Homework 3

1. Let p, q, r be atoms, and v a valuation. For each condition, give a CNF formula ϕ such that $v(\phi) = \mathsf{T}$ if and only if the condition holds.

   (a) $v(p) \neq v(q)$
   (b) *at least one* of $v(p), v(q)$ is T
   (c) *at most one* of $v(p), v(q)$ is T
   (d) *exactly one* of $v(p), v(q)$ is T
   (e) *at least* one of $v(p), v(q), v(r)$ is T
   (f) *at least* one of $v(p), v(q), v(r)$ is F
   (g) *at most* one of $v(p), v(q), v(r)$ is T
   (h) *exactly one* of $v(p), v(q), v(r)$ is T
   (i) *at least two* of $v(p), v(q), v(r)$ is T
   (j) *at most two* of $v(p), v(q), v(r)$ is T
   (k) *exactly two* of $v(p), v(q), v(r)$ is T

2. Suppose that there are two adjacent cities, namely City A and City B, on a map. We have two colors, black and white, and have to assign different colors to the two cities on the map. This is trivial, but let us overkill this problem using a SAT solver. Let $b_\mathsf{A}, w_\mathsf{A}, b_\mathsf{B}, w_\mathsf{B}$ be atoms. Each atom reads as follows:

   - b_A: City A is black.
   - w_A: City A is white.
   - b_B: City B is black.
   - w_B: City B is white.

   Next we encode the constraints into a CNF formula. Such a formula has to state the following:

   i. City A is black or white, *but not both*.
   ii. City B is black or white, *but not both*.
   iii. City A and City B have different colors.

   Now it is your turn.

   (a) Encode the constraints to a CNF formula.
   (b) Feed the CNF formula to a SAT solver, and verify that the output meets the constraints.

3. Figure 1 is the edge of Noto Peninsula, and we have to color Suzu, Noto, Wajima, and Anamizu. Again, we have to assign different colors to adjacent cities, and now we have three colors, red, green, and blue. Here we don't have to color the sea and other cities.

   (a) Sketch out an idea of CNF encoding.
   (b) Encode the constraints to an actual CNF formula. Now that the formula can be huge, you may write a program for this.
   (c) Feed the CNF formula to a SAT solver, and verify that the output meets the constraints.

Figure 1: Suzu, Noto, Wajima, and Anamizu. The original image attributes to Lincun (CC-BY SA 3.0).