1.2 Brief review of integration

Let $f : \mathbb{R} \to \mathbb{R}$ be an integrable function. Recall that $F(x)$ is the antiderivative of $f(x)$ if and only if $F'(x) = f(x)$, i.e.,

$$F(x) = \int f(x)dx \quad \Leftrightarrow \quad F'(x) = f(x)$$

Theorem 1.4. (Fundamental Theorem of Calculus.)

Let $f(x)$ be a continuous function on the interval $[a, b]$, and let $F(x)$ be the antiderivative of $f(x)$. Then

$$\int_a^b f(x)dx = F(x)|_a^b = F(b) - F(a)$$

Theorem 1.5. (Integration by parts.)

Let $f(x)$ and $g(x)$ be continuous functions. Then

$$\int f(x)g(x)dx = F(x)g(x) - \int F(x)g'(x)dx$$

Where $F(x) = \int f(x)dx$ is the antiderivative of $f(x)$. For definite integrals,

$$\int_a^b f(x)g(x)dx = F(b)g(b) - F(a)g(a) - \int_a^b F(x)g'(x)dx$$

Theorem 1.6. (Integration by substitution.)

Let $f(x)$ be an integrable function. Assume that $g(u)$ is an invertible and continuously differentiable function. The substitution $x = g(u)$ changes the integration variable from x to u as follows:

$$\int f(x)dx = \int f(g(u))g'(u)du$$

For definite integrals,

$$\int_a^b f(x)dx = \int_{g^{-1}(a)}^{g^{-1}(b)} f(g(u))g'(u)du$$

Examples:

$$\int \ln(1 + x)dx =$$