The Aging Tax on Potential Growth in Asia *

Quang-Thanh Tran †

January 09, 2022

Preliminary and Incomplete.
Please do not cite or circulate.

Abstract

Population aging is becoming a prominent issue in Asia, especially for developing countries where demographic changes have asserted a downward pressure on the rate of growth. This paper refers to such potential unwanted effects as an "aging tax" and analytically examines them from a neoclassical perspective, using a Diamond-type overlapping generations model with endogenous retirement, survival rate, and old-age productivity. Based on this setup, negative impacts exist if too many old workers who are sufficiently unproductive choose to defer retirement under the aging pressure, which drains resources from future generations. Numerical simulations show that an aging tax can reduce the potential per capita growth rate (technology-adjusted) by up to 0.12 percentage points annually for some countries in Asia. Our results highlight that countries with sufficiently large labor shares (due to a high ratio of self-employment or manual labor-centric production) and inadequate educational attainment are potentially the most sensitive and vulnerable to population aging.

Keywords: Overlapping Generations Model, Population Aging, Old Worker Productivity

JEL Classification: E20, E27, J11, J26

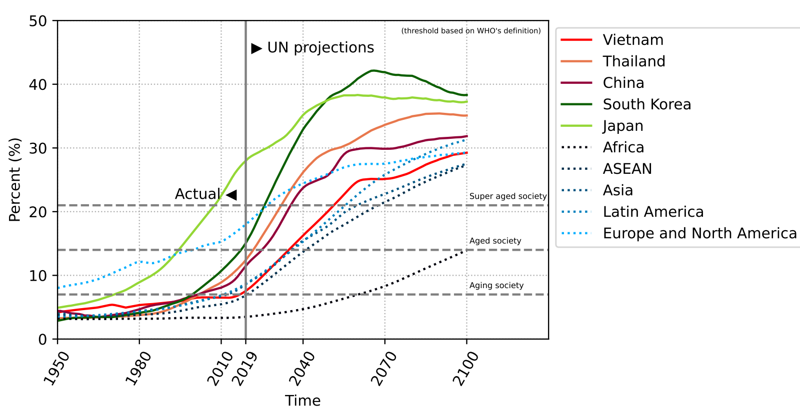
*This work was supported by JST SPRING, Grant Number JPMJSP2114. The author thanks Kitagawa Akiomi and Akita Jiro for their helpful comments and suggestions.

†Graduate School of Economics and Management, Tohoku University. (Email: tran.quang.thanh.p1@dc.tohoku.ac.jp)

1 Introduction

Recent demographic changes reflected in declining birth rates and rising life expectancy, also known as population aging, are becoming a major issue for many countries in Asia. Fig. 1 shows that Asia is currently and will continue to be the most rapidly aging region. In a historical context, rapid aging is not unprecedented, as various "Asian miracles" countries, such as Japan, Singapore, and South Korea, have faced a comparable demographic transition. However, what puts current developing nations in a riskier position than those "Asian miracles" is that, before rapid population aging begins, they have a relatively lower level of educational attainment accompanied by a higher life expectancy. From here, we consider the following three implications. First, since life expectancy is already high, there will be a drastic surge in the number of workers getting old, so it may drain a substantial amount of resources from future generations. Second, due to limited education, the current working population may not be productive enough and will become a burden if they work longer. Third, many of these countries still rely on cheap labor, agriculture, and self-employment, which means that the real adjusted labor share can be very high. At such a stage, growth tends to be sensitive to demographic changes. Thus, we postulate that the current state of rapid demographic change may slow down growth for various economies in Asia, especially developing countries. To what extent this potentially negatively affects growth and which key factors determine such an impact are the central questions for which this paper tries to find answers.

Figure 1: Percentage of Population Aged over 64. Data from United Nations (2019)



The overlapping generations model (OLG) developed by Diamond (1965) provides an essential tool to study the effects of demography on growth. However, due to its simplicity, extensions are needed to capture more essential factors. In this model, we employ the survival rate featured in Fanti and Gori (2008) and d'Autume (2003) to capture the change in life expectancy, and endogenous retirement choice from Hu (1979) and Michel and Pestieau (2013) to study how aging alters individuals' working preferences. Although these key features are necessary for a typical analysis in an OLG framework with population aging, one aspect that has not been considered thoroughly is old worker productivity. We argue that this is one of the most important elements that differentiate the state of aging in Asia currently from those in the past. In the papers of Miyazaki (2019) and Cipriani and Fioroni (2018), the old worker productivity is considered a constant parameter relative to the young worker productivity. While making the analysis less complicated, it implies that a rise in life expectancy, which is often associated with better living conditions, has no effect on productivity. We think that such an assumption is too strong and may underestimate the potential effects of population aging. Based on Aísa et al. (2012), we developed a more detailed treatment of old worker productivity. In particular, old-age productivity should depend (positively) on life expectancy and educational attainment when young. If an individual gets healthier and has better living conditions, he or she should be able to work longer upon getting old, and hence has higher productivity than the previous generations with less favorable conditions, as shown by Bound et al. (1999) and Bielecki et al. (2016). Related to the educational aspect, Verhaeghen and Salthouse (1997) and Goldin and Katz (2007) empirically show that knowledge (often reflected as educational attainment in the model) is one of the key factors that determine old age productivity since education-based productivity tends to stay steady in later life, despite gradual declines in cognitive and physiological components. We will later formulate a functional form for old worker productivity that captures these points.

In the next section, we present the theoretical setup for the model. In Section 3, we analyze the equilibrium dynamics and derive the steady-state solutions. In Section 4, we produce theoretical results regarding the effects of population aging on retirement and capital growth. Here, we identify the main factors that determine how severe the aging tax can be. Finally, in Section 5, we perform numerical simulations to calculate the aging tax on growth for selected countries in Asia, using the official projections (United Nations, 2019) toward 2055.

2 The Model

2.1 Household

Time is discrete and runs forever. The population grows at a constant rate $n \in (-1, 1)$, so that at each time t, $N_t = (1+n)N_{t-1}$ new young agents are born with homogeneous preferences. Each agent lives for 2 periods, each of which has a length of unity. All agents work inelastically in the first period of life, but only a portion $\pi \in (0, 1]$ survive to the second period. The survivors are then considered old and see their productivity decline. They work for a portion $(1-l)$ of the total time endowment, then retire, and die.

An individual born at t is assumed to have the following lifetime utility:

$$U_t = \ln(c_t) + \pi\beta \ln(d_{t+1}) + \pi\gamma \ln(l_{t+1}) \tag{1}$$

where c_t and d_{t+1} are consumption during the first and second period, while $l_{t+1} \in [0, 1]$ is the portion of time for retirement, thus making $(1 - l_{t+1})$ the portion of time agents work after getting old. Parameters β and γ are subjective discount factors weighting future consumption and leisure from retirement respectively. In addition, $\pi \in (0,1]$ is the survival rate, calibrated as life expectancy over the model's maximum length. As people live longer, a higher survival rate π implies higher utility from future consumption and retirement.

The budget constraint for the first period is:

$$c_t + s_t = (1-\tau)w_t \tag{2}$$

where s_t, w_t, and τ are saving, wage and social contribution tax rates, respectively.

The budget constraint for the second period is:

$$d_{t+1} = \frac{R_{t+1}}{\pi} s_t + (1-l_{t+1})(1-\tau)\varepsilon w_{t+1} + p_{t+1} l_{t+1} \tag{3}$$

where R_{t+1} is the interest rate factor (assuming a full depreciation rate per period), and p_{t+1} is the pension benefit. Similar to Blanchard (1985), we assume that there exists a perfect annuity market. Financial intermediaries in the economy take the uncertain survival rate into account and embed it into the real interest rate payment. Thus, a higher survival rate means lower rewards for those who made it into the second period. The second term is the after-tax income earned from working in the second period of life. Here, parameter ε governs the productivity of the old relative to that of the young (henceforth referred to as old-age productivity), so $\varepsilon \in [0,1]$. In reality, wages normally stop increasing when an individual reaches a certain age and then decrease due to a decline in senile productivity.

In defining the old-age productivity ε, based on Aísa et al. (2012)'s original work, we propose the following functional form:

$$\varepsilon = \pi^{1-\eta} \tag{4}$$

where $\eta \in [0,1]$ is the Education Index measured by the United Nations (UNDP, 2020). In this way, it implies that old-age productivity depends on 2 factors: life expectancy and education. Given the same age range, old workers who have healthier living conditions and higher life expectancy (higher π) must be more productive (higher ε) than the previous generation's old workers with lower quality of life. In addition, if people are better educated in their early life, their productivity should not degrade intensively when they get old. We choose the Education Index to calibrate η because it is a universally reasonable indicator for education, and ranges between 0 and 1 so that the value of ε stays true to our assumption.

After maximizing Eq. (1) with respect to Eqs. (2), (3) and (4), the agent arrives at the following optimal solution:

$$d_{t+1} = \beta R_{t+1} c_t \tag{5}$$

$$\frac{l_{t+1}}{\gamma} \leq \frac{R_{t+1} c_t}{(1-\tau)\varepsilon w_{t+1} - p_{t+1}} \tag{6}$$

(with equality holds if $l_{t+1} < 1$, otherwise $l_{t+1} = 1$)

The intertemporal consumption allocation is standard. Eq. (6) helps us anticipate that l tends to decrease in an aging environment. So long as the equality holds in equilibrium, a lower fertility rate reduces the prospective pension, which forces old workers to defer retirement. A longer lifespan (reflected by a higher π) also increases the average old-age productivity ε, which incentivizes old workers to stay longer in the job market.

2.2 Production and Government

We employ a standard Cobb-Douglas production function with labor-augmenting technological progress. Inputs are physical capital and skilled workers. There is a single good produced by a representative firm.

$$Y_t = F(K_t, E_t H_t) = K_t^{\alpha}(E_t.H_t)^{1-\alpha}$$

where $\alpha \in (0,1)$ is the capital share, K is the aggregate physical capital, E is the labor-augmenting technology and H represents aggregated skilled workers. Following Bils and Klenow (2000), we assume that individuals accumulate skills by spending time studying at school. Such a process is: $H_t = L_t.e^{\varphi z}$, where L is the amount of unskilled labor, φ represents the returns from education (Mincer, 1974), and z is the mean years of schooling.

Let g be the exogenous technological growth rate, we can rewrite the production in terms of per effective skilled worker (denoted by a tilde):

$$\tilde{y}_t = \tilde{k}_t^{\alpha}$$

where:

$$A_t = E_t e^{\varphi z}, \quad E_{t+1} = (1+g)E_t, \quad \tilde{y}_t = \frac{Y_t}{A_t L_t}\ , \quad \tilde{k}_t = \frac{K_t}{A_t L_t}$$

By this setup, the production follows standard literature but is expressed at the technology adjusted level. The Inada conditions: $\lim_{\tilde{k}\to+\infty} f'(\tilde{k}) = 0$ and $\lim_{\tilde{k}\to 0} f'(\tilde{k}) = +\infty$ are imposed to secure a balanced growth path and nonzero equilibrium. The problem for the firm is to maximize its profit as follows.

$$\max_{K_t, L_t} F(K_t, A_t L_t) - R_t K_t - w_t L_t \tag{7}$$

Under competitive factor markets, the first-order conditions are:

$$R_t = f'(\tilde{k}_t) = \alpha \tilde{k}_t^{\alpha-1} \tag{8}$$

$$w_t = A_t\left[f(\tilde{k}_t) - \tilde{k}_t f'(\tilde{k}_t)\right] = (1-\alpha)A_t \tilde{k}_t^{\alpha} \tag{9}$$

To close the model, we assume that the government runs a pay-as-you-go (PAYG) social security system that balances its budget every period. At any time t, the government's behavior is characterized as follows.

$$\tau w_t \left[N_t + (1-l_t)\pi\varepsilon N_{t-1}\right] = \pi p_t l_t N_{t-1} \tag{10}$$

where the left-hand side is the amount of tax the government receives from both young and old workers, and the right-hand side indicates the total pension it has to pay the retirees born in $t-1$.

3 Market Equilibrium and Dynamics

In this model, at each period t, the labor force consists of all the young agents born at t and a portion of old workers born at $t-1$. Therefore, the labor market clears according to:

$$L_t = \left[1 + \frac{(1-l_t)\pi\varepsilon}{1+n}\right] N_t \tag{11}$$

While older people are allowed to work in the second period of life, they do not save. Thus, the capital formation in period t comprises of only the savings from the young. For convenience, let us use the superscript y to denote the per effective young and skilled worker term, such that:

$$\tilde{k}_t^{\,y} = \frac{K_t}{A_t N_t} = \tilde{k}_t \left[1 + \frac{(1-l_t)\pi\varepsilon}{1+n}\right] \tag{12}$$

The capital market clears according to:

$$K_{t+1} = K^y_{t+1} = N_t s_t \tag{13}$$

Dividing both sides to $A_{t+1}N_{t+1}$ to get:

$$\tilde{k}^y_{t+1} = \frac{s_t}{(1+n)(1+g)A_t} \tag{14}$$

We use Eq. (12) to convert capital back in terms of per effective skilled worker.

$$\tilde{k}_{t+1} = \tilde{k}^y_{t+1}\left[\frac{1+n}{1+n+(1-l_{t+1})\pi\varepsilon}\right] = \frac{s_t}{(1+g)[1+n+(1-l_{t+1})\pi\varepsilon]A_t} \tag{15}$$

We now have sufficient ingredients to define the market equilibrium.

Definition 1 (Intertemporal Equilibrium)**.** Given the initial capital per effective skilled worker $\tilde{k}_1 = \frac{s_0}{(1+n)(1+g)A_0}$, social security contribution tax τ, old workers' productivity ε, fertility rate n, technological growth g and interperiod survival rate π, and a competitive equilibrium is a sequence of factor prices $\{r_t, w_t\}_{t=1}^{\infty}$; a sequence of allocations for agent's consumption when young, saving, consumption when old and retirement choice $(d_1, l_1\{c_t, s_t, d_{t+1}, l_{t+1}\}_{t=1}^{\infty})$; a sequence of allocations for firms' production $\{K_t, L_t\}_{t=1}^{\infty}$ and a sequence of social security benefits $\{p_t\}_{t=1}^{\infty}$ such that:
1. With $\tilde{k}_t = K_t/A_tL_t$, factor prices are determined by Eqs. (8) and (9).

2. Given the factor prices $\{w_1, r_1\}$, social security contribution tax rate τ and benefit p_1, (d_1, l_1) solves the following problem for the initial old:

$$\max_{d_1,l_1} U_1 = \pi\beta\ln(d_1) + \pi\gamma ln(l_1)$$
$$\text{s.t.} \quad d_1 = \frac{R_1}{\pi}s_0 + (1-l_1)(1-\tau)\varepsilon w_1 + l_1 p_1$$
$$1 - l_1 \geq 0$$

And $(c_t, s_t, d_{t+1}, l_{t+1})_{t=1}^{\infty}$ solves generation t's decision problem in Eq. (1) subject to Eqs. (2), (3) and (4).
3. Given factor prices $\{w_t, r_t\}_{t=1}^{\infty}$: $(K_t, L_t)_{t=1}^{\infty}$ solve the firm's problem in Eq. (7).
4. The goods market clears according to the feasibility constraint:

$$N_t c_t + N_{t-1}\pi d_t + K_{t+1} \leq F(K_t, A_t L_t)$$

5. The government budget is balanced according to Eq. (10).
6. The labor market clears according to Eq. (11).
7. The capital market clears according to Eq. (13).

Since a household's optimal retirement choice governed by Eq. (6) has more than one possible solution, there exist multiple types of equilibria. Under a specific condition, the economy can converge to a full-retirement equilibrium (such that $l = 1$) or a partial retirement equilibrium (such that $l < 1$). In the former, agents do not work in the second period of life, so the dynamics are identical to the Diamond model. However, as population aging intensifies, agents tend to defer retirement and work longer in the second period of life. As a result, the latter case becomes more likely in our research (and the simulations, later on, confirm this assertion). Therefore, we focus on the partial retirement equilibrium state and use it to analyze the effects of population aging.

Before assessing how population aging affects individuals' choices and the economy in general, let us find the steady-state capital per effective skilled worker, based on Definition 1. As the feasibility constraint holds for every period by the Walras' Law, the dynamics of capital is as follows.

Proposition 1. For any generation t, if the following condition is satisfied:

$$\tau < \frac{\beta\pi\varepsilon(1-\alpha) - (1+n)\gamma\alpha}{(1-\alpha)\left[(1+n)(\gamma+\beta) + \beta\pi\varepsilon\right]}$$

Then the equilibrium retirement choice is characterized by:

$$l_{t+1} = \frac{(1-\tau)[(1+n)\alpha + \pi\varepsilon]\gamma + (1+n+\pi\varepsilon)\left[(1-\alpha)\beta + \gamma\right]\tau}{\pi\varepsilon\left[(1-\alpha)\beta + \gamma\right]} < 1 \tag{16}$$

so that agents choose to retire only partially in the second period of life and decide to work for a portion of $(1 - l_{t+1})$.

The dynamics of capital per effective skilled worker are as follows:

$$\tilde{k}_{t+1} = \frac{\pi\alpha\left[(1-\alpha)\beta+\gamma\right]}{(1+g)\left[(1+n)(1+\pi\beta+\pi\gamma)\alpha+\pi\varepsilon(1+\pi\beta\alpha)\right]}\tilde{k}_t^{\alpha} \tag{17}$$

Thus, there exists a nonzero steady-state capital per effective skilled worker:

$$\tilde{k}_{ss} = \left[\frac{\pi\alpha\left[(1-\alpha)\beta+\gamma\right]}{(1+g)\left[(1+n)(1+\pi\beta+\pi\gamma)\alpha+\pi\varepsilon(1+\pi\beta\alpha)\right]}\right]^{\frac{1}{1-\alpha}} \tag{18}$$

Proof: See A.

Note that in this closed-form solution, the tax rate τ does not appear in the steady state. In other words, as long as an individual's optimal choice is to work for a proportion of time in the second period, the PAYG pension should not distort capital accumulation. What it does in our model is to determine which equilibrium state (full or partial retirement) prevails and to affect the redistribution of wealth. From Eq. (18), we can now analyze the effects of demographic aging (a decline in birth rate n and a rise in life expectancy π) on the economy and which factors play the central roles.

4 Theoretical Results: Impacts of Population Aging

First, let us examine how population aging affects individuals' retirement choices. From Eq. (16), it is easy to see that $\partial l/\partial n$ is positive and $\partial l/\partial \pi$ is negative. Consequently, the combination of a rise in life expectancy and a fall in birth rate always leads to a lower l, meaning that people will choose to work longer and delay retirement.

This incentive comes from the fact that agents will put more weight on the utility gained from the second-period consumption knowing that they will live longer. In addition, a higher survival rate reduces the extra benefits from uncertain mortality, so that other sources of income become more important. The decline in newborn agents also indicates that there will be fewer young workers contributing to the social security funds. Financial pressure forces old agents to prolong their working time in their second period of life. This trend has been observed to be true for the cases of Japan and South Korea, the 2 leading economies and also among the most rapidly aging societies in Asia. With similar Asian demographic cultures and immigration rules, it is reasonable to assume that many developing countries in Asia will soon follow the same path as they experience population aging.

However, the effect of demographic changes on capital is more complicated. On the one hand, $\partial k/\partial n$ is negative. Hence, a lower fertility rate induces higher capital per effective skilled worker via capital dilution. On the other hand, the effect of an increase in survival rate is rather ambiguous under our model's specification. For instance, suppose that ε is a constant and does not change over time regardless of population aging, we can see that $\partial k/\partial \pi$ will always be positive (since $\pi \in (0,1), \varepsilon \in (0,1), n \in (0,1)$, $\beta < 1$). Thus, it appears that an increase in the survival rate or life longevity surely increases savings and capital, as concluded in former research such as Prettner (2013) and Cipriani and Fioroni (2018). However, as we explained before, such an assumption may not always be reasonable, especially in developing countries where demography is in the transitional phase. With a more complex functional form of old worker productivity as indicated in Eq. (4), the effects of population aging can be explained more thoroughly.

Proposition 2. The effect of an increase in survival rate π (life expectancy) on capital $\tilde{k}$ is ambiguous if old-age productivity is nonlinear and takes the following form: $\varepsilon = \pi^{1-\eta}$ where $\pi, \eta \in (0,1)$. Specifically:

$$\frac{\partial \tilde{k}_{ss}}{\partial \pi} \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} 0 \Leftrightarrow \alpha(1+n) - (1-\eta)\pi^{2-\eta} - \eta\alpha\beta\pi^{3-\eta} \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} 0 \tag{19}$$

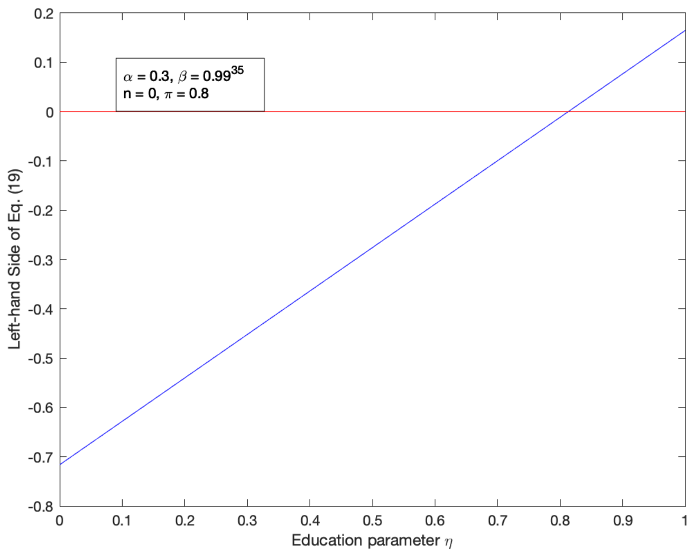
Proof: see Appendix B.

A straightforward observation from condition (19) is the influence of capital share α. If this parameter is smaller than a certain threshold, specifically $\frac{(1-\eta)\pi^{2-\eta}}{1+n+\eta\beta\pi^{3-\eta}}$, then the increase in life expectancy may hamper capital growth. As aging occurs in countries where production depends heavily on human labor (hence a small α), these economies have to spend more resources to pay the extra old workers, which results in a slower rate of capital accumulation. This feature is particularly true for many developing countries in Asia (such as Vietnam) where production mainly relies on cheap manual labor and/or the agriculture sector. Furthermore, due to a large portion of the self-employed population, the labor share can be relatively higher than reported. As a result, changes in the demographic structure are expected to have large impacts on the economy. On the other hand, if a country has a sufficiently high capital share α, production is relatively less sensitive to demographic changes, and the effects of population aging on potential growth are limited. In some cases, as we will show in our simulation, aging can even boost capital growth because it fosters more savings, and such an extra savings increase is proportionally larger than the addition of old workers.

Education also plays an important role here because it partially determines how productive old workers become. As it is difficult to point out directly from Eq. (19), we rely on the numerical assessment described in Fig. 2. Here, parameters α and β take the standard values of 0.3 and 0.99^{35}, the survival rate π is reasonably set at 0.8. Moreover, we let $n = 0$ to isolate the influence of population growth. The figure illustrates the relationship between different values of η and the left-hand side of Eq. (19).

We can see that if the education parameter η is high enough, the left-hand side of Eq. (19) (the blue line) is always positive, which means that a rise in life expectancy

Figure 2: Role of Education η in Population Aging



Notes: If η is located at the region where the blue line lies above the red line, the left-hand side of Eq. (19) is > 0, indicating a positive effect of aging on capital stock. Otherwise, the relationship is negative.

will boost capital accumulation. On the other hand, a sufficiently low education status implies an opposite effect. We conclude that under a specific demographic environment, the higher the education level is, the less likely population aging will have negative effects on growth. Note that by setting η to 1, the left-hand side is always positive; hence, population aging does not negatively affect per worker capital growth as long as the population is not decreasing. In general, retirement postponement may seem to be an optimal choice for individuals facing population aging, but the aggregate effect on growth is not guaranteed to be positive. In this regard, our theoretical result concurs with Matsuyama (2008), but we show more explicitly the factors that influence such an outcome, which are labor share and old-age productivity.

Let us explain this point further. In the model, we assume that less-educated workers suffer a greater decline in old age productivity than their highly educated counterparts. Since population aging naturally leads to retirement deferment, the labor force will expand with the extra participation of old workers. Such an addition in the labor force demands a larger amount of capital, which depends on young workers' savings. How much that amount can increase in turn depends on how much the wage being paid increases. If the education level is high enough, old workers are sufficiently productive (reflected in the efficient wage) so that when their wages are added up with the young workers', it results in a higher overall wage paid per effective skilled worker unit. Such a rise in wages has an income effect, which allows a net increase in capital per effective skilled worker. On the other hand, if the education level is sufficiently low, the additional old workers joining the labor force turn out to be a drag that pulls down the overall wage per effective skilled worker. This is also known as the "team effect". When young workers are put to work together with sufficiently unproductive old workers, they have to spend more time and resources helping the old. Consequently, the output may not be well maintained as before, which causes a reduction of the overall wage (in efficiency unit) and an insufficient savings increase. As a result, the capital per effective skilled worker must fall. Nevertheless, when we combine aging with fertility, the total effect can be ambiguous. Assume that an increase in lifespan reduces capital accumulation, while a decrease in the fertility rate facilitates capital dilution, even if the latter prevails and guarantees a positive capital growth, the existence of rapid population aging, by itself has already asserted a drag on the accumulation process, making the economy unable to reach its maximal potential growth rate. This can be vital for many developing countries in Asia where the growth rate is one of their most important targets.

5 Numerical Simulations

This section aims to simulate the effects of the demographic changes on several Asian economies. We restrict our model to countries that are running a PAYG social security program and are projected to have young population decreases in the next 35 years. The selected countries for our simulation are Vietnam (VNM), Sri Lanka (LKA), Thailand (THA), China (CHN), South Korea (KOR), and Japan (JPN). It is reported that these are the top 6 most rapidly aging countries in Asia [1].

5.1 Calibration

The benchmark fertility rate and life expectancy follow the official forecast (United Nations, 2019) (medium scenario). Holding the fertility rate unchanged, we generate different aging scenarios based on different magnitudes of change in life expectancy, which is used as a proxy to calibrate the survival rate π (d'Autume, 2003). The baseline projection of a change in life expectancy is called "moderate aging" (MA). Depending on the demographic status of each country, "fast aging" (FA) (more rapid change in life expectancy than MA) and "slow aging" (SA) (slower change in life expectancy than MA) are generated.

Since many developing countries in Asia retain a large portion of the self-employed population, we adopt the adjusted labor share retrieved from Gollin (2002) and Guerriero (2019). The calibration of labor-augmenting productivity growth g follows Jones (1997)'s assumption, where the long-run technological growth is assumed to be the same for all countries (also known as the "technological frontier"). This is a reasonable assumption given the current trend of globalization and technological transfer by

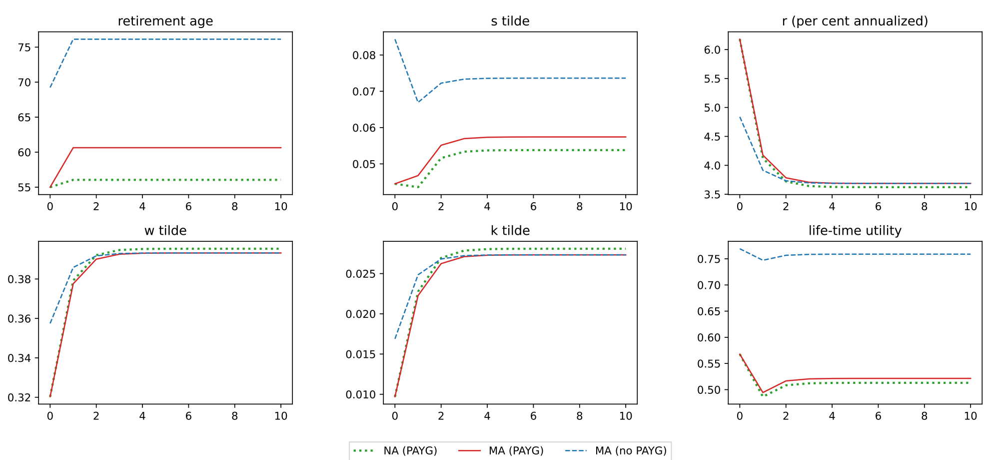
[1] https://www.thejakartapost.com/news/2018/02/14/how-asias-population-is-aging-2015-2030-scenario.html

various multinational firms in Asia. Thus, according to Jones (2002), the estimation of global technological frontier growth is 7.5% annually (taking the US economy as the frontier) so that an individual country has a yearly growth rate of $7.5 - \delta$ (%) where δ is the annual depreciation rate, obtained from Feenstra et al. (2015). We also assume that each year of schooling increases the wage rate by 10%, implying $\varphi = 0.1$ (Psacharopoulos, 1994). Parameter β follows the standard literature of 0.99^T where T is the length of the second period (35 years in real life). Parameter η follows the human capital data obtained from UNDP (2020). The social security contribution tax rate τ is based on ISSA (2019). Perhaps the most difficult parameter to estimate is the weight on leisure from retirement (γ) as there is little empirical research globally on this variable, especially for developing countries. A common practice is to use an estimation for the US by Kitao (2014), where $\gamma = 0.5123\beta$. However, in some cases, we adjust this parameter in an *ad hoc* manner so that the retirement choice in the model resembles the effective retirement age in reality. The detailed calibration for all countries is reported in Table 1.

5.2 The Results

We first use the Vietnamese economy as an example to show how rapid population aging affects growth. The dynamics of selected variables are shown in Fig. 3. The young population growth rate is fixed at medium fertility in every scenario. All variables are technology-adjusted.

Figure 3: Effects of Population Aging for Vietnam



NA(PAYG): No aging scenario (unchanged life expectancy) with the PAYG pension system. MA represents the benchmark moderate aging scenario

Let us analyze the role of pensions in the context of aging. Without a pension plan, individuals value working in the second period more seriously and want to delay retirement as long as possible. A lower interest rate and mortality during the aging process also discourage people from saving. In other words, without social security,

Table 1: Calibrated Parameters

	Description	VNM	LKA	THA	CHN	KOR	JPN
α	capital share	0.198*	0.14**	0.22**	0.41**	0.204*	0.275*
τ	social security tax	0.22	0.10	0.07	0.28	0.09	0.183
β	subjective discount	0.99	0.99	0.99	0.99	0.99	0.99
γ	leisure weight	0.38	0.38	0.38	0.38	0.30	0.21
g	technological growth	1.07	0.43	0.52	1.22	1.22	2.08
η	education index	0.64	0.74	0.68	0.66	0.86	0.85
n	young population growth	1.21	0.41	0.59	0.60	0.37	-0.12
π	survival rate	0.58	0.63	0.63	0.63	0.80	0.85
ε	old-age productivity	0.82	0.89	0.86	0.85	0.97	0.98
	life expectancy	75.4	76.98	77.15	76.91	83.03	84.63
	retirement age***	55	55	60	54	72	70
	demographic type****	LD	LD	PD	PD	PD	PD
		Forecast toward 2055 (35-year period)					
n	young population growth	-0.09	-0.09	-0.29	-0.29	-0.44	-0.35
	life expectancy *****						
	moderate aging (MA)	80.39	82.65	83.02	82.5	87.52	88.68
	slow aging (SA)	78.39	80.65	81.02	80.5	86.52	87.68
	fast aging (FA)	82.39	84.65	85.02	84.5	88.52	89.68

Notes:

* Gollin (2002)

** Guerriero (2019)

*** The average effective retirement age. Souces: VNM[2], CHN[3], KOR & JPN (OECD, 2018); for LKA and THA we rely on the legal retirement age as a benchmark since official data are not available.

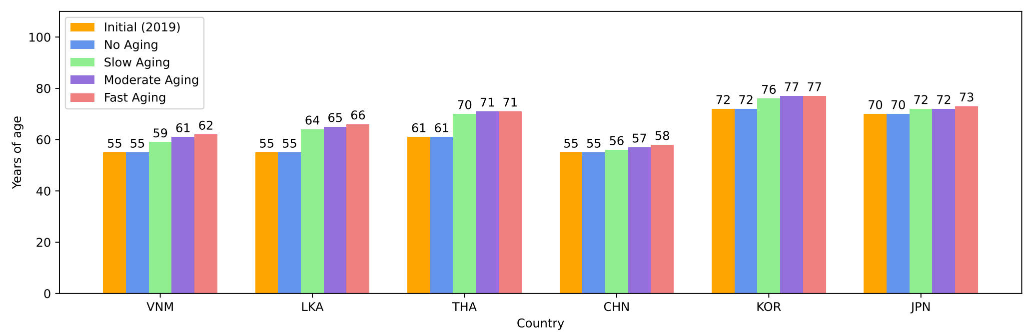
**** The classification of demographic dividends follows Salgado (2017). "LD", and "PD" are late- and post-demographic dividend respectively. PD means a total fertility rate 30 years earlier below 2.1 and a shrinking working-age population share over the subsequent 15 years or a shrinking absolute working-age population. LD implies a total fertility rate 30 years earlier above 2.1 and a shrinking working-age population share over the subsequent 15 years.

***** In the MA case, life expectancy follows the medium fertility scenario forecast (United Nations, 2019). In SA, it equals the MA case - 2 (for LD) or - 1 (for PD), whereas in FA, it equals the MA case + 2 (for LD) or + 1 (for PD).

working longer is a better choice to finance old age, which enables individuals to reduce savings and enjoy higher consumption when young. Such behavior is translated into a higher overall attainable utility in the no-pension case compared to the PAYG case. However, since the capital per worker in both cases is the same, we postulate that the justification of a PAYG pension system in an aging environment eventually boils down to social preference, and the long-run effect on economic growth can be seen as inessential. It is thus important to see and compare how the economy performs in a moderate-aging environment relative to a non-aging environment. Consistent with earlier analysis, population aging encourages people to work longer (to be retired optimally at age 61) and to save more. However, due to a sufficiently high labor share and not so high educational attainment, the extra old workers are sufficiently unproductive and proven to be a burden for such an economy. The technology-adjusted wages and capital per effective worker are lower in the aging case compared to the non-aging counterpart because of the "team effect". Overall, under the current status, aging reduces potential growth for the Vietnamese economy. In that sense, an aging tax is used to describe this gap between the dynamics of $\tilde{k}$ in the case of aging versus non-aging.

From the case of Vietnam, we now expand the analysis to other Asian economies. Fig. 4 predicts at what age people want to retire in the coming years. The initial values predicted by the model match closely with the observed average retirement age reported in Table 1. Looking further, we can see that the Asian population will stay in the job market longer in the future due to aging. As a result, raising the legal retirement age is unlikely to face opposition from the public. Note that the desire to work is stronger in countries with lower social security contribution taxes (such as Sri Lanka or Thailand), while in more socialist countries, such as Vietnam and China, the incentive to work is weaker due to the relatively generous pension schemes. In more aged societies such as Japan and South Korea, the changes are less dramatic but remain among the highest in Asia and the world.

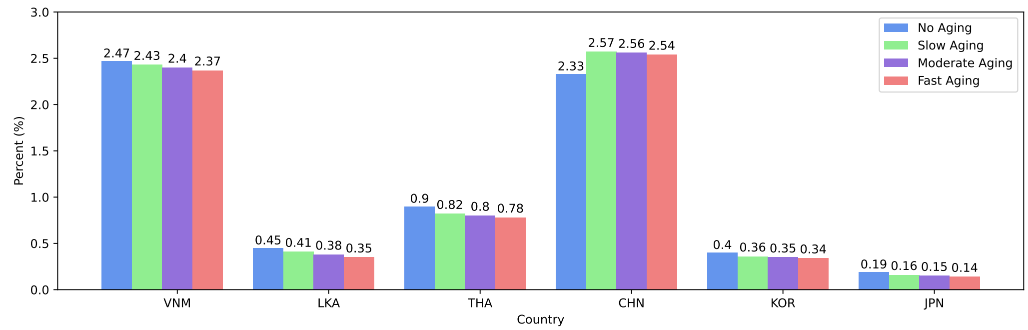
Figure 4: Effects of Population Aging on Retirement Choice



We then examine the growth rate of capital per effective skilled worker (technology-adjusted) under aging. Fig. 5 reports the annualized growth rate after the demographic change (annualized) under different aging scenarios. It is obvious that for the majority of countries, the capital growth rate is higher without population aging. In developing countries such as Sri Lanka, Vietnam, and Thailand, the current education level is not high enough to guarantee the generation of sufficiently productive old workers. Thus, the “team effect” occurs that drives down growth. Moreover, the fact that these countries have relatively high labor shares (having accounted for self-employment) is likely a contributor to the negative effect. Indeed, looking at Japan and South Korea, where the levels of educational attainment are so high that the decline in old worker productivity is negligible, aging still negatively affects capital accumulation. This reduction in growth is largely due to a high labor share since the production is sensitive to structural changes in the demography.

Our simulation raises an interesting exception: China. Here, population aging is beneficial for growth. A comparison with Thailand’s economy generates interesting insights. The two countries share fairly similar life expectancy, population growth rates, and educational attainment. However, when aging speeds up, China sees a higher rate of capital growth while Thailand suffers. Such a difference must be due to

Figure 5: Effects of Population Aging on Capital's Growth per effective skilled worker (annualized)

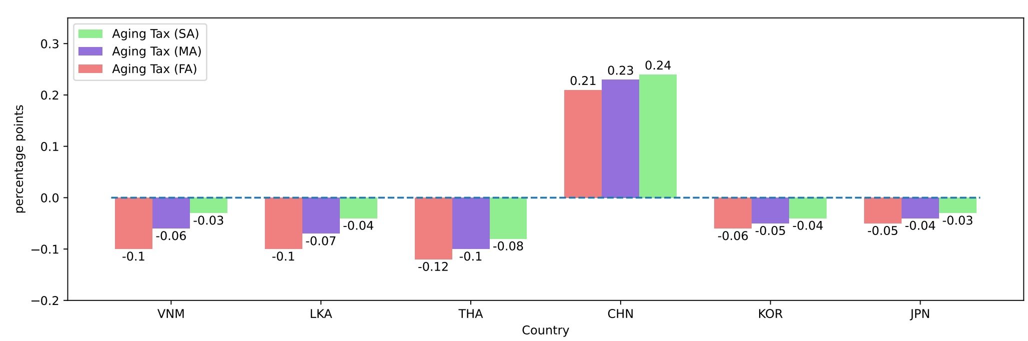


Notes: The rate of growth is calculated as $\nu_{1,i} = [(\tilde{k_{1,i}}/\tilde{k_{0,i}})^{-1/35} - 1] \times 100(\%)$ where $i \in$ {'no aging', 'slow aging', 'moderate aging', 'fast aging'}

the labor share. In China, the labor share is essentially lower than that of Thailand, so that the production is not too sensitive to structural changes in worker composition. Furthermore, the relatively low labor share also frees up more resources for capital accumulation in China, while old workers in Thailand appear to create a burden because production relies too heavily on human labor.

From this simulation, we begin to calculate the aging tax on growth for different Asian economies by taking the difference in the technology-adjusted capital growth rate (annualized) between the aging and non-aging cases. The results are reported in Fig. 6.

Figure 6: Aging Tax on Capital's Growth per effective skilled worker (annualized)



Note: The aging tax is calculated as $\nu_{1,j} - \nu_{1,\text{'no aging'}}$ where $j \in$ {'slow aging', 'moderate aging', 'fast aging'}.

As a benchmark case, moderate population aging (medium mortality) can reduce the growth rate of capital per effective skilled worker by as little as 0.04 percentage points (South Korea) to as much as 0.10 percentage points (Thailand). This decline in the potential growth rate is particularly crucial for developing countries such as Vietnam, Sri Lanka, and Thailand. As the young generation stops increasing in shares, population aging becomes a burden on capital accumulation. With limited resources and income, these countries may face difficulties in allocating resources to improve the economic structure (raising capital share in production) and workers' education (to limit the decline in the productivity of the future old workers). If population aging is faster than the benchmark case (the fast aging scenario), the reduction in growth can be even larger, at greater than 0.10 percentage points annually for many countries. In contrast, with sufficiently high educational attainment and low labor share, population aging has the potential to boost China's capital growth by 0.23 percentage points annually.

6 Conclusion

In this paper, we use a Diamond-type overlapping generations model to analyze the long-run economic growth for some Asian economies with a PAYG pension system. The model is developed to incorporate endogenous retirement and a reasonable calibration of old-age productivity. Under the current projection of demographic trends, we conclude that population aging hurts economic growth most severely in countries with a sufficiently high labor income share (self-employment adjusted) and low educational attainment. Meanwhile, for exceptional cases (such as China) where production is not very sensitive to human labor, population aging can benefit growth by allowing more productive old workers to join the labor force.

Our calibration and the numerical simulation show that population aging in Asia may reduce the annualized growth rate of capital per effective skilled worker by .06 to .10 percentage points for developing economies and approximately 0.05 percentage points for advanced economies. If aging accelerates more rapidly than the moderate scenario, such a potential reduction in growth can reach higher values, ranging from 0.10 to 0.12 percentage points. On another note, these results are comparable to a study by Otsu and Shibayama (2016) where the authors concluded a 0.55 percentage-point reduction in per capita growth for Asia. The difference in findings is because our paper allows individuals to work upon getting old and focuses on the team effect generated by sufficiently unproductive old workers. Furthermore, we analyzed the selected countries individually, instead of through a stylized economy. In general, our model can be regarded as a more complete treatment.

Several extensions from this model are possible. First, as we are assuming individuals work inelastically in both periods, a more realistic approach is to augment labor input by hours worked. Thus, the agents will optimize not only the retirement age but also the hours to be worked in a given time endowment (Hansen and Lønstrup, 2009). One of the most obvious improvements is to endogenize individuals' fertility and choice of education. Such a model has been developed by de la Croix and Matthias (2003). Applying these characteristics can make the model more realistic. One can postulate that when fertility decreases, a larger portion of resources is now liberated to be invested in educational attainment, which in turn can improve old worker productivity and reduce the impact of the team effect. The boost in education can also be enabled by introducing productive government expenditures.

References

Aísa, R., Pueyo, F., Sanso, M., 2012. Life expectancy and labor supply of the elderly. Journal of Population Economics 25, 545–568.

Bielecki, M., Goraus, K., Hagemejer, J., Tyrowicz, J., 2016. Decreasing fertility vs increasing longevity: Raising the retirement age in the context of ageing processes. Economic Modelling 52, 125–143.

Bils, M., Klenow, P.J., 2000. Does schooling cause growth? American economic review 90, 1160–1183.

Blanchard, O.J., 1985. Debt, deficits, and finite horizons. Journal of political economy 93, 223–247.

Bound, J., Schoenbaum, M., Stinebrickner, T.R., Waidmann, T., 1999. The dynamic effects of health on the labor force transitions of older workers. Labour Economics 6, 179–202.

Cipriani, G.P., Fioroni, T., 2018. Endogenous demographic change, retirement, and social security. Macroeconomic Dynamics 22, 609–631.

d'Autume, A., 2003. Ageing and retirement age. what can we learn from the overlapping generations model? 84, 13–20.

de la Croix, D., Matthias, D., 2003. Inequality and growth: Why differential fertility matters. American Economic Review 93, 1091–1113.

Diamond, P.A., 1965. National debt in a neoclassical growth model. The American Economic Review 55, 1126–1150.

Fanti, L., Gori, L., 2008. Longevity and payg pension systems sustainability. Economics Bulletin 10, 1–8.

Feenstra, R.C., Inklaar, R., Timmer, M.P., 2015. The next generation of the penn world table. American economic review 105, 3150–3182.

Goldin, C., Katz, L., 2007. Long-run changes in the u.s. wage structure: Narrowing, widening, polarizing.

Gollin, D., 2002. Getting income shares right. Journal of political Economy 110, 458–474.

Guerriero, M., 2019. The labor share of income around the world: Evidence from a panel dataset , 39–79.

Hansen, C.W., Lønstrup, L., 2009. The optimal legal retirement age in an olg model with endogenous labour supply .

Hu, S.C., 1979. Social security, the supply of labor, and capital accumulation. The American Economic Review 69, 274–283.

ISSA, 2019. Social security programs throughout the world: Asia and the Pacific, 2018. Technical Report. International Social Security Association.

Jones, C., 2002. Introduction to economic growth. W.W. Norton, New York.

Jones, C.I., 1997. On the evolution of the world income distribution. Journal of Economic Perspectives 11, 19–36.

Kitao, S., 2014. Sustainable social security: Four options. Review of Economic Dynamics 17, 756–779.

Matsuyama, K., 2008. A one-sector neoclassical growth model with endogenous retirement. The Japanese Economic Review 59, 139–155.

Michel, P., Pestieau, P., 2013. Social security and early retirement in an overlapping-generations growth model. Annals of Economics and Finance 14, 705–719.

Mincer, J., 1974. Schooling, Experience, and Earnings. volume 29. Natonal Bureau of Economic Research.

Miyazaki, K., 2019. Optimal pay-as-you-go social security with endogenous retirement. Macroeconomic Dynamics 23, 870–887.

OECD, 2018. Ageing and employment policies—statistics on average effective age of retirement. OECD Publishing.

Otsu, K., Shibayama, K., 2016. Population aging and potential growth in asia. Asian Development Review 33, 56–73.

Prettner, K., 2013. Population aging and endogenous economic growth. Journal of population economics 26, 811–834.

Psacharopoulos, G., 1994. Returns to investment in education: A global update. World development 22, 1325–1343.

Salgado, R., 2017. 2. asia: At risk of growing old before becoming rich?, in: Regional Economic Outlook, April 2017, Asia and Pacific: Preparing for Choppy Seas. International Monetary Fund.

UNDP, 2020. Human Development Report 2020. 2020 ed., United Nations.

United Nations, 2019. World Population Prospects, The 2019 Revision - Volume I: Comprehensive Tables. United Nations New York.

Verhaeghen, P., Salthouse, T.A., 1997. Meta-analyses of age–cognition relations in adulthood: Estimates of linear and nonlinear age effects and structural models. Psychological bulletin 122, 231–249.

A Proof of Proposition 1

A.1 Generation t

A representative agent's problem:

$$\max_{c_t, d_{t+1}, l_{t+1}} U_t := \ln(c_t) + \beta\pi \ln(d_{t+1}) + \gamma\pi \ln(l_{t+1})$$

$$\text{s.t. } (1-\tau)w_t + \frac{\pi}{R_{t+1}}\left[(1-l_{t+1})(1-\tau)\varepsilon w_{t+1} + l_{t+1}p_{t+1}\right] - c_t - \frac{\pi}{R_{t+1}}d_{t+1} = 0 \tag{20}$$

$$1 - l_{t+1} \geq 0$$

The KKT first-order conditions:

$$d_{t+1} = \beta R_{t+1} c_t \tag{21}$$

$$\frac{\gamma}{l_{t+1}} \geq \frac{(1-\tau)\varepsilon w_{t+1} - p_{t+1}}{R_{t+1}c_t} \tag{22}$$

with equality holds if $l_{t+1} < 1$

Suppose that:

$$\gamma < \frac{(1-\tau)\varepsilon w_{t+1} - p_{t+1}}{R_{t+1}c_t} \tag{23}$$

so that $l_{t+1} < 1$ and the economy converges to a partial retirement equilibrium. Plugging Eqs. (21) and (22) with equality holding into Eq. (20), we get:

$$c_t + \pi\beta c_t = (1-\tau)w_t + \frac{\pi}{R_{t+1}}\left[(1-l_{t+1})(1-\tau)\varepsilon w_{t+1} + l_{t+1}p_{t+1}\right]$$

$$\Rightarrow c_t = \frac{1-\tau}{1+\pi\beta+\pi\gamma}\left[w_t + \frac{\pi\varepsilon w_{t+1}}{R_{t+1}}\right] \tag{24}$$

Thus, the saving function becomes:

$$s_t = (1-\tau)w_t - c_t = \frac{(1-\tau)\pi}{1+\pi\beta+\pi\gamma}\left[(\beta+\gamma)w_t - \frac{\varepsilon w_{t+1}}{R_{t+1}}\right] \tag{25}$$

Plugging Eq. (24) into Eq. (22) (equality holds) to derive l_{t+1} as follows.

$$l_{t+1} = \frac{\gamma(1-\tau)}{1+\pi\beta+\pi\gamma}\left[\frac{w_t R_{t+1} + \pi\varepsilon w_{t+1}}{(1-\tau)\varepsilon w_{t+1} - p_{t+1}}\right] \tag{26}$$

From the budget balancing at Eq. (10), we have:

$$(1-\tau)\varepsilon w_{t+1} - p_{t+1} = w_{t+1}\left[\varepsilon - \frac{\tau(1+n+\pi\varepsilon)}{\pi l_{t+1}}\right]$$

Plugging back into Eq. (26), we get:

$$l_{t+1} = \frac{\gamma\pi(1-\tau)[w_t \frac{R_{t+1}}{w_{t+1}} + \pi\varepsilon]}{\pi\varepsilon(1+\pi\beta+\pi\gamma)} + \frac{\tau(1+n+\pi\varepsilon)}{\varepsilon\pi} \tag{27}$$

Retirement choice now depends only on w_t, R_{t+1} and w_{t+1}, similar to the saving function at Eq. (25). Using factor prices at Eqs. (8) & (9), we have:

$$w_t \frac{R_{t+1}}{w_{t+1}} = (1-\alpha)A_t \tilde{k}_t^{\alpha} \frac{\alpha \tilde{k}_{t+1}^{\alpha-1}}{(1-\alpha)(1+g)A_t \tilde{k}_{t+1}^{\alpha}} = \frac{\alpha \tilde{k}_t^{\alpha}}{(1+g)\tilde{k}_{t+1}} \tag{28}$$

$$\frac{w_{t+1}}{R_{t+1}} = \frac{(1-\alpha)(1+g)A_t \tilde{k}_{t+1}^{\alpha}}{\alpha \tilde{k}_{t+1}^{\alpha-1}} = \frac{(1-\alpha)(1+g)A_t \tilde{k}_{t+1}}{\alpha} \tag{29}$$

Plugging Eq. (28) into Eq. (27), l_{t+1} can be expressed in terms of $\tilde{k}_t$ and $\tilde{k}_{t+1}$:

$$l_{t+1} = \frac{(1-\tau)\gamma\pi\alpha}{\varepsilon\pi(1+g)(1+\pi\beta+\pi\gamma)} \frac{\tilde{k}_t^{\alpha}}{\tilde{k}_{t+1}} + \left[\frac{(1-\tau)\gamma\pi}{1+\pi\beta+\pi\gamma} + \frac{\tau(1+n+\pi\varepsilon)}{\varepsilon\pi}\right] \tag{30}$$

Similarly, plugging Eqs. (9), (29) into the saving function Eq. (25), s_t can also be expressed in terms of $\tilde{k}_t$ and $\tilde{k}_{t+1}$:

$$s_t = \frac{(1-\tau)(1-\alpha)\pi A_t}{\alpha(1+\pi\beta+\pi\gamma)} \left[(\beta+\gamma)\alpha\tilde{k}_t^{\alpha} - \varepsilon(1+g)\tilde{k}_{t+1}\right] \tag{31}$$

With Eqs. (30) and (31), the capital accumulation at Eq. (15) becomes:

$$\tilde{k}_{t+1} = \frac{\pi\alpha\left[(1-\alpha)\beta+\gamma\right]}{(1+g)\left[(1+n)(1+\pi\beta+\pi\gamma)\alpha + \pi\varepsilon(1+\pi\beta\alpha)\right]} \tilde{k}_t^{\alpha} \tag{32}$$

Expanding Eq. (26), we obtain a fully parameterized retirement decision:

$$l_{t+1} = \frac{(1-\tau)[(1+n)\alpha+\pi\varepsilon]\gamma + (1+n+\pi\varepsilon)\left[(1-\alpha)\beta+\gamma\right]\tau}{\pi\varepsilon\left[(1-\alpha)\beta+\gamma\right]} \tag{33}$$

From Eq. (32), we derive a unique nonzero steady state of capital per effective skilled worker.

$$\tilde{k}_{ss} = \left[\frac{\pi\alpha\left[(1-\alpha)\beta+\gamma\right]}{(1+g)\left[(1+n)(1+\pi\beta+\pi\gamma)\alpha + \pi\varepsilon(1+\pi\beta\alpha)\right]}\right]^{\frac{1}{1-\alpha}}$$

The condition (23) for a partial retirement equilibrium can now be explicitly parameterized as:

$$\tau < \frac{\beta\pi\varepsilon(1-\alpha) - (1+n)\gamma\alpha}{(1-\alpha)\left[(1+n)(\gamma+\beta)+\beta\pi\varepsilon\right]} \qquad \blacksquare$$

A.2 Generation 0 (the initial old)

A representative initial old agent has to solve the following problem:

$$\begin{aligned}
&\max_{d_1, l_1} U_1 = \pi\beta\ln(d_1) + \pi\gamma ln(l_1) \\
&\text{s.t.} \quad d_1 = \frac{R_1}{\pi}s_0 + (1-l_1)(1-\tau)\varepsilon w_1 + l_1 p_1 \\
&\qquad\quad 1 - l_1 \geq 0 \\
&\qquad\quad c_0, s_0 > 0 \quad \text{are given}
\end{aligned}$$

Substituting d_1 into the objective function, the problem is reduced to finding an optimal choice of l_1:

$$\max_{l_1} U_1 = \pi\beta \ln\left(\frac{R_1}{\pi}s_0 + (1-\tau)\varepsilon w_1 - (1-\tau)\varepsilon w_1 l_t + l_1 p_1\right) + \pi\gamma\ln(l_1)$$
$$\text{s.t. } 1 - l_1 \geq 0$$

The KKT first-order conditions:

$$l_1 \leq \frac{\gamma\frac{R_1}{\pi}s_0 + (1-\tau)\varepsilon w_1\gamma}{(\beta+\gamma)[(1-\tau)\varepsilon w_1 - p_1]} \tag{34}$$
$$\text{with equality holds if } l_1 < 1\text{, otherwise } l_1 = 1$$

In the previous sections, we know that:

$$\begin{aligned}
\tilde{k}_1 &= \frac{s_0}{(1+g)[1+n+(1-l_1)\pi\varepsilon]A_0}\\
R_1 &= \alpha\tilde{k}_1^{\alpha-1}\\
w_1 &= (1-\alpha)A_1\tilde{k}_1^{\alpha}\\
(1-\tau)\varepsilon w_1 - p_1 &= w_1\left[\varepsilon - \frac{\tau(1+n+\pi\varepsilon)}{\pi l_1}\right]
\end{aligned}$$

Plugging all of the above equations into Eq. (34), we get:

$$\begin{aligned}
&l_1 \leq \frac{\gamma w_1\left[\frac{\alpha(1+n+(1-l_1)\pi\varepsilon)}{(1-\alpha)\pi} + (1-\tau)\varepsilon\right]}{(\beta+\gamma)w_1\left[\varepsilon - \frac{\tau(1+n+\pi\varepsilon)}{\pi l_1}\right]}\\
&\Rightarrow l_1 \leq \frac{(1-\tau)\left[(1+n)\alpha+\pi\varepsilon\right]\gamma + (1+n+\pi\varepsilon)\left[(1-\alpha)\beta+\gamma\right]\tau}{\pi\varepsilon\left[(1-\alpha)\beta+\gamma\right]}\\
&\text{with equality holds if } l_1 < 1\text{, otherwise } l_1 = 1
\end{aligned}$$

This is identical to Eq. (33), which means that retirement choice is consistent through all generations. ■

B Proof of Proposition 2

We divide both the numerator and denominator of Eq. (18) to $\pi\alpha$. Since $\varepsilon = \pi^{1-\eta}$, the steady state capital per effective skilled worker is written as:

$$\tilde{k}_{ss} = \left\{\frac{(1-\alpha)\beta+\gamma}{(1+g)(1+n)}\frac{1}{\left[\frac{1}{\pi}+\beta+\gamma+\frac{\pi^{1-\eta}}{\alpha(1+n)}+\frac{\beta\pi^{2-\eta}}{1+n}\right]}\right\}^{\frac{1}{1-\alpha}}$$

Let $\left[\frac{1}{\pi}+\beta+\gamma+\frac{\pi^{1-\eta}}{\alpha(1+n)}+\frac{\beta\pi^{2-\eta}}{1+n}\right]$ be $f(\pi)>0$ and $h \circ f$ be $f^{-1}>0$, then we have: $\tilde{k}_{ss}=\mathscr{C}\left[h(f(\pi))\right]^{\frac{1}{1-\alpha}}$ where $\mathscr{C}=\left[\frac{(1-\alpha)\beta+\gamma}{(1+g)(1+n)}\right]^{\frac{1}{1-\alpha}}$ is a positive constant.

Using the chain rule, taking the derivative of $\tilde{k}_{ss}$ with respect to π:

$$\frac{\partial \tilde{k}_{ss}}{\partial \pi}=\frac{\mathscr{C}h(f(\pi))^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)f^2(\pi)}\left[\frac{\alpha(1+n)-(1-\eta)\pi^{2-\eta}-(2-\eta)\alpha\beta\pi^{3-\eta}}{\alpha(1+n)\pi^2}\right]$$

On the right-hand side, the first term is always positive since $0<\alpha<1$. In addition, functions f, h and the denominator of the second term are all positive. Thus, the sign of $\partial k_{ss}/\partial\pi$ depends only on the sign of the numerator of the second term. Specifically:

$$\frac{\partial \tilde{k}_{ss}}{\partial \pi}\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \quad \Leftrightarrow \quad \alpha(1+n)-(1-\eta)\pi^{2-\eta}-\eta\alpha\beta\pi^{3-\eta}\begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \quad \blacksquare$$