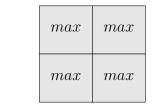
Pooling Layer Backpropagation

Maik Schmidt

14. März 2017

1 Single Kernel

$f_{0,0}$	$f_{0,1}$	$f_{0,2}$	$f_{0,3}$
$f_{1,0}$	$f_{1,1}$	$f_{1,2}$	$f_{1,3}$
$f_{2,0}$	$f_{2,1}$	$f_{2,2}$	$f_{2,3}$
$f_{3,0}$	$f_{3,1}$	$f_{3,2}$	$f_{3,3}$



$f_{0,0}$	$f_{0,1}$
$f_{1,0}$	$f_{1,1}$

$s_{0,0}$	$s_{0,1}$
$s_{1,0}$	$s_{1,1}$

Forward:

$$\begin{split} f_{i,j}^l &= \max_{\substack{i' = i \cdot \text{kernel} \\ \text{height}}, \dots, \binom{(i+1) \cdot \text{kernel}}{\text{height-1}},} f_{i',j'}^{l-1} \\ j' &= j \cdot \text{kernel}, \dots, \binom{(j+1) \cdot \text{kernel}}{\text{width-1}} \end{split}$$

$$\begin{split} s_{i,j}^l &= \underset{i'=i \text{ kernel }, \dots, \binom{(i+1) \cdot \text{kernel }}{\text{height }, \dots, \binom{(j+1) \cdot \text{kernel }}{\text{width }, \dots, \binom{(j+1) \cdot \text{kernel }}{\text{width }-1}}, \\ j' &= j \cdot \underset{\text{width }}{j \cdot \text{kernel }, \dots, \binom{(j+1) \cdot \text{kernel }}{\text{width }-1}} \end{split}$$

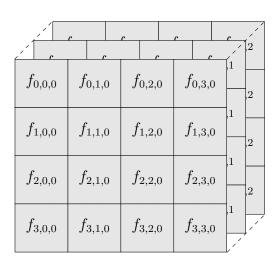
with zero padding:

$$f_{x,y}^{l-1} := 0 \quad \text{if } x \notin [0, \text{input height}] \text{ or } y \notin [0, \text{input width}]$$

Backward:

$$\delta_{i,j}^l = \left\{ \begin{array}{cc} \delta_{i/\text{kernel j/kernel}}^{l+1} & \text{if } s_{i/\text{kernel j/kernel}}^{l+1} = (i,j) \\ \text{height 'width} & \text{height 'width} \\ 0 & \text{else} \end{array} \right.$$

2 Multiple Kernels



	max	max	
*	max	max	

=

	<u> </u>	, , ,
$f_{0,0,0}$	$f_{0,1,0}$,1
9 0,0,0	0,1,0	
$f_{1,0,0}$	$f_{1,1,0}$,1
9 -, •, •	V =,=,=	/

			, , '
$s_{0,0,0}$	s	0,1,0	0
\$1,0,0	s	1,1,0	0

Forward:

$$\begin{split} f_{i,j,f}^l = \max_{\substack{i' = i \cdot \text{kernel} \\ \text{height}, \cdots, \, (i+1) \cdot \text{kernel} \\ \text{height-1}}}, f_{i',j',f}^{l-1} \\ j' = j \cdot \text{kernel}, \cdots, (j+1) \cdot \text{kernel} \\ \text{width-1} \end{split}$$

$$\begin{split} s_{i,j,f}^l = \mathop{\arg\max}_{\substack{i'=i\text{\cdot}\text{kernel},\cdots,\binom{(i+1)\cdot\text{kernel}}{\text{height-1}},\\ j'=j\text{\cdot}\text{kernel},\cdots,\binom{(j+1)\cdot\text{kernel}}{\text{width-1}}} f_{i',j',f}^{l-1} \end{split}$$

with zero padding:

$$f_{x,y,z}^{l-1} := 0 \quad \text{if } x \notin [0, \text{input height}] \text{ or } y \notin [0, \text{input width}]$$

Backward:

$$\delta_{i,j,f}^{l} = \begin{cases} & \delta_{i/\text{kernel j/kernel j/k$$