Practice homework 1.1

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1.

a) $s[n] = \frac{1}{2^n} + j\frac{1}{3^n}$

$$\sum_{n=1}^{\infty} s[n] = \sum_{n=1}^{\infty} \frac{1}{2^n} + j\sum_{n=1}^{\infty}\frac{1}{3^n}$$

$$= \sum_{n=1}^{\infty}(2^{-1})^n + j\sum_{n=1}^{\infty}(3^{-1})^n$$

$$= \frac{2^{-1}}{1-2^{-1}} + j\frac{3^{-1}}{1-3^{-1}}$$

$$= 1 + \frac{1}{2}j$$

b) $s[n] = \left(\frac{j}{3}\right)^n$

$$\sum_{n=1}^{\infty} s[n] = \sum_{n=1}^{\infty}\left(\frac{j}{3}\right)^n = \sum_{n=0}^{\infty}\frac{j}{3^{(4n+1)}} - \sum_{n=0}^{\infty}\frac{1}{3^{(4n+2)}} - \sum_{n=0}^{\infty}\frac{j}{3^{(4n+3)}} + \sum_{n=0}^{\infty}\frac{1}{3^{(4n+4)}}$$

$$= \frac{j}{3}\sum_{n=0}^{\infty}(81^{-1})^n - \frac{1}{9}\sum_{n=0}^{\infty}(81^{-1})^n - \frac{j}{27}\sum_{n=0}^{\infty}(81^{-1})^n + \frac{1}{81}\sum_{n=0}^{\infty}(81^{-1})^n$$

$$= \left(\frac{j}{3} - \frac{1}{9} - \frac{j}{27} + \frac{1}{81}\right)\frac{1}{1-81^{-1}}$$

$$= \left(\frac{8j}{27} - \frac{8}{81}\right)\frac{81}{80}$$

$$= \frac{-1}{10} + \frac{3}{10}j$$

c) The set of complex number satisfied $z^* = z^{-1}$

Let $z = a + jb \Rightarrow \begin{cases} z^* = a - jb \\ z^{-1} = \frac{1}{a+jb}\end{cases}$

$$z^* = z^{-1} \Leftrightarrow a - jb = \frac{1}{a+jb}$$

$$\Leftrightarrow (a-jb)(a+jb) = 1$$

$$\Leftrightarrow a^2 - (jb)^2 = 1$$

$$\Leftrightarrow a^2 + b^2 = 1$$

The set of complex number inside the unit circle satisfied $z^* = z^{-1}$

d) Find $\{z_0, z_1, z_2\}$ s.t $z_i^3 = 1$

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Let $z = a + jb$

+ $z^3 = (a+jb)^3 = a^3 + j3a^2b - 3ab^2 - jb^3$
+ $z^3 = 1 \Leftrightarrow \begin{cases} a^3 - 3ab^2 = 1 & (1) \\ 3a^2b - b^3 = 0 & (2) \end{cases}$

$(2) \Leftrightarrow b(3a^2 - b^2) = 0$

$\Leftrightarrow b(a\sqrt{3} - b)(a\sqrt{3} + b) = 0$

$\Leftrightarrow \left[\begin{array}{l} b = 0 \\ b = a\sqrt{3} \\ b = -a\sqrt{3} \end{array}\right.$

+ $b = 0$, $(1) \Leftrightarrow a^3 = 1 \Leftrightarrow a = 1$. We have $z_0 = 1$
+ $b = a\sqrt{3}$, $(1) \Leftrightarrow a^3 - 9a^3 = 1$

$\Leftrightarrow a^3 = \frac{-1}{8}$

$\Leftrightarrow a = \frac{-1}{2}$ and $b = \frac{-\sqrt{3}}{2}$. We have $z_1 = \frac{-1}{2} - j\frac{\sqrt{3}}{2}$

+ $b = -a\sqrt{3}$, $(1) \Leftrightarrow a = \frac{-1}{2}$ and $b = \frac{\sqrt{3}}{2}$. We have $z_2 = \frac{-1}{2} + j\frac{\sqrt{3}}{2}$

e) $\prod_{n=1}^{\infty} e^{j\frac{\pi}{2^n}} = \exp\left(\sum_{n=1}^{\infty} j\frac{\pi}{2^n}\right)$

$= \exp(j\pi) = e^{j\pi} = -1$

2. Same as homework 1.1 question 8
3. Same as homework 1.1 question 7
4. Same as homework 1.1 question 9