SpringCamp 2024

Some Special Topics in Micro-founded Macroeconomics

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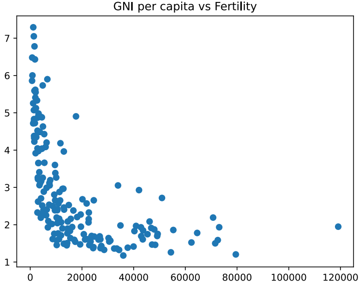
Child Quality-Quantity Tradeoff

Reality



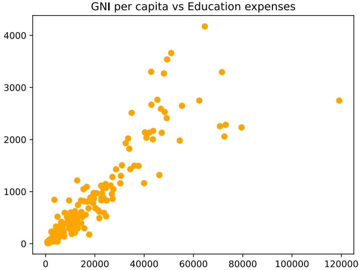


Figure: World Development Indicators. Average: 2009–2019.

Rich countries have lower fertility rates and higher educational expenses, while poorer countries have more children and spend less on education.

Model (de la Croix, 2013)

Preferences

$$\ln(c) + \gamma \ln(nh(e)),$$

Human capital follows

$$h = \mu(\theta + e)^{\eta},$$

Budget constraint

$$c + ne = w(1 - \phi n),$$

Production

$$Y = vL,$$

where v is labor productivity, L is total labor input. Hence, we have the wage rate is

$$w = Y'(L) = v.$$

Proposition

If $w > \theta/(\eta\phi)$ *then*

$$e = \frac{\eta\phi w - \theta}{1-\eta}, \qquad n = \frac{(1-\eta)\gamma w}{(\phi w - \theta)(1+\gamma)}.$$

Otherwise,

$$e = 0, \qquad n = \frac{\gamma}{\phi(1+\gamma)}.$$

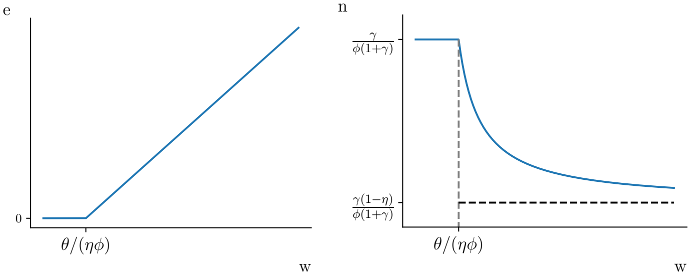


Figure: Illustration for Proposition 1

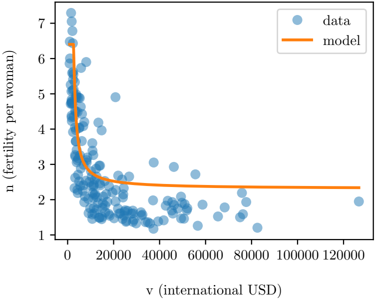
Model Variable	Description	Real-world Equivalence
n	Fertility (net)	Fertility $\times$ (1 - infant mortality /1000)
y	Productivity	GNI per person
$e + \theta$	Education expenses	Adjusted savings: edu $\times$ y
v	Wage	implied from $y = v(1 - \phi n)$

Table: Estimation Strategy.

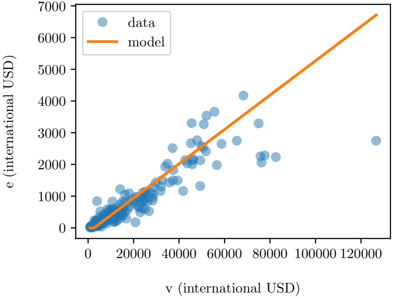
Use MLE

Parameter	Guess (1998-2002 average)	Calibrated Value
η	0.572	0.639
ϕ	0.039	0.031
θ	51.61	51.80
γ	0.103	0.243

Table: Calibrated Parameters.



(a) Fertility vs Productivity. Fitness: $R^2 = 0.657$, t-stats = 18.654, p-val=0

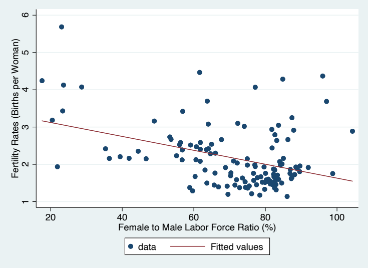


(b) Education vs Productivity. Fitness: $R^2 = 0.796$, t-stats = 26.687, p-val = 0.00

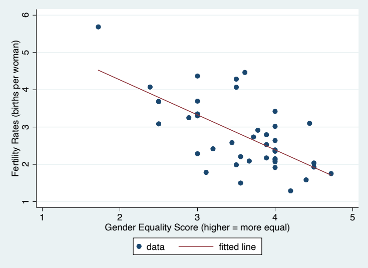
Figure: Model performance vs real-world data

Gender Equality

Reality



(a) Female to Male Labor Participation



(b) CPIA gender equality rating

Figure: Average WDI data from 2008 to 2017. Excluding Gulf and African countries.

Model (Doepke et al., 2023)

Wife's time to work in the market is

$$l_f = 1 - \phi n$$

Men do not spend time taking care of children. A couple's budget constraint is

$$c = w_m + l_f w_f$$

A couple's utility

$$\ln(c) + \delta \ln(n)$$

Prove the following

Proposition

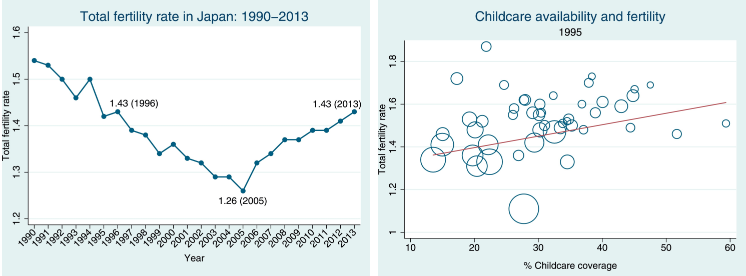
The optimal fertility choice for the couple is

$$n = \frac{\delta}{1+\delta} \frac{1}{\phi} \left(1 + \frac{w_m}{w_f}\right)$$

Show that when the gender pay gap closes, the fertility declines.

Marketization of Childcare

Female participation in the labor market does not necessarily lead to declining birth rates. Fukai (2017) shows that the availability of childcare availability can support women in balancing childcare and work.



(a) TFR in Japan. (b) Childcare service & fertility: 1995.

Figure: Empirical Evidence from Fukai (2017).

Instead of spending time on childcare directly, a couple can buy on the childcare market under the price $p_s > 0$. We denote by $s \in [0, \bar{s}]$ the share of childcare they can buy with $\bar{s}$ is the maximum amount of childcare that can be outsourced. A couple's choice problem is

$$\max_{c,n,s} u := \ln(c) + \delta \ln(n)$$
$$\text{s.t. } c + \psi n + s p_s n \phi = w_m + w_f[1 - (1 - s)n\phi].$$

Show that the optimal fertility choice is

$$n = \frac{\delta}{1+\delta} \cdot \frac{w_m + w_f}{\psi + [s p_s + (1-s) w_f]\phi}$$

Show that the utility u can be expressed as a function of s as follows

$$u(s) = \ln\left(\delta^\delta \left[\frac{w_m + w_f}{1+\delta}\right]^{1+\delta}\right) - \delta \ln(\psi + [s p_s + (1-s) w_f]\phi)$$

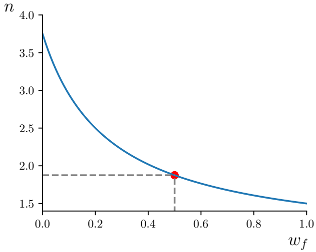
Proposition

Assume that the good cost of childrearing is smaller than the time cost $\psi < w_m\phi$. *Let*

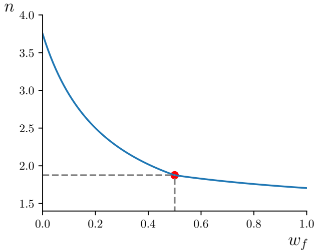
$$\hat{s} = \frac{w_m\phi - \psi}{(p_s + w_m)\phi},$$

then we have

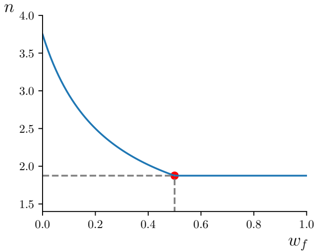
$$\frac{\partial n}{\partial w_f} \begin{cases} < 0 & \text{if } s < \hat{s}, \\ = 0 & \text{if } s = \hat{s}, \\ > 0 & \text{if } s > \hat{s} \end{cases}$$



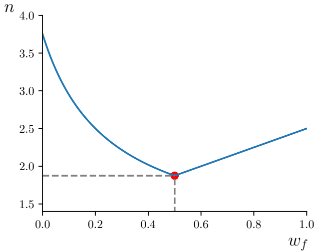
(a) $\bar{s} = 0$



(b) $\bar{s} < \hat{s}$



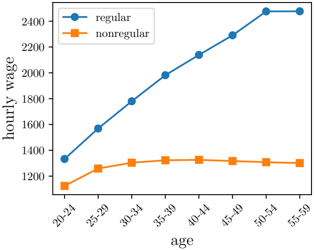
(c) $1 > \bar{s} = \hat{s}$



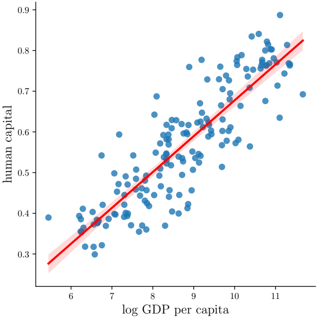
(d) $\bar{s} = 1$

Human Capital

Reality



(a) Full-time vs Part-time Wage (Japan, 2019, Source: https://www.mhlw.go.jp/stf/wp/hakusyo/kousei/19/backdata/01-01-03-23.html)



(b) log GDP per capita and human capital in 2018. Source: WDI

Figure: Some Indication of Human Capital.

Model (Ceroni, 2001)

Preferences are

$$\ln(c_t) + \delta h_{t+1}$$

subject to

$$h_{t+1} = f(e_{t+1}) = \ln(\gamma e_{t+1} + v),$$
$$c_t = w_t - e_{t+1}$$

Labor productivity is now h_t instead of ν, hence:

$$w_t = h_t(e_t)$$

Now, prove the following proposition.

Proposition

Let $\bar{h} = v/\delta\gamma$, then

$$e_{t+1} = \begin{cases} 0 \text{ if } h_t < \bar{h}, \\ \dfrac{\delta}{1+\delta} h_t - \dfrac{v}{\gamma(1+\delta)} \text{ if } h_t \geq \bar{h} \end{cases}$$

Show the law of motion of human capital accumulation

$$h_{t+1} = \phi(h_t) = \begin{cases} \ln(v) \text{ if } h_t < \bar{h}, \\ \ln\left[\dfrac{\delta(\gamma h_t + v)}{1+\delta}\right] \text{ if } h_t \geq \bar{h} \end{cases}$$

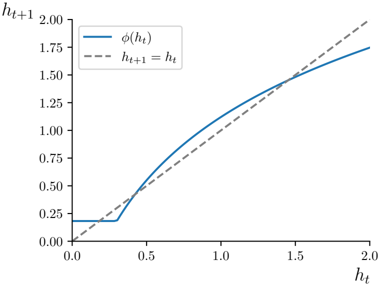
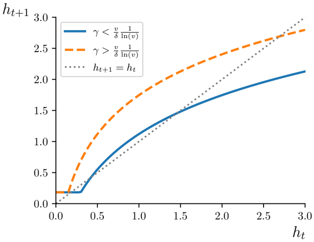
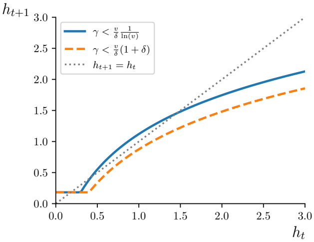


Figure: Dynamics of $\phi(h_t)$ with $\gamma = 8, \delta = 0.5, v = 1.2$.

Counterfactual

1. What happens if $\gamma > \frac{v}{\delta} \frac{1}{\ln(v)}$? How do you know?
2. What happens if $\gamma < \frac{v}{\delta}(1 + \delta)$? How do you know?





Career Choice

Reality

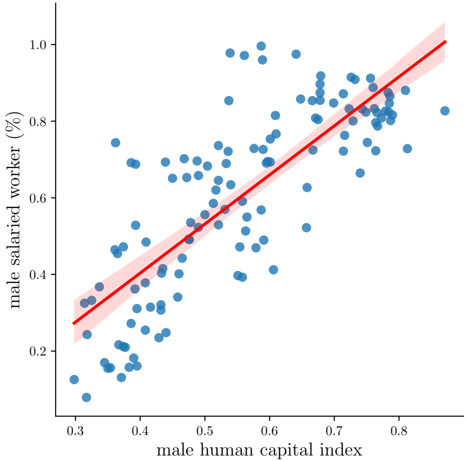


Figure: Human Capital Index and Salaried Workers Ratio (% of employment) for Male. Global data from WDI in 2017.

Model (Kimura and Yasui, 2007)

Production

$$Y_t = A[K_t^{\alpha} H_t^{1-\alpha} + bL_t],$$

where $A > 0, b > 0, \alpha \in (0, 1)$. Capital is skill-complimentary. Time spent on study is τ, and childcare is z. Market time

$$(H): 1 - \tau - zn_t^H,$$
$$(L): 1 - zn_t^L$$

ϕ is the portion of skilled workers / whole population. The labor supply is

$$(H): H = (1 - \tau - zn_t^H)\phi_t N_t,$$
$$(L): L = (1 - zn_t^L)(1 - \phi_t)N_t$$

Input factor prices under a competitive market are

$$w_t^H = \frac{\partial Y_t}{\partial H_t} = A(1-\alpha)\left[\frac{k_t}{(1 - \tau - zn_t^H)\phi_t}\right]^{\alpha}, \tag{1}$$

$$w_t^L = \frac{\partial Y_t}{\partial L_t} = Ab, \tag{2}$$

where $k_t = K_t/N_t$ is the capital per capita.

Utility (regardless of type)

$$u_t = \gamma \ln(n_t) + (1-\gamma)\ln(c_{t+1}).$$

The budget constraint for a skilled worker is

$$s_t^H = w_t^H(1-\tau-zn_t^H),$$
$$c_{t+1}^H = (1+r_{t+1})s_t$$

and for an unskilled worker is

$$s_t^L = w_t^L(1-zn_t^L),$$
$$c_{t+1}^L = (1+r_{t+1})s_t^L$$

FOCs

$$n_t^H = \frac{\gamma(1-\tau)}{z}, \tag{3}$$
$$s_t^H = w_t^H(1-\tau)(1-\gamma) \tag{4}$$

and

$$n_t^L = \frac{\gamma}{z}, \tag{5}$$
$$s_t^L = w_t^L(1-\gamma). \tag{6}$$

Equilibrium

In equilibrium, agents are indifferent between becoming skilled or unskilled. Thus, the following condition must hold

$$u_t^H = u_t^R.$$

By substituting the FOC conditions for the utility function, we have

$$\frac{w_t^L}{w_t^H} = (1-\tau)^{1/(1-\gamma)} \tag{7}$$

From (1), (2), (3), (5), one can derive

$$\phi(k_t) = \frac{(1-\tau)^{\frac{1}{\alpha(1-\gamma)}-1}}{1-\gamma}\left(\frac{1-\alpha}{b}\right)^{\frac{1}{\alpha}} k_t \equiv \theta k_t. \tag{8}$$

Since ϕ cannot be larger than 1, define $\bar{k}$ such that $\phi(\bar{k}) = 1$, then

$$\bar{k} = \theta^{-1}.$$

We have

$$\phi(k_t) = \begin{cases} \theta k_t \text{ if } k_t < \bar{k}, \\ 1 \text{ if } k_t \geq \bar{k} \end{cases}$$

Let the total fertility rate at time t be m_t, then from (3) and (5)

$$m_t = \phi_t n_t^H + (1-\phi_t)n_t^L = (1-\tau\phi_t)\frac{\gamma}{z}$$

Note that $N_{t+1} = m_t N_t$. Using (8), it can be written as a function of k_t

$$m(k_t) = \begin{cases} (1-\tau\theta k_t)\dfrac{\gamma}{z} \text{ if } k_t < \bar{k}, \\ (1-\tau)\dfrac{\gamma}{z} \text{ if } k_t \geq \bar{k} \end{cases}$$

The dynamics of the model can now be pinned down solely on the dynamics of k. The capital accumulates according to

$$K_{t+1} = [\phi_t s_t^H + (1-\phi_t)s_t^L]N_t$$

Dividing both sides by N_{t+1} and use the saving function from (4), (6), we have

$$k_{t+1} = \frac{z(1-\gamma)}{\gamma}\frac{\phi_t w_t^H(1-\tau) + (1-\phi_t)w_t^L}{1-\tau\phi_t}$$

Key dynamical function

$$k_{t+1} = \begin{cases} \dfrac{Az(1-\gamma)}{\gamma} \dfrac{1}{1-\tau\theta k_t} \left[\dfrac{(1-\alpha)(1-\tau)^{1-\alpha}\theta^{1-\alpha} k_t}{(1-\gamma)^{\alpha}} + (1-\theta k_t) b \right] & \text{if } k_t < \bar{k}, \\ \dfrac{Az(1-\gamma)}{\gamma} \dfrac{(1-\alpha)}{(1-\tau)^{\alpha}(1-\gamma)^{\alpha}} k_t^{\alpha} \end{cases} \tag{9}$$

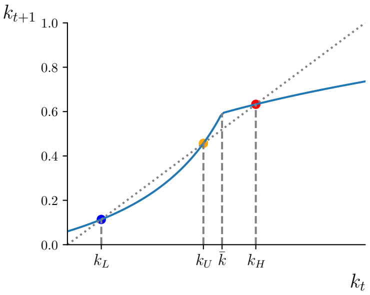


Figure: Development trap case with three equilibria where k_L, k_H are stable and k_U is unstable. Parameters: $\alpha = 0.33, \tau = 0.6, \gamma = 0.6, z = 0.2, b = 0.1, A = 4.5$.