

Manipulation motion planning

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A few examples



Definitions

A manipulation motion

- ▶ is the motion of
 - ▶ one or several robots and of
 - ▶ one or several objects
- ▶ such that each object
 - ▶ either is in a stable position, or
 - ▶ is moved by one or several robots.

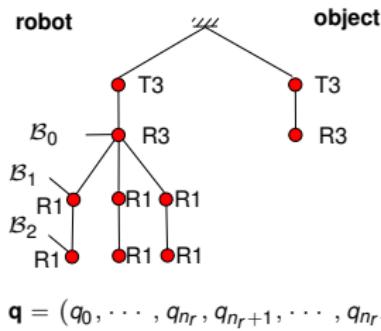
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Composite robot

Kinematic chain composed of each robot and of each object

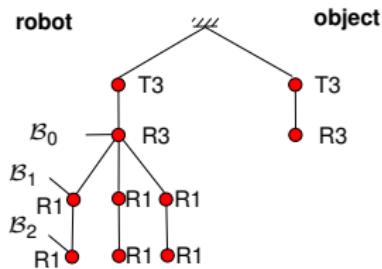


The configuration space of a composite robot is the cartesian product of the configuration spaces of each robot and object.

$$\mathcal{C} = \mathcal{C}_{r1} \times \mathcal{C}_{nb_robots} \times SE(3)^{nb_objets}$$

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$$\mathbf{q} = (q_0, \dots, q_{n_r}, q_{n_r+1}, \dots, q_{n_r+n_o})$$

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Numerical constraints

Constraints to which manipulation motions are subject can be expressed numerically.

- ▶ Numerical constraints:

$$f(\mathbf{q}) = 0, \quad m \in \mathbb{N}, \\ f \in C^1(\mathcal{C}, \mathbb{R}^m)$$

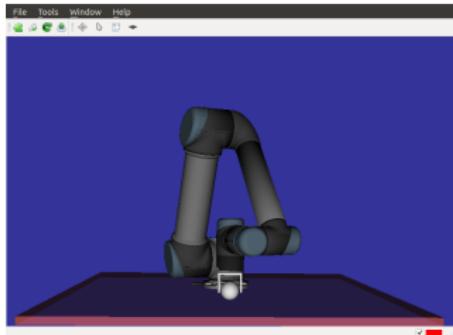
- ▶ `setConstantRightHandSide(True)`

- ▶ Parameterizable numerical constraints:

$$f(\mathbf{q}) = f_0, \quad m \in \mathbb{N}, \\ f \in C^1(\mathcal{C}, \mathbb{R}^m) \\ f_0 \in \mathbb{R}^m$$

- ▶ `setConstantRightHandSide(False)`

Example: robot manipulating a ball



$$\mathcal{C} = [-\pi, \pi]^6 \times \mathbb{R}^3 \quad (1)$$

$$\mathbf{q} = (q_0, \dots, q_5, x_b, y_b, z_b) \quad (2)$$

Two *states*:

- ▶ placement: the ball is lying on the table,
- ▶ grasp: the ball is held by the end-effector.

Example: robot manipulating a ball

Each state is defined by a numerical constraint

- ▶ placement

$$z_b = 0$$

- ▶ grasp

$$\mathbf{x}_{gripper}(q_0, \dots, q_5) - \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} = 0$$

Each state is a sub-manifold of the configuration space

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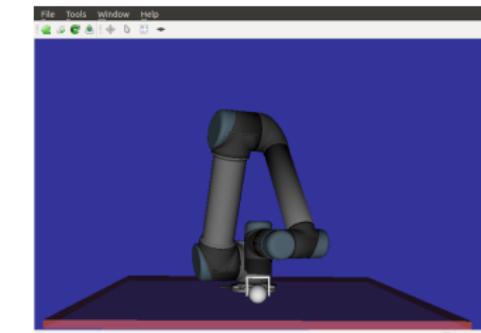
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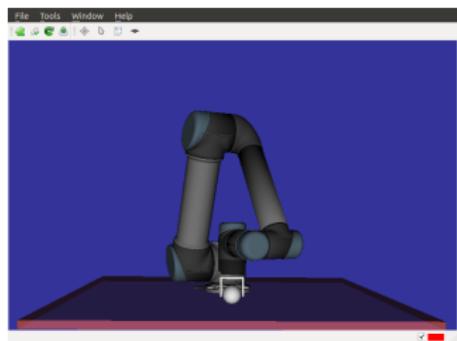
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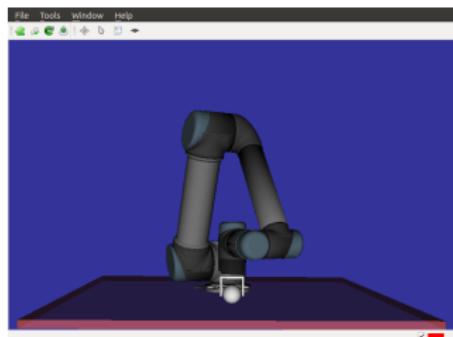


Two types of motion:

- ▶ **transit:** the ball is lying and **fixed** on the table,
- ▶ **transfer:** the ball moves with the end-effector.

Example: robot manipulating a ball

Motion constraints



► transit

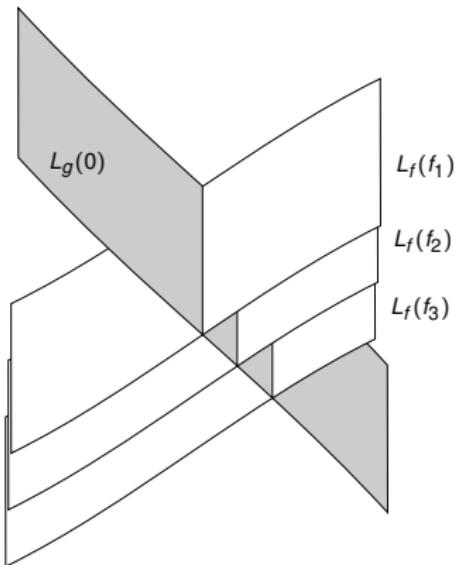
$$\begin{aligned}x_b &= x_0 \\y_b &= y_0 \\z_b &= 0\end{aligned}\quad \begin{array}{l}\} \text{ parameterizable} \\ \} \text{ simple}\end{array}$$

► transfer

$$\mathbf{x}_{gripper}(q_0, \dots, q_5) - \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} = 0$$

Foliation

Motion constraints define a foliation of the admissible configuration space ($\text{grasp} \cup \text{placement}$).



- ▶ f : position of the ball

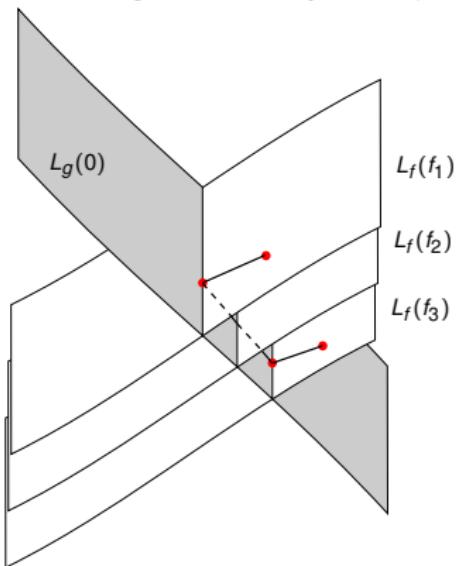
$$L_f(f_1) = \{\mathbf{q} \in \mathcal{C}, f(\mathbf{q}) = f_1\}$$

- ▶ g : grasp of the ball

$$L_g(0) = \{\mathbf{q} \in \mathcal{C}, g(\mathbf{q}) = 0\}$$

Foliation

Motion constraints define a foliation of the admissible configuration space ($\text{grasp} \cup \text{placement}$).



Solution to a manipulation planning problem is a concatenation of *transit* and *transfer* paths.

General case

In a manipulation problem,

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 - ▶ numerical constraints
- ▶ trajectories of the system are subject to
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 - ▶ parameterizable numerical constraints.

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 - ▶ parameter value is constant along trajectories.

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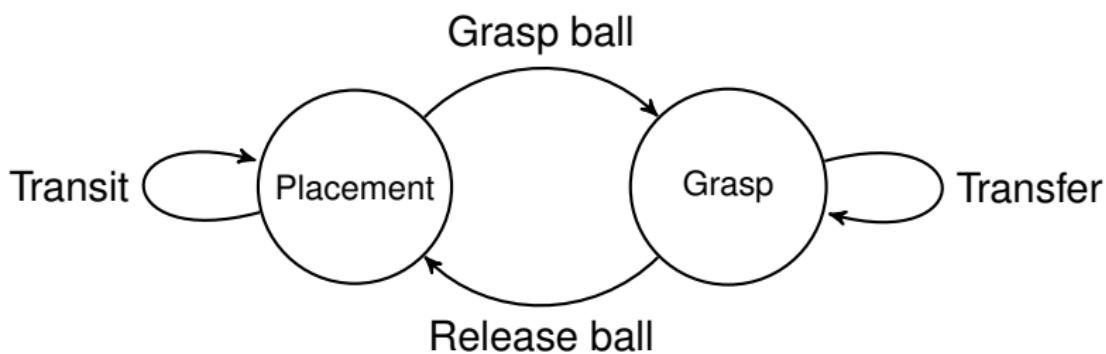
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Constraint graph

A manipulation planning problem can be represented by a *manipulation graph*.

- ▶ **Nodes** or *states* are numerical constraints.
- ▶ **Edges** or *transitions* are parameterizable numerical constraints.



Projecting configuration on constraint

Newton-Raphson algorithm

- ▶ \mathbf{q}_0 configuration,
- ▶ $f(\mathbf{q}) = 0$ non-linear constraint,
- ▶ ϵ numerical tolerance

Projection (\mathbf{q}_0, f):

$$\mathbf{q} = \mathbf{q}_0; \alpha = 0.95$$

for i from 1 to max_iter:

$$\mathbf{q} = \mathbf{q} - \alpha \left(\frac{\partial f}{\partial \mathbf{q}}(\mathbf{q}) \right)^+ f(\mathbf{q})$$

if $\|f(\mathbf{q})\| < \epsilon$: return \mathbf{q}

return failure

Steering method

Mapping \mathcal{SM} from $\mathcal{C} \times \mathcal{C}$ to $C^1([0, 1], \mathcal{C})$ such that

$$\mathcal{SM}(\mathbf{q}_0, \mathbf{q}_1)(0) = \mathbf{q}_0$$

$$\mathcal{SM}(\mathbf{q}_0, \mathbf{q}_1)(1) = \mathbf{q}_1$$

Constrained steering method

Let

- ▶ \mathcal{SM} be a steering method
- ▶ $f \in C^1(\mathcal{C}, \mathbb{R}^m)$ be a numerical constraint.

A constrained steering method $\bar{\mathcal{SM}}$ of constraint f is a steering method such that

$$\forall t \in [0, 1], f(\bar{\mathcal{SM}}(t)) = 0$$

It can be defined by projection

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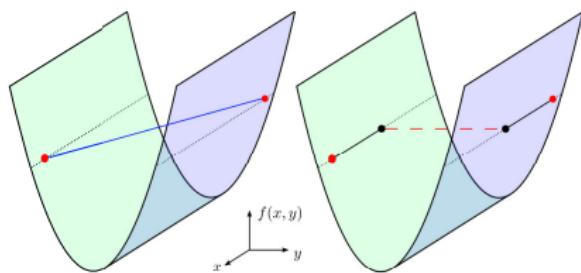
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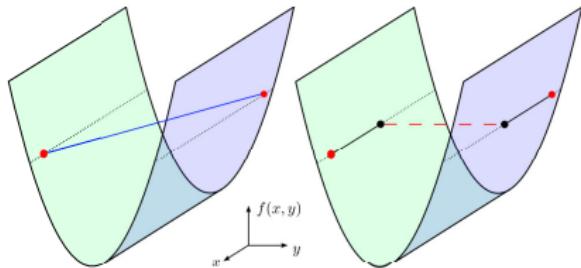
Projecting path on constraint

- ▶ path : mapping from $[0, 1]$ to \mathcal{C}
- ▶ $f(\mathbf{q}) = 0$ non-linear constraint,

Applying Newton Raphson at each point may result in a discontinuous path



Discontinuous Projection



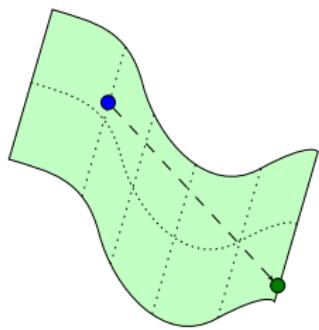
$$\mathcal{C} = \mathbb{R}^2, f(x, y) = y^2 - 1$$

$$\frac{\partial f}{\partial \mathbf{q}} = \begin{pmatrix} 0 & 2y \end{pmatrix}, \quad \frac{\partial f}{\partial \mathbf{q}}^+ = \begin{pmatrix} 0 \\ \frac{1}{2y} \end{pmatrix} \text{ ou } \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$y_{i+1} = y_i + \frac{1 - y_i^2}{2y_i}$$

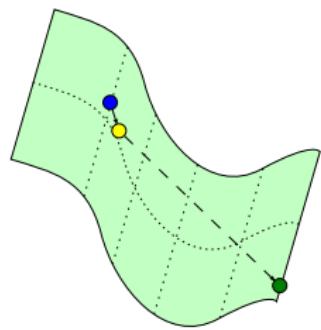
Testing projection continuity

- ▶ The initial path is sampled and successive samples are projected,
- ▶
- ▶



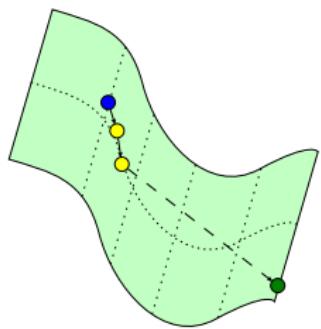
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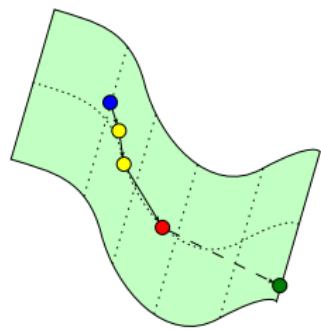
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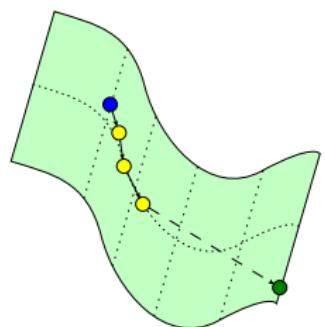
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- ▶



Testing projection continuity

- ▶ The initial path is sampled and successive samples are projected,
- ▶ if 2 successive projections are too far away, an intermediate sample is selected.
- ▶ Choosing appropriate sampling ensures us continuity of the projection.



Algorithm

Manipulation RRT

Manipulation RRT

$q_{rand} = \text{shoot_random_config}()$

for each connected component:

$q_{near} = \text{nearest_neighbor}(q_{rand}, roadmap)$

$T = \text{select_transition}(q_{near})$

$q_{proj} = \text{generate_target_config}(q_{near}, q_{rand}, T)$

$q_{new} = \text{extend}(q_{near}, q_{proj}, T)$

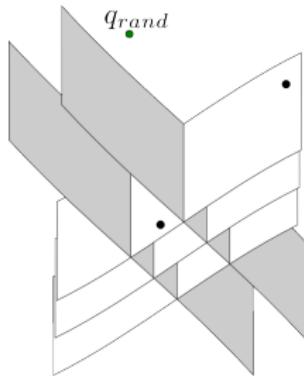
$\text{roadmap.insert_node}(q_{new})$

$\text{roadmap.insert_edge}(T, q_{near}, q_{new})$

$\text{new_nodes.append}(q_{new})$

for $q \in (q_{new}^1, \dots, q_{new}^{n_{cc}})$:

connect (q , roadmap)



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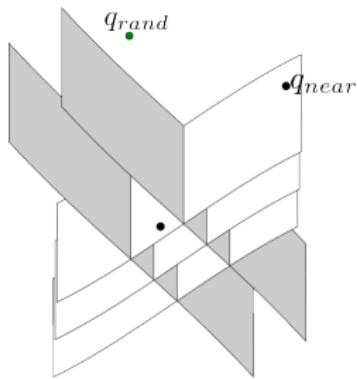
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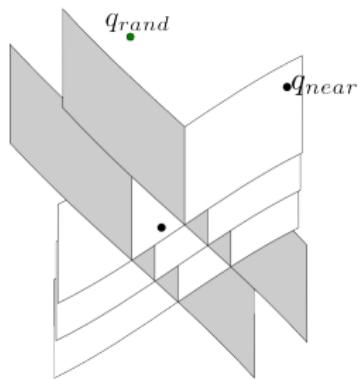
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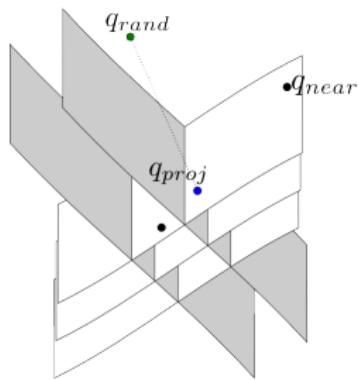
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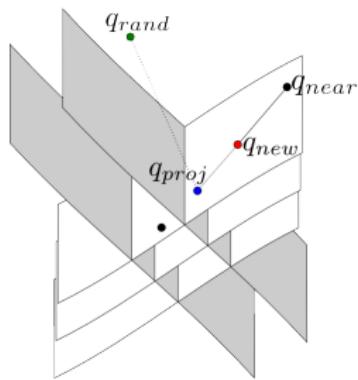
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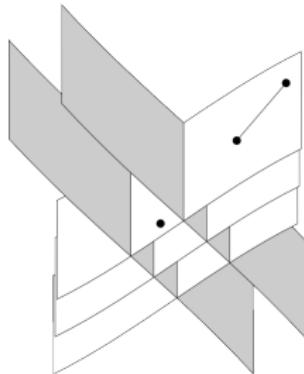
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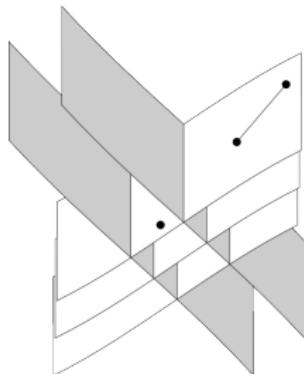
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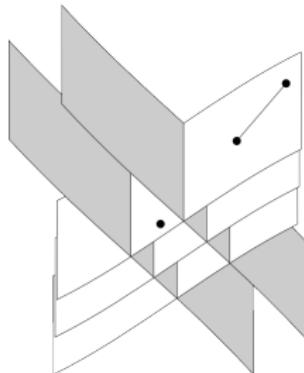
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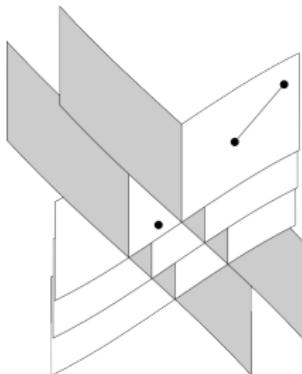
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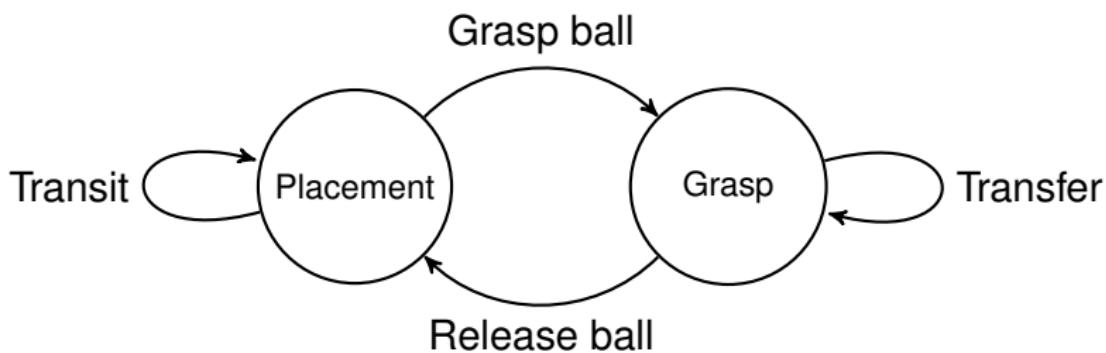
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Select transition

$$T = \text{select_transition}(\mathbf{q}_{near})$$

Outward transitions of each state are given a probability distribution. The transition from a state to another state is chosen by random sampling.



Generate target configuration

$$\mathbf{q}_{proj} = \text{generate_target_config}(\mathbf{q}_{near}, \mathbf{q}_{rand}, T)$$

Once transition T has been selected, \mathbf{q}_{rand} is *projected* onto the destination state S_{dest} in a configuration reachable by \mathbf{q}_{near} .

$$f_T(\mathbf{q}_{proj}) = f_T(\mathbf{q}_{near})$$

$$f_{S_{dest}}(\mathbf{q}_{proj}) = 0$$

Extend

$$\mathbf{q}_{new} = \text{extend}(\mathbf{q}_{near}, \mathbf{q}_{proj}, T)$$

Project straight path $[\mathbf{q}_{near}, \mathbf{q}_{proj}]$ on transition constraint:

- ▶ if projection successful and projected path collision free

$$\mathbf{q}_{new} \leftarrow \mathbf{q}_{proj}$$

- ▶ otherwise $(\mathbf{q}_{near}, \mathbf{q}_{new}) \leftarrow$ largest path interval tested as collision-free with successful projection.

$$\forall \mathbf{q} \in (\mathbf{q}_{near}, \mathbf{q}_{new}), f_T(\mathbf{q}) = f_T(\mathbf{q}_{near})$$

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Connect

connect (\mathbf{q} , roadmap)

for each connected component cc not containing \mathbf{q} :

for all n closest config \mathbf{q}_1 to \mathbf{q} in cc :

- ▶ connect (\mathbf{q}, \mathbf{q}_1)

Connect

connect (\mathbf{q}_0 , \mathbf{q}_1):

$S_0 = \text{state } (\mathbf{q}_0)$

$S_1 = \text{state } (\mathbf{q}_1)$

$T = \text{transition } (S_0, S_1)$

if T and $f_T(\mathbf{q}_0) == f_T(\mathbf{q}_1)$:

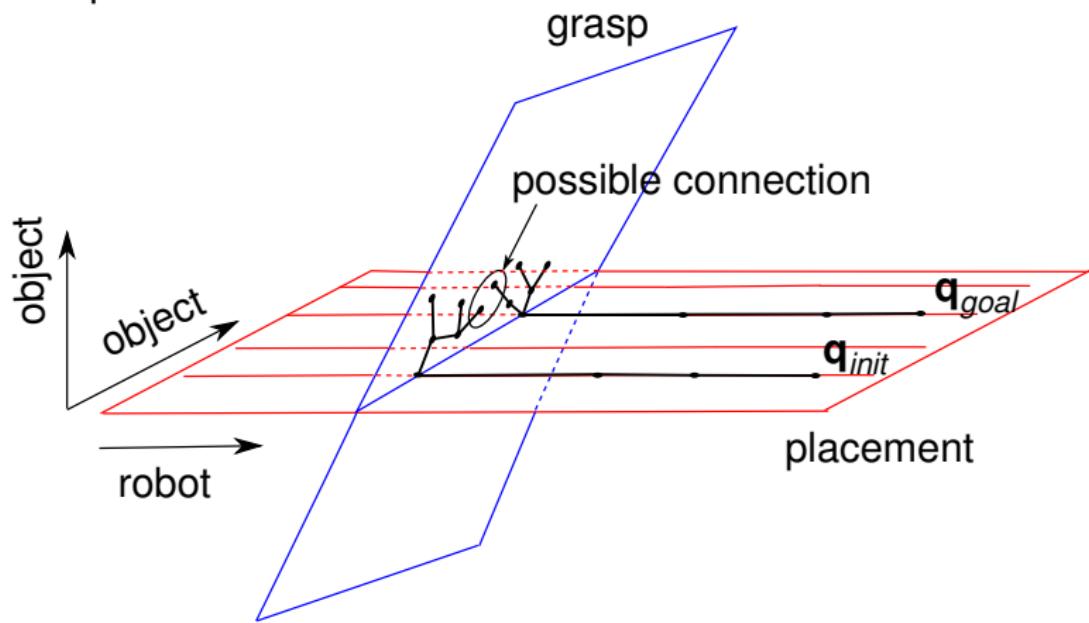
if $p = \text{projected_path } (T, \mathbf{q}_0, \mathbf{q}_1)$ collision-free:
 $\text{roadmap.insert_edge } (T, \mathbf{q}_0, \mathbf{q}_1)$

return

Connecting trees

Manipulation RRT is initialized with \mathbf{q}_{init} , \mathbf{q}_{goal} .

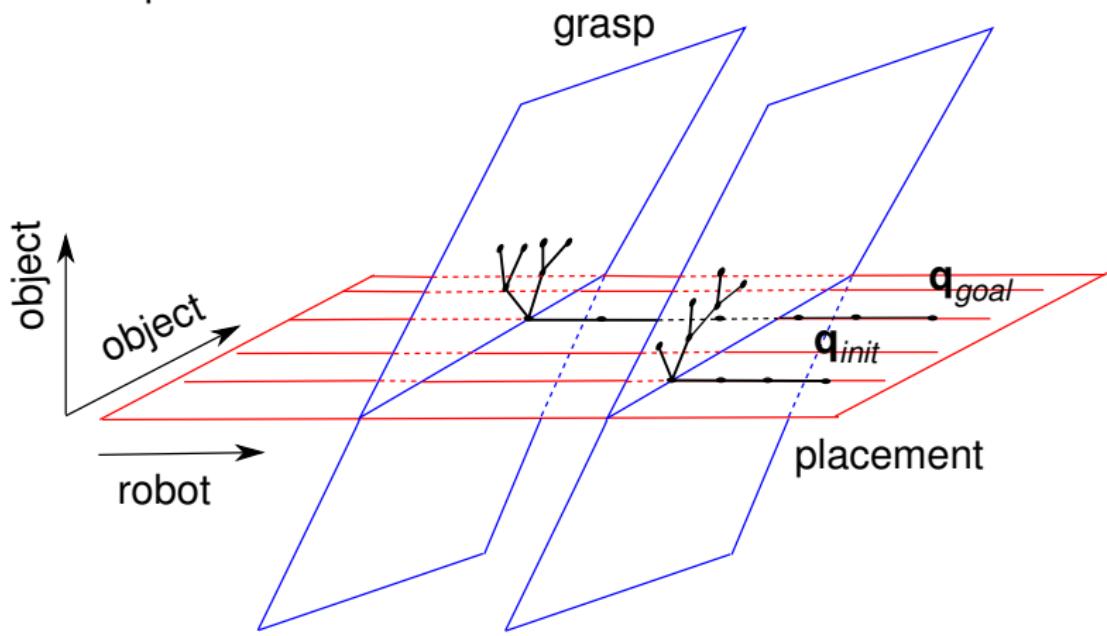
- ▶ 2 connected components.
- ▶ possible connection.



Connecting trees: general case

Manipulation RRT is initialized with \mathbf{q}_{init} , \mathbf{q}_{goal} .

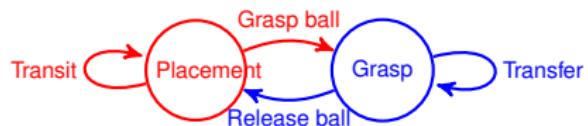
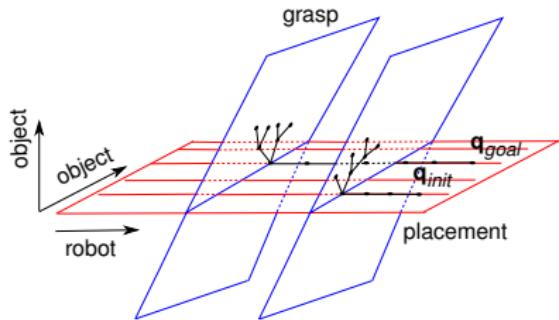
- ▶ 2 connected components,
- ▶ no possible connection.



Connecting trees: general case

Manipulation RRT is initialized with \mathbf{q}_{init} , \mathbf{q}_{goal} .

- ▶ 2 connected components,
- ▶ no possible connection.



Crossed foliation transition: generate target configuration

$\mathbf{q}_{proj} =$
generate_target_config(\mathbf{q}_{near} , \mathbf{q}_{rand} , T)

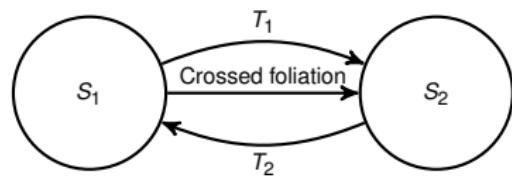
$\mathbf{q}_1 \leftarrow$ pick configuration

- ▶ in state S_1 ,
- ▶ not in same connected component as \mathbf{q}_{near}

$$f_{T_1}(\mathbf{q}_{proj}) = f_{T_1}(\mathbf{q}_{near})$$

$$f_{T_2}(\mathbf{q}_{proj}) = f_{T_2}(\mathbf{q}_1)$$

$$f_{S_2}(\mathbf{q}_{proj}) = 0$$



Crossed foliation transition: extend

$$\mathbf{q}_{new} = \text{extend}(\mathbf{q}_{near}, \mathbf{q}_{proj}, T_1)$$

Project straight path $[\mathbf{q}_{near}, \mathbf{q}_{proj}]$ on T_1 constraint:

- ▶ if projection successful and projected path collision free

$$\mathbf{q}_2 \leftarrow \mathbf{q}_{proj}$$

$$f_{T_2}(\mathbf{q}_2) = f_{T_2}(\mathbf{q}_1)$$

$$f_{S_2}(\mathbf{q}_2) = 0$$

- ▶ \mathbf{q}_2 is connectable to \mathbf{q}_1 via T_2 .

Crossed foliation transition: extend

$$\mathbf{q}_{new} = \text{extend}(\mathbf{q}_{near}, \mathbf{q}_{proj}, T_1)$$

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- ▶ \mathbf{q}_2 is connectable to \mathbf{q}_1 via T_2 .

Relative positions as numerical constraints

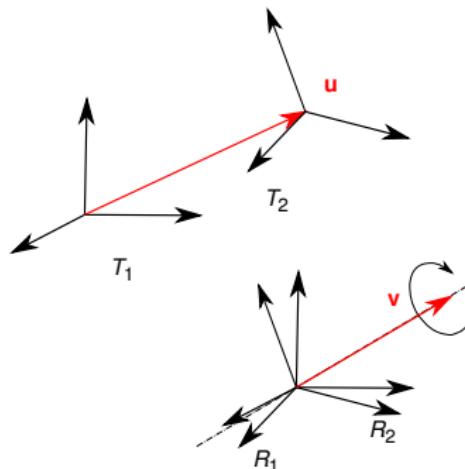
- ▶ $T_1 = T_{(R_1, t_1)} \in SE(3)$,
- ▶ $T_2 = T_{(R_2, t_2)} \in SE(3)$.
- ▶ $T_{2/1} = T_1^{-1} \circ T_2$ can be represented by a vector of dimension 6:

$$\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}$$

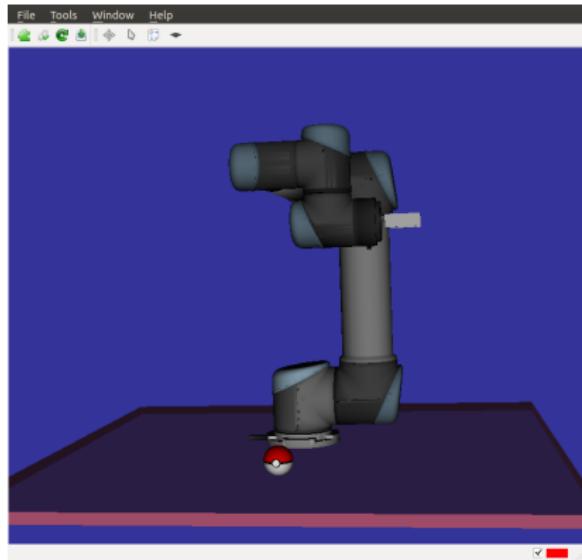
where

$$\mathbf{u} = R_1^T(t_2 - t_1)$$

$R_1^T R_2$ matrix of the rotation around axis $\mathbf{v}/\|\mathbf{v}\|$ and of angles $\|\mathbf{v}\|$.



A few words about the BE



- ▶ script/grasp_ball.py