# SpringCamp 2024

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# Child Quality-Quantity Tradeoff

# Reality

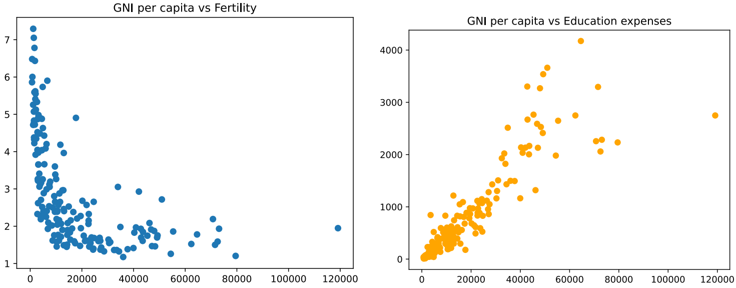



Figure: World Development Indicators. Average: 2009–2019.

Rich countries have lower fertility rates and higher educational expenses, while poorer countries have more children and spend less on education.

## The Model

Preferences

$$\ln(c) + \gamma \ln(n_t h(e)),$$

Human capital follows

$$h = \mu(\theta + e)^{\eta},$$

Budget constraint

$$c + ne = w(1 - \phi n),$$

Production

$$Y = vL,$$

where $v$ is labor productivity, $L$ is total labor input. Hence, we have the wage rate is

$$w = Y'(L) = v.$$

## Proposition

*If $w > \theta/(\eta\phi)$ then*

$$e = \frac{\eta\phi w - \theta}{1-\eta}, \qquad n = \frac{(1-\eta)\gamma w}{(\phi w - \theta)(1+\gamma)}.$$

*Otherwise,*

$$e = 0, \qquad n = \frac{\gamma}{\phi(1+\gamma)}.$$

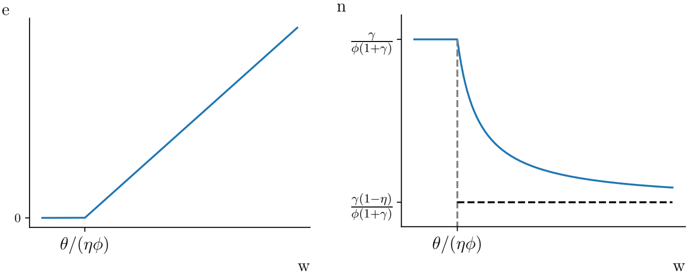



Figure: Illustration for Proposition 1

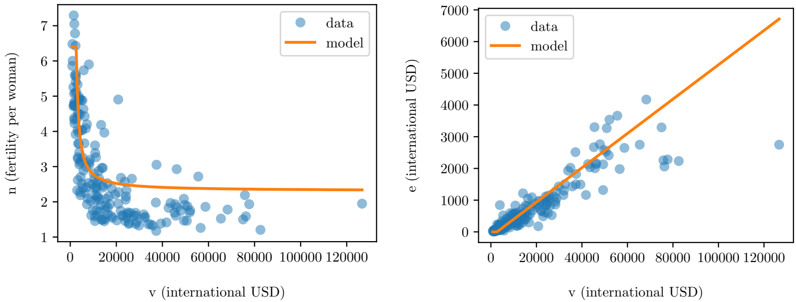
| Model Variable | Description | Real-world Equivalence |
|---|---|---|
| $n$ | Fertility (net) | Fertility $\times$ ( 1 - infant mortality /1000 ) |
| $y$ | Productivity | GNI per person |
| $e+\theta$ | Education expenses | Adjusted savings: edu $\times$ y |
| $v$ | Wage | implied from $y = v(1-\phi n)$ |

Table: Estimation Strategy.

Use MLE

| Parameter | Guess (1998-2002 average) | Calibrated Value |
|---|---|---|
| $\eta$ | 0.572 | 0.639 |
| $\phi$ | 0.039 | 0.031 |
| $\theta$ | 51.61 | 51.80 |
| $\gamma$ | 0.103 | 0.243 |

Table: Calibrated Parameters.



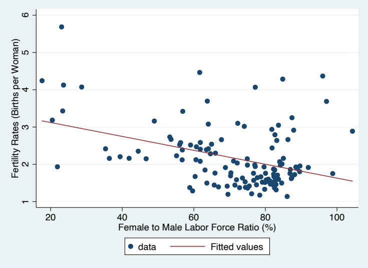

(a) Fertility vs Productivity. Fitness: $R^2 = 0.657$, t-stats = 18.654, p-val=0

(b) Education vs Productivity. Fitness: $R^2 = 0.796$, t-stats = 26.687, p-val = 0.00

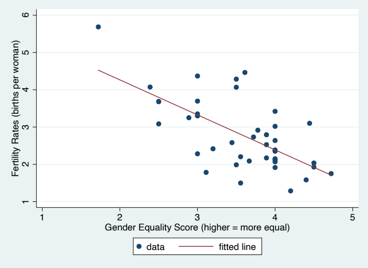
Figure: Model performance vs real-world data

# Gender Equality

# Reality




(a) Female to Male Labor Participation




(b) CPIA gender equality rating

Figure: Average WDI data from 2008 to 2017. Excluding Gulf and African countries.

# Model

Wife's time to work in the market is

$$l_f = 1 - \phi n$$

Men do not spend time taking care of children. A couple's budget constraint is

$$c = w_m + l_f w_f$$

A couple's utility

$$\ln(c) + \delta \ln(n)$$

Prove the following

**Proposition**

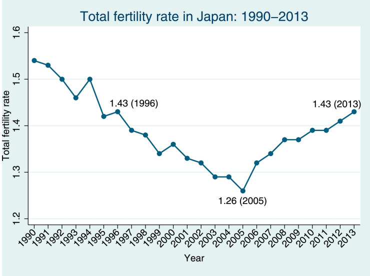
*The optimal fertility choice for the couple is*

$$n = \frac{\delta}{1+\delta} \frac{1}{\phi} \left(1 + \frac{w_m}{w_f}\right)$$

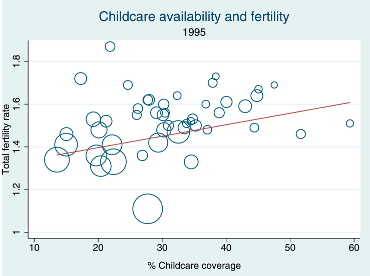
Show that when the gender pay gap closes, the fertility declines.

# Marketization of Childcare

Female participation in the labor market does not necessarily lead to declining birth rates. A paper by **fukai2017childcare** shows that the availability of childcare availability can support women in balancing childcare and work.




(a) TFR in Japan.




(b) Childcare service & fertility: 1995.

Figure: Empirical Evidence from **fukai2017childcare**.

Instead of spending time on childcare directly, a couple can buy on the childcare market under the price $p_s > 0$. We denote by $s \in [0, \bar{s}]$ the share of childcare they can buy with $\bar{s}$ is the maximum amount of childcare that can be outsourced. A couple's choice problem is

$$\max_{c,n,s} u := \ln(c) + \delta \ln(n)$$
$$\text{s.t. } c + \psi n + s p_s n \phi = w_m + w_f[1 - (1 - s)n\phi].$$

Show that the optimal fertility choice is

$$n = \frac{\delta}{1+\delta} \cdot \frac{w_m + w_f}{\psi + [s p_s + (1-s) w_f]\phi}$$

Show that the utility $u$ can be expressed as a function of $s$ as follows

$$u(s) = \ln\left(\delta^{\delta}\left[\frac{w_m + w_f}{1+\delta}\right]^{1+\delta}\right) - \delta \ln(\psi + [s p_s + (1-s) w_f]\phi)$$

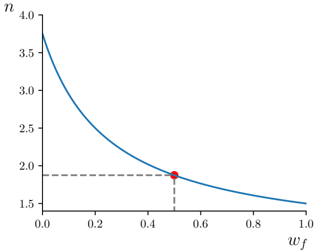
## Proposition

*Assume that the good cost of childrearing is smaller than the time cost* $\psi < w_m\phi$. *Let*

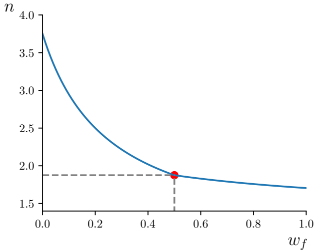
$$\hat{s} = \frac{w_m\phi - \psi}{(p_s + w_m)\phi},$$

*then we have*

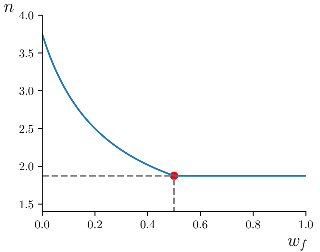
$$\frac{\partial n}{\partial w_f} \begin{cases} < 0 & \text{if } s < \hat{s}, \\ = 0 & \text{if } s = \hat{s}, \\ > 0 & \text{if } s > \hat{s} \end{cases}$$



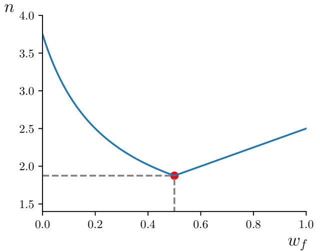

(a) $\bar{s} = 0$




(b) $\bar{s} < \hat{s}$



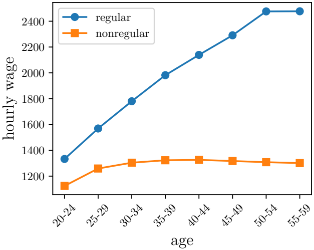

(c) $1 > \bar{s} > \hat{s}$



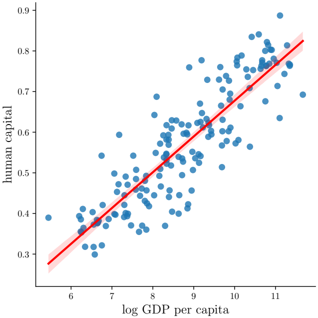

(d) $\bar{s} = 1$

# Human Capital

# Reality




(a) Full-time vs Part-time Wage (Japan, 2019, Source: https://www.mhlw.go.jp/stf/wp/hakusyo/kousei/19/backdata/01-01-03-23.html)




(b) log GDP per capita and human capital in 2018. Source: WDI

Figure: Some Indication of Human Capital.

## Model

Preferences are

$$\ln(c_t) + \delta h_{t+1}$$

subject to

$$h_{t+1} = f(e_{t+1}) = \ln(\gamma e_{t+1} + v),$$
$$c_t = w_t - e_t$$

Labor productivity is now $h_t$ instead of $\nu$, hence:

$$w_t = h_t(e_t)$$

Now, prove the following proposition.

**Proposition**

*Let $\bar{h} = v/\delta\gamma$, then*

$$e_{t+1} = \begin{cases} 0 \text{ if } h_t < \bar{h}, \\ \dfrac{\delta}{1+\delta} h_t - \dfrac{v}{\gamma(1+\delta)} \text{ if } h_t \geq \bar{h} \end{cases}$$

Show the law of motion of human capital accumulation

$$h_{t+1} = \phi(h_t) = \begin{cases} \ln(v) \text{ if } h_t < \bar{h}, \\ \ln\left[\dfrac{\delta(\gamma h_t + v)}{1+\delta}\right] \text{ if } h_t \geq \bar{h} \end{cases}$$

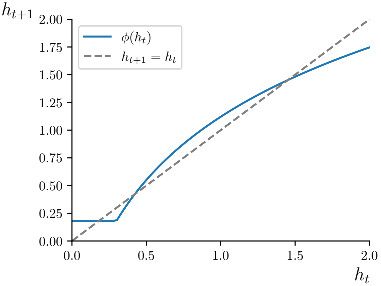
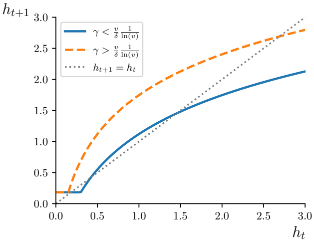
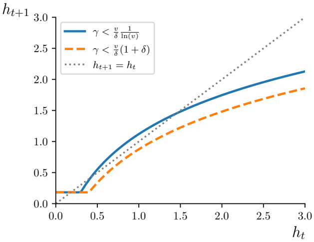



Figure: Dynamics of $\phi(h_t)$ with $\gamma = 8, \delta = 0.5, v = 1.2$.

# Counterfactual

1. What happens if $\gamma > \frac{v}{\delta}\frac{1}{\ln(v)}$? How do you know?
2. What happens if $\gamma < \frac{v}{\delta}(1+\delta)$? How do you know?



# Career Choice

## Reality

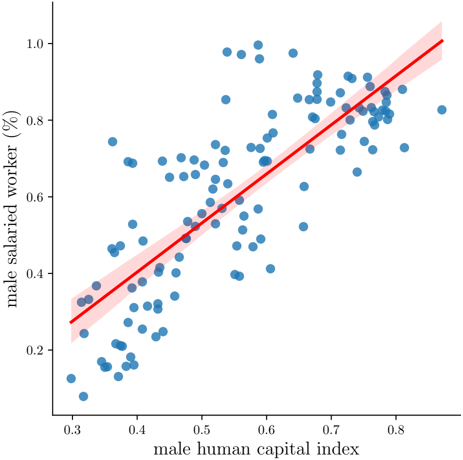



Figure: Human Capital Index and Salaried Workers Ratio (% of employment) for Male. Global data from WDI in 2017.

# Model

Production

$$Y_t = A[K_t^{\alpha} H_t^{1-\alpha} + bL_t],$$

where $A > 0, b > 0, \alpha \in (0,1)$. Capital is skill-complimentary. Time spent on study is $\tau$, and childcare is $z$. Market time

$$(H):1 - \tau - zn_t^H,$$
$$(L):1 - zn_t^L$$

$\phi$ is the portion of skilled workers / whole population. The labor supply is

$$(H):H = (1 - \tau - zn_t^H)\phi_t N_t^H,$$
$$(L):L = (1 - zn_t^L)(1 - \phi_t)N_t^L$$

Input factor prices under a competitive market are

$$w_t^H = \frac{\partial Y_t}{\partial H_t} = A(1-\alpha)\left[\frac{k_t}{(1-\tau - zn_t^H)\phi_t}\right]^{\alpha}, \quad (1)$$

$$w_t^L = \frac{\partial Y_t}{\partial L_t} = Ab, \quad (2)$$

where $k_t = K_t/N_t$ is the capital per capita.

Utility (regardless of type)

$$u_t = \gamma \ln(n_t) + (1-\gamma)\ln(c_{t+1}).$$

The budget constraint for a skilled worker is

$$\begin{aligned} n_t^H + s_t^H &= w_t^H(1-\tau-zn_t^H), \\ c_{t+1}^H &= (1+r_{t+1})s_t \end{aligned}$$

and for an unskilled worker is

$$\begin{aligned} n_t^L + s_t^L &= w_t^L(1-zn_t^L), \\ c_{t+1}^L &= (1+r_{t+1})s_t^L \end{aligned}$$

FOCs

$$n_t^H = \frac{\gamma(1-\tau)}{z}, \tag{3}$$

$$s_t^H = w_t^H(1-\tau)(1-\gamma) \tag{4}$$

and

$$n_t^L = \frac{\gamma}{z}, \tag{5}$$

$$s_t^L = w_t^L(1-\gamma). \tag{6}$$

# Equilibrium

In equilibrium, agents are indifferent between becoming skilled or unskilled. Thus, the following condition must hold

$$u_t^H = u_t^R.$$

By substituting the FOC conditions for the utility function, we have

$$\frac{w_t^L}{w_t^H} = (1-\tau)^{1/(1-\gamma)} \tag{7}$$

From (1), (2), (3), (5), one can derive

$$\phi(k_t) = \frac{(1-\tau)^{\frac{1}{\alpha(1-\gamma)}-1}}{1-\gamma}\left(\frac{1-\alpha}{b}\right)^{\frac{1}{\alpha}} k_t \equiv \theta k_t. \tag{8}$$

Since $\phi$ cannot be larger than 1, define $\bar{k}$ such that $\phi(\bar{k}) = 1$, then

$$\bar{k} = \theta^{-1}.$$

We have

$$\phi(k_t) = \begin{cases} \theta k_t \text{ if } k_t < \bar{k}, \\ 1 \text{ if } k_t \geq \bar{k} \end{cases}$$

Let the total fertility rate at time $t$ be $m_t$, then from (3) and (5)

$$m_t = \phi_t n_t^H + (1 - \phi_t) n_t^L = (1 - \tau\phi_t)\frac{\gamma}{z}$$

Note that $N_{t+1} = m_t N_t$. Using (8), it can be written as a function of $k_t$

$$m(k_t) = \begin{cases} (1 - \tau\theta k_t)\frac{\gamma}{z} \text{ if } k_t < \bar{k}, \\ (1 - \tau)\frac{\gamma}{z} \text{ if } k_t \geq \bar{k} \end{cases}$$

The dynamics of the model can now be pinned down solely on the dynamics of $k$. The capital accumulates according to

$$K_{t+1} = [\phi_t s_t^H + (1 - \phi_t) s_t^L] N_t$$

Dividing both sides by $N_{t+1}$ and use the saving function from (4), (6), we have

$$k_{t+1} = \frac{z(1-\gamma)}{\gamma} \frac{\phi_t w_t^H (1-\tau) + (1-\phi_t) w_t^L}{1 - \tau\phi_t}$$

Key dynamical function

$$k_{t+1} = \begin{cases} \dfrac{Az(1-\gamma)}{\gamma} \dfrac{1}{1-\tau\theta k_t} \left[ \dfrac{(1-\alpha)(1-\tau)^{1-\alpha}\theta^{1-\alpha} k_t}{(1-\gamma)^{\alpha}} + (1-\theta k_t) b \right] & \text{if } k_t < \bar{k}, \\ \dfrac{Az(1-\gamma)}{\gamma} \dfrac{(1-\alpha)}{(1-\tau)^{\alpha}(1-\gamma)^{\alpha}} k_t^{\alpha} \end{cases} \tag{9}$$

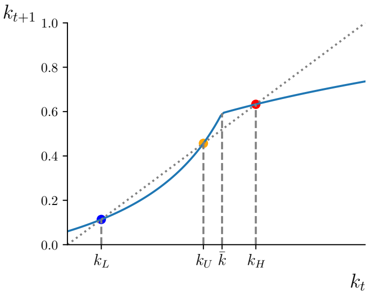



Figure: Development trap case with three equilibria where $k_L, k_H$ are stable and $k_U$ is unstable. Parameters: $\alpha = 0.33, \tau = 0.6, \gamma = 0.6, z = 0.2, b = 0.1, A = 4.5$.