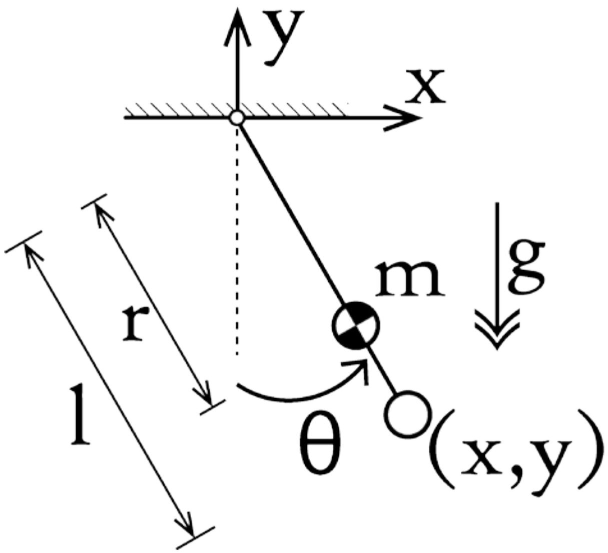
Parameter Identification:
Given some **observable** data, estimate the **fixed parameters** of the system

Given n measurements of (x, y)

- $2n$ equations: x, y
- n unknown states: θ
- Three unknown parameters: T_x, T_y, l

PI: FREE CASE



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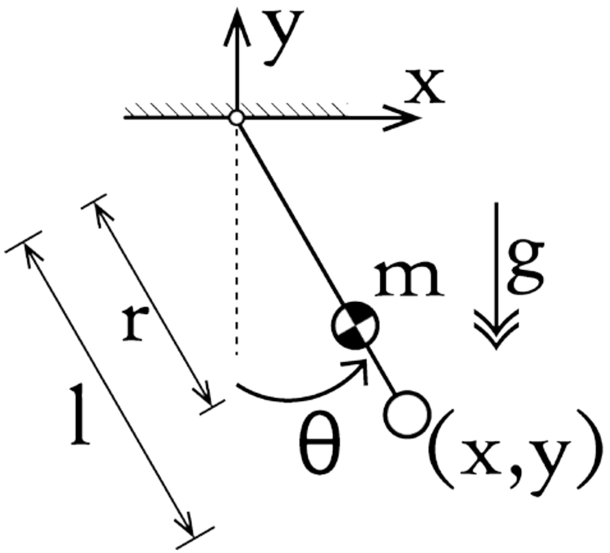
$\Rightarrow 2n$ equations , $n + 3$ unknowns

PI: FREE CASE



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$\Rightarrow 2n$ equations , $n + 3$ unknowns

$\Rightarrow$ solvable for $n \geq 3$

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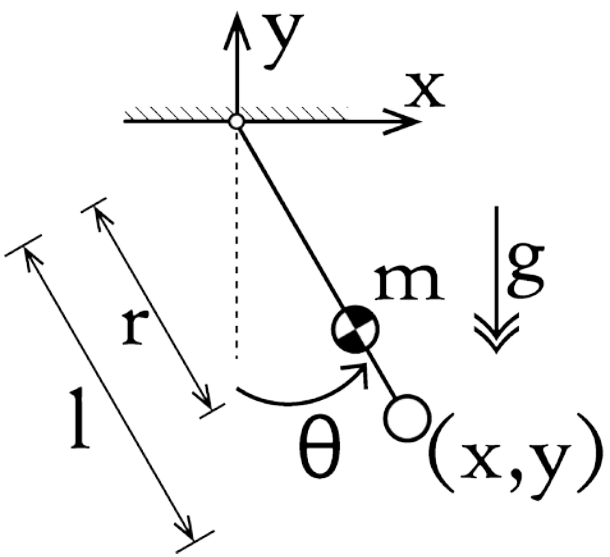
PI: FREE CASE



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- $2n$ equations: x, y
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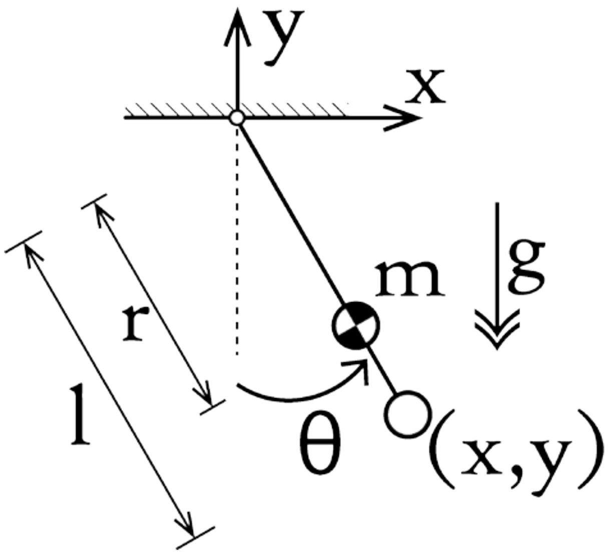
⇒ $2n$ equations , $n + 3$ unknowns
⇒ solvable for $n \geq 3$

BUT! Non-linear

PI: FREE CASE



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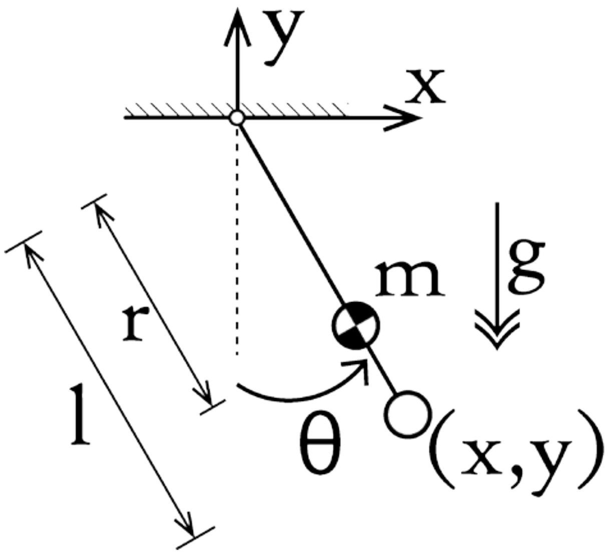
Solution method: **non-linear least squares** (python: *lmfit*) (MATLAB: *lsqnonlin*)

$$\min_\nu (f_1(\nu)^2 + \cdots + f_n(\nu)^2)$$

PI: FREE CASE



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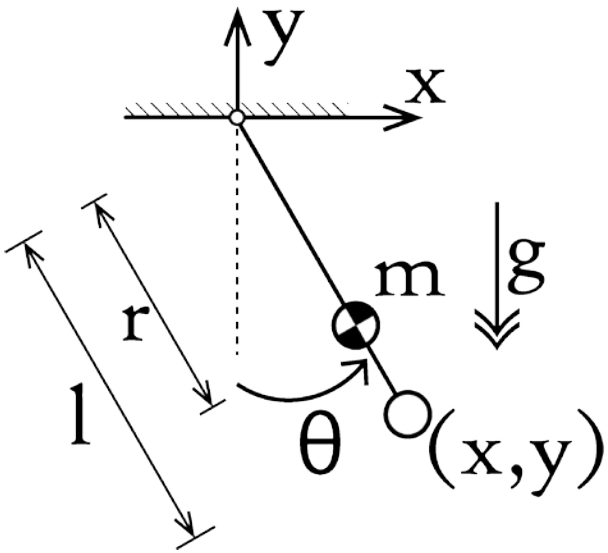
Solution method: **non-linear least squares** (python: *lmfit*) (MATLAB: *lsqnonlin*)

$$\min_{\nu}\left(f_1(\nu)^2 + \cdots + f_n(\nu)^2\right)$$

ν are the parameters to be found

$f_i(\nu)^2$ is an associated cost function

PI: FREE CASE



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$$\ddot{\theta} = \frac{1}{mr^2}\tau - \frac{g}{r} s_\theta$$

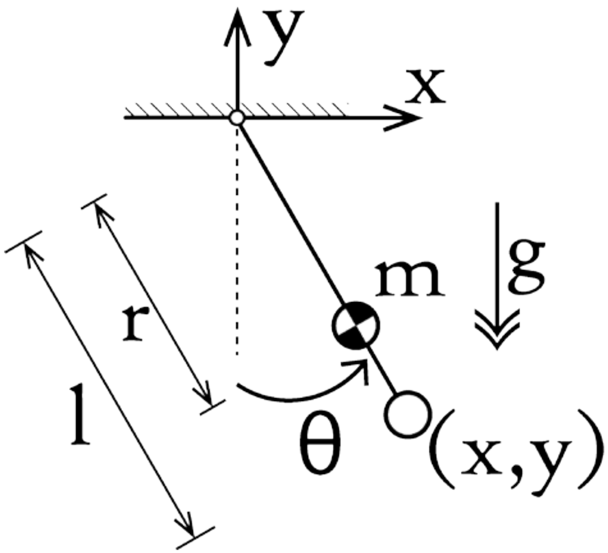
$$\min_\nu (f_1(\nu)^2 + \cdots + f_n(\nu)^2)$$

Given observed datapoint x_i, the predicted location is given by:

$$c_{\theta_i} + l s_{\theta_i} + T_x$$

PI: FREE CASE

$$\min_{\nu}(f_1(\nu)^2 + \cdots + f_n(\nu)^2)$$



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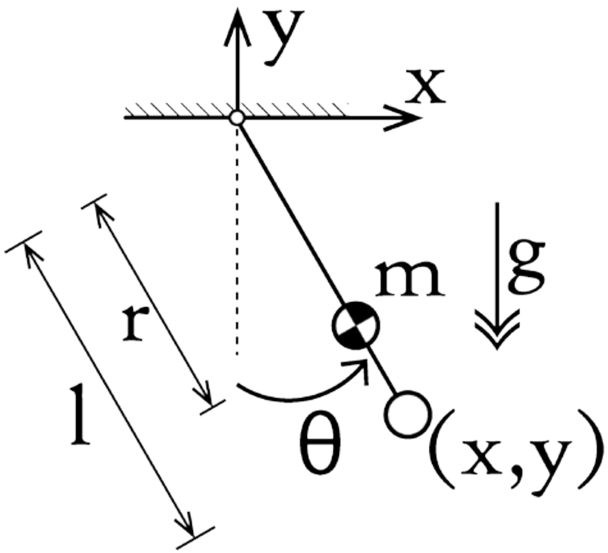

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We want to minimise the error between the observed and predicted points:

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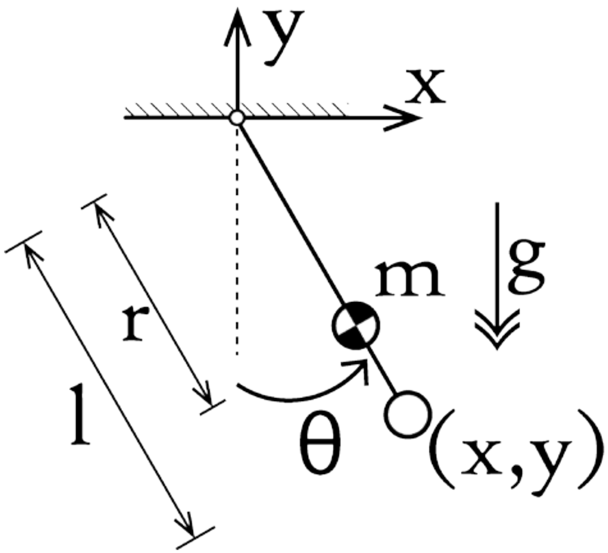
PI: FREE CASE

$$\min_{\nu}(f_1(\nu)^2 + \cdots + f_n(\nu)^2)$$



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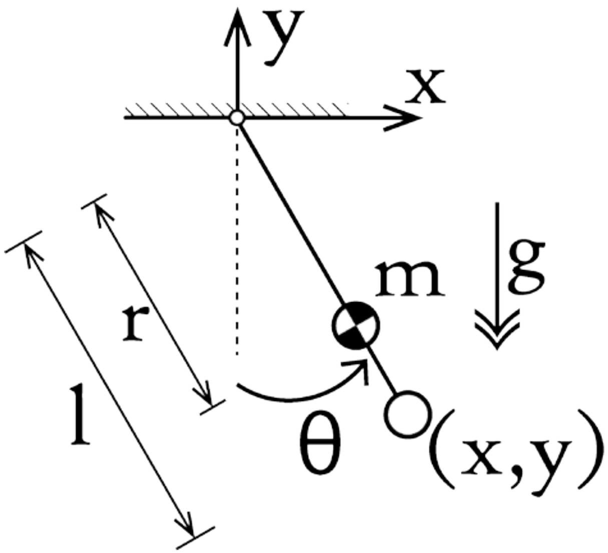
We want to minimise the error between the observed and predicted points:

$$\epsilon_i = x_i - \left(c_{\theta_i} + l s_{\theta_i} + T_x\right)$$

PI: FREE CASE

$$\min_{\nu}\left(f_1(\nu)^2 + \cdots + f_n(\nu)^2\right)$$



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$$\ddot{\boldsymbol{\theta}} = \frac{1}{mr^2}\boldsymbol{\tau} - \frac{g}{r}s_{\boldsymbol{\theta}}$$

Given observed datapoint $\boldsymbol{x_i}$, the predicted location is given by:

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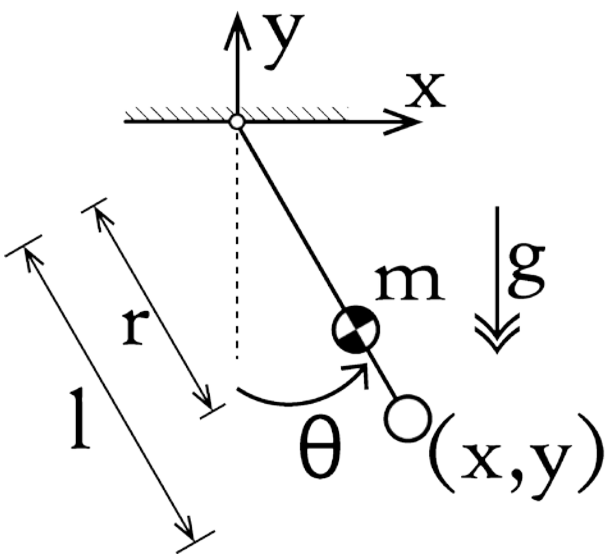
We want to minimise the error between the observed and predicted points:

$$\epsilon_i = \boldsymbol{x_i} - \left(c_{\boldsymbol{\theta_i}} + \boldsymbol{l}\boldsymbol{s}_{\boldsymbol{\theta_i}} + \boldsymbol{T_x}\right)$$

Therefore we perform the minimisation:

$$\min_{\nu}\left({\epsilon_1}^2 + \cdots + {\epsilon_n}^2\right)$$

PI: FREE CASE



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$$\min_{\nu} \left(f_1(\nu)^2 + \cdots + f_n(\nu)^2 \right)$$

Given observed datapoint x_i, the predicted location is given by:

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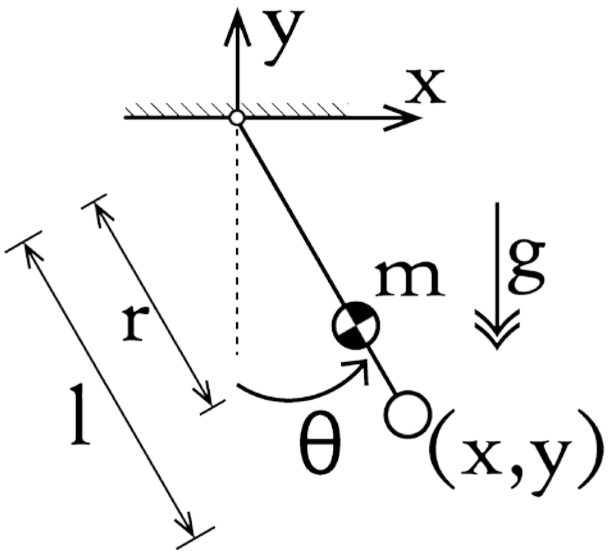
$$\nu = [[T_x \quad T_y \quad l] \quad [\theta_1 \quad \ldots \quad \theta_n]]$$

PI: FREE CASE



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$$\nu = [[T_x \quad T_y \quad l] \quad [\theta_1 \quad \ldots \quad \theta_n]]$$

$\Rightarrow$ We can solve for the parameters: T_x, T_y, l

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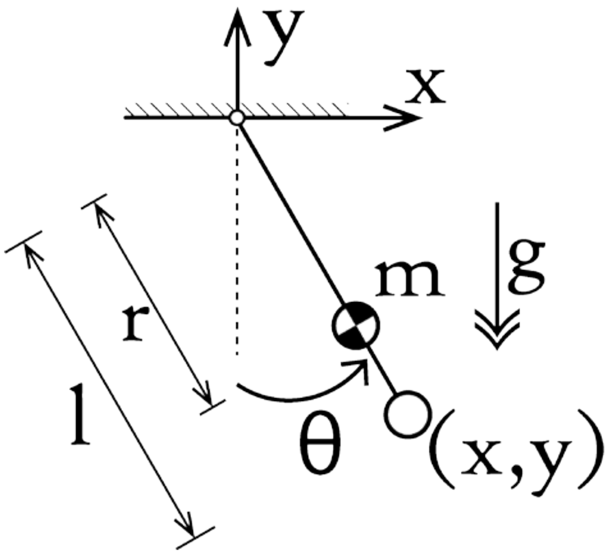
PI: FREE CASE



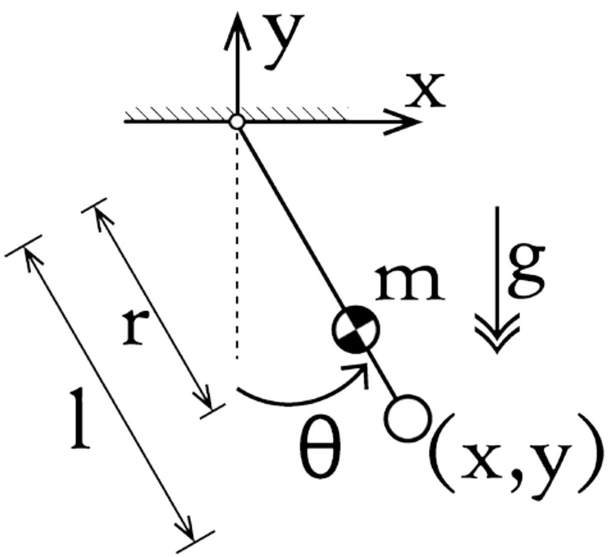
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c_{\boldsymbol{\theta}} & -s_{\boldsymbol{\theta}} & \boldsymbol{T}_x \\ s_{\boldsymbol{\theta}} & c_{\boldsymbol{\theta}} & \boldsymbol{T}_y \\ \multicolumn{2}{c}{\mathbf{0}} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -\boldsymbol{l} \\ 1 \end{bmatrix}$$

$$\ddot{\boldsymbol{\theta}} = \frac{1}{mr^2}\boldsymbol{\tau} - \frac{g}{r} s_{\boldsymbol{\theta}}$$

How many samples?

System is solvable for $\boldsymbol{n} \geq \mathbf{3}$

PI: FREE CASE



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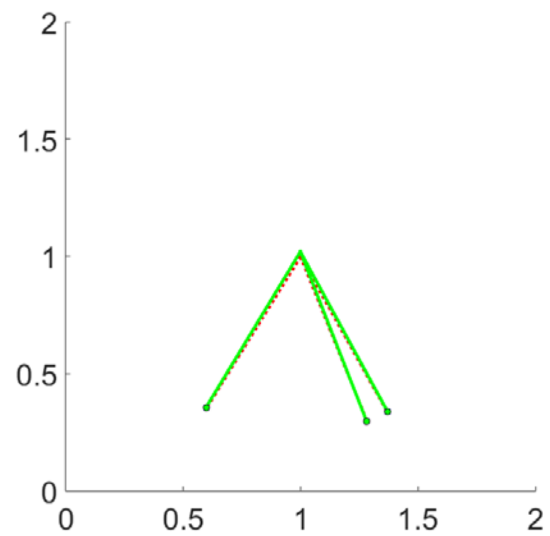
$$\ddot{\theta} = \frac{1}{mr^2}\tau - \frac{g}{r}s_\theta$$

How many samples?

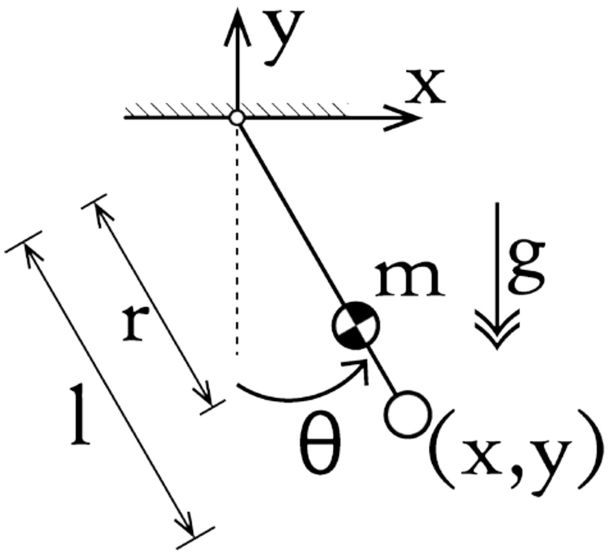
System is solvable for $n \geq 3$

$n = 3$ Solve time: 0.05s



$T_x = 1.00 \quad T_y = 1.02 \quad l = 0.78$

PI: FREE CASE



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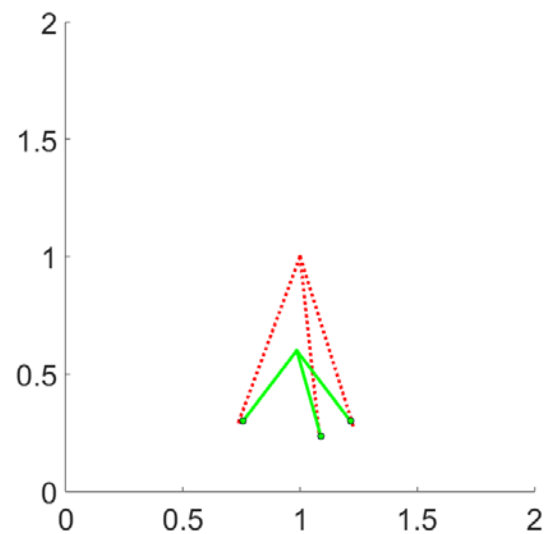
$$\ddot{\theta} = \frac{1}{mr^2}\tau - \frac{g}{r} s_\theta$$

How many samples?

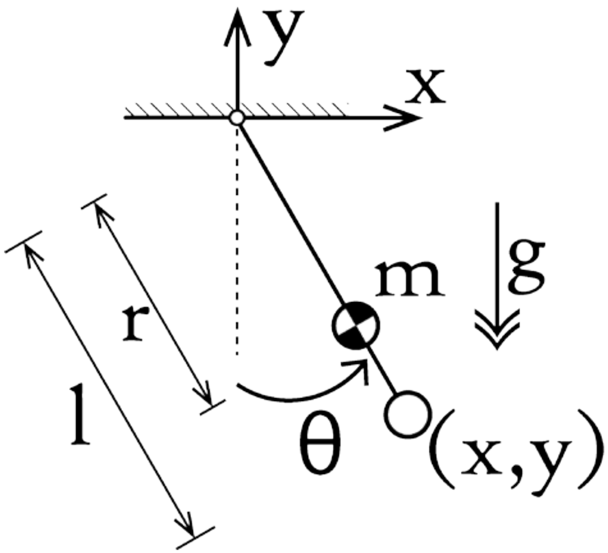
System is solvable for $n \geq 3$

$n = 3$ Solve time: 0.06s



$T_x = 0.99 \quad T_y = 0.60 \quad l = 0.38$

PI: Free Case



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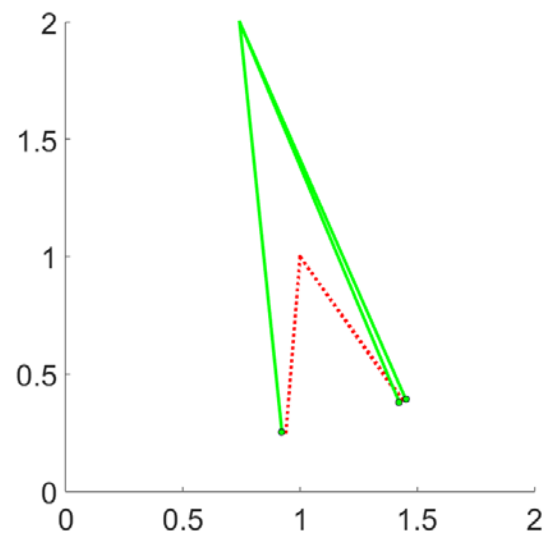
How many samples?

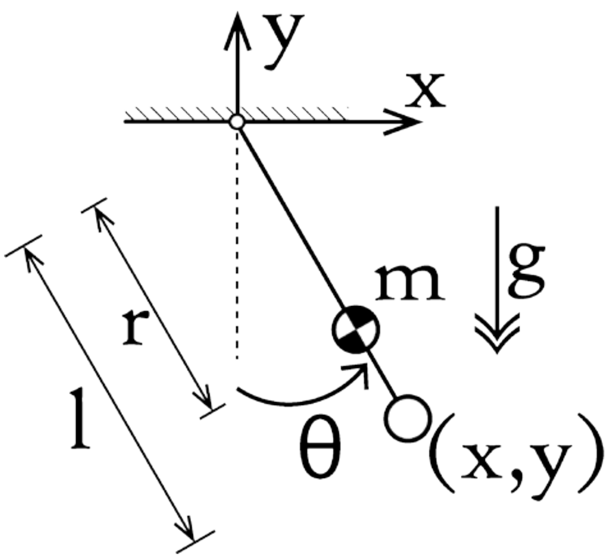
System is solvable for $n \geq 3$

$n = 3$ Solve time: 297.80s



$T_x = 0.74$ $T_y = 2.00$ $l = 1.76$

PI: FREE CASE



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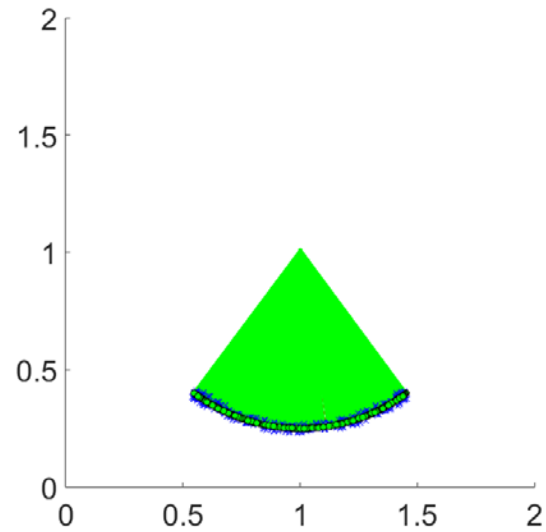
$$\ddot{\theta} = \frac{1}{mr^2}\tau - \frac{g}{r}s_\theta$$

How many samples?

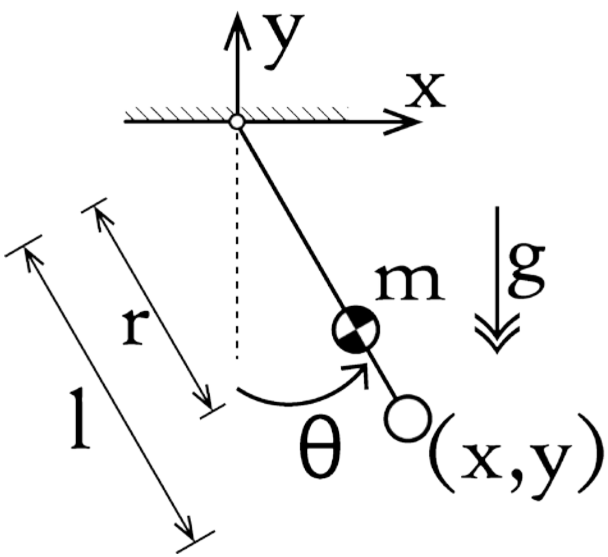
System is solvable for $n \geq 3$

$n = 1000$ Solve time: 297.80s



$T_x = 1.00$ $T_y = 1.00$ $l = 0.76$

PI: FREE CASE



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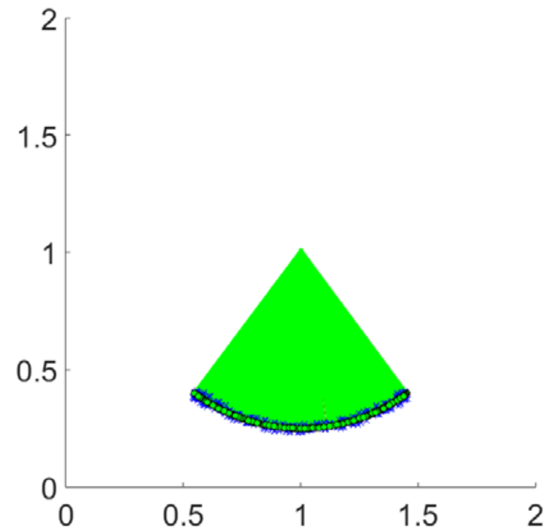
How many samples?

System is solvable for $n \geq 3$

$n = 500$ Solve time: 87.58s



$T_x = 1.00$ $T_y = 1.00$ $l = 0.75$

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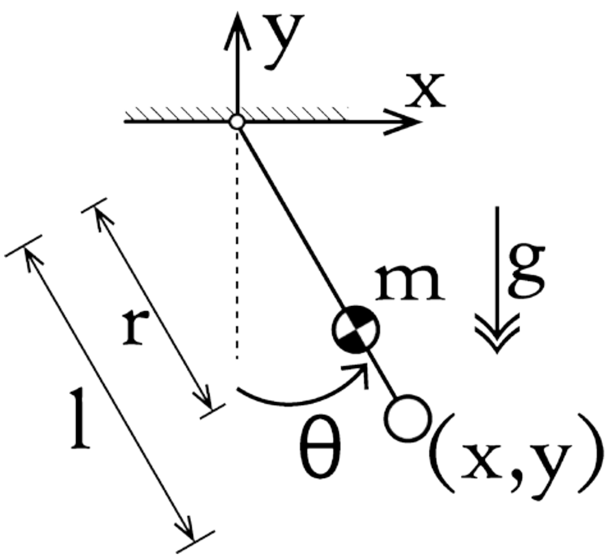
PI: FREE CASE



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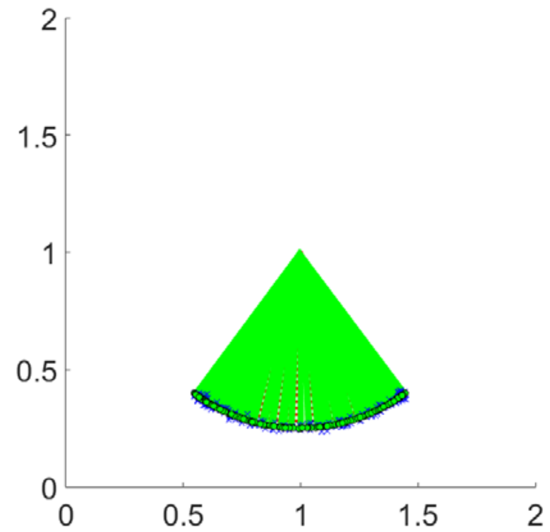
$$\ddot{\theta} = \frac{1}{mr^2}\tau - \frac{g}{r} s_\theta$$

How many samples?

System is solvable for $n \geq 3$

$n = 250$ Solve time: 20.18s



$T_x = 1.00$ $T_y = 1.00$ $l = 0.75$

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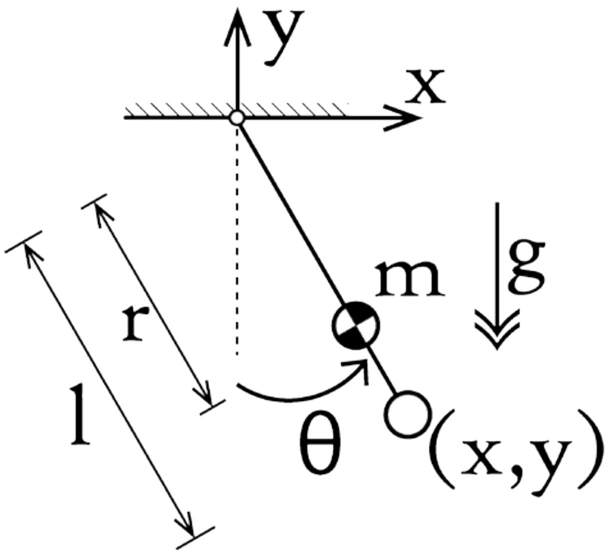
PI: FREE CASE



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$$\ddot{\theta} = \frac{1}{mr^2}\tau - \frac{g}{r} s_\theta$$

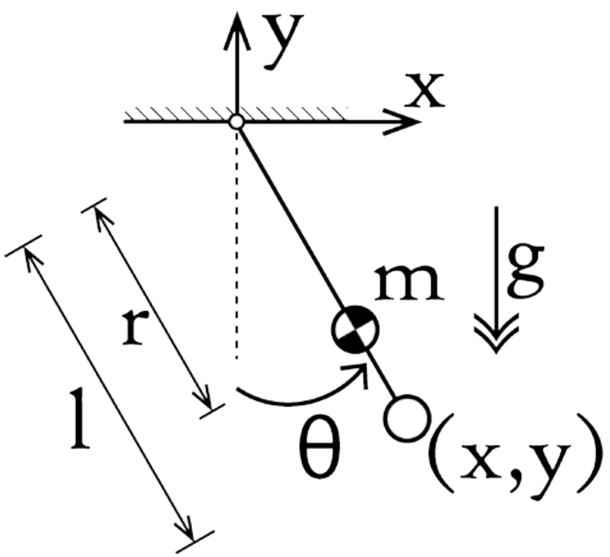
How many samples?

System is solvable for $n \geq 3$

Words of Wisdom:

- Try multiple samples
- Try lots of samples first
- Reduce until negligible variability

PI: FREE CASE



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How many samples?

System is solvable for $n \geq 3$

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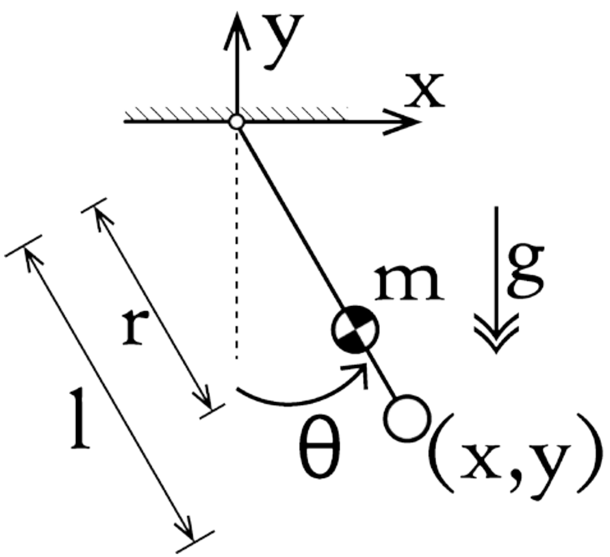
- Try lots of samples first
- Reduce until negligible variability

$$T_x = 1.00 \quad T_y = 1.00 \quad l = 0.75$$

FREE CASE



$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c_\theta & -s_\theta & T_x \\ s_\theta & c_\theta & T_y \\ & \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -l \\ 1 \end{bmatrix}$$

$$\ddot{\theta} = \frac{1}{mr^2}\tau - \frac{g}{r} s_\theta$$

Parameter Identification:

Given some **observable** data, estimate the **fixed parameters** of the system

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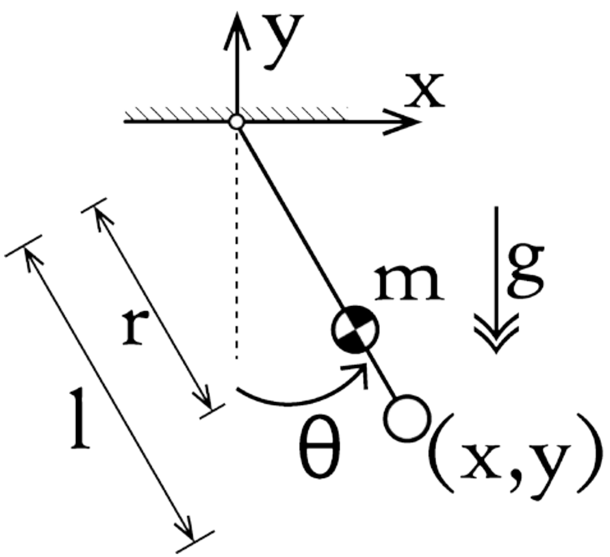
FREE CASE



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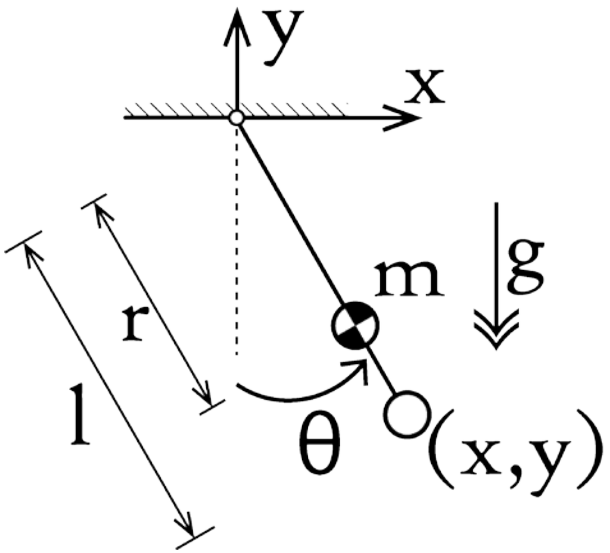
Parameter Identification:
Given some **observable** data, estimate the **fixed parameters** of the system

State Estimation:
Given some **observable** data, estimate the **time varying states** of the system

FREE CASE



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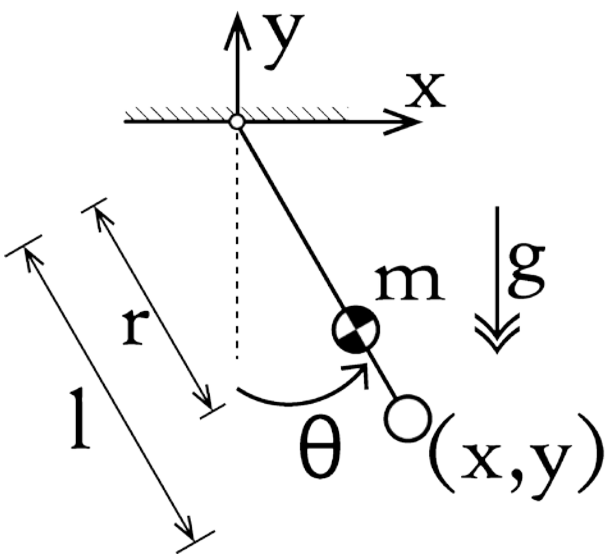
FREE CASE



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Given some **observable** data, estimate the **time varying states** of the system

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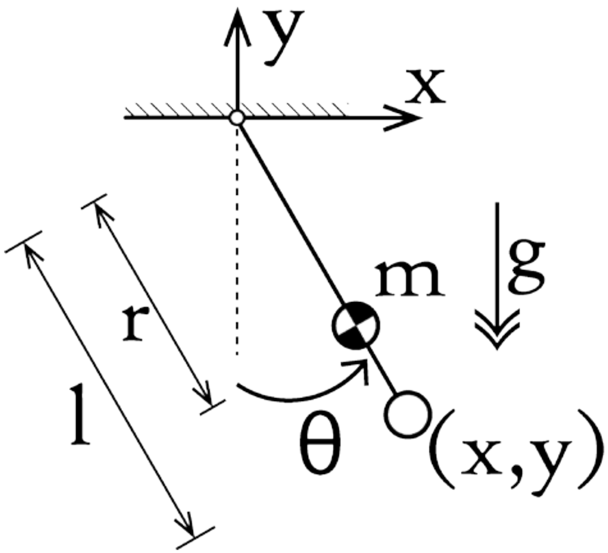
$$\begin{bmatrix} \dot{\theta} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{g}{r} s_\theta \end{bmatrix}$$

SE: FREE CASE



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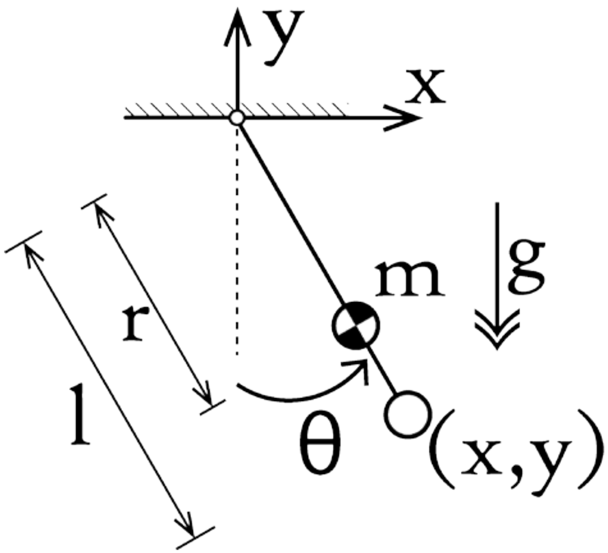

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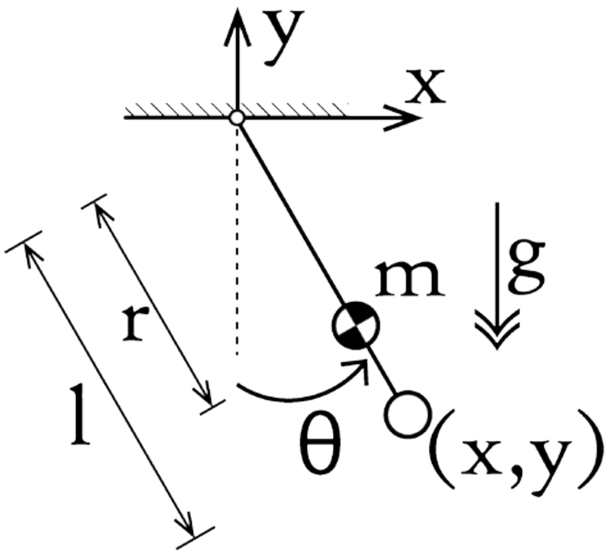
⇒ solvable using a Kalman filter?

SE: FREE CASE



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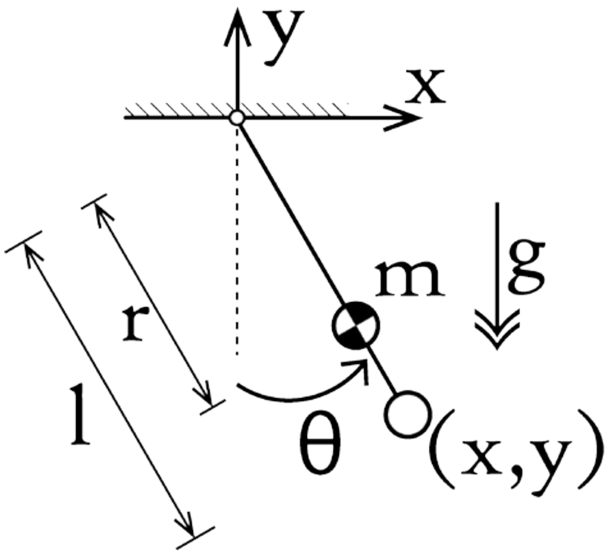
⇒ solvable using a Kalman filter?

BUT! Non-linear

SE: FREE CASE



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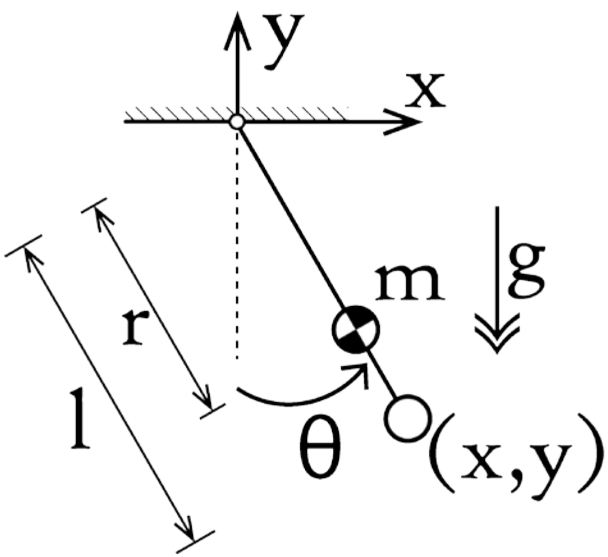
⇒ solvable using a Kalman filter?

BUT! Non-linear

⇒ solvable using an Unscented Kalman filter

SE: FREE CASE



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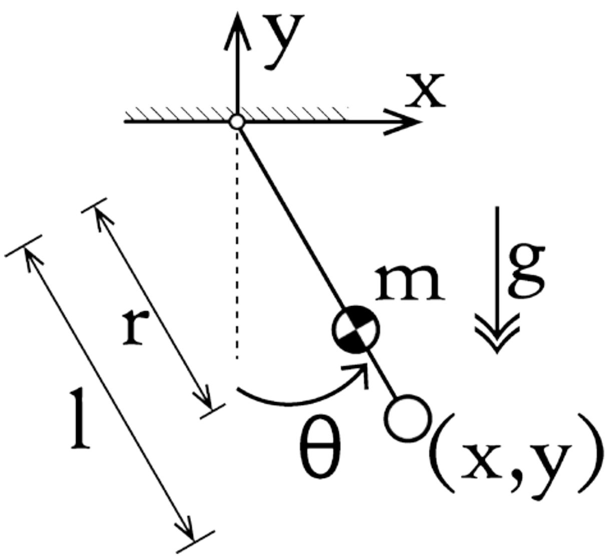
Solution method: **Unscented Kalman Filter**
(python: *pykalman*)
(MATLAB: *Learning the Unscented Kalman Filter*)

SE: FREE CASE



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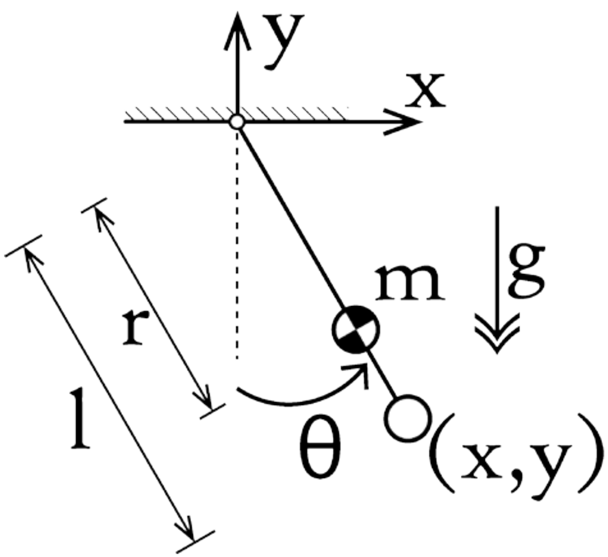
(python: *pykalman*)
(MATLAB: *Learning the Unscented Kalman Filter*)

$$\begin{aligned} X_{k+1} &= f(X_k) + w_k \\ Y_k &= h(X_k) + v_k \end{aligned}$$

SE: FREE CASE



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X_k Current state
Y_k Current observation

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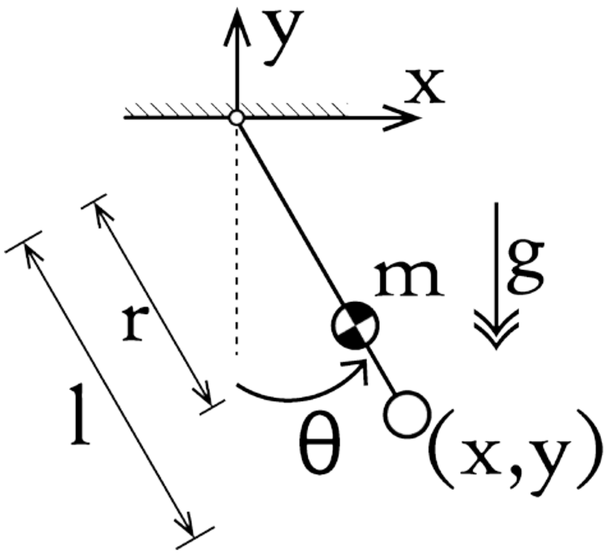
SE: FREE CASE



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X_k Current state
Y_k Current observation

w_k State error
v_k Observation error

SE: FREE CASE

$$X_{k+1} = f(X_k) + w_k$$
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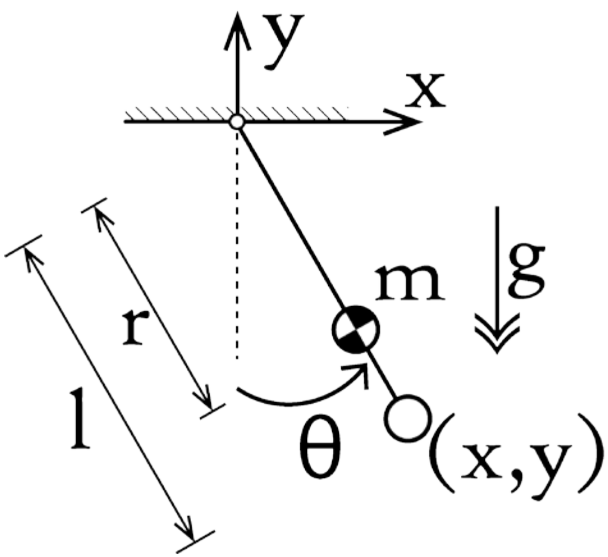
Need discrete time equations:



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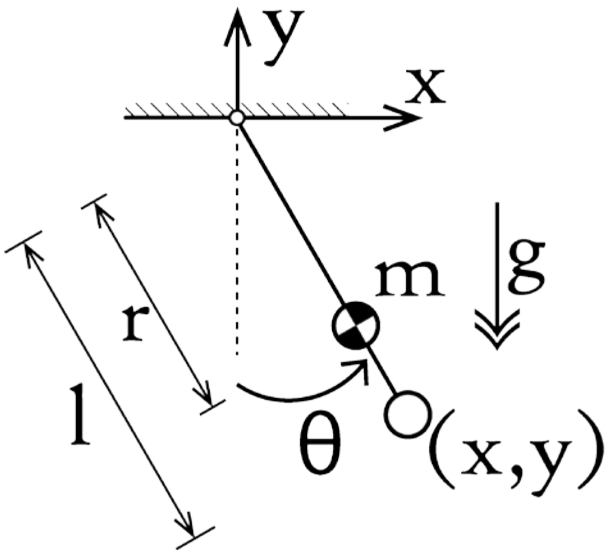

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Need discrete time equations:

$$\frac{\theta_{k+1} - \theta_k}{\partial t} = \dot{\theta}_k$$

∂t is the time step

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SE: FREE CASE

$$X_{k+1} = f(X_k) + w_k$$
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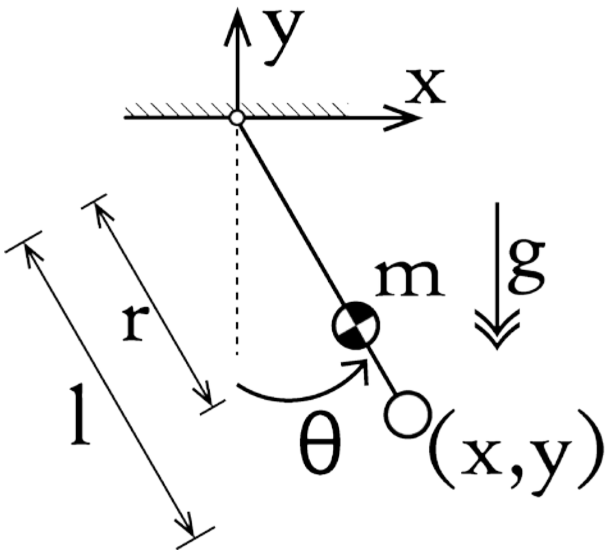

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$$\frac{\theta_{k+1} - \theta_k}{\partial t} = \dot{\theta}_k \qquad \theta_{k+1} = \partial t \dot{\theta}_k + \theta_k$$

∂t is the time step

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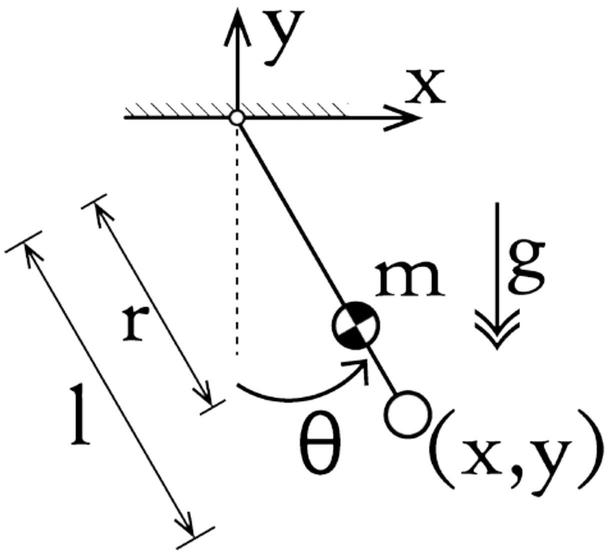

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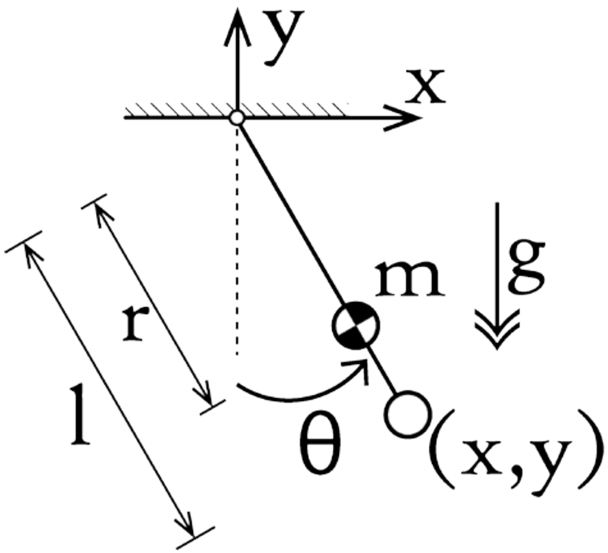

Need discrete time equations:

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∂t is the time step

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c_\theta & -s_\theta & T_x \\ s_\theta & c_\theta & T_y \\ \multicolumn{2}{c}{\mathbf{0}} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -l \\ 1 \end{bmatrix}$$

$$\frac{\dot{\theta}_{k+1} - \dot{\theta}_k}{\partial t} = \ddot{\theta}_k = -\frac{g}{r} s_{\theta_k}$$

$$\dot{\theta}_{k+1} = -\partial t \frac{g}{r} s_{\theta_k} + \dot{\theta}_k$$

$$\ddot{\theta} = \frac{1}{mr^2}\tau - \frac{g}{r} s_\theta$$

SE: FREE CASE

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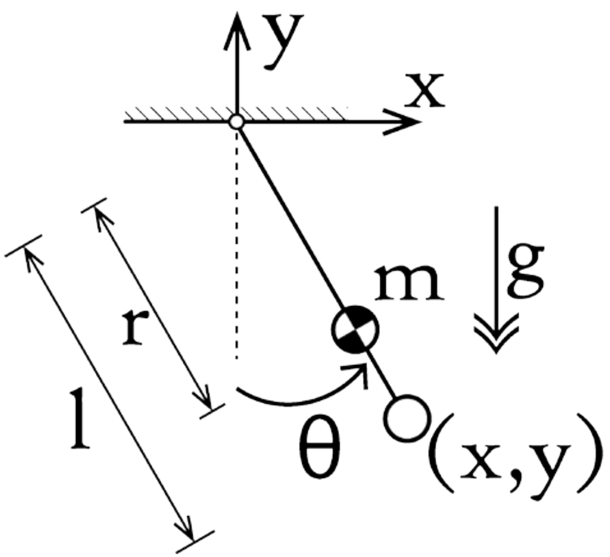

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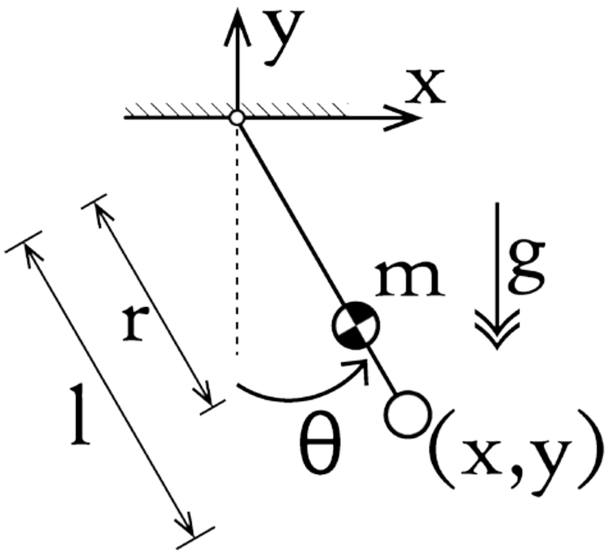

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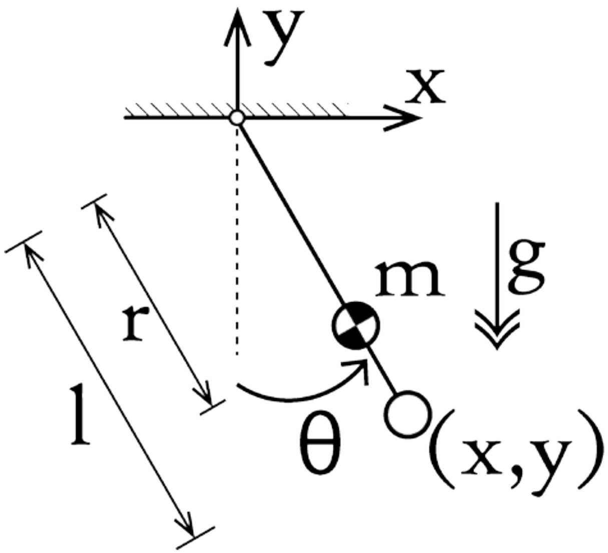

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$$\begin{bmatrix} x \\ y \end{bmatrix}_k = \begin{bmatrix} c_{\theta_k} & -s_{\theta_k} & T_x \\ s_{\theta_k} & c_{\theta_k} & T_y \end{bmatrix} \begin{bmatrix} 0 \\ -l \\ 1 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_k$$

SE: FREE CASE

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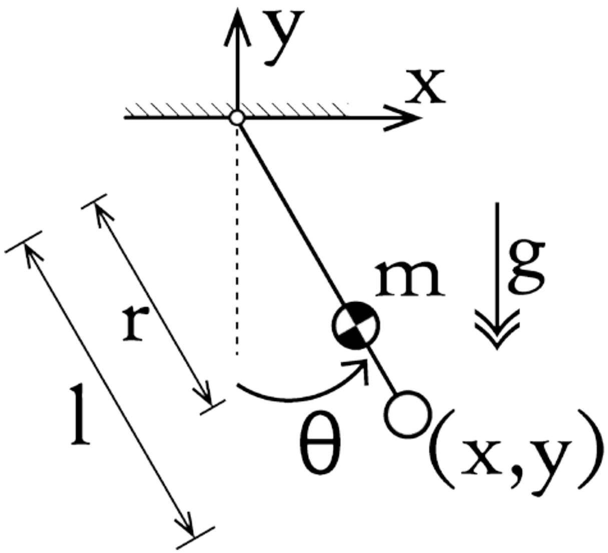
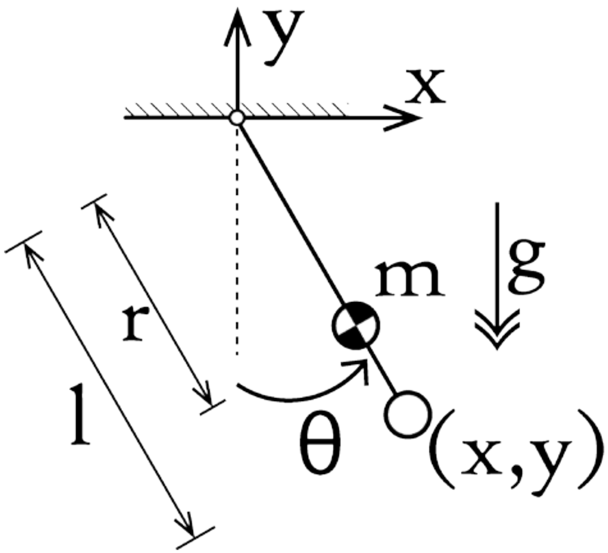


How do you account for $\boldsymbol{r}$?

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c_{\boldsymbol{\theta}} & -s_{\boldsymbol{\theta}} & \boldsymbol{T_x} \\ s_{\boldsymbol{\theta}} & c_{\boldsymbol{\theta}} & \boldsymbol{T_y} \\ & \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -\boldsymbol{l} \\ 1 \end{bmatrix}$$

$$\ddot{\boldsymbol{\theta}} = \frac{1}{\boldsymbol{mr}^2}\boldsymbol{\tau} - \frac{g}{\boldsymbol{r}} s_{\boldsymbol{\theta}}$$

SE: FREE CASE



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$$X_{k+1} = f(X_k) + w_k$$
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How do you account for r?

Add an additional state! $X_3 = \frac{1}{r}$

SE: FREE CASE

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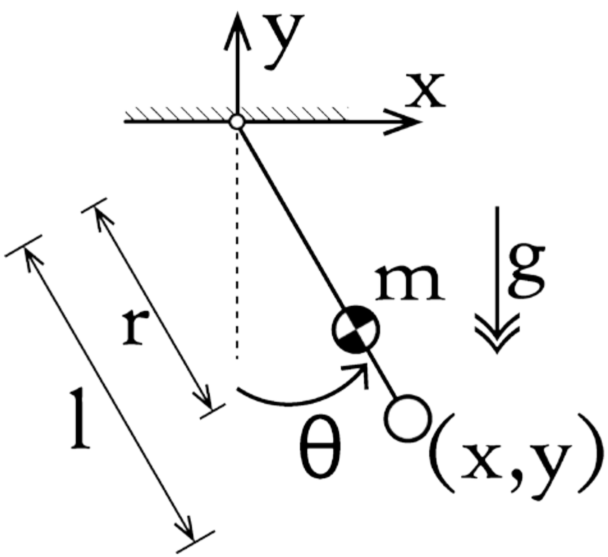

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Add an additional state! $X_3 = \frac{1}{r}$

$$\begin{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \\ X_3 \end{bmatrix}_{k+1} = \begin{bmatrix} \begin{bmatrix} 1 & \partial t \\ 0 & 1 \end{bmatrix} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \\ X_3 \end{bmatrix}_k + \begin{bmatrix} \begin{bmatrix} 0 \\ -\partial t \frac{g}{r} s_{\theta_k} \end{bmatrix} \\ 0 \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \\ 0 \end{bmatrix}_k$$

SE: Free Case

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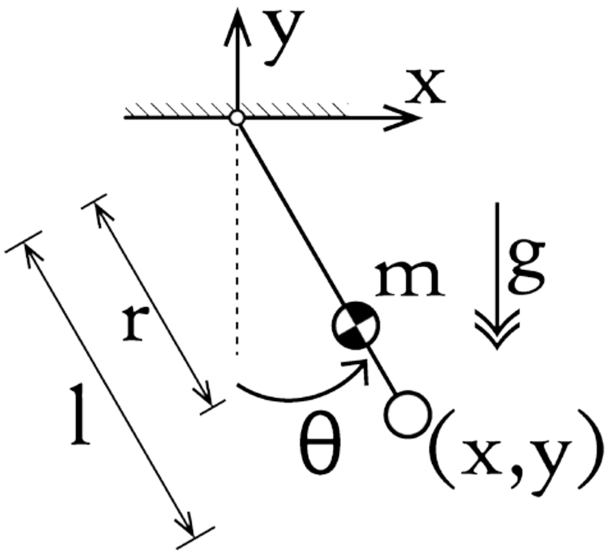

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How do you account for r?

Add an additional state! $X_3 = \frac{1}{r}$

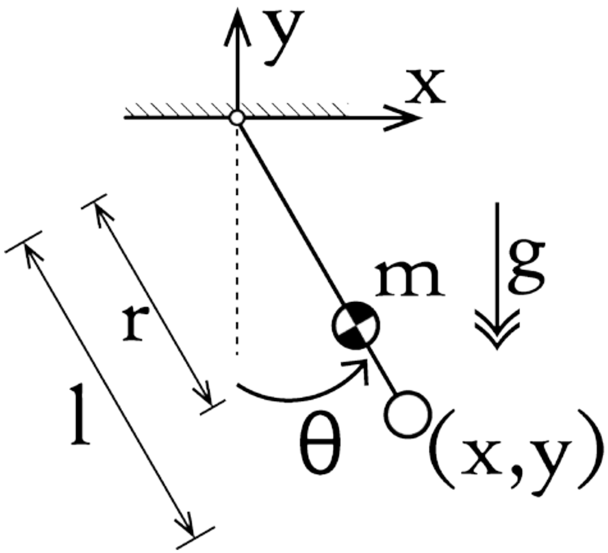
$$\begin{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \\ X_3 \end{bmatrix}_{k+1} = \begin{bmatrix} \begin{bmatrix} 1 & \partial t \\ 0 & 1 \end{bmatrix} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \\ X_3 \end{bmatrix}_k + \begin{bmatrix} \begin{bmatrix} 0 \\ -\partial t \frac{g}{r} s_{\theta_k} \end{bmatrix} \\ 0 \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \\ 0 \end{bmatrix}_k$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_k = \begin{bmatrix} c_{\theta_k} & -s_{\theta_k} & T_x \\ s_{\theta_k} & c_{\theta_k} & T_y \end{bmatrix} \begin{bmatrix} 0 \\ -l \\ 1 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_k$$

SE: FREE CASE

$$X_{k+1} = f(X_k) + w_k$$
$$Y_k = h(X_k) + v_k$$

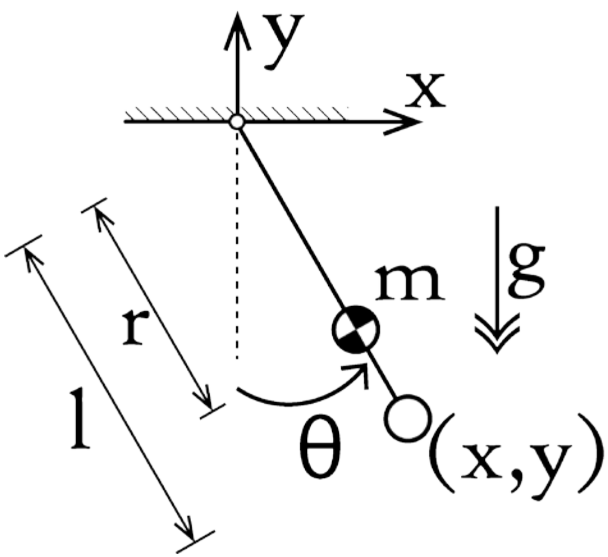
What are the noise terms w, v?



$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c_\theta & -s_\theta & T_x \\ s_\theta & c_\theta & T_y \\ & \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -l \\ 1 \end{bmatrix}$$

$$\ddot{\theta} = \frac{1}{mr^2}\tau - \frac{g}{r} s_\theta$$

SE: FREE CASE



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What are the noise terms w, v?

The noise is modelled as two covariance matrices:

Q Process noise covariance (w_k)

R Measurement noise covariance (v_k)

SE: FREE CASE

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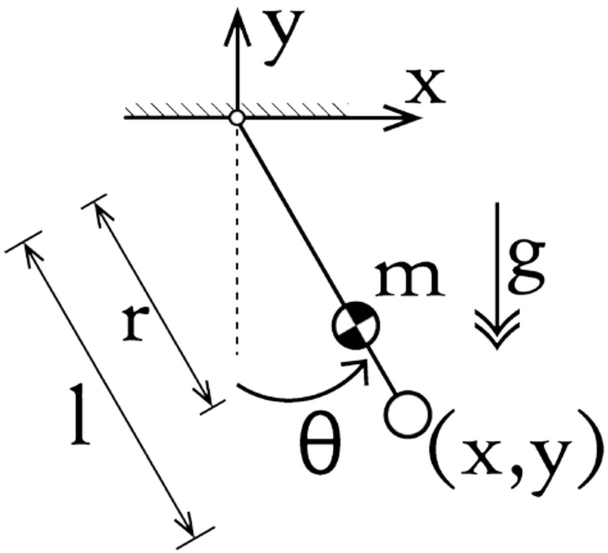

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c_\theta & -s_\theta & T_x \\ s_\theta & c_\theta & T_y \\ \multicolumn{2}{c}{\mathbf{0}} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -l \\ 1 \end{bmatrix}$$

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What are the noise terms w, v?

The noise is modelled as two covariance matrices:

Q Process noise covariance (w_k)

R Measurement noise covariance (v_k)

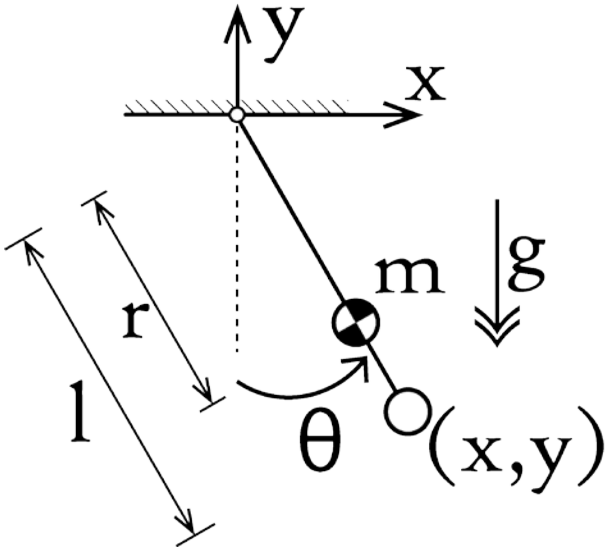
They essentially weight how much errors in observations to their predicted values effect the state.

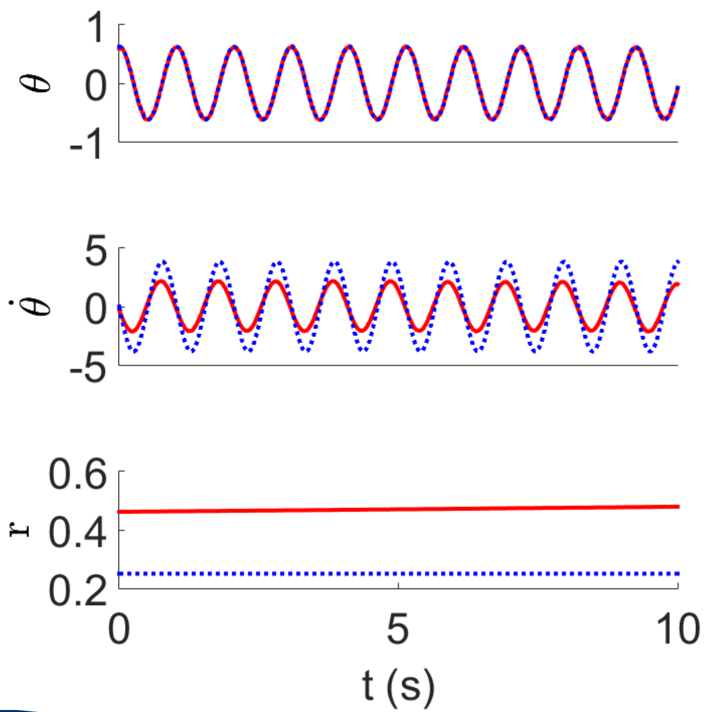
SE: FREE CASE

Q Process noise covariance (w_k)

R Measurement noise covariance (v_k)

$$Q = \begin{bmatrix} \sigma_w^2 & 0 & 0 \\ 0 & \sigma_w^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \sigma_w = 1e^{-1} \qquad P = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}, \sigma_v = 1e^{-3}$$





$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c_\theta & -s_\theta & T_x \\ s_\theta & c_\theta & T_y \\ & \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -l \\ 1 \end{bmatrix}$$

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SE: FREE CASE

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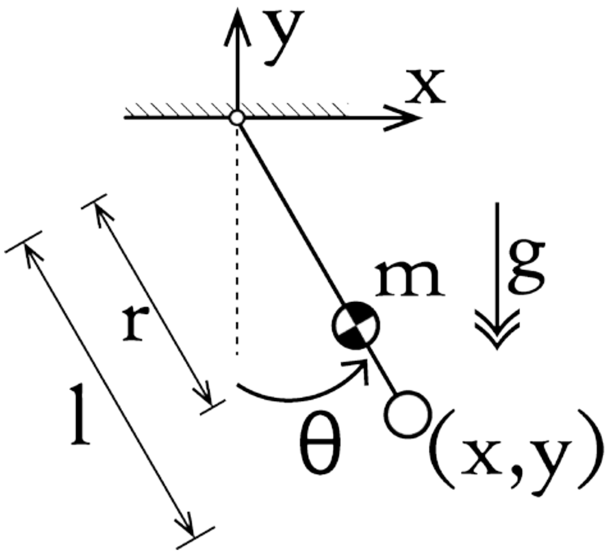

$$Q = \begin{bmatrix} \sigma_w^2 & 0 & 0 \\ 0 & \sigma_w^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \sigma_w = 1e^{-3} \quad P = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}, \sigma_v = 1e^{-1}$$

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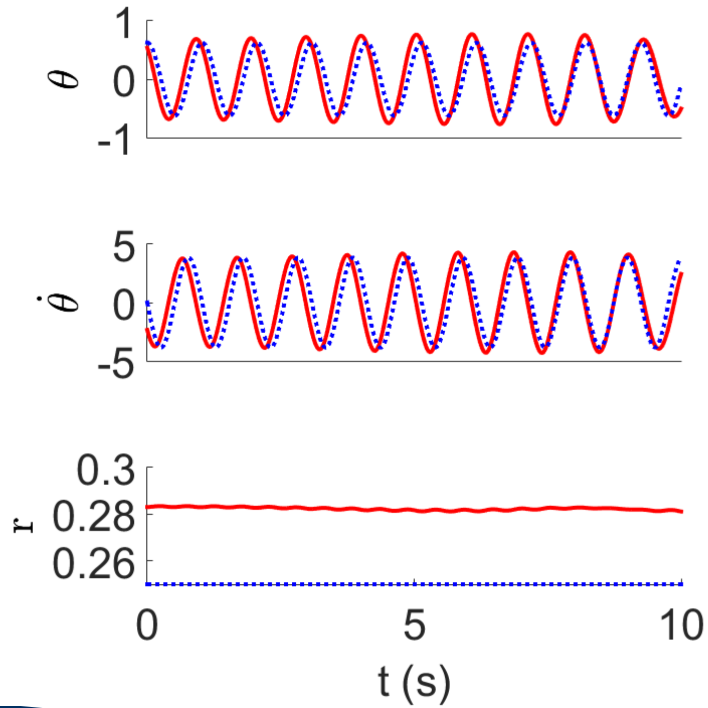

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SE: FREE CASE

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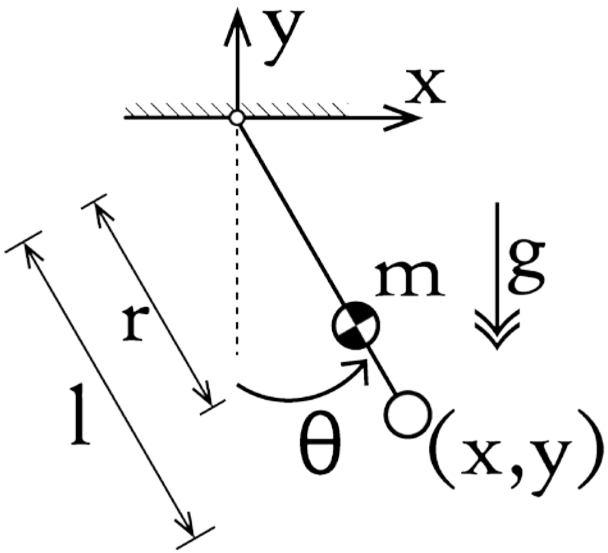

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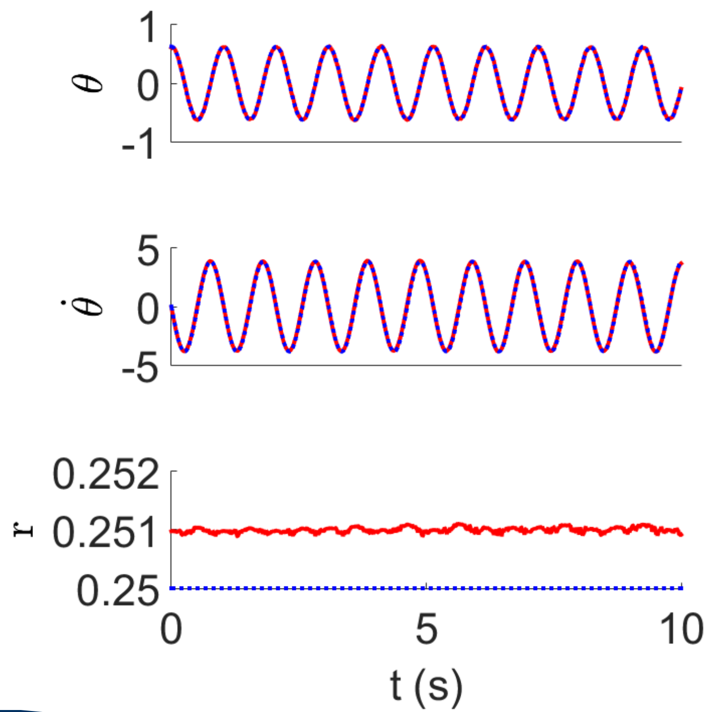

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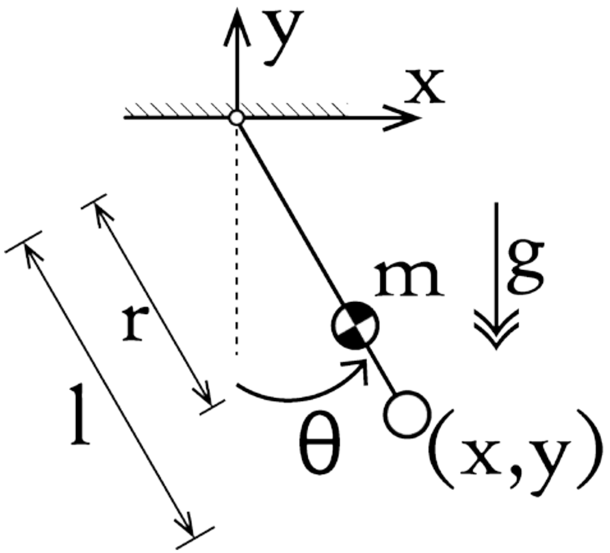
SE: FREE CASE



$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c_\theta & -s_\theta & T_x \\ s_\theta & c_\theta & T_y \\ & \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -l \\ 1 \end{bmatrix}$$

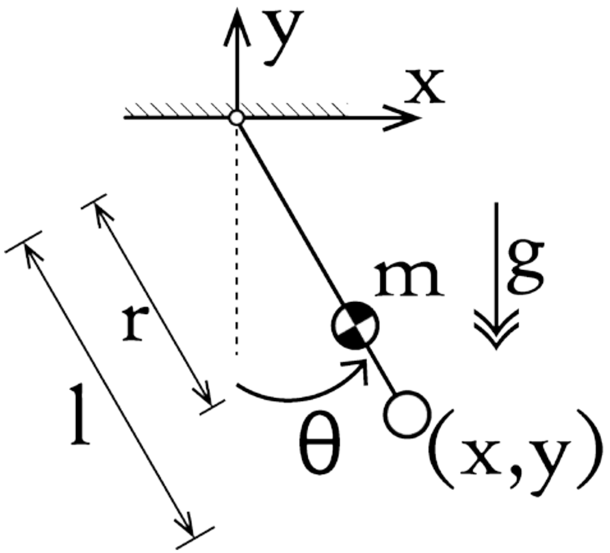
$$\ddot{\theta} = \frac{1}{mr^2}\tau - \frac{g}{r} s_\theta$$

State Estimation:

Given some **observable** data, estimate the **time varying states** of the system

Recovered states θ, $\dot{\theta}$ and length l .

SE: FREE CASE



$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c_\theta & -s_\theta & T_x \\ s_\theta & c_\theta & T_y \\ & \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -l \\ 1 \end{bmatrix}$$

$$\ddot{\theta} = \frac{1}{mr^2}\tau - \frac{g}{r}s_\theta$$

Parameter Identification:

Given some **observable** data, estimate the **fixed parameters** of the system

Recovered parameters T_x, T_y, and l.

State Estimation:

Given some **observable** data, estimate the **time varying states** of the system

Recovered states θ, $\dot{\theta}$ and radius $\mathbf{r}$.

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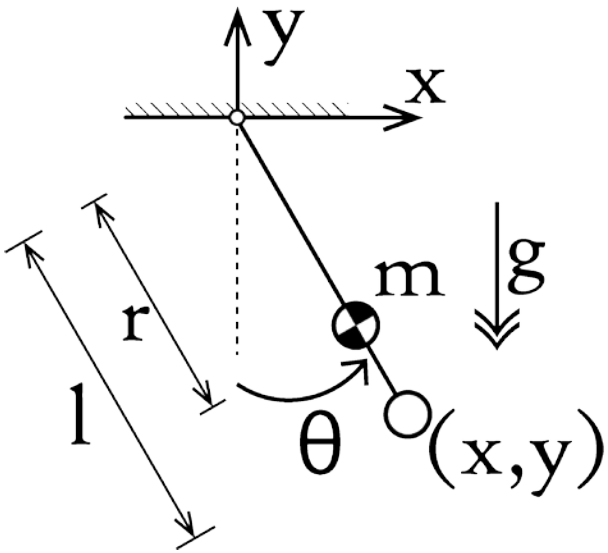
PROBLEM STATEMENT



$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c_{\theta} & -s_{\theta} & T_x \\ s_{\theta} & c_{\theta} & T_y \\ \multicolumn{2}{c}{\mathbf{0}} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -l \\ 1 \end{bmatrix}$$

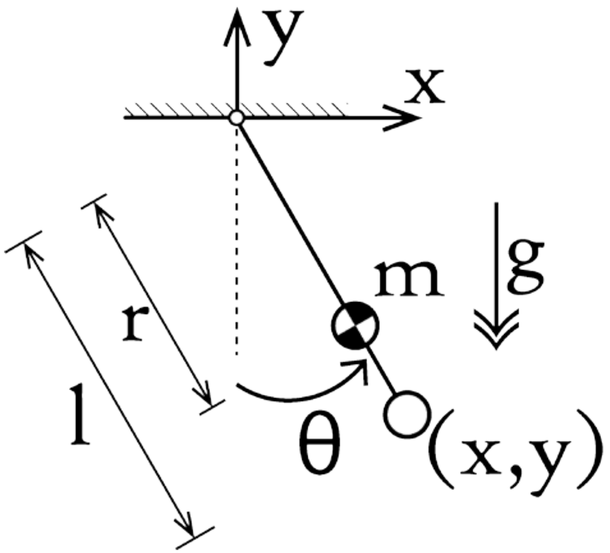
$$\ddot{\theta} = \frac{1}{mr^2}\tau - \frac{g}{r} s_{\theta}$$

Control:

Specify some joint torque τ that will achieve some desired trajectory

For this example, develop a controller to stabilise the system at some desired angle θ_d.

PROBLEM STATEMENT



$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c_\theta & -s_\theta & T_x \\ s_\theta & c_\theta & T_y \\ & \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -l \\ 1 \end{bmatrix}$$

$$\ddot{\theta} = \frac{1}{mr^2}\tau - \frac{g}{r} s_\theta$$

Solution method: **change system dynamics**

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{g}{r} s_\theta \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{mr^2} \end{bmatrix} \tau$$

Given some θ_d:

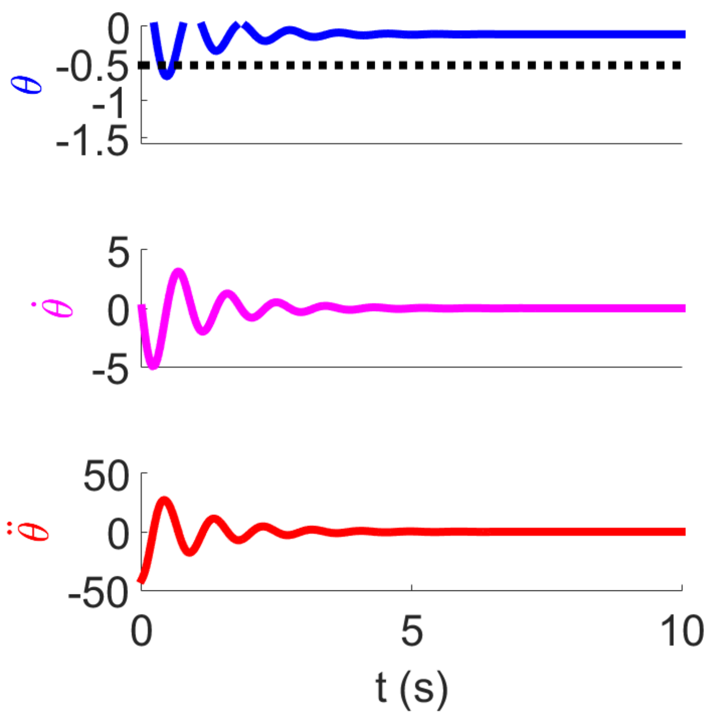
$$\tau = -mr^2(\alpha(\theta - \theta_d) + \beta\dot{\theta})$$

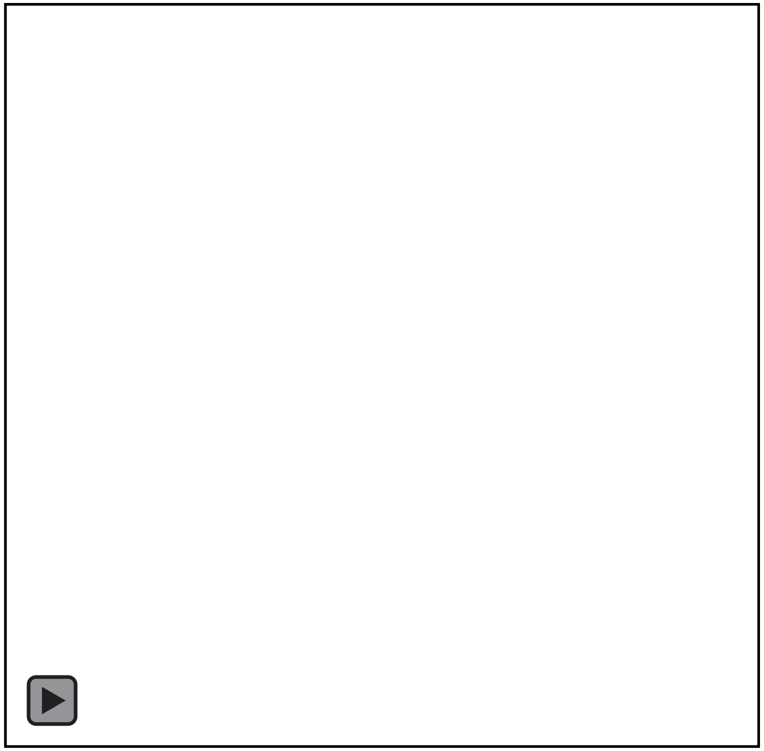
PROBLEM STATEMENT

Given some θ_d:

$\tau = -mr^2(\alpha(\theta - \theta_d) + \beta\dot{\theta})$

$\alpha = 10, \beta = 2$





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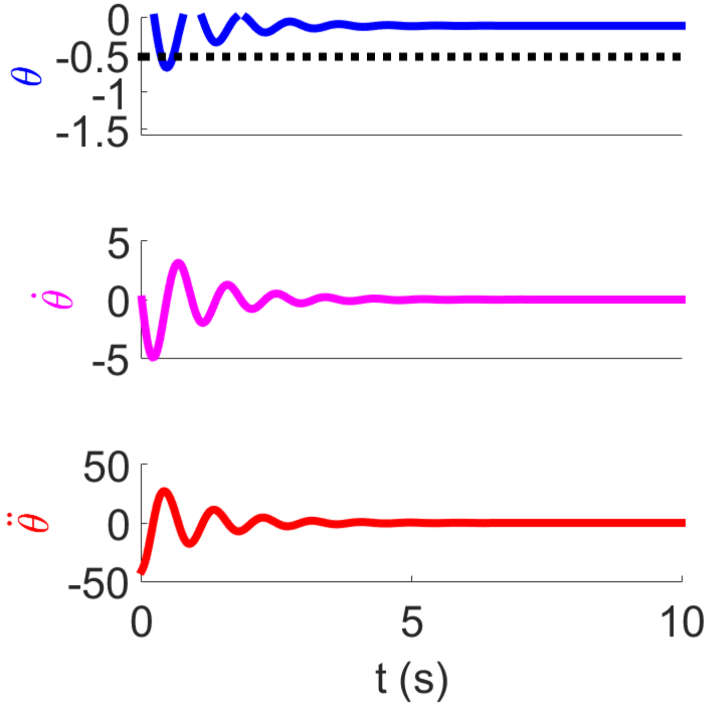
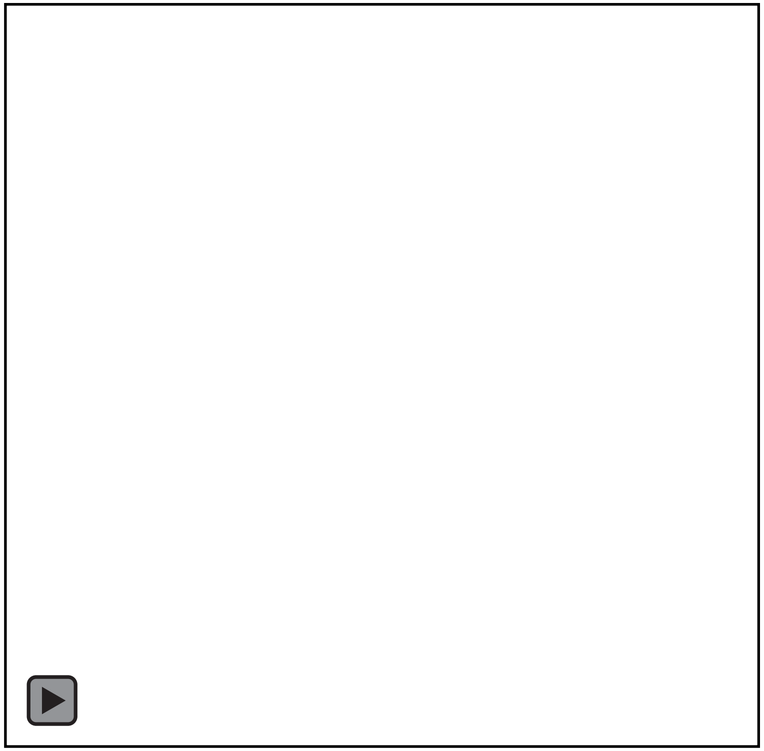




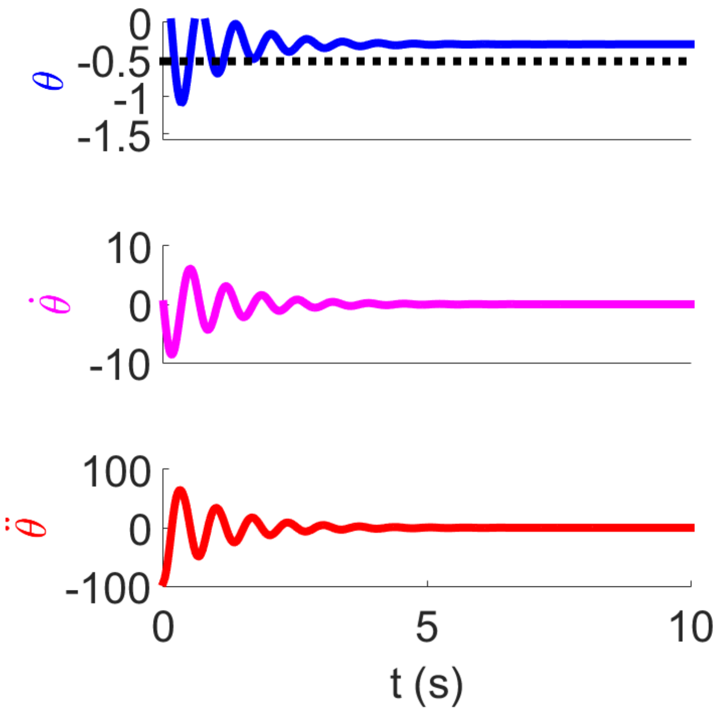
Insufficient gain α

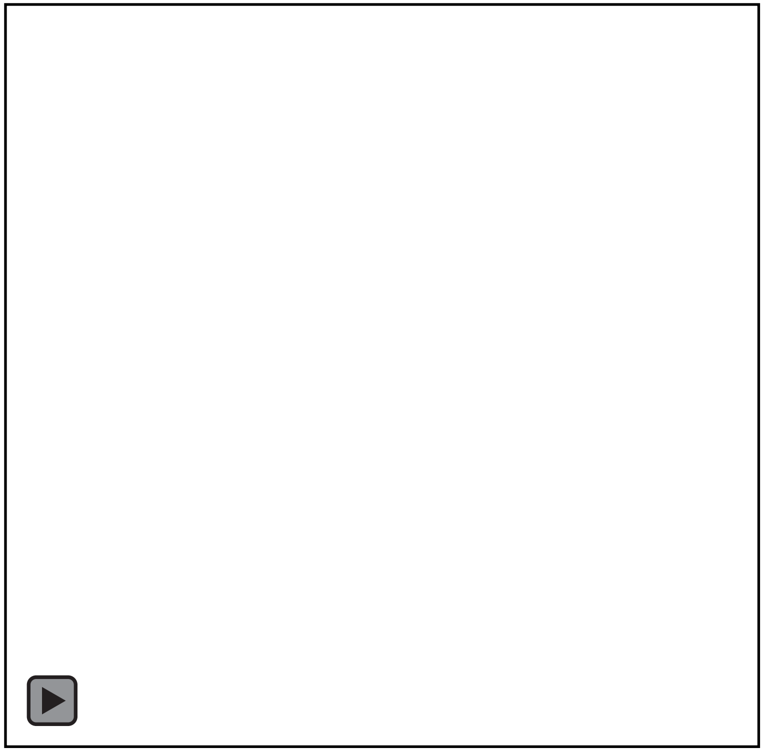
PROBLEM STATEMENT

Given some θ_d:

$$\boldsymbol{\tau} = -mr^2(\alpha(\theta - \theta_d) + \beta\dot{\theta})$$

$$\alpha = 50, \beta = 2$$





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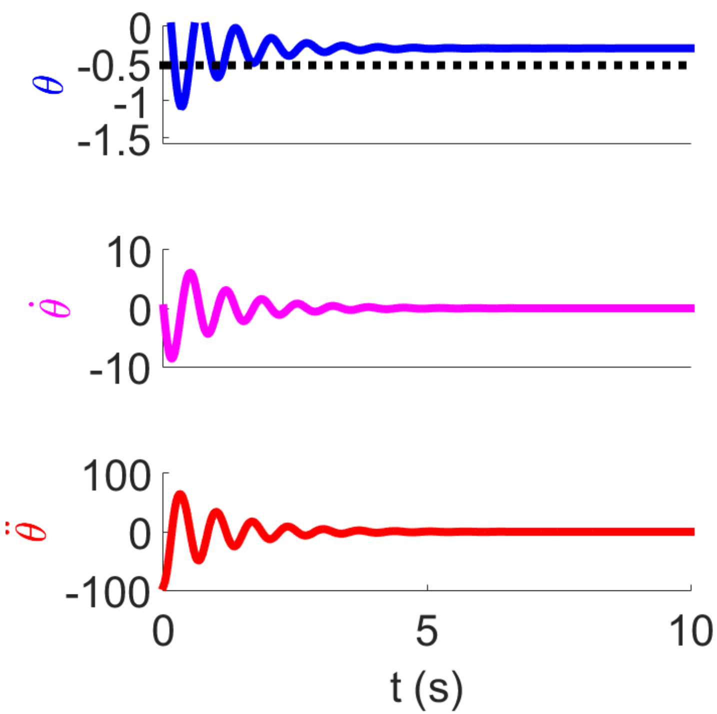
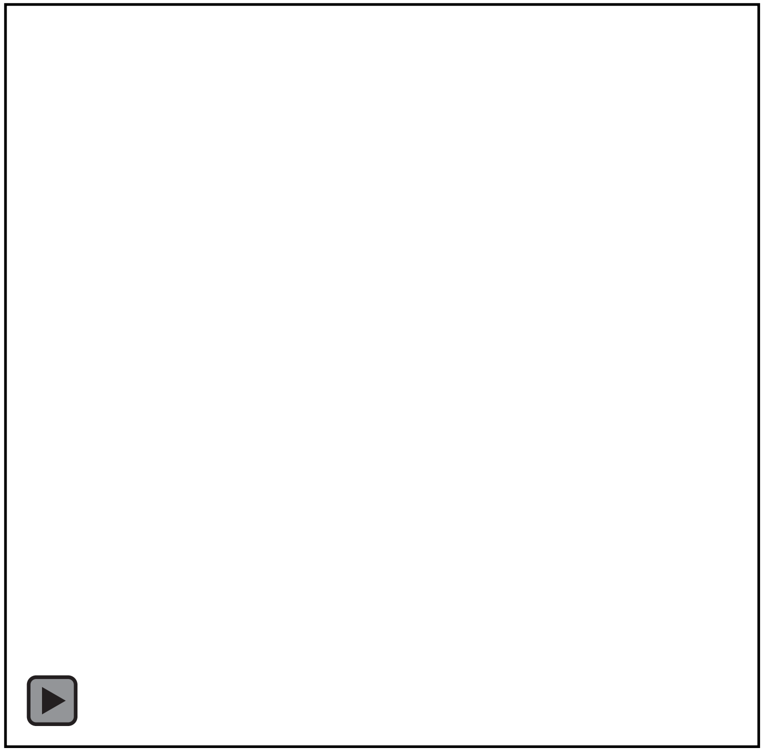




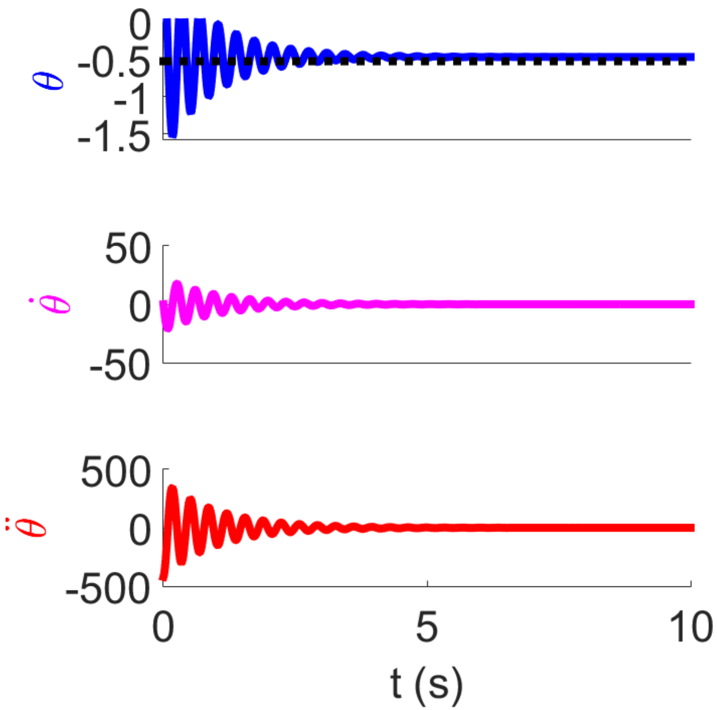
Insufficient gain α

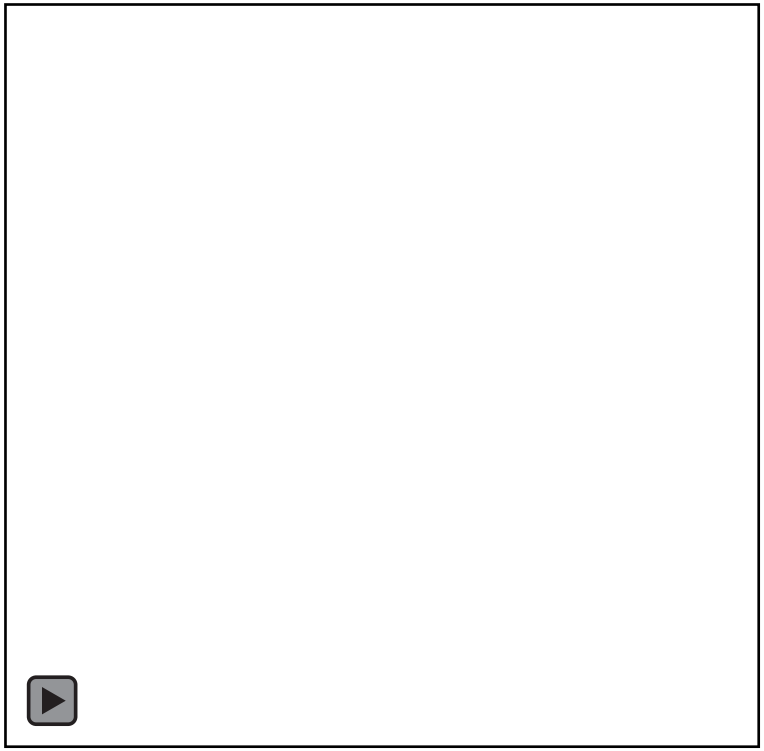
PROBLEM STATEMENT

Given some θ_d:

$\boldsymbol{\tau} = -mr^2(\alpha(\theta - \theta_d) + \beta\dot{\theta})$

$\alpha = 300, \beta = 2$





PROBLEM STATEMENT

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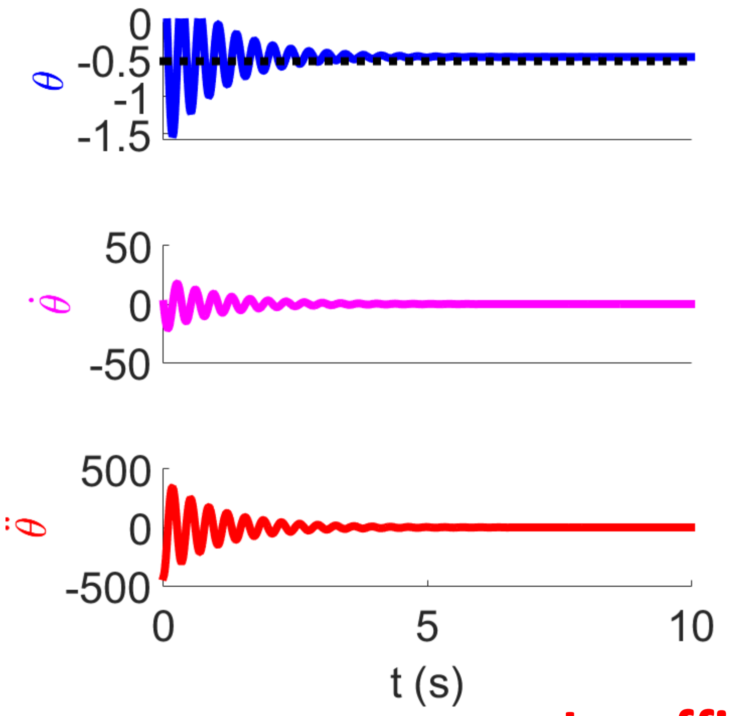

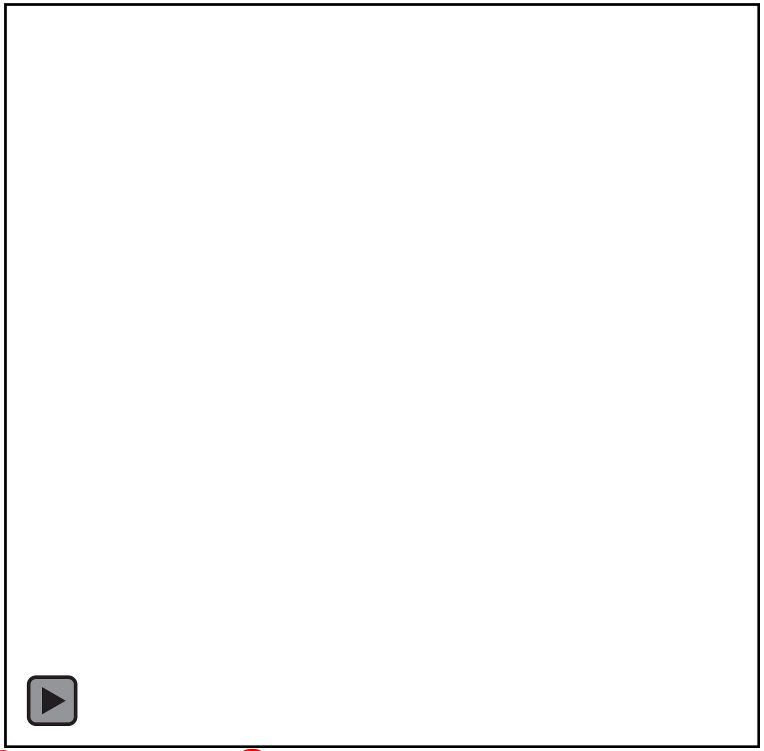

Insufficient damping β

PROBLEM STATEMENT

Given some θ_d:

$$\boldsymbol{\tau} = -mr^2(\alpha(\theta - \theta_d) + \beta\dot{\theta})$$

$$\alpha = 300, \beta = 5$$