# Pooling Layer Backpropagation

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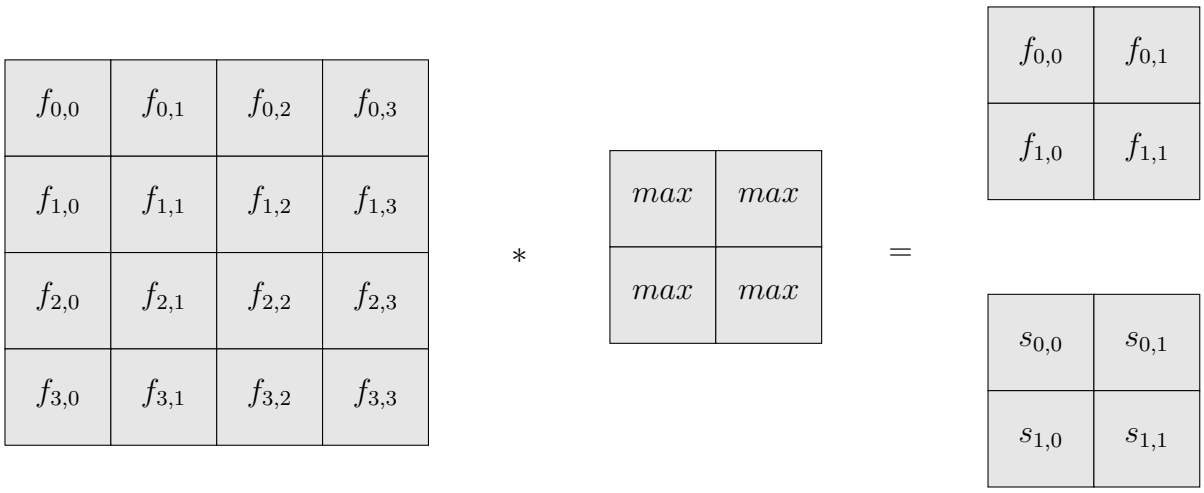
14. März 2017

## 1 Single Kernel




**Forward:**

$$f_{i,j}^l = \max_{\substack{i'=\frac{i\cdot\text{kernel}}{\text{height}},\ldots,\frac{(i+1)\cdot\text{kernel}}{\text{height-1}},\\ j'=\frac{j\cdot\text{kernel}}{\text{width}},\ldots,\frac{(j+1)\cdot\text{kernel}}{\text{width-1}}}} f_{i',j'}^{l-1}$$

$$s_{i,j}^l = \arg\max_{\substack{i'=\frac{i\cdot\text{kernel}}{\text{height}},\ldots,\frac{(i+1)\cdot\text{kernel}}{\text{height-1}},\\ j'=\frac{j\cdot\text{kernel}}{\text{width}},\ldots,\frac{(j+1)\cdot\text{kernel}}{\text{width-1}}}} f_{i',j'}^{l-1}$$

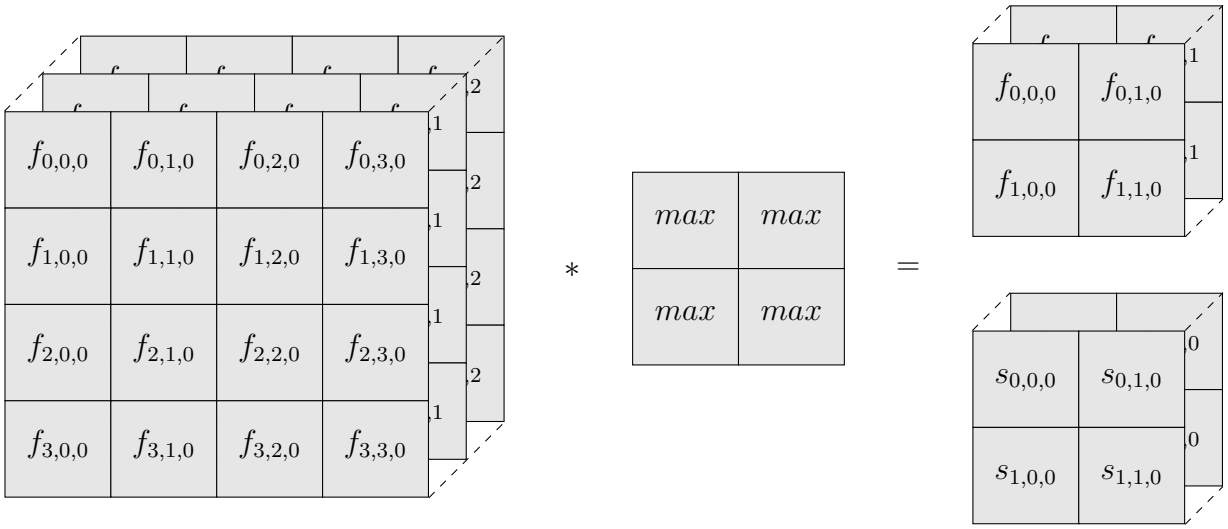
with *zero padding*:

$$f_{x,y}^{l-1} := 0 \quad \text{if } x \notin [0, \text{input height}] \text{ or } y \notin [0, \text{input width}]$$

**Backward:**

$$\delta_{i,j}^l = \begin{cases} \delta_{\frac{\text{i/kernel}}{\text{height}},\frac{\text{j/kernel}}{\text{width}}}^{l+1} & \text{if } s_{\frac{\text{i/kernel}}{\text{height}},\frac{\text{j/kernel}}{\text{width}}}^{l+1} = (i,j) \\ 0 & \text{else} \end{cases}$$

# 2 Multiple Kernels




**Forward:**

$$f^l_{i,j,f} = \max_{\substack{i'=\frac{i\cdot\text{kernel}}{\text{height}},\dots,\frac{(i+1)\cdot\text{kernel}}{\text{height-1}},\\ j'=\frac{j\cdot\text{kernel}}{\text{width}},\dots,\frac{(j+1)\cdot\text{kernel}}{\text{width-1}}}} f^{l-1}_{i',j',f}$$

$$s^l_{i,j,f} = \arg\max_{\substack{i'=\frac{i\cdot\text{kernel}}{\text{height}},\dots,\frac{(i+1)\cdot\text{kernel}}{\text{height-1}},\\ j'=\frac{j\cdot\text{kernel}}{\text{width}},\dots,\frac{(j+1)\cdot\text{kernel}}{\text{width-1}}}} f^{l-1}_{i',j',f}$$

with *zero padding*:

$$f^{l-1}_{x,y,z} := 0 \quad \text{if } x \notin [0, \text{input height}] \text{ or } y \notin [0, \text{input width}]$$

**Backward:**

$$\delta^l_{i,j,f} = \begin{cases} \delta^{l+1}_{\frac{\text{i/kernel}}{\text{height}},\frac{\text{j/kernel}}{\text{width}},f} & \text{if } s^{l+1}_{\frac{\text{i/kernel}}{\text{height}},\frac{\text{j/kernel}}{\text{width}},f} = (i,j) \\ 0 & \text{else} \end{cases}$$