0.1 Even and odd functions

- **Even functions**: symmetric through where $x = 0$

The function $f : \mathbb{R} \to \mathbb{R}$ is an even function and only if:

$$f(-x) = f(x), \quad \forall x \in \mathbb{R}$$

Let $f(x)$ be an integrable even function. Then,

$$\int_{-a}^{0} f(x) \,\mathrm{d}x = \int_{0}^{a} f(x) \,\mathrm{d}x, \quad \forall a \in \mathbb{R}$$

and therefore

$$\int_{-a}^{a} f(x) \,\mathrm{d}x = \int_{0}^{a} f(x) \,\mathrm{d}x, \quad \forall a \in \mathbb{R}$$

Moreover, if $\int_0^\infty f(x) \,\mathrm{d}x$ exists, then

$$\int_{-\infty}^{0} f(x) \,\mathrm{d}x = \int_{0}^{\infty} f(x) \,\mathrm{d}x$$

and

$$\int_{-\infty}^{\infty} f(x) \,\mathrm{d}x = 2 \int_{0}^{\infty} f(x) \,\mathrm{d}x$$

- **Odd functions**: symmetric and "flip" through where $x = 0$

The function $f : \mathbb{R} \to \mathbb{R}$ is an odd function and only if:

$$f(-x) = -f(x), \quad \forall x \in \mathbb{R}$$

Let $f(x)$ be an integrable even function. Then,

$$\int_{-a}^{a} f(x) \,\mathrm{d}x = 0, \quad \forall a \in \mathbb{R}$$

Moreover, if $\int_0^\infty f(x) \,\mathrm{d}x$ exists, then

$$\int_{-\infty}^{\infty} f(x) \,\mathrm{d}x = 0$$