VELOCITIES & JACOBIANS

INTRODUCTION TO ROBOTICS

EE106A/206A

SLIDES BY ROBERT PETER MATTHEW

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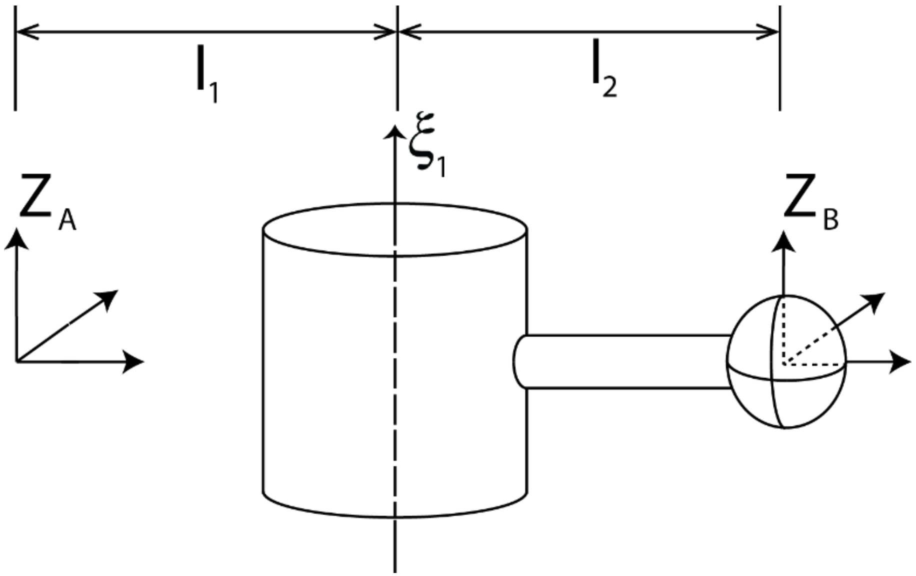
VELOCITIES

What are the velocities of the point ***q*** at the center of the sphere?



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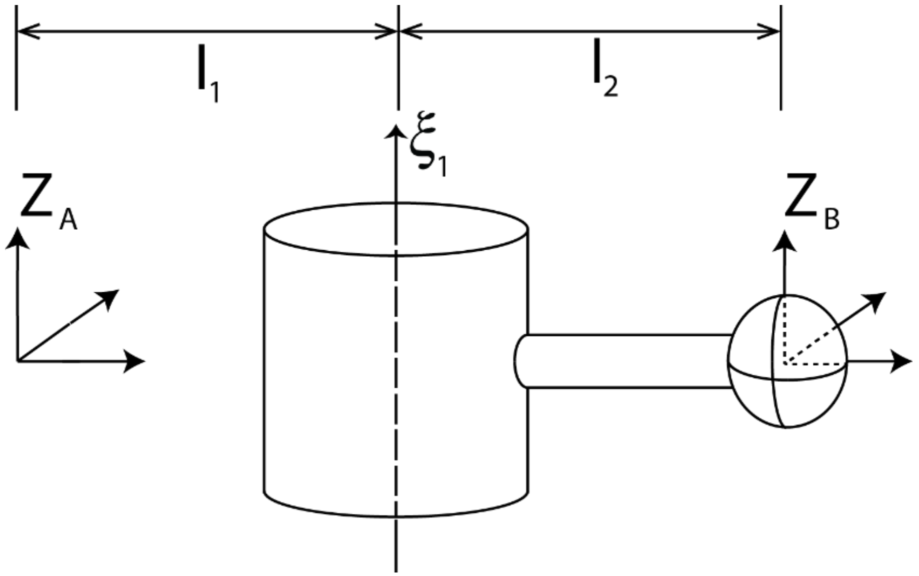

$$v_{q_A} = \begin{bmatrix} \begin{bmatrix} -l_2 s_1 \\ l_2 c_1 \\ 0 \end{bmatrix} \\ 0 \end{bmatrix} \dot{\theta}_1$$

$$v_{q_B} = \begin{bmatrix} \begin{bmatrix} 0 \\ l_2 \\ 0 \end{bmatrix} \\ 0 \end{bmatrix} \dot{\theta}_1$$

We need to define which frame of reference!

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VELOCITIES

Goal: Find two matrices ($\hat{V}_{AB}^{A}$, $\hat{V}_{AB}^{B}$) that will give the velocities of the points in either frame A or frame B:

$$v_{q_A} = \hat{V}_{AB}^{A} q_A \qquad\qquad v_{q_B} = \hat{V}_{AB}^{B} q_B$$

q_A, q_B: The coordinates of point q given in frames A and B
v_{q_A}, v_{q_B}: The velocities of point q given in frames A and B

DEFINITIONS OF $SO(3)$ VELOCITIES

Consider the point q with coordinates q_A, q_B as given in frames A and B. These coordinates are related by the rotation R_{AB}:

$$q_A = R_{AB} q_B$$

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DEFINITIONS OF $SO(3)$ VELOCITIES

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The velocity of point q_A is written as v_{q_A} and has the form:

$$v_{q_A} = \dot{q}_A = \dot{R}_{AB} q_B$$

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The velocity of point q_A is written as v_{q_A} and has the form:

$$v_{q_A} = \dot{q}_A = \dot{R}_{AB} q_B$$

This velocity can be given in relation to the coordinates in the A frame:

$$v_{q_A} = \dot{R}_{AB} \mathbb{I} q_B = \dot{R}_{AB} R_{AB}^{-1} R_{AB} q_B = \dot{R}_{AB} R_{AB}^{-1} q_A$$

DEFINITIONS OF $SO(3)$ VELOCITIES

We can similarly define the velocity with respect to the B frame as:

$$q_B = R_{AB}^{-1} q_A$$

$$v_{q_B} = R_{AB}^{-1} \dot{q}_A = R_{AB}^{-1} \dot{R}_{AB} q_B$$

Definitions of $SO(3)$ Velocities

We can similarly define the velocity with respect to the B frame as:

$$q_B = R_{AB}^{-1} q_A$$
$$v_{q_B} = R_{AB}^{-1} \dot{q}_A = R_{AB}^{-1} \dot{R}_{AB} q_B$$

This gives us two expressions for velocity:

$$v_{q_A} = \dot{R}_{AB} R_{AB}^{-1} q_A \qquad\qquad v_{q_B} = R_{AB}^{-1} \dot{R}_{AB} q_B$$

DEFINITIONS OF $SO(3)$ VELOCITIES

$$v_{q_A} = \dot{R}_{AB} R_{AB}^{-1} q_A \qquad v_{q_B} = R_{AB}^{-1} \dot{R}_{AB} q_B$$

DEFINITIONS OF $SO(3)$ VELOCITIES

$$v_{q_A} = \dot{R}_{AB} R_{AB}^{-1} q_A \qquad v_{q_B} = R_{AB}^{-1} \dot{R}_{AB} q_B$$

$\dot{R}_{AB} R_{AB}^{-1}$ operates in frame A: $\dot{R}_{AB} R_{BA}$

DEFINITIONS OF $SO(3)$ VELOCITIES

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$R_{AB}^{-1} \dot{R}_{AB}$ operates in frame B: $R_{BA} \dot{R}_{AB}$

Definitions of $SO(3)$ Velocities

$$v_{q_A} = \dot{R}_{AB} R_{AB}^{-1} q_A \qquad v_{q_B} = R_{AB}^{-1} \dot{R}_{AB} q_B$$

$\dot{R}_{AB} R_{AB}^{-1}$ operates in frame A: $\dot{R}_{AB} R_{BA}$

$R_{AB}^{-1} \dot{R}_{AB}$ operates in frame B: $R_{BA} \dot{R}_{AB}$

$\dot{R}_{AB} R_{AB}^{-1}$ and $R_{AB}^{-1} \dot{R}_{AB}$ are skew-symmetric (MLS lemma 2.12), allowing us to write them in terms of the hat operator:

$$v_{q_A} = \widehat{\omega}_{AB}^{A} q_A \qquad v_{q_B} = \widehat{\omega}_{AB}^{B} q_B$$

DEFINITIONS OF $SO(3)$ VELOCITIES

$$v_{q_A} = \hat{\omega}_{AB}^{A} q_A \qquad v_{q_B} = \hat{\omega}_{AB}^{B} q_B$$

We name these two velocities as the spatial and body velocities

Definitions of $SO(3)$ Velocities

$$v_{q_A} = \widehat{\omega}_{AB}^{A} q_A \qquad v_{q_B} = \widehat{\omega}_{AB}^{B} q_B$$

We name these two velocities as the spatial and body velocities

The spatial velocity s is referenced to the fixed frame A
The body velocity b is referenced to the rotating frame B

$$\widehat{\omega}_{AB}^{s} = \widehat{\omega}_{AB}^{A}$$
$$\widehat{\omega}_{AB}^{b} = \widehat{\omega}_{AB}^{B}$$

DEFINITIONS OF $SE(3)$ VELOCITIES

For pure rotations we have the relations:

$$v_{q_A} = \dot{R}_{AB} R_{AB}^{-1} q_A \qquad v_{q_B} = R_{AB}^{-1} \dot{R}_{AB} q_B$$

$$v_{q_A} = \widehat{\omega}_{AB}^{s} q_A \qquad v_{q_B} = \widehat{\omega}_{AB}^{b} q_B$$

DEFINITIONS OF $SE(3)$ VELOCITIES

For pure rotations we have the relations:

$$v_{q_A} = \dot{R}_{AB} R_{AB}^{-1} q_A \qquad v_{q_B} = R_{AB}^{-1} \dot{R}_{AB} q_B$$

$$v_{q_A} = \widehat{\omega}_{AB}^{s} q_A \qquad v_{q_B} = \widehat{\omega}_{AB}^{b} q_B$$

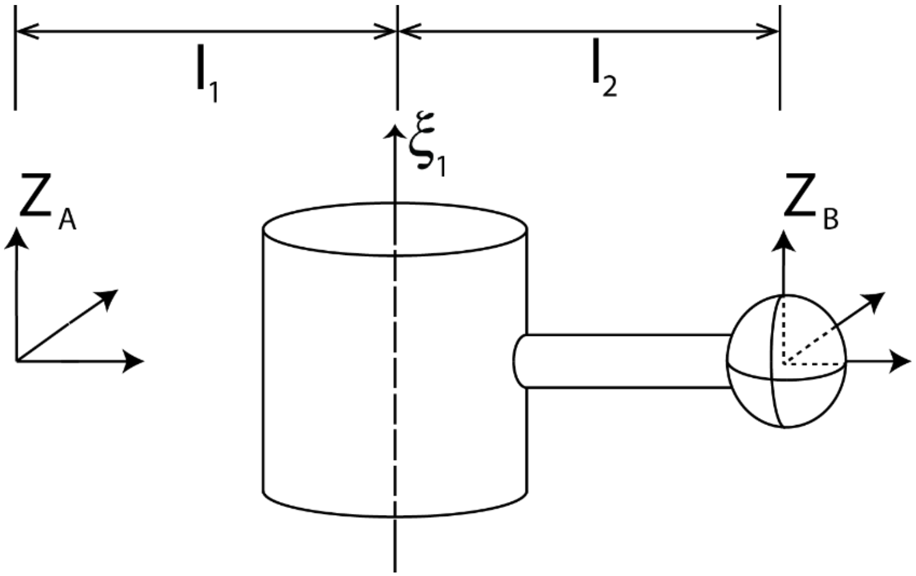
Through similar derivation, we can obtain the rigid body relations:

$$v_{q_A} = \dot{g}_{AB} g_{AB}^{-1} q_A \qquad v_{q_B} = g_{AB}^{-1} \dot{g}_{AB} q_B$$

$$v_{q_A} = \hat{V}_{AB}^{s} q_A \qquad v_{q_B} = \hat{V}_{AB}^{b} q_B$$

Example of $SE(3)$ Velocities

$$v_{q_A} = \hat{V}^s_{AB} q_A \qquad v_{q_B} = \hat{V}^b_{AB} q_B$$
$$v_{q_A} = \dot{g}_{AB} g^{-1}_{AB} q_A \qquad v_{q_B} = g^{-1}_{AB} \dot{g}_{AB} q_B$$



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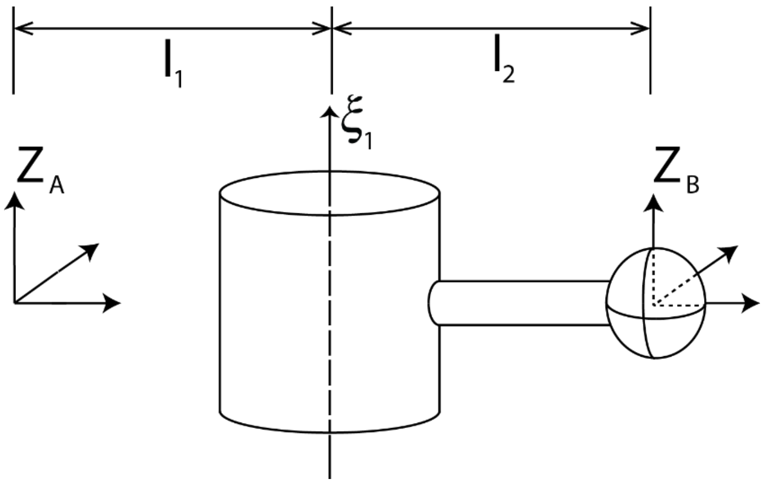
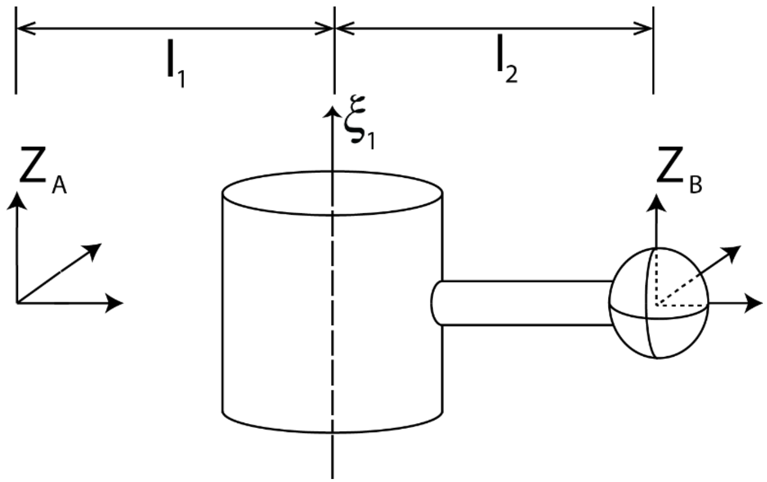
EXAMPLE OF $SE(3)$ VELOCITIES

$$g_{AB} = \begin{bmatrix} [R_z(\theta_1)] & \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix} \begin{bmatrix} [\mathbb{I}_3] & \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix}$$



Example of $SE(3)$ Velocities

$$
\begin{aligned}
g_{AB} &= \begin{bmatrix} [R_z(\theta_1)] & \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix} \begin{bmatrix} [\mathbb{I}_3] & \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix} \\
&= \begin{bmatrix} \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} l_1 + l_2 c_1 \\ l_2 s_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix}
\end{aligned}
$$



Example of $SE(3)$ Velocities

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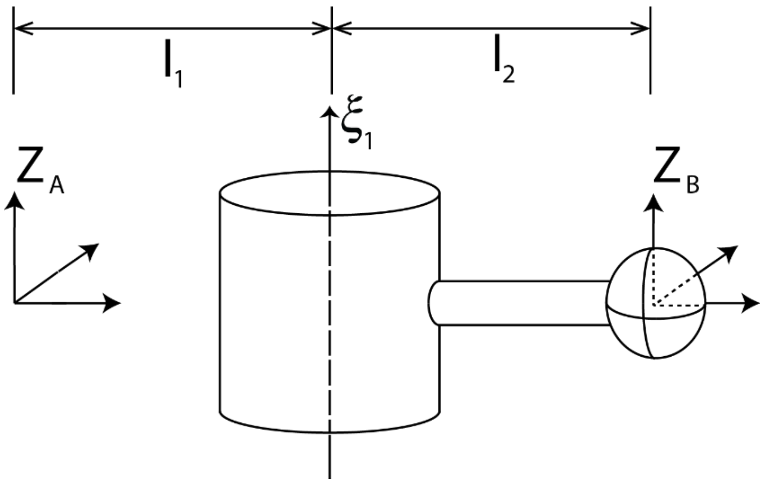
$$= \begin{bmatrix} \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} l_1 + l_2 c_1 \\ l_2 s_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix}$$

$$\dot{g}_{AB} = \begin{bmatrix} \begin{bmatrix} -s_1 & -c_1 & 0 \\ c_1 & -s_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} -l_2 s_1 \\ l_2 c_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 0 \end{bmatrix} \dot{\theta}_1$$



Example of $SE(3)$ Velocities

$$\dot{g}_{AB} g_{AB}^{-1} = \begin{bmatrix} \begin{bmatrix} -\dot{\theta}_1 s_1 & -\dot{\theta}_1 c_1 & 0 \\ \dot{\theta}_1 c_1 & -\dot{\theta}_1 s_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} -l_2 \dot{\theta}_1 s_1 \\ l_2 \dot{\theta}_1 c_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 0 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} -l_1 c_1 - l_2 \\ l_1 s_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ -l_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 0 \end{bmatrix} \dot{\theta}_1$$

EXAMPLE OF $SE(3)$ VELOCITIES

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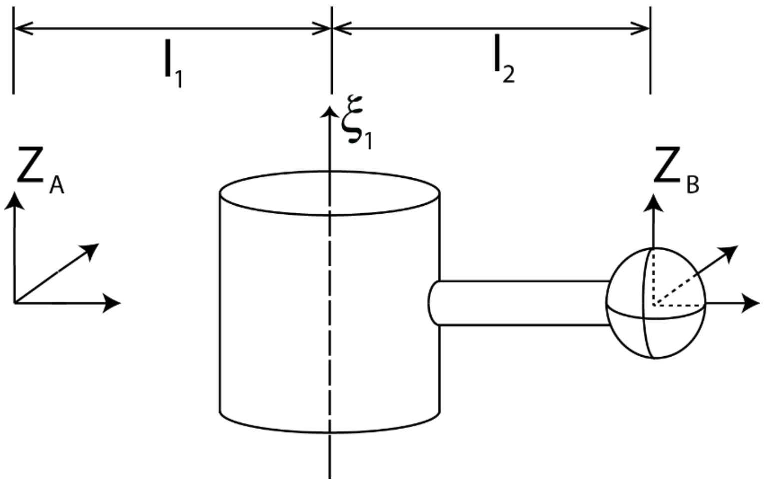
$$= \begin{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ -l_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 0 \end{bmatrix} \dot{\theta}_1$$

$$V_{AB}^{s} = (\dot{g}_{AB} g_{AB}^{-1})^{\vee}$$
$$= [[0 \quad -l_1 \quad 0] \quad [0 \quad 0 \quad 1]]^T \dot{\theta}_1$$

EXAMPLE OF $SE(3)$ VELOCITIES

$$
\begin{aligned}
v_{q_A} &= \dot{g}_{AB} g_{AB}^{-1} q_A \\
&= \begin{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ -l_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 0 \end{bmatrix} \dot{\theta}_1 \begin{bmatrix} \begin{bmatrix} l_1 + l_2 c_1 \\ l_2 s_1 \\ 0 \end{bmatrix} \\ 1 \end{bmatrix}
\end{aligned}
$$



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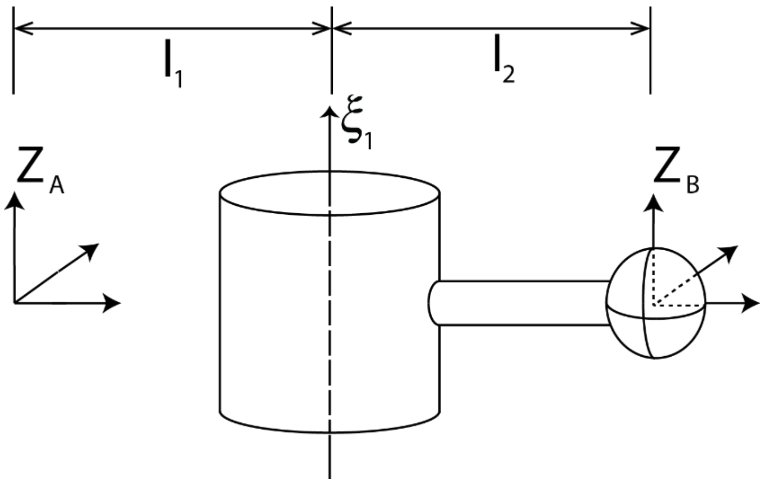
&= \begin{bmatrix} \begin{bmatrix} -l_2 s_1 \\ l_2 c_1 \\ 0 \end{bmatrix} \\ 0 \end{bmatrix} \dot{\theta}_1
\end{aligned}
$$



Example of $SE(3)$ Velocities

$$g_{AB}^{-1}\dot{g}_{AB} = \begin{bmatrix} \begin{bmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} -l_1 c_1 - l_2 \\ l_1 s_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} -\dot{\theta}_1 s_1 & -\dot{\theta}_1 c_1 & 0 \\ \dot{\theta}_1 c_1 & -\dot{\theta}_1 s_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} -l_2 \dot{\theta}_1 s_1 \\ l_2 \dot{\theta}_1 c_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ l_2 \\ 0 \end{bmatrix} \\ \mathbb{0} & 0 \end{bmatrix} \dot{\theta}_1$$

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Example of $SE(3)$ Velocities

$$g_{AB}^{-1}\dot{g}_{AB} = \begin{bmatrix} \begin{bmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} -l_1 c_1 - l_2 \\ l_1 s_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} -\dot{\theta}_1 s_1 & -\dot{\theta}_1 c_1 & 0 \\ \dot{\theta}_1 c_1 & -\dot{\theta}_1 s_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} -l_2 \dot{\theta}_1 s_1 \\ l_2 \dot{\theta}_1 c_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 0 \end{bmatrix}$$

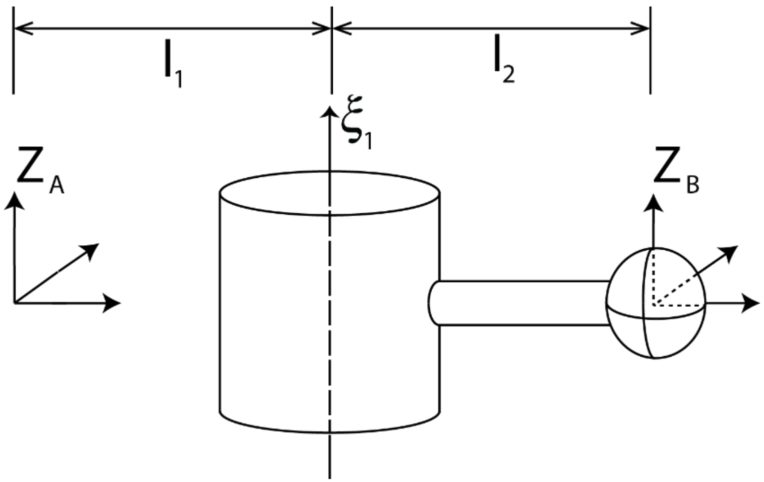
$$= \begin{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ l_2 \\ 0 \end{bmatrix} \\ \mathbb{0} & 0 \end{bmatrix} \dot{\theta}_1$$

$$V_{AB}^{b} = (g_{AB}^{-1}\dot{g}_{AB})^{\vee}$$

$$= [[0 \quad l_2 \quad 0] \quad [0 \quad 0 \quad 1]]^T \, \dot{\theta}_1$$

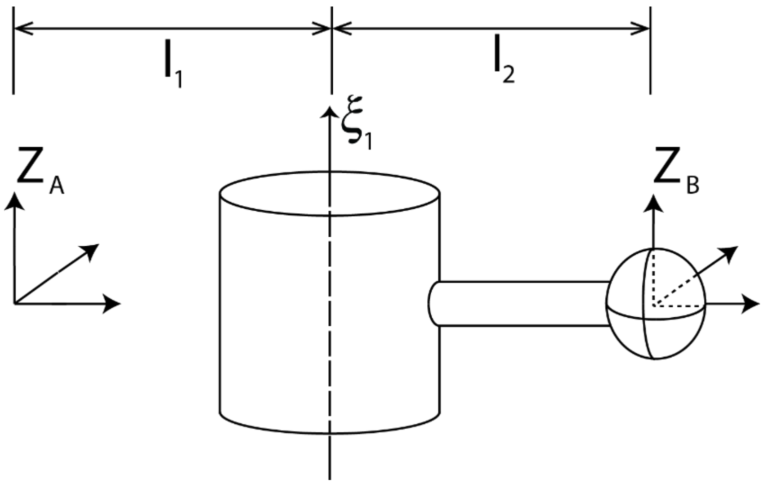
Example of $SE(3)$ Velocities

$$
\begin{aligned}
v_{q_B} &= g_{AB}^{-1}\dot{g}_{AB}q_B \\
&= \begin{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ l_2 \\ 0 \end{bmatrix} \\ \mathbb{0} & 0 \end{bmatrix} \dot{\theta}_1 \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\ 1 \end{bmatrix}
\end{aligned}
$$

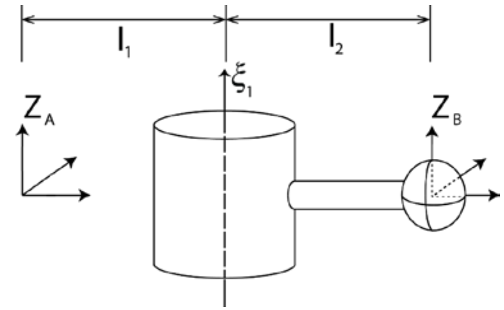


EXAMPLE OF $SE(3)$ VELOCITIES

$$
\begin{aligned}
v_{q_B} &= g_{AB}^{-1}\dot{g}_{AB}q_B \\
&= \begin{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ l_2 \\ 0 \end{bmatrix} \\ \mathbb{0} & 0 \end{bmatrix} \dot{\theta}_1 \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} \begin{bmatrix} 0 \\ l_2 \\ 0 \end{bmatrix} \\ 0 \end{bmatrix} \dot{\theta}_1
\end{aligned}
$$



Example of $SE(3)$ Velocities



$$V_{AB}^{s} = [[0 \quad -l_1 \quad 0] \quad [0 \quad 0 \quad 1]]^T \dot{\theta}_1$$
$$V_{AB}^{b} = [[0 \quad l_2 \quad 0] \quad [0 \quad 0 \quad 1]]^T \dot{\theta}_1$$

Example of $SE(3)$ Velocities

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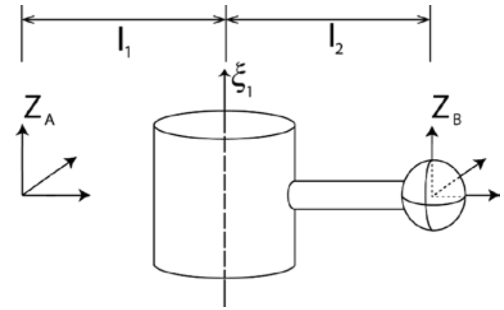
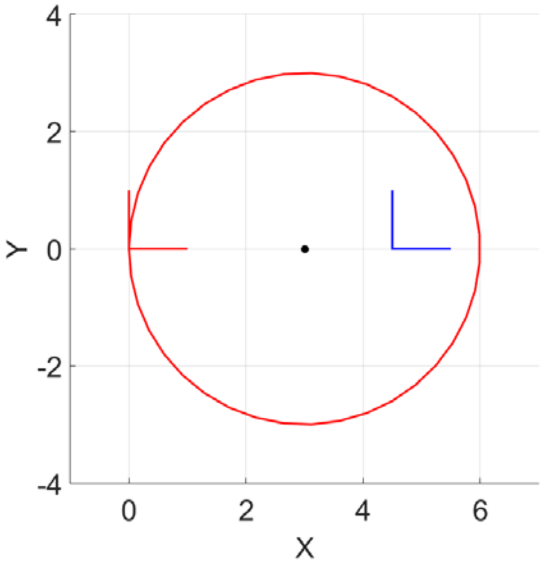


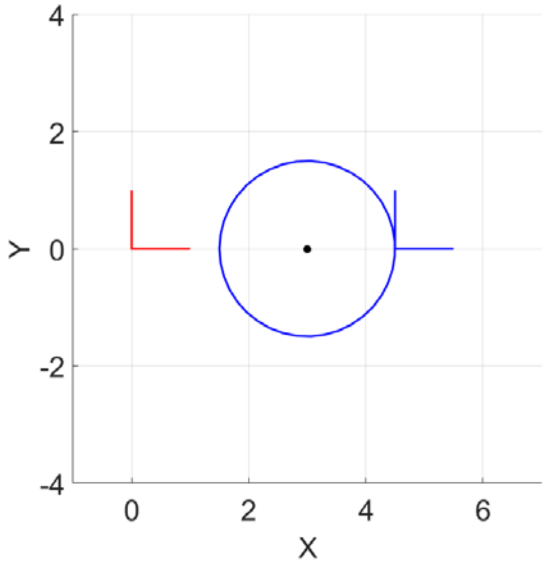
V_{AB}^{s}: A circle centered at the joint, passing through the origin

V_{AB}^{b}: A circle centered at the joint, passing through the end effector point

Spatial Velocities

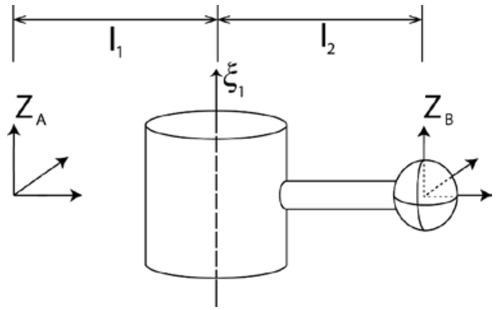


Body Velocities



Example of $SE(3)$ Velocities

$$V_{AB}^{s} = [[0 \quad -l_1 \quad 0] \quad [0 \quad 0 \quad 1]]^T \dot{\theta}_1$$
$$V_{AB}^{b} = [[0 \quad l_2 \quad 0] \quad [0 \quad 0 \quad 1]]^T \dot{\theta}_1$$



Spatial Velocities

Body Velocities

Example of $SE(3)$ Velocities

$$V_{AB}^{s} = [[0 \quad -l_1 \quad 0] \quad [0 \quad 0 \quad 1]]^T \dot{\theta}_1$$
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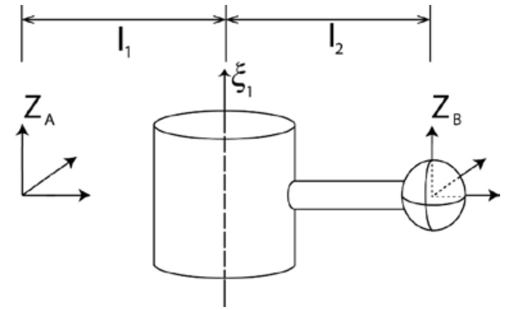

Spatial Velocities　　　**Body** Velocities

ADJOINT

The spatial and body velocities are related via the *Adjoint* relation:

$$V_{AB}^{s} = Ad_{g_{AB}} V_{AB}^{b}$$

where:

$$Ad_{g_{AB}} = \begin{bmatrix} R_{g_{AB}} & \hat{p}_{g_{AB}} R_{g_{AB}} \\ 0 & R_{g_{AB}} \end{bmatrix}$$

(proof part of HW4)

Adjoint: Example

$$Ad_{g_{AB}} = \begin{bmatrix} R_{g_{AB}} & \hat{p}_{g_{AB}} R_{g_{AB}} \\ 0 & R_{g_{AB}} \end{bmatrix}$$

$$g_{AB} = \begin{bmatrix} \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} l_1 + l_2 c_1 \\ l_2 s_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix}$$

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Adjoint: Example

$$Ad_{g_{AB}} = \begin{bmatrix} R_{g_{AB}} & \hat{p}_{g_{AB}} R_{g_{AB}} \\ 0 & R_{g_{AB}} \end{bmatrix} \qquad g_{AB} = \begin{bmatrix} \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} l_1 + l_2 c_1 \\ l_2 s_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix}$$

$$\mathrm{Ad}_{\mathrm{g}_{AB}} = \begin{bmatrix} \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 & l_2 s_1 \\ 0 & 0 & -l_1 - l_2 c_1 \\ -l_2 s_1 & l_1 + l_2 c_1 & 0 \end{bmatrix} \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \mathbb{0} & \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{bmatrix}$$

Adjoint: Example

$$Ad_{g_{AB}} = \begin{bmatrix} R_{g_{AB}} & \hat{p}_{g_{AB}} R_{g_{AB}} \\ 0 & R_{g_{AB}} \end{bmatrix} \qquad g_{AB} = \begin{bmatrix} \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} l_1 + l_2 c_1 \\ l_2 s_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix}$$

$$Ad_{g_{AB}} = \begin{bmatrix} \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 & l_2 s_1 \\ 0 & 0 & -l_1 - l_2 c_1 \\ -l_2 s_1 & l_1 + l_2 c_1 & 0 \end{bmatrix} \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \mathbb{0} & \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{bmatrix}$$

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Adjoint: Example

$$V_{AB}^{s} = Ad_{g_{AB}} V_{AB}^{b}$$

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$$= \begin{bmatrix} 0 \\ -l_1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \dot{\theta}_1 = V_{AB}^{s}$$

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The Adjoint allows for the velocities of multi-joint systems to be calculated:

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$$\xi_i^{\dagger} = Ad_{\left(e^{\hat{\xi}_i\theta_i} \dots e^{\hat{\xi}_n\theta_n} g_{WT}(\mathbf{0})\right)^{-1}} \xi_i$$

ADJOINT

For every joint in the system, the representation of the twist can be found in the spatial and body frames:

$$\xi_i' = Ad_{e^{\hat{\xi}_1\theta_1}\dots e^{\hat{\xi}_{i-1}\theta_{i-1}}}\xi_i \qquad \xi_i^{\dagger} = Ad_{\left(e^{\hat{\xi}_i\theta_i}\dots e^{\hat{\xi}_n\theta_n}g_{WT}(\mathbf{0})\right)^{-1}}\xi_i$$

These can be stacked together to give a matrix form for the velocities of a manipulator:

$$J_{WT}^{s}(\boldsymbol{\theta}) = [\xi_1 \quad \xi_2' \quad \dots \quad \xi_n'] \qquad J_{WT}^{b}(\boldsymbol{\theta}) = [\xi_1^{\dagger} \quad \xi_2^{\dagger} \quad \dots \quad \xi_n^{\dagger}]$$

These matrices are called the spatial and body Jacobians

Jacobians

The Jacobians give a relation between the joints and the end effector:

$$J_{WT}^{s}(\boldsymbol{\theta}) = [\xi_1 \quad \xi_2' \quad \dots \quad \xi_n']$$

$$V_{WT}^{s}(\boldsymbol{\theta}) = J_{WT}^{s}(\boldsymbol{\theta}) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}$$

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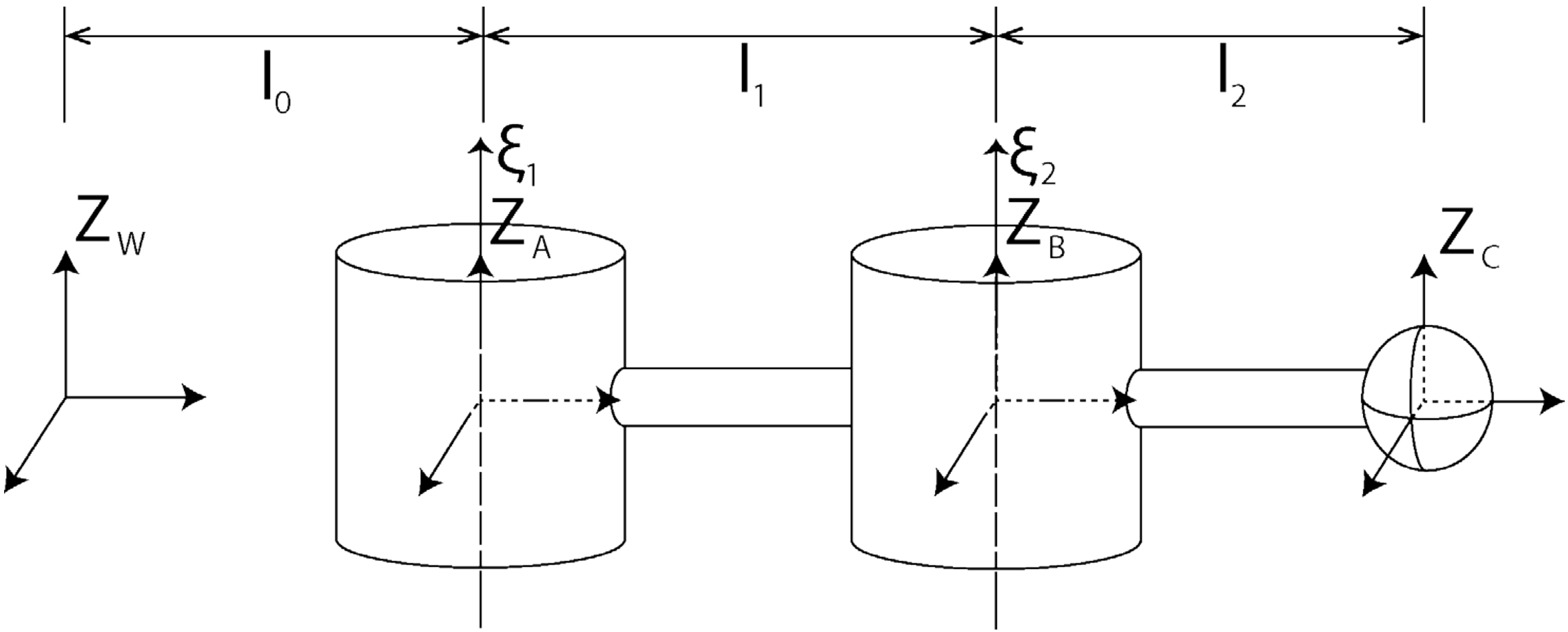
where:

$$J_{WT}^{s}(\boldsymbol{\theta}) = Ad_{g_{WT}(\boldsymbol{\theta})} J_{WT}^{b}(\boldsymbol{\theta})$$

JACOBIANS: EXAMPLE

Consider the multi-joint manipulator:



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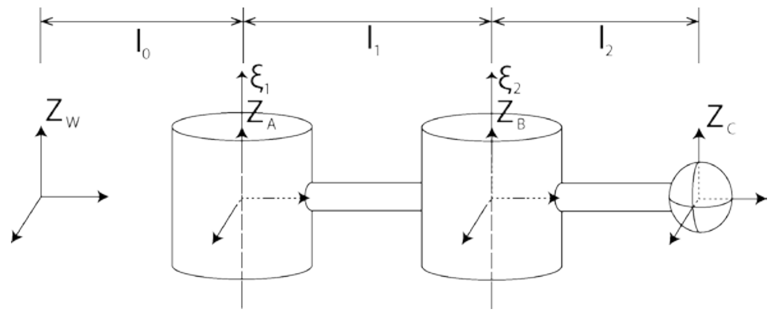

$$g_{WA}(\mathbf{0}) = \begin{bmatrix} [\mathbb{I}_3] & \begin{bmatrix} 0 \\ l_0 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix}$$

$$g_{WB}(\mathbf{0}) = \begin{bmatrix} [\mathbb{I}_3] & \begin{bmatrix} 0 \\ l_0 + l_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix}$$

$$g_{WC}(\mathbf{0}) = \begin{bmatrix} [\mathbb{I}_3] & \begin{bmatrix} 0 \\ l_0 + l_1 + l_2 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix}$$

$$\xi_1 = \begin{bmatrix} -\widehat{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} \begin{bmatrix} 0 \\ l_0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

$$\xi_2 = \begin{bmatrix} -\widehat{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} \begin{bmatrix} 0 \\ l_0 + l_1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

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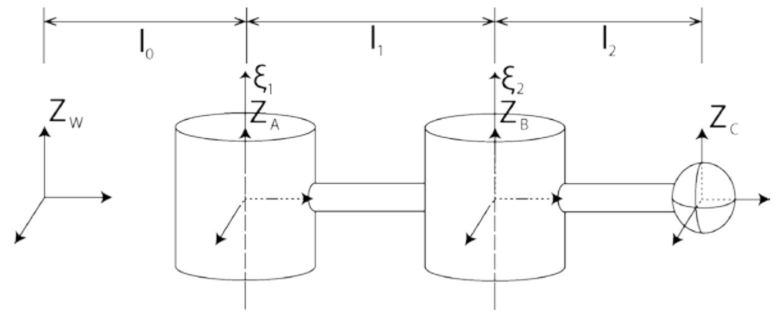

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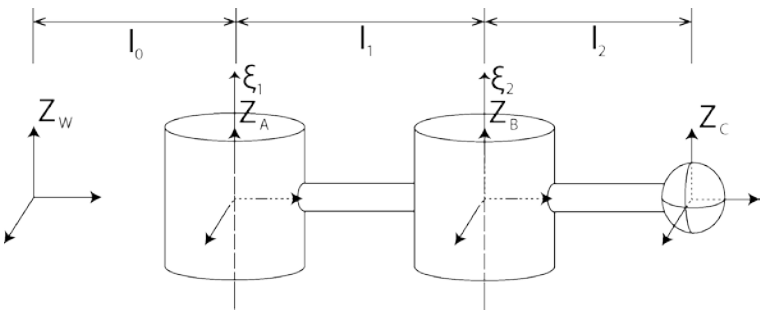

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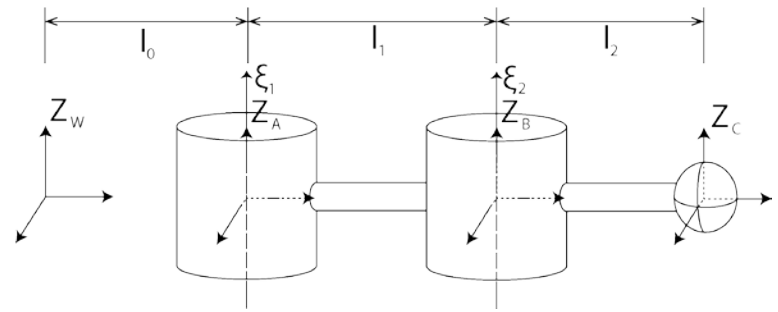

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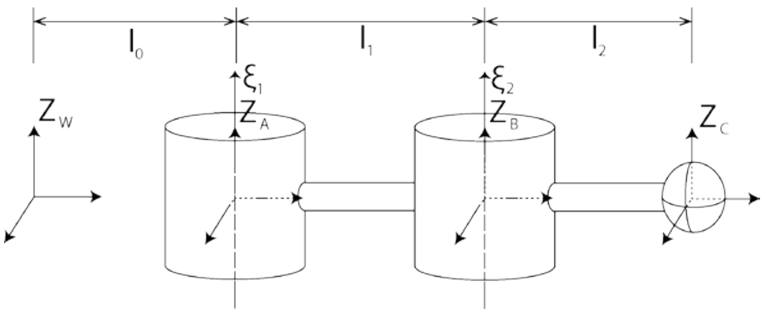

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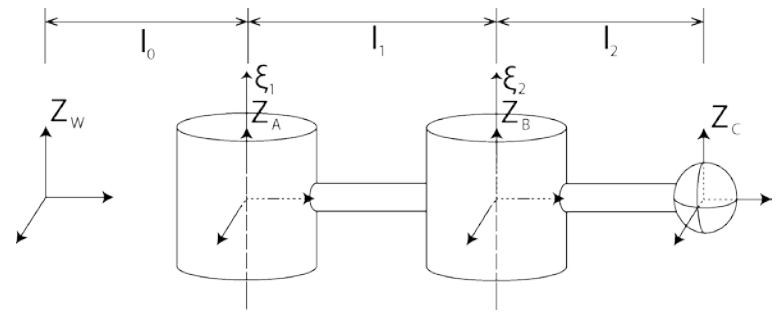

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$$\xi_2' = Ad_{e^{\hat{\xi}_1\theta_1}}\xi_2$$

$$\xi_2' = \begin{bmatrix} \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 & -l_0(c_1-1) \\ 0 & 0 & -l_0 s_1 \\ -l_0(c_1-1) & l_0 s_1 & 0 \end{bmatrix} \\ \mathbb{0} & \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} l_0 + l_1 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

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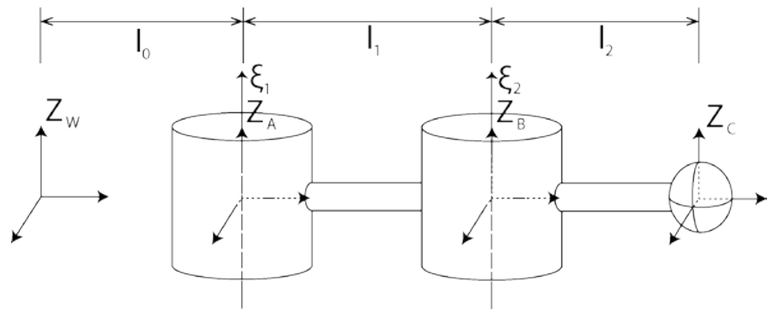

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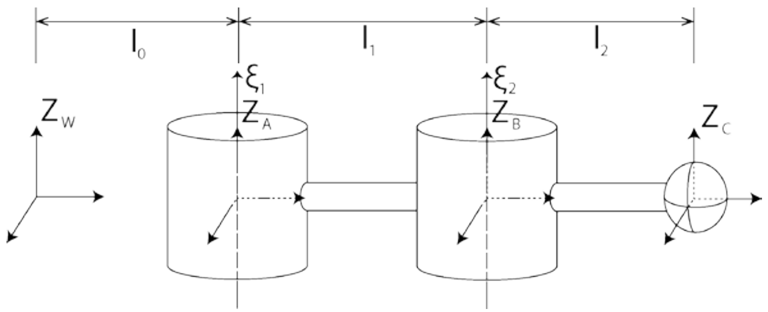
$$\xi_2' = \begin{bmatrix} \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 & -l_0(c_1-1) \\ 0 & 0 & -l_0 s_1 \\ -l_0(c_1-1) & l_0 s_1 & 0 \end{bmatrix} \\ \mathbb{0} & \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} l_0 + l_1 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

$$\xi_1' = [[l_0 \quad 0 \quad 0] \quad [0 \quad 0 \quad 1]]^T$$

$$\xi_2' = [[l_0 + l_1 c_1 \quad l_1 s_1 \quad 0] \quad [0 \quad 0 \quad 1]]^T$$

JACOBIANS: EXAMPLE

Consider the multi-joint manipulator:

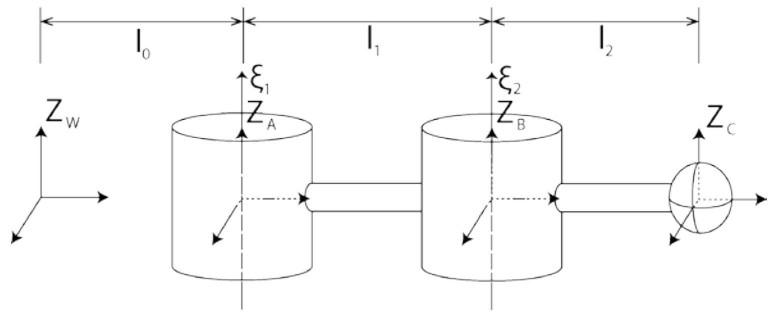


$$J_{WT}^{b}(\boldsymbol{\theta}) = \begin{bmatrix} \xi_1^{\dagger} & \xi_2^{\dagger} & \dots & \xi_n^{\dagger} \end{bmatrix}$$

$$\xi_i^{\dagger} = Ad_{\left(e^{\hat{\xi}_i\theta_i}\dots e^{\hat{\xi}_n\theta_n} g_{WT}(\mathbf{0})\right)^{-1}} \xi_i$$

Jacobians: Example

Consider the multi-joint manipulator:



$$J_{WT}^{b}(\boldsymbol{\theta}) = \begin{bmatrix} \xi_1^{\dagger} & \xi_2^{\dagger} & \dots & \xi_n^{\dagger} \end{bmatrix}$$

$$\xi_i^{\dagger} = Ad_{\left(e^{\hat{\xi}_i \theta_i} \dots e^{\hat{\xi}_n \theta_n} g_{WT}(\mathbf{0})\right)^{-1}} \xi_i$$

$$\xi_1^{\dagger} = Ad_{\left(e^{\hat{\xi}_1 \theta_1} e^{\hat{\xi}_2 \theta_2} g_{WC}(\mathbf{0})\right)^{-1}} \xi_1$$

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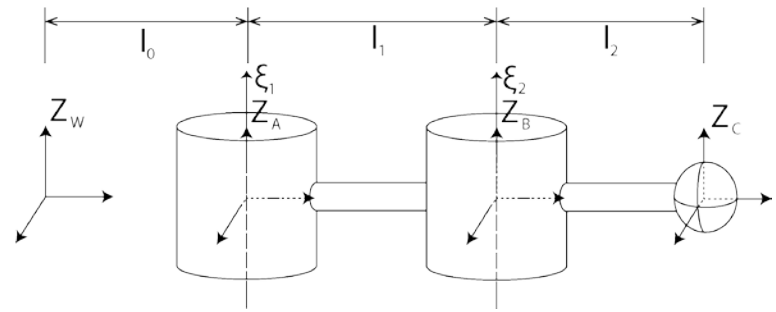
JACOBIANS: EXAMPLE



Consider the multi-joint manipulator:

$$J_{WT}^b(\boldsymbol{\theta}) = \begin{bmatrix} \xi_1^\dagger & \xi_2^\dagger & \dots & \xi_n^\dagger \end{bmatrix}$$

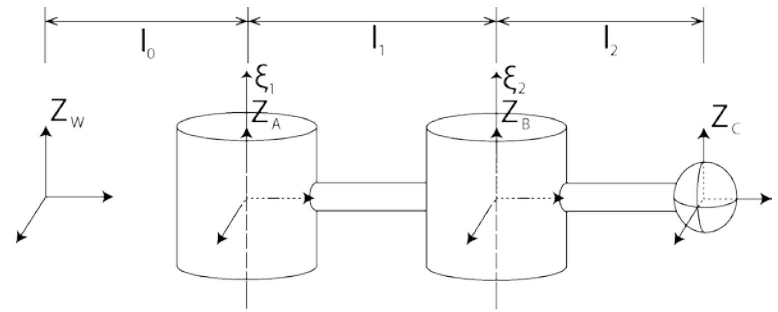
$$\xi_i^\dagger = Ad_{\left(e^{\hat{\xi}_i\theta_i}\dots e^{\hat{\xi}_n\theta_n} g_{WT}(\mathbf{0})\right)}^{-1} \xi_i$$

$$\xi_1^\dagger = Ad_{\left(e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} g_{WC}(\mathbf{0})\right)}^{-1} \xi_1$$

$$\xi_1^\dagger = \begin{bmatrix} \begin{bmatrix} c_{1+2} & s_{1+2} & 0 \\ -s_{1+2} & c_{1+2} & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 & -l_0 c_{1+2} - l_1 c_2 - l_2 \\ 0 & 0 & l_0 s_{1+2} + l_1 s_2 \\ l_0 + l_1 c_1 + l_2 c_{1+2} & l_1 s_1 + l_2 s_{1+2} & 0 \end{bmatrix} \\ \mathbb{0} & \begin{bmatrix} c_{1+2} & s_{1+2} & 0 \\ -s_{1+2} & c_{1+2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} l_0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

Jacobians: Example

Consider the multi-joint manipulator:



$J_{WT}^{b}(\boldsymbol{\theta}) = [\xi_1^\dagger \quad \xi_2^\dagger \quad \dots \quad \xi_n^\dagger]$

$\xi_i^\dagger = Ad_{\left(e^{\hat{\xi}_i\theta_i}\dots e^{\hat{\xi}_n\theta_n} g_{WT}(\mathbf{0})\right)^{-1}} \xi_i$

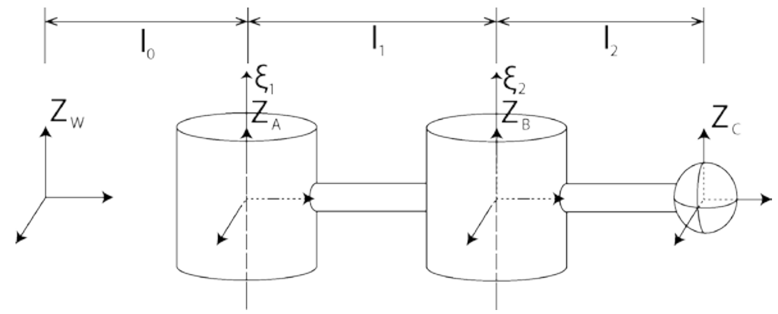
$$\xi_1^\dagger = Ad_{\left(e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} g_{WC}(\mathbf{0})\right)^{-1}} \xi_1$$

$$\xi_1^\dagger = \begin{bmatrix} \begin{bmatrix} c_{1+2} & s_{1+2} & 0 \\ -s_{1+2} & c_{1+2} & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 & -l_0 c_{1+2} - l_1 c_2 - l_2 \\ 0 & 0 & l_0 s_{1+2} + l_1 s_2 \\ l_0 + l_1 c_1 + l_2 c_{1+2} & l_1 s_1 + l_2 s_{1+2} & 0 \end{bmatrix} \\ \mathbb{0} & \begin{bmatrix} c_{1+2} & s_{1+2} & 0 \\ -s_{1+2} & c_{1+2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} l_0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

$$\xi_1^\dagger = [[-l_1 c_2 - l_2 \quad l_1 s_2 \quad 0] \quad [0 \quad 0 \quad 1]]^T$$

JACOBIANS: EXAMPLE

Consider the multi-joint manipulator:



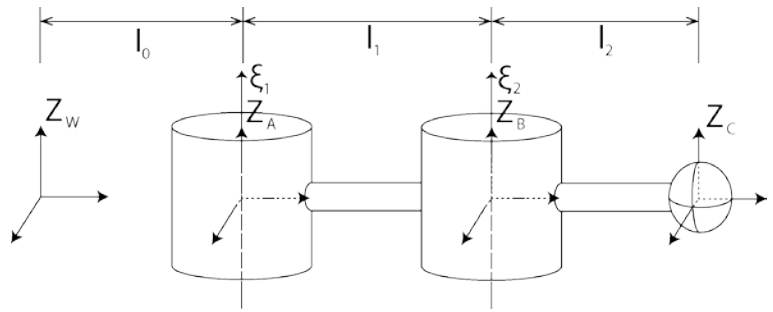
$$J^b_{WT}(\boldsymbol{\theta}) = \begin{bmatrix} \xi_1^\dagger & \xi_2^\dagger & \dots & \xi_n^\dagger \end{bmatrix}$$

$$\xi_i^\dagger = Ad_{\left(e^{\hat{\xi}_i\theta_i}\dots e^{\hat{\xi}_n\theta_n} g_{WT}(\mathbf{0})\right)}^{-1}\xi_i$$

$$\xi_2^\dagger = Ad_{\left(e^{\hat{\xi}_2\theta_2} g_{WC}(\mathbf{0})\right)}^{-1}\xi_1$$

JACOBIANS: EXAMPLE

Consider the multi-joint manipulator:

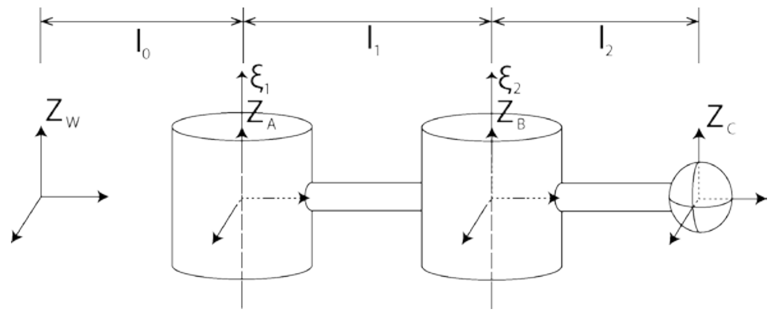


$$J^b_{WT}(\boldsymbol{\theta}) = \left[\xi_1^\dagger \quad \xi_2^\dagger \quad \dots \quad \xi_n^\dagger\right]$$

$$\xi_i^\dagger = Ad_{\left(e^{\hat{\xi}_i\theta_i}\dots e^{\hat{\xi}_n\theta_n} g_{WT}(\mathbf{0})\right)^{-1}}\xi_i$$

Jacobians: Example

Consider the multi-joint manipulator:



$$J_{WT}^{b}(\boldsymbol{\theta}) = \begin{bmatrix} \xi_1^{\dagger} & \xi_2^{\dagger} & \dots & \xi_n^{\dagger} \end{bmatrix}$$

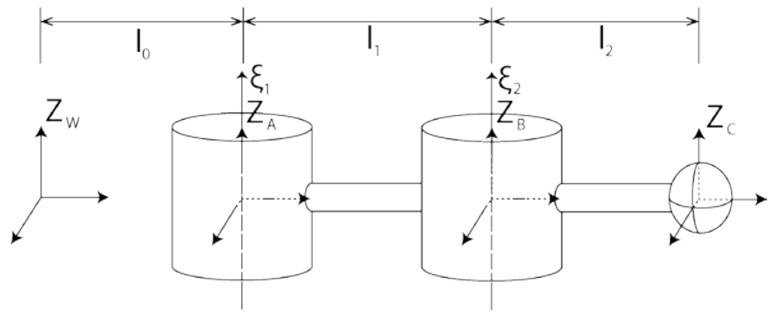
$$\xi_i^{\dagger} = Ad_{\left(e^{\hat{\xi}_i\theta_i} \dots e^{\hat{\xi}_n\theta_n} g_{WT}(\mathbf{0})\right)}^{-1} \xi_i$$

$$\xi_2^{\dagger} = Ad_{\left(e^{\hat{\xi}_2\theta_2} g_{WC}(\mathbf{0})\right)}^{-1} \xi_1$$

$$\xi_2^{\dagger} = \begin{bmatrix} \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 & -l_0c_2 - l_1c_2 - l_2 \\ 0 & 0 & (l_0 + l_1)s_2 \\ l_0 + l_1 + l_2c_2 & l_2s_2 & 0 \end{bmatrix} \\ \mathbb{0} & \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} l_0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

JACOBIANS: EXAMPLE

Consider the multi-joint manipulator:



$$J_{WT}^{b}(\boldsymbol{\theta}) = [\xi_1^{\dagger} \quad \xi_2^{\dagger} \quad \dots \quad \xi_n^{\dagger}]$$

$$\xi_i^{\dagger} = Ad_{\left(e^{\hat{\xi}_i\theta_i}\dots e^{\hat{\xi}_n\theta_n} g_{WT}(\mathbf{0})\right)}^{-1}\xi_i$$

$$\xi_2^{\dagger} = Ad_{\left(e^{\hat{\xi}_2\theta_2} g_{WC}(\mathbf{0})\right)}^{-1}\xi_1$$

$$\xi_2^{\dagger} = \begin{bmatrix} \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 & -l_0c_2 - l_1c_2 - l_2 \\ 0 & 0 & (l_0 + l_1)s_2 \\ l_0 + l_1 + l_2c_2 & l_2s_2 & 0 \end{bmatrix} \\ \mathbb{0} & \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} l_0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

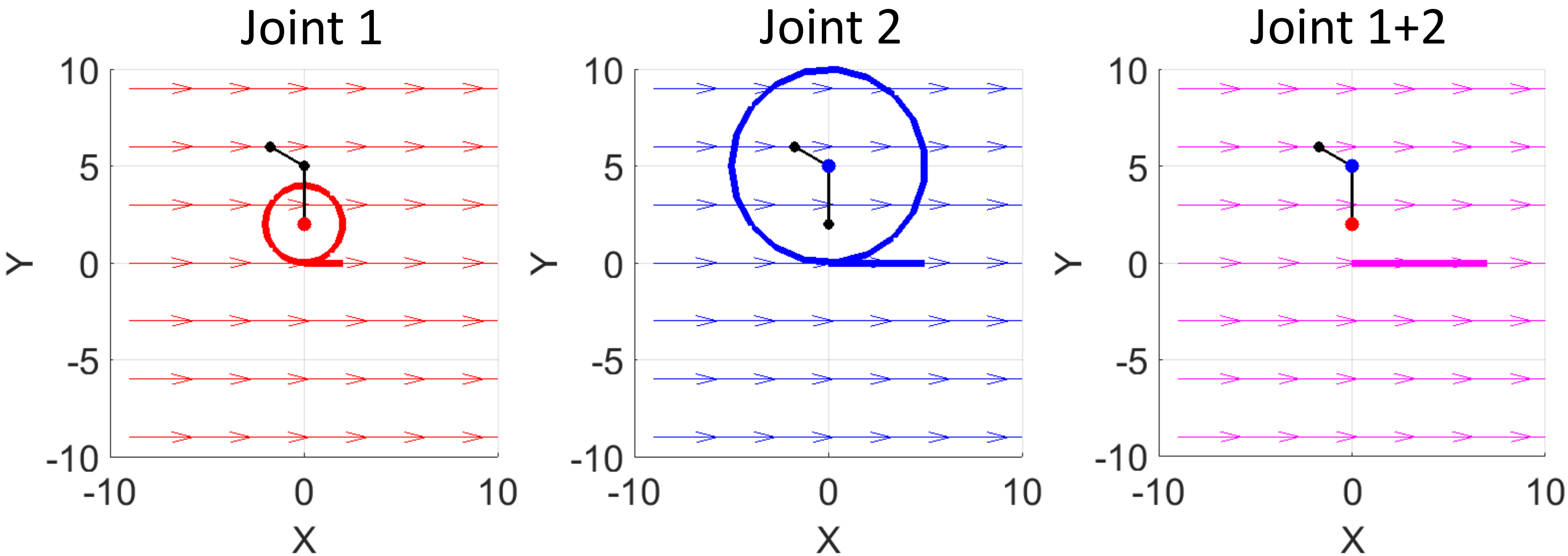
$$\xi_2^{\dagger} = [[-l_2 \quad 0 \quad 0] \quad [0 \quad 0 \quad 1]]^T$$

JACOBIANS: EXAMPLE

Consider the multi-joint manipulator:

$$J_{WT}^{s}(\boldsymbol{\theta}) = \left[\begin{bmatrix}\begin{bmatrix} l_0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\end{bmatrix} \begin{bmatrix}\begin{bmatrix} l_0 + l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\end{bmatrix}\right]$$

Spatial Velocities



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Jacobians: Example

Consider the multi-joint manipulator:

$$J_{WT}^{s}(\boldsymbol{\theta}) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} l_0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} \begin{bmatrix} l_0 + l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

Spatial Velocities

Joint 1 Joint 2 Joint 1+2

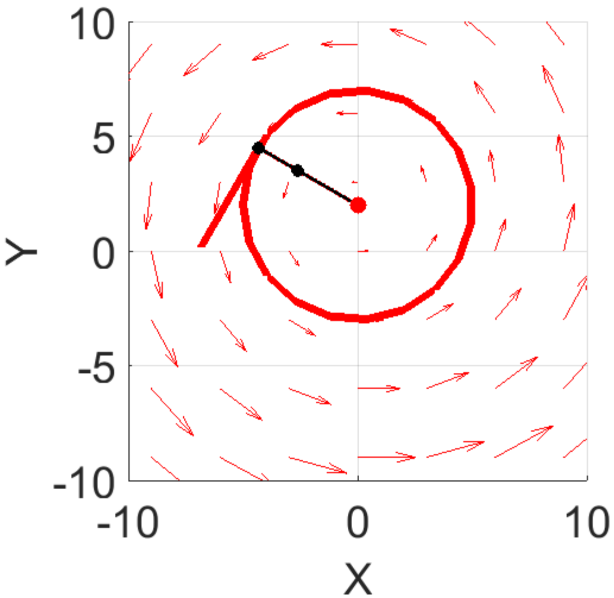
Jacobians: Example

Consider the multi-joint manipulator:

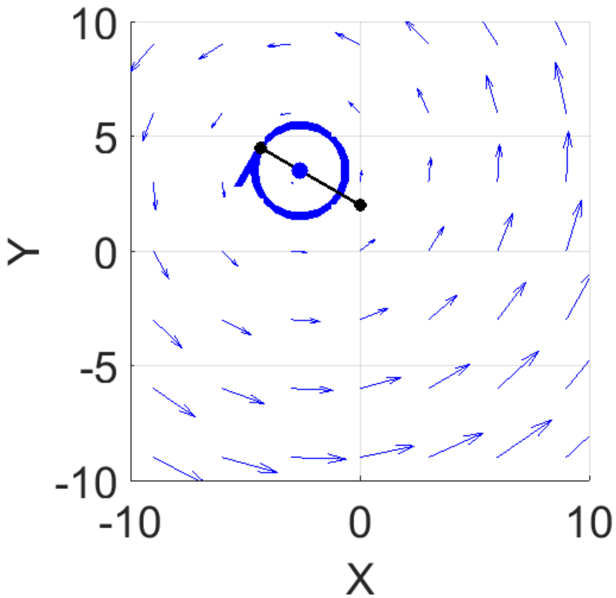
$$J_{WT}^{b}(\boldsymbol{\theta}) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} -l_1 c_2 - l_2 \\ l_1 s_2 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} \begin{bmatrix} -l_2 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

Body Velocities

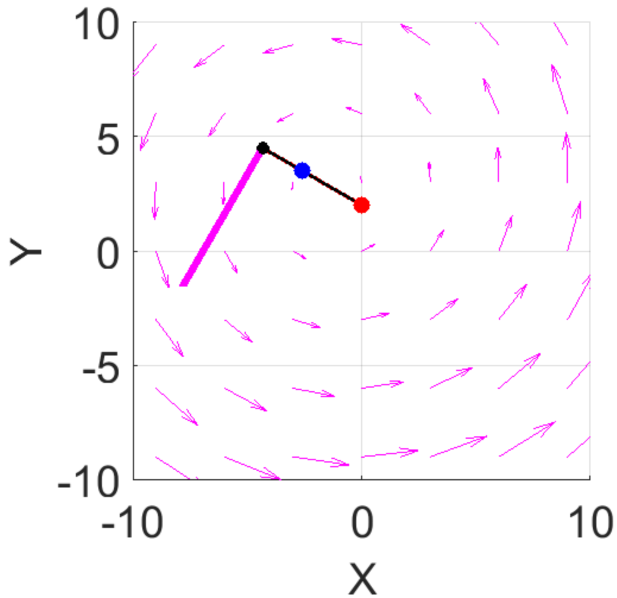
Joint 1



Joint 2



Joint 1+2



Jacobians: Example

Consider the multi-joint manipulator:

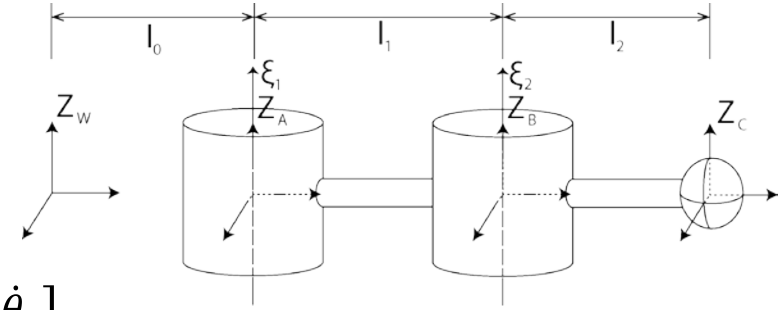
$$J_{WT}^{b}(\boldsymbol{\theta}) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} -l_1 c_2 - l_2 \\ l_1 s_2 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} \begin{bmatrix} -l_2 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

Body Velocities

Joint 1 | Joint 2 | Joint 1+2

JACOBIANS: EXAMPLE

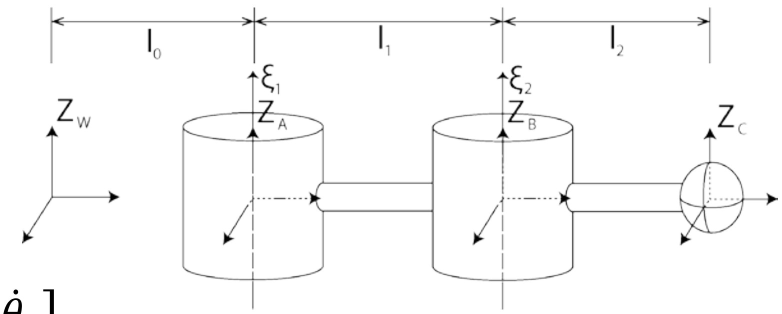
Consider the multi-joint manipulator:



$$J_{WT}^{s}(\boldsymbol{\theta}) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} l_0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} \begin{bmatrix} l_0 + l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \end{bmatrix} \qquad V_{WT}^{s}(\boldsymbol{\theta}) = J_{WT}^{s}(\boldsymbol{\theta}) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

JACOBIANS: EXAMPLE

Consider the multi-joint manipulator:

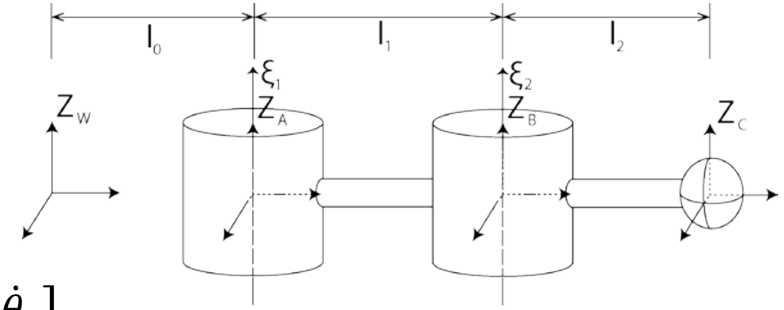


$$J_{WT}^{s}(\boldsymbol{\theta}) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} l_0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} \begin{bmatrix} l_0 + l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \end{bmatrix} \qquad V_{WT}^{s}(\boldsymbol{\theta}) = J_{WT}^{s}(\boldsymbol{\theta}) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$v_{q_W} = \hat{V}_{WT}^{s} q_W$$

JACOBIANS: EXAMPLE

Consider the multi-joint manipulator:



$$J_{WT}^{s}(\boldsymbol{\theta}) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} l_0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} \begin{bmatrix} l_0 + l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \end{bmatrix} \qquad V_{WT}^{s}(\boldsymbol{\theta}) = J_{WT}^{s}(\boldsymbol{\theta}) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$v_{q_W} = \hat{V}_{WT}^{s} q_W \qquad \hat{V}_{WT}^{s} = \begin{bmatrix} \begin{bmatrix} 0 & -\dot{\theta}_1 - \dot{\theta}_2 & 0 \\ \dot{\theta}_1 + \dot{\theta}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} l_0\left(\dot{\theta}_1 + \dot{\theta}_2\right) + l_1 c_1 \dot{\theta}_2 \\ l_1 s_1 \dot{\theta}_2 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix}$$

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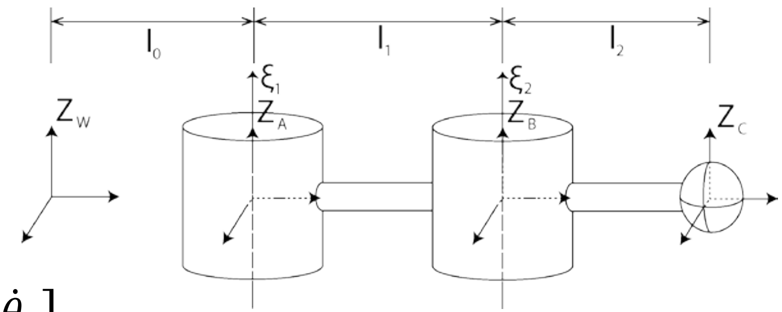
Jacobians: Example

Consider the multi-joint manipulator:



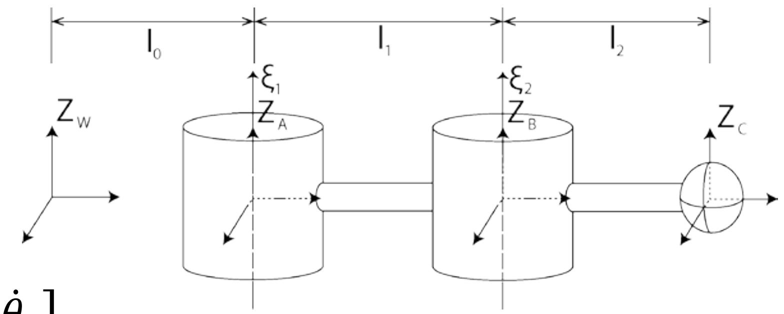
$$J^s_{WT}(\boldsymbol{\theta}) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} l_0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} \begin{bmatrix} l_0 + l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \end{bmatrix} \qquad V^s_{WT}(\boldsymbol{\theta}) = J^s_{WT}(\boldsymbol{\theta}) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$v_{q_W} = \hat{V}^s_{WT} q_W \qquad \hat{V}^s_{WT} = \begin{bmatrix} \begin{bmatrix} 0 & -\dot{\theta}_1 - \dot{\theta}_2 & 0 \\ \dot{\theta}_1 + \dot{\theta}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} l_0(\dot{\theta}_1 + \dot{\theta}_2) + l_1 c_1 \dot{\theta}_2 \\ l_1 s_1 \dot{\theta}_2 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix}$$

$$v_{q_W}(0,\cdot) = \hat{V}^s_{WT} \begin{bmatrix} \begin{bmatrix} 0 \\ l_0 + l_1 + l_2 \\ 0 \end{bmatrix} \\ 1 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} -(l_1 + l_2)\dot{\theta}_1 - l_2 \dot{\theta}_2 \\ 0 \\ 0 \end{bmatrix} \\ 1 \end{bmatrix}$$

Jacobians: Example

Consider the multi-joint manipulator:



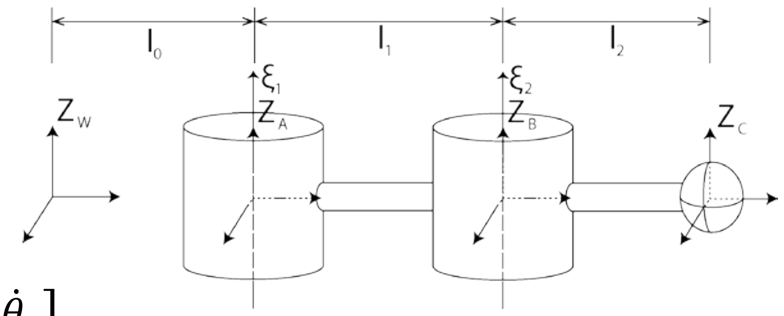
$$J_{WT}^{s}(\boldsymbol{\theta}) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} l_0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} \begin{bmatrix} l_0 + l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \end{bmatrix} \qquad V_{WT}^{s}(\boldsymbol{\theta}) = J_{WT}^{s}(\boldsymbol{\theta}) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$v_{q_W} = \hat{V}_{WT}^{s} q_W \qquad \hat{V}_{WT}^{s} = \begin{bmatrix} \begin{bmatrix} 0 & -\dot{\theta}_1 - \dot{\theta}_2 & 0 \\ \dot{\theta}_1 + \dot{\theta}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} l_0\left(\dot{\theta}_1 + \dot{\theta}_2\right) + l_1 c_1 \dot{\theta}_2 \\ l_1 s_1 \dot{\theta}_2 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix}$$

$$v_{q_W}\left(\frac{\pi}{2}, \cdot\right) = \hat{V}_{WT}^{s} \begin{bmatrix} \begin{bmatrix} -l_1 - l_2 \\ l_0 \\ 0 \end{bmatrix} \\ 1 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 \\ -(l_1 + l_2)\dot{\theta}_1 - l_2 \dot{\theta}_2 \\ 0 \end{bmatrix} \\ 1 \end{bmatrix}$$

Jacobians: Example

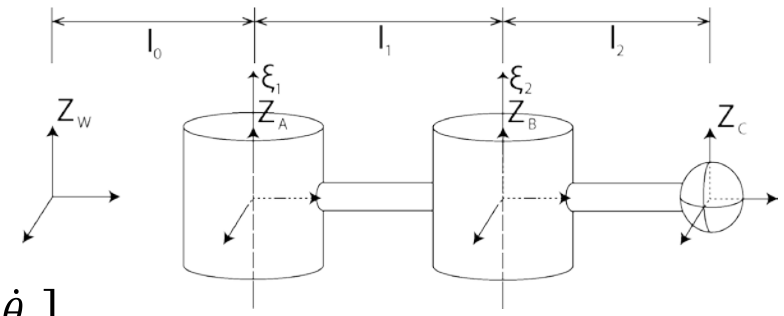
Consider the multi-joint manipulator:



$$J_{WT}^b(\boldsymbol{\theta}) = \left[\left[\begin{bmatrix} -l_1 c_2 - l_2 \\ l_1 s_2 \\ 0 \end{bmatrix} \atop \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right] \quad \left[\begin{bmatrix} -l_2 \\ 0 \\ 0 \end{bmatrix} \atop \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right]\right] V_{WT}^b(\boldsymbol{\theta}) = J_{WT}^s(\boldsymbol{\theta}) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

JACOBIANS: EXAMPLE

Consider the multi-joint manipulator:

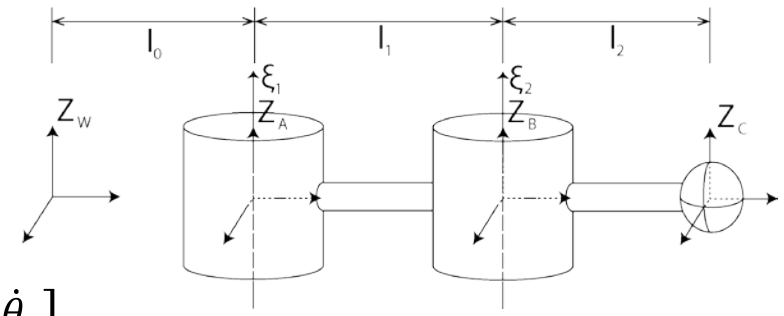


$$J^b_{WT}(\boldsymbol{\theta}) = \left[\left[\begin{bmatrix} -l_1 c_2 - l_2 \\ l_1 s_2 \\ 0 \end{bmatrix} \atop \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right] \left[\begin{bmatrix} -l_2 \\ 0 \\ 0 \end{bmatrix} \atop \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right]\right] \quad V^b_{WT}(\boldsymbol{\theta}) = J^s_{WT}(\boldsymbol{\theta}) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$v_{q_C} = \hat{V}^b_{WT} q_C$$

JACOBIANS: EXAMPLE

Consider the multi-joint manipulator:

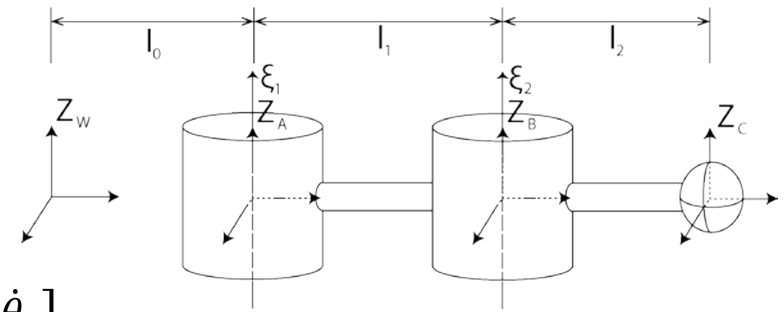


$$J_{WT}^{b}(\boldsymbol{\theta}) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} -l_1 c_2 - l_2 \\ l_1 s_2 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} \begin{bmatrix} -l_2 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \end{bmatrix} \quad V_{WT}^{b}(\boldsymbol{\theta}) = J_{WT}^{s}(\boldsymbol{\theta}) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$v_{q_C} = \hat{V}_{WT}^{b} q_C \qquad \hat{V}_{WT}^{b} = \begin{bmatrix} \begin{bmatrix} 0 & -\dot{\theta}_1 - \dot{\theta}_2 & 0 \\ \dot{\theta}_1 + \dot{\theta}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} -l_1 c_2 \dot{\theta}_1 - l_2(\dot{\theta}_1 + \dot{\theta}_2) \\ -l_1 s_2 \dot{\theta}_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix}$$

JACOBIANS: EXAMPLE

Consider the multi-joint manipulator:



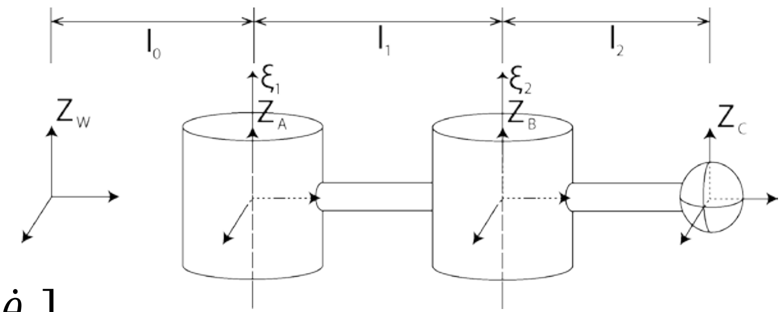
$$J^b_{WT}(\boldsymbol{\theta}) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} -l_1 c_2 - l_2 \\ l_1 s_2 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} \begin{bmatrix} -l_2 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \end{bmatrix} \qquad V^b_{WT}(\boldsymbol{\theta}) = J^s_{WT}(\boldsymbol{\theta}) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$v_{q_C} = \hat{V}^b_{WT} q_C \qquad \hat{V}^b_{WT} = \begin{bmatrix} \begin{bmatrix} 0 & -\dot{\theta}_1 - \dot{\theta}_2 & 0 \\ \dot{\theta}_1 + \dot{\theta}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} -l_1 c_2 \dot{\theta}_1 - l_2(\dot{\theta}_1 + \dot{\theta}_2) \\ -l_1 s_2 \dot{\theta}_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix}$$

$$v_{q_C}(\cdot, 0) = \hat{V}^b_{WT} \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\ 1 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} -l_1 \dot{\theta}_1 - l_2(\dot{\theta}_1 + \dot{\theta}_2) \\ 0 \\ 0 \end{bmatrix} \\ 1 \end{bmatrix}$$

JACOBIANS: EXAMPLE

Consider the multi-joint manipulator:



$$J_{WT}^{b}(\boldsymbol{\theta}) = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} -l_1 c_2 - l_2 \\ l_1 s_2 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} & \begin{bmatrix} \begin{bmatrix} -l_2 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix} \end{bmatrix} \quad V_{WT}^{b}(\boldsymbol{\theta}) = J_{WT}^{s}(\boldsymbol{\theta}) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$v_{q_C} = \hat{V}_{WT}^{b} q_C \qquad \hat{V}_{WT}^{b} = \begin{bmatrix} \begin{bmatrix} 0 & -\dot{\theta}_1 - \dot{\theta}_2 & 0 \\ \dot{\theta}_1 + \dot{\theta}_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} -l_1 c_2 \dot{\theta}_1 - l_2(\dot{\theta}_1 + \dot{\theta}_2) \\ -l_1 s_2 \dot{\theta}_1 \\ 0 \end{bmatrix} \\ \mathbb{0} & 1 \end{bmatrix}$$

$$v_{q_C}\left(\cdot, \frac{\pi}{2}\right) = \hat{V}_{WT}^{b} \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\ 1 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} -l_2(\dot{\theta}_1 + \dot{\theta}_2) \\ -l_1 \dot{\theta}_1 \\ 0 \end{bmatrix} \\ 1 \end{bmatrix}$$

Summary

Spatial and body velocities

$$v_{q_A} = \hat{V}^s_{AB} q_A \qquad v_{q_B} = \hat{V}^b_{AB} q_B$$

$$v_{q_A} = \dot{g}_{AB} g_{AB}^{-1} q_A \qquad v_{q_B} = g_{AB}^{-1} \dot{g}_{AB} q_B$$

Adjoint Relationships

$$V^s_{AB} = Ad_{g_{AB}} V^b_{AB}$$

$$V^s_{AC} = V^s_{AB} + Ad_{g_{AB}} V^s_{BC} \qquad V^b_{AC} = Ad_{g_{BC}^{-1}} V^b_{AB} + V^b_{BC}$$

$$Ad_{g_{AB}} = \begin{bmatrix} R_{g_{AB}} & \hat{p}_{g_{AB}} R_{g_{AB}} \\ 0 & R_{g_{AB}} \end{bmatrix}$$

SUMMARY

Jacobian maps joint velocities to end-effector velocities:

$$\xi_i' = Ad_{e^{\hat{\xi}_1\theta_1}\dots e^{\hat{\xi}_{i-1}\theta_{i-1}}}\xi_i \qquad \xi_i^\dagger = Ad_{g_{WT}^{-1}(\mathbf{0})e^{-\hat{\xi}_n\theta_n}\dots e^{-\hat{\xi}_i\theta_i}}\xi_i$$

$$J_{WT}^s(\boldsymbol{\theta}) = \begin{bmatrix}\xi_1 & \xi_2' & \dots & \xi_n'\end{bmatrix} \qquad J_{WT}^b(\boldsymbol{\theta}) = \begin{bmatrix}\xi_1^\dagger & \xi_2^\dagger & \dots & \xi_n^\dagger\end{bmatrix}$$

$$V_{WT}^s(\boldsymbol{\theta}) = J_{WT}^s(\boldsymbol{\theta})\begin{bmatrix}\dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n\end{bmatrix} \qquad V_{WT}^b(\boldsymbol{\theta}) = J_{WT}^b(\boldsymbol{\theta})\begin{bmatrix}\dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n\end{bmatrix}$$

where:

$$J_{WT}^s(\boldsymbol{\theta}) = Ad_{g_{WT}(\boldsymbol{\theta})}J_{WT}^b(\boldsymbol{\theta})$$

Summary

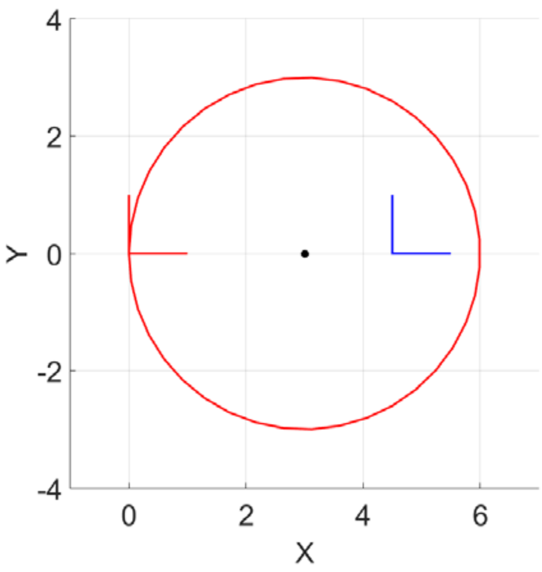
We can determine the velocities and Jacobian for a manipulator at a given configuration with the circle-drawing method

$$V_{AB}^{s} = [[0 \quad -l_1 \quad 0] \quad [0 \quad 0 \quad 1]]^T \dot{\theta}_1$$
$$V_{AB}^{b} = [[0 \quad l_2 \quad 0] \quad [0 \quad 0 \quad 1]]^T \dot{\theta}_1$$

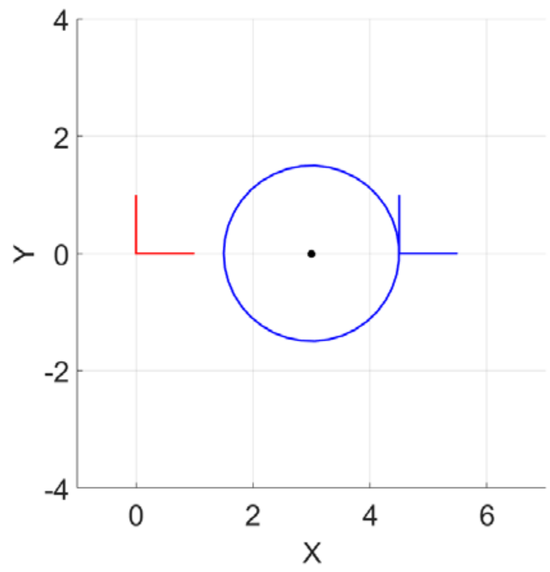
V_{AB}^{s}: A circle centered at the joint, passing through the origin

V_{AB}^{b}: A circle centered at the joint, passing through the end effector point

Spatial Velocities



Body Velocities



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