

The Bisection Method is a numerical technique used to find the root (or zero) of a continuous function. It is based on the Intermediate Value Theorem, which states that if a continuous function changes sign over an interval, then there exists at least one root in that interval.

Here's how the Bisection Method works:

1. Choose an initial interval: You need to select two points, say a and b , such that the function values at these points have opposite signs, i.e., $f(a) \cdot f(b) < 0$. This implies there is at least one root between a and b due to the Intermediate Value Theorem.

2. Find the midpoint: Compute the midpoint of the interval:

$$m = \frac{a + b}{2}$$

3. Evaluate the function at the midpoint: Calculate $f(m)$, the value of the function at the midpoint.

4. Check the sign:

- If $f(m) = 0$, then m is the root, and the process is complete.
- If $f(m) \cdot f(a) < 0$, the root lies in the interval $[a, m]$, so we update the interval to a and m .
- If $f(m) \cdot f(b) < 0$, the root lies in the interval $[m, b]$, so we update the interval to m and b .

5. Repeat: Repeat steps 2 through 4 until the width of the interval is sufficiently small, i.e., $|b - a|$ is smaller than some predetermined tolerance ϵ . At this point, m is considered an approximation of the root.
6. Convergence: The method converges linearly, meaning that the number of correct digits in the estimate increases slowly with each iteration. The convergence rate depends on how close the initial interval is to the actual root.
7. Advantages of the Bisection Method:
 - Guaranteed Convergence: As long as the function is continuous and the initial interval has opposite signs at the endpoints, the method will always find a root.
 - Simplicity: The method is easy to implement and understand.
8. Disadvantages:
 - Slow Convergence: It converges relatively slowly compared to methods like Newton's method or the secant method.
 - Requires a Bracketing Interval: You need to know two initial points where the function has opposite signs.

In summary, the Bisection Method is a reliable and simple way to approximate the roots of a function when you can identify a suitable bracketing interval.