# 02 Useful sums with interesting proofs

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \quad 1.$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \quad 2.$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2 \quad 3.$$

We can prove them using induction. For example, with the first equation:

- If $n = 1$, both sides are equal to 1
- Assuming that the equation holds for $n$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \quad 4.$$

- Applying the equation to $n + 1$ means

$$\sum_{k=1}^{n+1} k = \frac{(n+1)(n+2)}{2} \quad 5.$$

- We can transform the right side:

$$\begin{aligned} \frac{(n+1)(n+2)}{2} &= \frac{n(n+1)+2(n+1)}{2} \\ &= \frac{n(n+1)}{2} + (n+1) \end{aligned} \quad 6.$$

- The left side can also be transformed:

$$\sum_{k=1}^{n+1} k = \sum_{k=1}^{n} +(n+1) \quad 7.$$

- Combining Equation 6 and Equation 7, we see that it's the same as adding $n + 1$ to both sides of Equation 4:

$$\sum_{k=1}^{n} k + (n+1) = \frac{n(n+1)}{2} + (n+1) \quad 8.$$