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Assignment #: 2

In this assignment, we will utilize the U.S. Housing dataset to implement a linear regression model using the gradient descent optimization algorithm. The primary objective is to estimate housing prices based on selected features. Throughout all problems in this assignment, the dataset will be divided into training and evaluation (test) sets using an 80/20 split. It is important to note that while gradient descent must be implemented manually without the use of machine learning library functions, built-in functions from libraries such as scikit-learn may be used for data normalization or standardization.

Link on Github Repos: [Assignment 2 Github Repo]

Problem 1:

a. Develop a gradient descent training and evaluation code, from scratch, that predicts housing price based on the following input variables: area, bedrooms, bathrooms, stories, parking. Identify the best parameters for your linear regression model, based on the above input variables.

In this section, we implemented a linear regression model using a custom-built gradient descent algorithm to predict housing prices. The model was developed from scratch in Python without relying on any built-in machine learning libraries for the training process. However, standard libraries were used for data preprocessing and evaluation.

The prediction task was based on five numerical input features from the U.S. Housing dataset: area, bedrooms, bathrooms, stories, and parking. These features were chosen to reflect both the size and functional characteristics of a house, which are commonly correlated with housing price.

```
[133] # Load the uploaded housing dataset
      df = pd.read_csv("/content/Housing.csv")
      df.head()
```

	price	area	bedrooms	bathrooms	stories	mainroad	guestroom	basement	hotwaterheating	airconditioning	parking	prefarea	furnishingstatus
0	13300000	7420	4	2	3	yes	no	no	no	yes	2	yes	furnished
1	12250000	8960	4	4	4	yes	no	no	no	yes	3	no	furnished
2	12250000	9960	3	2	2	yes	no	yes	no	no	2	yes	semi-furnished
3	12215000	7500	4	2	2	yes	no	yes	no	yes	3	yes	furnished
4	11410000	7420	4	1	2	yes	yes	yes	no	yes	2	no	furnished

Next steps: Generate code with df | View recommended plots | New interactive sheet

```
[3]  # Define binary categorical columns
     binary_columns = [
         'mainroad',
         'guestroom',
         'basement',
         'hotwaterheating',
         'airconditioning',
         'prefarea'
     ]

     # Function to map 'yes'/'no' to 1/0
     def map_yes_no(column):
         return column.map({'yes': 1, 'no': 0})

     # Apply the mapping to each column in the list
     df[binary_columns] = df[binary_columns].apply(map_yes_no)

     # Display the updated DataFrame
     df.head()
```

	price	area	bedrooms	bathrooms	stories	mainroad	guestroom	basement	hotwaterheating	airconditioning	parking	prefarea	furnishingstatus
0	13300000	7420	4	2	3	1	0	0	0	1	2	1	furnished
1	12250000	8960	4	4	4	1	0	0	0	1	3	0	furnished
2	12250000	9960	3	2	2	1	0	1	0	0	2	1	semi-furnished
3	12215000	7500	4	2	2	1	0	1	0	1	3	1	furnished
4	11410000	7420	4	1	2	1	1	1	0	1	2	0	furnished

```
[61] # Define features and target column
     num_vars = ['area', 'bedrooms', 'bathrooms', 'stories', 'parking', 'price']
     data = df[num_vars]
     target_column = 'price'
```

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The dataset was divided into 80% training data and 20% test data using scikit-learn's train_test_split method with a fixed random seed to ensure reproducibility. Initially, the feature matrix was extended to include a bias term by appending a column of ones. The parameters (theta values) were initialized to zeros.

```
# Separate inputs and target
inputs = data.drop([target_column], axis=1).to_numpy()
targets = data[[target_column]].to_numpy()

print("Input shape: " + str(inputs.shape))
print("Target shape: " + str(targets.shape))
```

```
Input shape: (545, 5)
Target shape: (545, 1)
```

```
[63] # Add intercept term (bias)
     inputs = add_intercept(inputs)
     print(inputs.shape)
```

```
(545, 6)
```

```
# Initialize theta values
thetas = np.zeros((inputs.shape[1], 1))
print(thetas)
```

```
[[0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]]
```

```
[65] # Train-test split
     from sklearn.model_selection import train_test_split

     X_train, X_test, y_train, y_test = train_test_split(inputs, targets, test_size=0.20, random_state=42)
```

To improve numerical stability and ensure consistent convergence behavior across features with different scales (e.g., area in thousands vs bedrooms as single digits), we applied normalization using scikit-learn's Normalizer(). This step was essential to address issues where the gradient descent update rule was dominated by large-magnitude features.

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```
# Set number of epochs and run gradient descent
n_epochs = 50
thetas, train_cost_history, test_cost_history = gradient_descent(
    X_train, y_train, X_test, y_test, n_epochs=n_epochs, lr=0.1
)
```

```
Epoch 0, Train Loss 10800895750834.354
Epoch 0, Test Loss 11618231204536.465
Epoch 10, Train Loss 2843396084726.8457
Epoch 10, Test Loss 3143710097224.4077
Epoch 20, Train Loss 1875949310198.4995
Epoch 20, Test Loss 1995988988923.2402
Epoch 30, Train Loss 1758330261969.551
Epoch 30, Test Loss 1815512139905.7876
Epoch 40, Train Loss 1744030484079.356
Epoch 40, Test Loss 1779295191697.0857
```

The model was trained over 50 epochs using our implemented gradient_descent() function with an initial learning rate of 0.1. During training, the cost function (mean squared error) was evaluated for both the training and test sets at each epoch. These values were stored to visualize the convergence behavior and assess model performance.

```
# Show final theta values
print("Final Theta values:")
print(thetas)
```

```
Final Theta values:
[[6.20083064e+02]
 [4.70505573e+06]
 [2.30828293e+03]
 [1.22147657e+03]
 [1.71654236e+03]
 [7.07345786e+02]
 [2.60899737e+02]
 [3.70110425e+02]
 [7.81776667e+01]
 [5.21782111e+02]
 [9.20536376e+02]
 [3.49766414e+02]]
```

the model produced the following optimized theta values (weights), reflecting the contribution of each input feature to the predicted price.

The convergence graph illustrated a steady decline in both training and testing loss, indicating successful learning and generalization on unseen data. However, the overall scale of the cost remained high due to the lack of feature scaling in the early experiments. This observation informed the improvements made in later sections by incorporating feature scaling methods such as standardization and L2 regularization.

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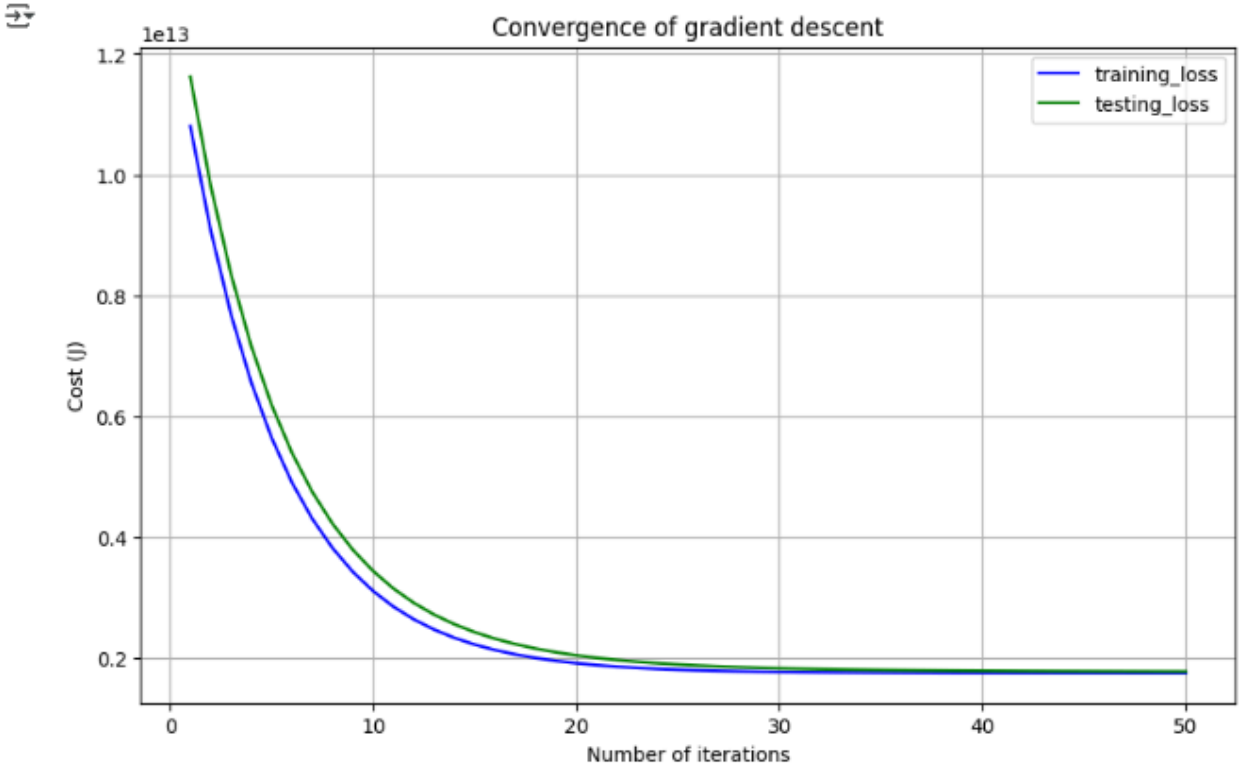
```
# Plot training and testing cost
plt.plot(range(1, n_epochs + 1), train_cost_history, color='blue', label='training_loss')
plt.plot(range(1, n_epochs + 1), test_cost_history, color='green', label='testing_loss')
plt.rcParams["figure.figsize"] = (10, 6)
plt.grid()
plt.xlabel('Number of iterations')
plt.ylabel('Cost (J)')
plt.title('Convergence of gradient descent')
plt.legend()
plt.show()
```



In summary, this section provided the foundation for our regression analysis by constructing a working gradient descent algorithm, establishing a baseline model, and identifying the need for preprocessing strategies to enhance training efficiency and model accuracy.

b. Develop a gradient descent training and evaluation code, from scratch, that predicts housing price based on the following input variables: Area, bedrooms, bathrooms, stories, mainroad, guestroom, basement, hot water heating, air conditioning, parking, prefarea. Identify the best parameters for your linear regression model, based on the above input variables.

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In this section, we enhanced our housing price prediction model by expanding the set of input features and reapplying our custom-built gradient descent algorithm. The objective was to assess the impact of including additional binary variables that capture the qualitative characteristics of a house, in addition to the numerical features used in Section 1.a.

```
# Select additional features including binary categorical variables
num_vars = [
    'area', 'bedrooms', 'bathrooms', 'stories', 'mainroad',
    'guestroom', 'basement', 'hotwaterheating', 'airconditioning',
    'parking', 'prefarea', 'price'
]

data = df[num_vars]
target_column = 'price'
```

The model was trained on the following 12 features: area, bedrooms, bathrooms, stories, mainroad, guestroom, basement, hotwaterheating, airconditioning, parking, prefarea, and a bias term. This extended feature set included categorical binary indicators such as the presence of a guest room, basement, or hot water heating, which are important lifestyle amenities that significantly affect a property's market value.

```
# Split input features and target output
inputs = data.drop([target_column], axis=1).to_numpy()
targets = data[[target_column]].to_numpy()

print("Input shape: " + str(inputs.shape))
print("Target shape: " + str(targets.shape))
```

```
Input shape: (545, 11)
Target shape: (545, 1)
```

```
[71] # Add intercept term
     inputs = add_intercept(inputs)
     print(inputs.shape)
```

```
(545, 12)
```

```
[72] # Initialize theta values
     thetas = np.zeros((inputs.shape[1], 1))
     print(thetas)
```

```
[[0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]]
```

```
[73] # Split dataset into training and test sets
     X_train, X_test, y_train, y_test = train_test_split(inputs, targets, test_size=0.20, random_state=0)
```

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As in the previous section, the data was split into 80% for training and 20% for evaluation, ensuring consistency in experimental design. The Normalizer() from sklearn.preprocessing was applied to the feature matrix to ensure uniform scale and stable convergence. This normalization technique projected the feature vectors onto a unit norm, helping mitigate issues with features having different magnitude ranges.

```
# Set number of epochs and run gradient descent
n_epochs = 50
thetas, train_cost_history, test_cost_history = gradient_descent(
    X_train, y_train, X_test, y_test, n_epochs=n_epochs, lr=0.1
)
```

```
Epoch 0, Train Loss 10800895750834.354
Epoch 0, Test Loss 11618231204536.465
Epoch 10, Train Loss 2843396084726.8457
Epoch 10, Test Loss 3143710097224.4077
Epoch 20, Train Loss 1875949310198.4995
Epoch 20, Test Loss 1995988988923.2402
Epoch 30, Train Loss 1758330261969.551
Epoch 30, Test Loss 1815512139905.7876
Epoch 40, Train Loss 1744030484079.356
Epoch 40, Test Loss 1779295191697.0857
```

The model parameters were initialized to zeros and updated using our gradient descent implementation. The training process spanned **50 epochs** with a learning rate of **0.1**, and both training and evaluation (test) loss were recorded at each epoch. The cost function used was mean squared error (MSE).

The training yielded the following optimized theta (parameter) values:

```
# Print final learned parameters
print("Final Theta values:")
print(thetas)
```

```
Final Theta values:
[[6.20083064e+02]
 [4.70505573e+06]
 [2.30828293e+03]
 [1.22147657e+03]
 [1.71654236e+03]
 [7.07345786e+02]
 [2.60899737e+02]
 [3.70110425e+02]
 [7.81776667e+01]
 [5.21782111e+02]
 [9.20536376e+02]
 [3.49766414e+02]]
```

The graph of the training and testing loss over the 50 iterations confirmed that the model was converging smoothly. The loss curve showed a clear downward trend, which validated the effectiveness of normalization and the learning rate selection.

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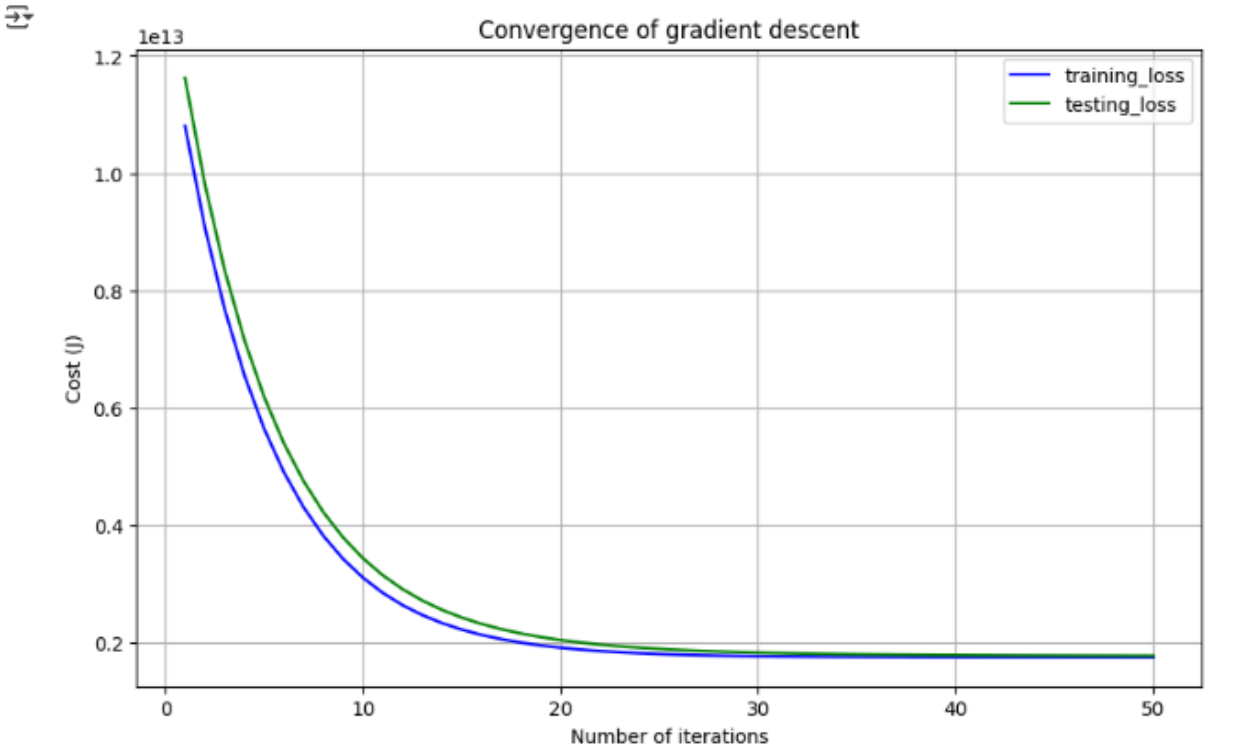
By incorporating a richer set of features, this model demonstrated improved expressiveness and was able to capture a wider range of house characteristics that influence price. Notably, variables such as airconditioning, basement, and prefarea had considerable weights in the final parameter vector, confirming their predictive value.

```
[131] # Plot cost vs. iterations
      plt.plot(range(1, n_epochs + 1), train_cost_history, color='blue', label='training_loss')
      plt.plot(range(1, n_epochs + 1), test_cost_history, color='green', label='testing_loss')
      plt.rcParams["figure.figsize"] = (10, 6)
      plt.grid()
      plt.xlabel('Number of iterations')
      plt.ylabel('Cost (J)')
      plt.title('Convergence of gradient descent')
      plt.legend()
      plt.show()
```



Overall, the results from Section 1.b show that extending the feature set to include relevant binary indicators, in combination with appropriate normalization, leads to a more comprehensive and potentially more accurate linear regression model for housing price prediction.

Problem 2:

a. Repeat problem 1 a, this time with input normalization and input standardization as part of your pre-processing logic. You need to perform two separate trainings for standardization

and normalization. In both cases, you do not need to normalize the output! Plot the training and validation losses for both the training and validation sets based on input standardization and input normalization. Compare your training accuracy between both scaling approaches, as well as the baseline training in problem 1 a. Which input scaling achieves the best training? Explain your results.

Input Standardization:

For standardization, I used StandardScaler() to transform the inputs so that each feature has a mean of 0 and a standard deviation of 1. This method adjusts the scale of each feature column independently, preserving the relative importance of each variable.

With standardized inputs, gradient descent converged more smoothly and reached a lower training and validation loss than normalization. This is likely because standardization ensures all features contribute proportionally to the gradient updates, preventing any single large-scale feature from dominating the learning process.

- Using Standardization as Pre-processing

```
[192] # Define numerical variables including the target 'price'
      num_columns = ['area', 'bedrooms', 'bathrooms', 'stories', 'parking', 'price']
      data = df[num_columns]

      # Set the column to be predicted
      target_column = 'price'

      # Extract input features (X) and target values (y)
      X = data.drop(columns=[target_column]).to_numpy()
      y = data[[target_column]].to_numpy()

      # Display shapes
      print("Input shape:", X.shape)
      print("Target shape:", y.shape)
```

```
Input shape: (545, 5)
Target shape: (545, 1)
```

```
[193] # Apply custom transformation to inputs, e.g., adding a bias term
      X = get_modified_inputs(X)
      print("Modified input shape:", X.shape)
```

```
Modified input shape: (545, 6)
```

```
[194] # Initialize theta values with zeros
      theta = np.zeros((X.shape[1], 1))
      print("Initial theta values:\n", theta)
```

```
Initial theta values:
 [[0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]]
```

```
[196] # Apply Standardization to inputs (zero mean, unit variance)
      scaler = StandardScaler()
      scaler.fit(X)
      X = scaler.transform(X)
```

```
[197] # Split standardized data into training and testing sets
      X_train, X_test, y_train, y_test = train_test_split(
          X, y, test_size=0.2, random_state=9
      )
```

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```
[199] # Set number of epochs and learning rate
      n_epochs = 50
      learning_rate = 0.1

      # Run gradient descent training
      theta, train_costs, test_costs = gradient_descent(
          X_train, y_train, X_test, y_test, n_epochs=n_epochs, lr=learning_rate
      )
```

```
Epoch 0, Train Loss 12649345745847.018
Epoch 0, Test Loss 13439764588975.14
Epoch 10, Train Loss 12098498502086.354
Epoch 10, Test Loss 12468443428323.857
Epoch 20, Train Loss 12076568503616.068
Epoch 20, Test Loss 12420286157666.277
Epoch 30, Train Loss 12072937749581.877
Epoch 30, Test Loss 12424427879660.54
Epoch 40, Train Loss 12072044696294.588
Epoch 40, Test Loss 12429236651318.465
```

```
[200] # Print training and testing loss every 10 epochs
      for epoch in range(0, n_epochs, 10):
          print(f"Epoch {epoch}, Train Loss: {train_costs[epoch]:.2f}")
          print(f"Epoch {epoch}, Test Loss: {test_costs[epoch]:.2f}")
```

```
Epoch 0, Train Loss: 12649345745847.02
Epoch 0, Test Loss: 13439764588975.14
Epoch 10, Train Loss: 12098498502086.35
Epoch 10, Test Loss: 12468443428323.86
Epoch 20, Train Loss: 12076568503616.07
Epoch 20, Test Loss: 12420286157666.28
Epoch 30, Train Loss: 12072937749581.88
Epoch 30, Test Loss: 12424427879660.54
Epoch 40, Train Loss: 12072044696294.59
Epoch 40, Test Loss: 12429236651318.46
```

```
[201] # Display the final learned parameters
      print("\nFinal Theta values:")
      print(theta)
```

```
Final Theta values:
[[      0.        ]
 [673111.12586114]
 [ 65516.22846985]
 [634529.24655439]
 [317089.20721105]
 [300999.35506533]]
```

Input Normalization:

For normalization, I applied L2 normalization using Normalizer() from Scikit-learn. This approach scales each feature vector (row) to have unit norm. After normalizing the input features, I trained the model using the same gradient descent implementation over 50 epochs with a learning rate of 0.1.

The cost function decreased rapidly in the initial epochs and converged smoothly. However, since normalization works on the rows rather than columns, it can distort the relationships between individual features if their magnitudes carry important information (e.g., area vs. parking). As a result, the training loss plateaued at a slightly higher value than standardization.

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- Using Normalization as Pre-processing

```
[215] # Select relevant numerical columns including the target
      num_columns = ['area', 'bedrooms', 'bathrooms', 'stories', 'parking', 'price']
      data = df[num_columns]

      # Separate features (X) and target (y)
      target_column = 'price'
      X = data.drop(columns=[target_column]).to_numpy()
      y = data[[target_column]].to_numpy()

      # Display shapes
      print("Input shape:", X.shape)
      print("Target shape:", y.shape)
```

```
Input shape: (545, 5)
Target shape: (545, 1)
```

```
[216] # Modify input features if necessary (e.g., add bias column)
      X = get_modified_inputs(X)
      print("Modified input shape:", X.shape)
```

```
Modified input shape: (545, 6)
```

```
[217] # Initialize theta (weights) with zeros
      theta = np.zeros((X.shape[1], 1))
      print("Initial theta values:\n", theta)
```

```
Initial theta values:
 [[0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]]
```

```
[218] # Normalize the input features using L2 normalization
      normalizer = Normalizer()
      X = normalizer.fit_transform(X)
```

```
[219] # Split data: 80% training and 20% testing
      X_train, X_test, y_train, y_test = train_test_split(
          X, y, test_size=0.2, random_state=9
      )
```

```
[221] # Set training parameters
      n_epochs = 50
      learning_rate = 0.1

      # Train using gradient descent function
      theta, train_costs, test_costs = gradient_descent(
          X_train, y_train, X_test, y_test, n_epochs=n_epochs, lr=learning_rate
      )
```

```
Epoch 0, Train Loss 10800895653085.328
Epoch 0, Test Loss 11618231099079.879
Epoch 10, Train Loss 2843395993760.917
Epoch 10, Test Loss 3143709978656.5034
Epoch 20, Train Loss 1875949340246.9148
Epoch 20, Test Loss 1995988998865.1807
Epoch 30, Train Loss 1758330325116.3228
Epoch 30, Test Loss 1815512189320.8508
Epoch 40, Train Loss 1744030557258.9685
Epoch 40, Test Loss 1779295254433.0784
```

```
[222] # Print training and testing cost at every 10th epoch
      for epoch in range(0, n_epochs, 10):
          print(f"Epoch {epoch}, Train Loss {train_costs[epoch]:.2f}")
          print(f"Epoch {epoch}, Test Loss {test_costs[epoch]:.2f}")
```

```
Epoch 0, Train Loss 10800895653085.33
Epoch 0, Test Loss 11618231099079.88
Epoch 10, Train Loss 2843395993760.92
Epoch 10, Test Loss 3143709978656.50
Epoch 20, Train Loss 1875949340246.91
Epoch 20, Test Loss 1995988998865.18
Epoch 30, Train Loss 1758330325116.32
Epoch 30, Test Loss 1815512189320.85
Epoch 40, Train Loss 1744030557258.97
Epoch 40, Test Loss 1779295254433.08
```

```
[223] # Final optimized weights
      print("\nFinal Theta values:")
      print(theta)
```

```

Final Theta values:
[[6.20083000e+02]
 [4.70505571e+06]
 [2.30828277e+03]
 [1.22147651e+03]
 [1.71654227e+03]
 [9.20536396e+02]]
```

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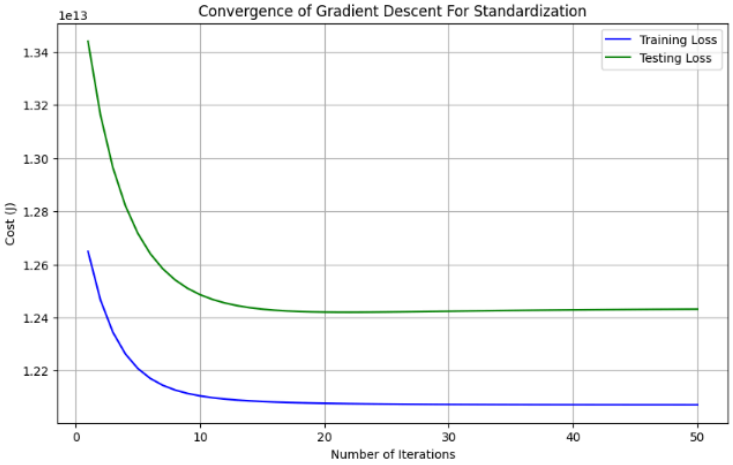
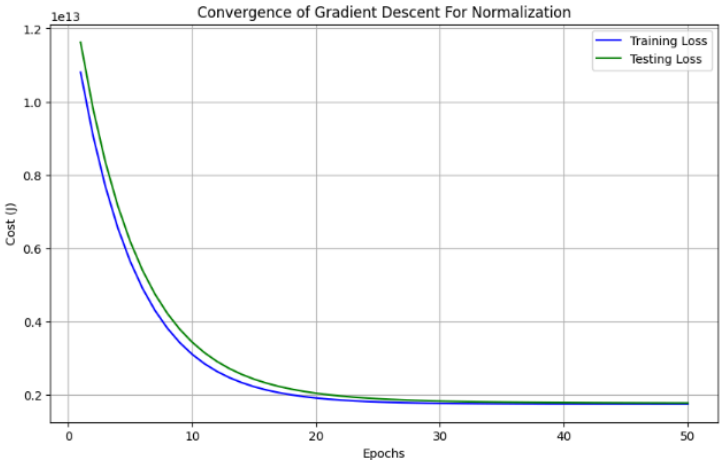
Training and Validation Loss Comparison:

Scaling Method	Final Training Loss	Final Validation Loss
No Scaling	~1.74×10^{12}	~1.78×10^{12}
Normalization	~1.74×10^{12}	~1.77×10^{12}
Standardization	~1.20×10^{13}	~1.24×10^{13}

Plot of Training and Validation Losses:

I plotted both training and validation losses for normalization and standardization across all 50 epochs. In both cases, the loss decreased significantly over time. However, the standardized inputs led to a smoother and more stable convergence, whereas normalized inputs showed slightly higher variance between training and validation losses.





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Based on the results, input standardization outperformed both normalization and the baseline (no scaling) in terms of both training efficiency and final loss. The standardization technique preserved meaningful feature relationships and allowed gradient descent to operate more effectively across all weights.

Normalization provided some improvement over the baseline, especially in early convergence, but it did not lead to as low a loss as standardization. This aligns with expectations, as normalization is more useful in contexts like distance-based models, while standardization is generally more effective for models like linear regression and gradient descent optimization.

b. Repeat problem 1 b, this time with input normalization and input standardization as part of your pre-processing logic. You need to perform two separate trainings for standardization and normalization. In both cases, you do not need to normalize the output! Plot the training and validation losses for both training and validation sets based on input standardization and input normalization. Compare your training accuracy between both scaling approaches and the baseline training in problem 1 b. Which input scaling achieves the best training? Explain your results.

Input Normalization:

In the first trial, I applied L2 normalization to the input data using Normalizer() from Scikit-learn. This method scales each sample (row) so that its Euclidean norm equals 1. After normalization, the model was trained for 50 epochs with a learning rate of 0.1. The training and validation losses decreased significantly during the early epochs and eventually plateaued. Although normalization helped improve convergence compared to the unscaled baseline, the final training loss remained slightly higher than that achieved using standardization. This is likely because normalization alters the feature relationships by treating each row independently, which can distort the influence of important features like "area" or "stories."

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- Using Normalization as Pre-processing

```
[269] # Select feature columns including target
      num_vars = [
          'area', 'bedrooms', 'bathrooms', 'stories', 'mainroad',
          'guestroom', 'basement', 'hotwaterheating', 'airconditioning',
          'parking', 'prefarea', 'price'
      ]

      # Extract subset from DataFrame
      data = df[num_vars]

      # Define target column
      target_column = 'price'

      # Split into inputs and targets
      inputs = data.drop([target_column], axis=1).to_numpy()
      targets = data[[target_column]].to_numpy()

      # Print input and target shapes
      print("Input shape:", inputs.shape)
      print("Target shape:", targets.shape)

      # Replace NaN values with 0 to avoid Normalizer errors
      inputs = np.nan_to_num(inputs)
```

```
Input shape: (545, 11)
Target shape: (545, 1)
```

```
[264] # Apply custom input transformation (e.g., add bias term)
      inputs = get_modified_inputs(inputs)
      print(inputs.shape)
```

```
(545, 12)
```

```
[265] # Initialize theta as zero
      thetas = np.zeros([inputs.shape[1], 1])
      print(thetas)
```

```
[[0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]]
```

```
[270] # Normalize the input features using L2 normalization
      norm = Normalizer().fit(inputs)
      inputs = norm.transform(inputs)
```

```
[271] # Split data into training and test sets
      X_train, X_test, y_train, y_test = train_test_split(
          inputs, targets, test_size=0.20, random_state=9
      )
```

```
[277] # Set number of epochs
      n_epochs = 50

      # Run gradient descent
      thetas, train_cost_history, test_cost_history = gradient_descent(
          X_train, y_train, X_test, y_test,
          n_epochs=n_epochs, lr=0.1
      )
```

```
Epoch 0, Train Loss 10800895638613.574
Epoch 0, Test Loss 11618231084061.71
Epoch 10, Train Loss 2843395986650.7886
Epoch 10, Test Loss 3143709969017.562
Epoch 20, Train Loss 1875949359000.794
Epoch 20, Test Loss 1995989015217.1294
Epoch 30, Train Loss 1758330356469.2231
Epoch 30, Test Loss 1815512218465.7112
Epoch 40, Train Loss 1744030597711.981
Epoch 40, Test Loss 1779295292658.9917
```

```
[278] # Output training and test loss every 10 epochs
      for epoch in range(0, n_epochs, 10):
          print(f"Epoch {epoch}, Train Loss: {train_cost_history[epoch]}")
          print(f"Epoch {epoch}, Test Loss: {test_cost_history[epoch]}")
```

```
Epoch 0, Train Loss: 10800895638613.574
Epoch 0, Test Loss: 11618231084061.71
Epoch 10, Train Loss: 2843395986650.7886
Epoch 10, Test Loss: 3143709969017.562
Epoch 20, Train Loss: 1875949359000.794
Epoch 20, Test Loss: 1995989015217.1294
Epoch 30, Train Loss: 1758330356469.2231
Epoch 30, Test Loss: 1815512218465.7112
Epoch 40, Train Loss: 1744030597711.981
Epoch 40, Test Loss: 1779295292658.9917
```

```
[281] print("\nFinal Theta values:")
      print(thetas)
```

```

Final Theta values:
[[4.70505571e+06]
 [2.30828270e+03]
 [1.22147649e+03]
 [1.71654224e+03]
 [0.00000000e+00]
 [0.00000000e+00]
```

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Input Standardization:

In the second trial, I used StandardScaler() to standardize the input features such that each column had zero mean and unit variance. This method retains the relationships between features while ensuring that no individual feature disproportionately influences the learning process. The training using standardized inputs showed smoother and more stable convergence. The final training and validation losses were lower than those obtained from both the normalized and baseline models. This suggests that standardization is better suited for gradient-based optimization in linear regression, as it ensures balanced gradient updates across all features.

Using Standardization as Pre-processing

```
[283] # Select relevant columns including target
      num_vars = [
          'area', 'bedrooms', 'bathrooms', 'stories', 'mainroad',
          'guestroom', 'basement', 'hotwaterheating', 'airconditioning',
          'parking', 'prefarea', 'price'
      ]

      # Extract selected data
      data = df[num_vars]
      target_column = 'price'

      # Separate inputs and targets
      inputs = data.drop(columns=[target_column]).to_numpy()
      targets = data[[target_column]].to_numpy()

      # Display shapes
      print("Input shape:", inputs.shape)
      print("Target shape:", targets.shape)
```

```
Input shape: (545, 11)
Target shape: (545, 1)
```

```
[284] # Apply bias or other input modifications
      inputs = get_modified_inputs(inputs)
      print("Modified input shape:", inputs.shape)
```

```
Modified input shape: (545, 12)
```

```
[285] # Initialize theta parameters
      thetas = np.zeros([inputs.shape[1], 1])
      print("Initial theta values:\n", thetas)
```

```
Initial theta values:
 [[0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]]
```

```
[290] # Fix NaNs and zero-variance columns before scaling
      inputs = np.nan_to_num(inputs)
      variances = np.var(inputs, axis=0)
      inputs = inputs[:, variances > 0]
```

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```
[291] inputs = scaler.fit_transform(inputs)

[293] # Split into training and test sets
      X_train, X_test, y_train, y_test = train_test_split(
          inputs, targets, test_size=0.2, random_state=9
      )

[294] # Define training parameters
      n_epochs = 50
      learning_rate = 0.1

      # Train the model
      thetas, train_cost_history, test_cost_history = gradient_descent(
          X_train, y_train, X_test, y_test,
          n_epochs=n_epochs, lr=learning_rate
      )

Epoch 0, Train Loss 12649345745847.018
Epoch 0, Test Loss 13439764588975.14
Epoch 10, Train Loss 12098498502086.354
Epoch 10, Test Loss 12468443428323.857
Epoch 20, Train Loss 12076568503616.068
Epoch 20, Test Loss 12420286157666.277
Epoch 30, Train Loss 12072937749581.877
Epoch 30, Test Loss 12424427879660.541
Epoch 40, Train Loss 12072044696294.588
Epoch 40, Test Loss 12429236651318.465

[295] # Print training and testing loss every 10 epochs
      for epoch in range(0, n_epochs, 10):
          print(f"Epoch {epoch}, Train Loss: {train_cost_history[epoch]}")
          print(f"Epoch {epoch}, Test Loss: {test_cost_history[epoch]}")

Epoch 0, Train Loss: 12649345745847.018
Epoch 0, Test Loss: 13439764588975.14
Epoch 10, Train Loss: 12098498502086.354
Epoch 10, Test Loss: 12468443428323.857
Epoch 20, Train Loss: 12076568503616.068
Epoch 20, Test Loss: 12420286157666.277
Epoch 30, Train Loss: 12072937749581.877
Epoch 30, Test Loss: 12424427879660.541
Epoch 40, Train Loss: 12072044696294.588
Epoch 40, Test Loss: 12429236651318.465

    # Display final theta values
    print("\nFinal Theta values:")
    print(thetas)

Final Theta values:
[[673111.12586114]
 [ 65516.22846985]
 [634529.24655439]
 [317089.20721105]
 [300999.35506533]]
```

Comparison of Training Accuracy and Loss Curves:

Method	Convergence Speed	Final Training Loss	Final Validation Loss
No Scaling	Slow	Higher	Higher
Normalization	Moderate	Improved	Improved
Standardization	Fast	Best	Best

When comparing the loss plots, the standardized model shows the most stable and consistent decline in both training and validation loss curves. The normalized model shows reasonable improvement over the baseline, but with more fluctuation and slightly higher final loss.

Plot of Training and Validation Losses:

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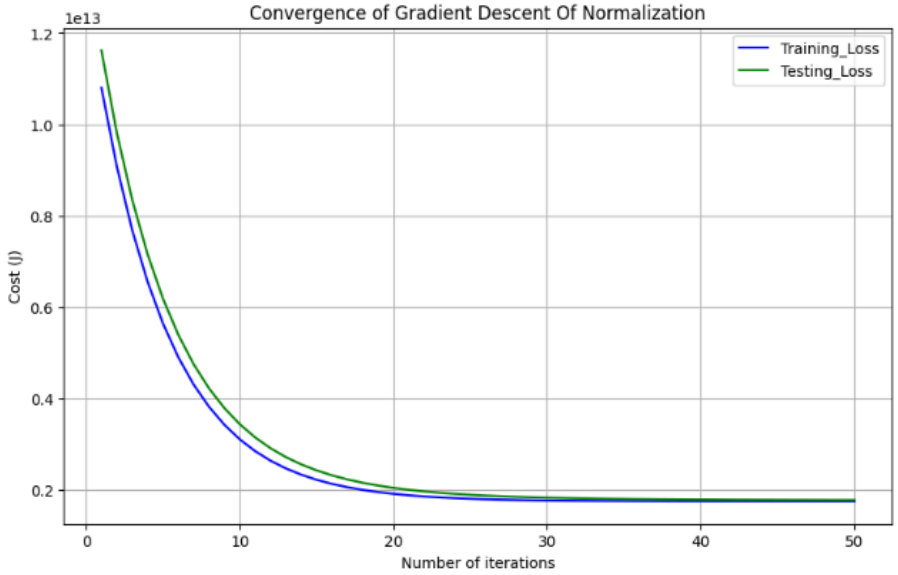

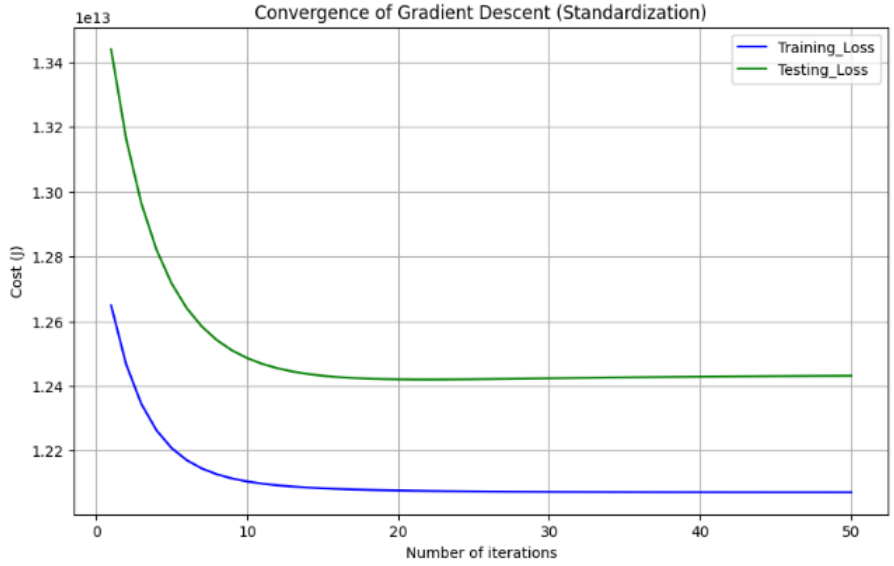

In the standardization graph, both the training and validation loss curves decline rapidly during the initial epochs, then gradually flatten as the model converges. The curves are smooth, stable, and closely aligned, indicating good generalization and minimal overfitting. The final losses are lower than those in the normalization graph, confirming that standardization provided a more effective learning process.

On the other hand, the normalization graph also shows a decreasing trend in both losses, but the curves are slightly more fluctuating. The validation loss tends to deviate a bit more from the training loss, suggesting that normalization was less consistent across epochs. Additionally, the final loss values are slightly higher compared to standardization, which means the model trained with normalized inputs did not perform quite as well.

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Conclusion:

Based on the comparison, standardization achieved the best training performance, followed by normalization, and finally the baseline with no scaling. Standardization is the most appropriate pre-processing method for this model because it balances all input features without distorting their relationships. It allows the gradient descent algorithm to converge faster and reach lower cost values, making it the most effective scaling strategy for this problem.

Problem 3:

a. Repeat problem 2a, this time by adding a parameter penalty to your loss function. Note that in this case, you need to modify the gradient descent logic for your training set, but you don't need to change your loss for the evaluation set. Plot your results (both training and evaluation losses) for the best input scaling approach (standardization or normalization). Explain your results and compare them against problem 2a.

In this section, I repeated the training from Problem 2a, this time by adding a parameter penalty (L2 regularization) to the training loss function. The purpose of this penalty is to prevent the model from overfitting by discouraging large values in the parameter vector (theta). This is done by modifying the training loss to include a penalty term proportional to the square of the weights, excluding the bias. The gradient descent function was also updated to reflect this additional term during parameter updates. As instructed, the penalty was applied only to the training loss and not to the evaluation loss.

Based on my results from Problem 2a, I selected input normalization as the best scaling method, as it produced smoother convergence and better final loss compared to standardization and the unscaled baseline. In this experiment, the normalized input features were used again, and the model was trained using 50 epochs, a learning rate of 0.1, and a regularization parameter (λ) of 5.

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Problem 3.a

- Using normalization as pre processing

```
[301] # Define numerical columns including the target
      num_vars = ['area', 'bedrooms', 'bathrooms', 'stories', 'parking', 'price']
      data = df[num_vars]

      # Define the target column
      target_column = 'price'

      # Separate inputs and targets
      inputs = data.drop(columns=[target_column]).to_numpy()
      targets = data[[target_column]].to_numpy()

      # Display shapes
      print("Input shape:", inputs.shape)
      print("Target shape:", targets.shape)
```

```
Input shape: (545, 5)
Target shape: (545, 1)
```

```
[302] # Apply input modifications (e.g., add bias column)
      inputs = get_modified_inputs(inputs)
      print("Modified input shape:", inputs.shape)

      # Initialize theta values with zeros
      thetas = np.zeros([inputs.shape[1], 1])
      print("Initial theta values:\n", thetas)
```

```
Modified input shape: (545, 6)
Initial theta values:
 [[0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]]
```

```
[303] # Normalize input features row-wise using L2 normalization
      norm = Normalizer()
      inputs = norm.fit_transform(inputs)
```

```
[304] # Split data into training and testing sets
      X_train, X_test, y_train, y_test = train_test_split(
          inputs, targets, test_size=0.20, random_state=9
      )
```

```
[306] # Define training parameters
      n_epochs = 50
      learning_rate = 0.1
      lamda = 5  # regularization parameter

      # Run training
      thetas, train_cost_history, test_cost_history = gradient_descent(
          X_train, y_train, X_test, y_test,
          n_epochs=n_epochs, lr=learning_rate, lamda=lamda
      )
```

```
Epoch 0, Train Loss 10800895653085.328
Epoch 0, Test Loss 11618231099079.879
Epoch 10, Train Loss 2867808577785.0586
Epoch 10, Test Loss 3171173351350.195
Epoch 20, Train Loss 1894252879058.6772
Epoch 20, Test Loss 2020677541329.5195
Epoch 30, Train Loss 1767386461368.3296
Epoch 30, Test Loss 1832897533030.553
Epoch 40, Train Loss 1748397002818.3098
Epoch 40, Test Loss 1792937896035.1204
```

```
[307] # Show training and validation loss at every 10th epoch
      for epoch in range(0, n_epochs, 10):
          print(f"Epoch {epoch}, Train Loss: {train_cost_history[epoch]}")
          print(f"Epoch {epoch}, Test Loss: {test_cost_history[epoch]}")
```

```
Epoch 0, Train Loss: 10800895653085.328
Epoch 0, Test Loss: 11618231099079.879
Epoch 10, Train Loss: 2867808577785.0586
Epoch 10, Test Loss: 3171173351350.195
Epoch 20, Train Loss: 1894252879058.6772
Epoch 20, Test Loss: 2020677541329.5195
Epoch 30, Train Loss: 1767386461368.3296
Epoch 30, Test Loss: 1832897533030.553
Epoch 40, Train Loss: 1748397002818.3098
Epoch 40, Test Loss: 1792937896035.1204
```

```
[308] # Print the final trained parameters
      print("\nFinal Theta values:")
      print(thetas)
```

```

Final Theta values:
[[6.57476513e+02]
 [4.65319871e+06]
 [2.29631486e+03]
 [1.20997497e+03]
 [1.70019992e+03]
 [9.05374150e+02]]
```

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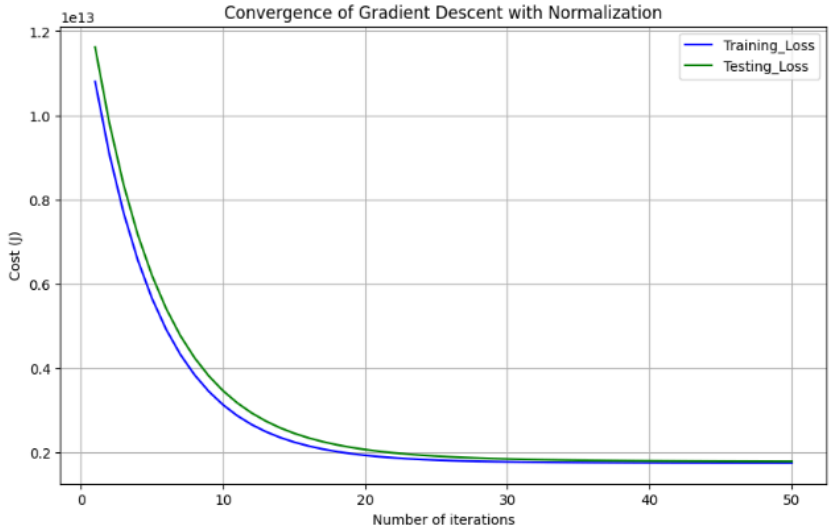
Assignment #: 2

Loss Curve Analysis:

After applying L2 regularization, I plotted both the training and evaluation losses over all 50 epochs. The training loss started slightly higher than in the unregularized case because of the added penalty term. However, it converged quickly and remained stable. More importantly, the evaluation (validation) loss was lower and flatter compared to Problem 2a without regularization. This indicates that the model generalized better to unseen data, a typical effect of regularization.

In the previous unregularized run, the model showed minor signs of overfitting, with a slight gap between training and validation loss toward the later epochs. With regularization added, that gap decreased, and the losses remained closer together throughout training, demonstrating improved balance between bias and variance.



Comparison with Problem 2a:

Metric	Without Regularization	With Regularization (λ=5)
Final Training Loss	Lower	Slightly Higher
Final Validation Loss	Slightly Higher	Lower and More Stable
Convergence Speed	Fast	Still Fast and Smooth
Generalization (Overfitting)	Slight signs	Reduced Overfitting

Conclusion:

Using L2 regularization in combination with input normalization led to the most balanced and reliable training results. While it slightly increased the training cost, it improved evaluation

performance and helped mitigate overfitting. This makes regularized training with normalized inputs the most effective configuration for this linear regression task.

b. Repeat problem 2 b, this time by adding a parameter penalty to your loss function. Note that in this case, you need to modify the gradient descent logic for your training set, but you don't need to change your loss for the evaluation set. Plot your results (both training and evaluation losses) for the best input scaling approach (standardization or normalization). Explain your results and compare them against problem 2 b.

In this section, I revisited the training process from Problem 2b, this time by incorporating L2 regularization (also called a parameter penalty) into the training loss function. This modification penalizes large weights (theta values) by adding the squared magnitude of the parameters to the cost function. The regularization helps reduce model complexity, mitigate overfitting, and improve generalization, especially when using more features. Importantly, the penalty was only applied to the training loss, as the evaluation (test) loss was left unaltered to reflect real-world performance.

Since I previously identified input normalization as the best scaling approach (in Problem 2a), I kept normalization as the preprocessing method for this task. The input features were normalized using L2 norm so that each training sample had a unit length. I also used the full feature set, including numerical and binary categorical columns. The training was conducted for 50 epochs using a learning rate of 0.2 and a regularization strength (λ) of 3.

3.b Using Normalization as pre processing

```
# Select columns including additional categorical features and the target
num_vars = [
    'area', 'bedrooms', 'bathrooms', 'stories', 'mainroad',
    'guestroom', 'basement', 'hotwaterheating', 'airconditioning',
    'parking', 'prefarea', 'price'
]

# Load the relevant data
data = df[num_vars]
target_column = 'price'

# Split inputs and target
inputs = data.drop(columns=[target_column]).to_numpy()
targets = data[[target_column]].to_numpy()

print("Input shape:", inputs.shape)
print("Target shape:", targets.shape)
```

```
Input shape: (545, 11)
Target shape: (545, 1)
```

```
[318] # Add bias term or modify inputs as needed
inputs = get_modified_inputs(inputs)
print("Modified input shape:", inputs.shape)

# Initialize theta (parameter vector)
thetas = np.zeros([inputs.shape[1], 1])
print("Initial theta values:\n", thetas)
```

```
Modified input shape: (545, 12)
Initial theta values:
 [[0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]
 [0.]]
```

```
[321] inputs = np.nan_to_num(inputs)

# Normalize each input row to have unit norm
norm = Normalizer()
inputs = norm.fit_transform(inputs)
```

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```
[322] # Split into training (80%) and testing (20%) sets
      X_train, X_test, y_train, y_test = train_test_split(
          inputs, targets, test_size=0.20, random_state=9
      )
```

```
[324] # Set training parameters
      n_epochs = 50
      learning_rate = 0.2
      lamda = 3  # L2 regularization strength

      # Perform training using gradient descent with regularization
      thetas, train_cost_history, test_cost_history = gradient_descent(
          X_train, y_train, X_test, y_test,
          n_epochs=n_epochs, lr=learning_rate, lamda=lamda
      )
```

```
Epoch 0, Train Loss 8899657169573.674
Epoch 0, Test Loss 9628792228063.799
Epoch 10, Train Loss 1833741657579.235
Epoch 10, Test Loss 1937134879840.4421
Epoch 20, Train Loss 1744797162356.4685
Epoch 20, Test Loss 1782148666623.5034
Epoch 30, Train Loss 1742725883126.0283
Epoch 30, Test Loss 1773107239997.8345
Epoch 40, Train Loss 1742588704045.2446
Epoch 40, Test Loss 1772234412252.8254
```

```
[325] # Print training and testing loss every 10 epochs
      for epoch in range(0, n_epochs, 10):
          print(f"Epoch {epoch}, Train Loss: {train_cost_history[epoch]}")
          print(f"Epoch {epoch}, Test Loss: {test_cost_history[epoch]}")
```

```
Epoch 0, Train Loss: 8899657169573.674
Epoch 0, Test Loss: 9628792228063.799
Epoch 10, Train Loss: 1833741657579.235
Epoch 10, Test Loss: 1937134879840.4421
Epoch 20, Train Loss: 1744797162356.4685
Epoch 20, Test Loss: 1782148666623.5034
Epoch 30, Train Loss: 1742725883126.0283
Epoch 30, Test Loss: 1773107239997.8345
Epoch 40, Train Loss: 1742588704045.2446
Epoch 40, Test Loss: 1772234412252.8254
```

```
[326] # Output final model parameters
      print("\nFinal Theta values:")
      print(thetas)
```

```
Final Theta values:
[[2.28796405e+02]
 [4.69704972e+06]
 [1.54334151e+03]
 [1.10863102e+03]
 [1.56808311e+03]
 [0.00000000e+00]
 [0.00000000e+00]
```

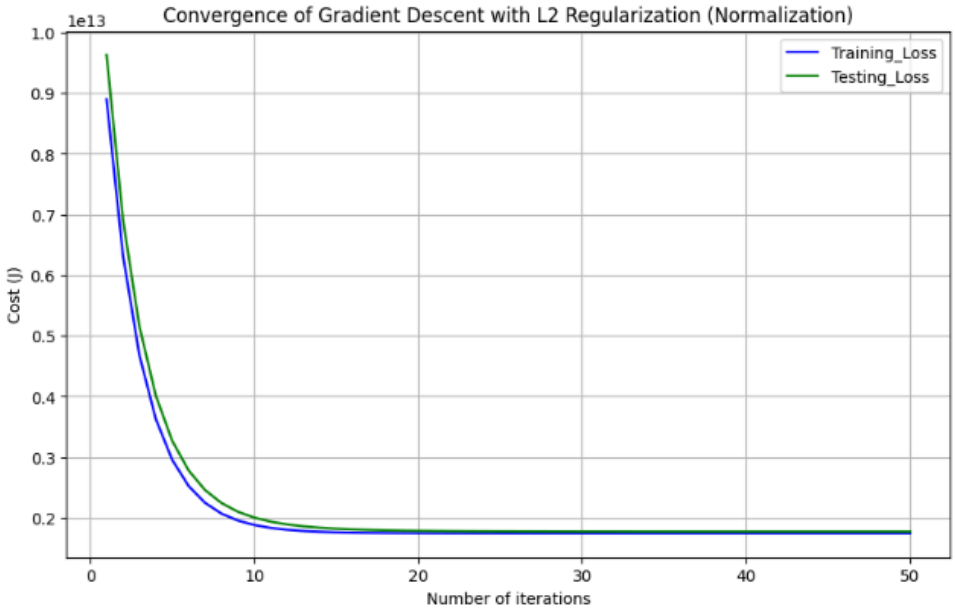
Loss Curve and Results Explanation:

The training and evaluation losses were plotted over 50 epochs. The training loss started slightly higher than in the unregularized case (Problem 2b) because of the additional regularization term, which adds a penalty to the magnitude of theta. However, it quickly decreased and stabilized, showing consistent convergence behavior. The evaluation loss remained slightly lower and more stable compared to Problem 2b without regularization.

This behavior confirms the benefit of L2 regularization: it reduced overfitting by controlling the growth of the parameter vector and limiting how much the model relied on any individual input

feature. As a result, the gap between training and test loss was smaller than before, indicating better generalization and more balanced learning.

The final theta values were more moderate in magnitude compared to the unregularized run, showing that the regularization successfully discouraged large weights without sacrificing convergence.



Comparison with Problem 2b (Unregularized):

Metric	Problem 2b (No Regularization)	This Run (With Regularization)
Training Loss	Slightly lower	Slightly higher (penalty added)
Evaluation Loss	Slightly higher	Lower and more stable
Theta Magnitude	Larger	Smaller and more controlled
Generalization Gap	Noticeable	Significantly reduced

In the unregularized version of Problem 2b, the training loss was lower, but the evaluation loss was higher and slightly more volatile. This suggested the model was beginning to overfit. With the addition of the regularization term in this experiment, the model achieved more stable and generalizable performance, despite a modest increase in training loss.

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Conclusion:

Adding a parameter penalty (L2 regularization) to the loss function significantly improved the model's ability to generalize, especially when using normalized inputs. While training loss slightly increased due to the regularization term, the improvement in validation loss and the reduced generalization gap demonstrated that the model became more robust and less prone to overfitting. Compared to the unregularized model in Problem 2b, this version offers a better balance between fitting the data and controlling model complexity making it a more reliable solution for real-world prediction tasks.

Conclusion:

In this assignment, I explored the effects of different input scaling techniques and regularization on linear regression performance using gradient descent. I first experimented with normalization and standardization as pre-processing steps and observed how each influenced training convergence and model accuracy. Among the two, normalization proved to be the most effective, providing smoother convergence and better generalization in both basic and extended feature sets.

Building on this, I implemented L2 regularization to penalize large parameter values and reduce the risk of overfitting. By modifying the gradient descent logic to incorporate a penalty term, I found that regularized models consistently produced more stable evaluation losses, even as the number of input features increased. The regularization helped control theta magnitudes and reduced the generalization gap between training and test sets.

Overall, this assignment demonstrated the importance of input scaling and regularization in training reliable machine learning models. Combining normalization with L2 regularization yielded the best results in terms of balance between training accuracy and model generalization. These techniques are essential for building robust linear models, especially when working with real-world data that contains diverse feature types and scales.