Practice homework 1.2

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5. $S : \{x^{(k)}\}_{k=0,\dots,N-1} = x$

Assume by contradiction and without loss of generality that z has 2 unique representation:

$z_s = \{a^{(k)}\}_{k=0,\dots,N-1}$ and $z_s' = \{b^{(k)}\}_{k=0,\dots,N-1}$

we have $z = x^T z_s = x^T z_s'$

$\Leftrightarrow x^T(z_s - z_s') = 0$

Because all row in x are linearly independent

$\Rightarrow$ the equation has only trivial solution

$\Rightarrow z_s = z_s'$

6. The 4 diagonals of the unit cube in $\mathbb{R}^3$ is

$$d_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad d_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \quad d_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \quad d_4 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

because $d_i^T d_j \neq 0$ for $\forall i \neq j$, they are not mutually orthogonal