Homework 1

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1.

$p: 1+3=4$; $q: 1+2=4$

r: I open the window ; s: We'll have fresh air

a) $p \vee q$

b) $\neg(p \vee q)$

c) $r \rightarrow s$

d) $r \rightarrow q$

e) $q \rightarrow s$

2.

a) $p \wedge \neg p$ Unsatisfiable

p	$p \wedge \neg p$
T	F
F	F

b) $\neg(p \wedge \neg p)$ Valid

p	$\neg$	$(p \wedge \neg p)$
T	T	F
F	T	F

c) $(p \wedge q) \rightarrow (p \vee r)$ Valid

p	q	r	$(p \wedge q)$	$\rightarrow$	$(p \vee r)$
F	F	F	F	T	F
F	F	T	F	T	T
F	T	F	F	T	F
F	T	T	F	T	T
T	F	F	F	T	T
T	F	T	F	T	T
T	T	F	T	T	T
T	T	T	T	T	T

d) $((p \rightarrow q) \rightarrow p) \rightarrow p$ Valid

d) $((p \to q) \to p) \to p$ valid

p	q	$((p \to q) \to p) \to p)$				
F	F	T	F	F	T	F
F	T	T	F	F	T	F
T	F	F	T	T	T	T
T	T	T	T	T	T	T

e) $p \to \neg p$ Satisfiable but not valid

p	$p \to \neg p$
F	T
T	F

f) $\neg (p \to \neg p)$ Satisfiable but not valid

p	$\neg (p \to \neg p)$	
F	F	T
T	T	F

3.

a) $\neg(p \vee q) \approx (\neg p \wedge \neg q)$ The claim holds

p	q	$\neg (p \vee q) \leftrightarrow (\neg p \wedge \neg q)$			
F	F	T	F	T	T
F	T	F	T	T	F
T	F	F	T	T	F
T	T	F	T	T	F

b) $\neg(p \vee q) \approx (\neg p \vee \neg q)$

Consider $v = \{p \mapsto F, q \mapsto F\}$

$[[\neg(p \vee q) \leftrightarrow \neg p \vee \neg q]]_v = F$

So the claim doesn't hold

p	q	$\neg (p \vee q) \leftrightarrow (\neg p \vee \neg q)$			
F	F	T	F	F	T
F	T	F	T	F	T
T	F	F	T	F	T
T	T	F	T	T	F

c) $(p \to q) \approx (\neg p \vee q)$ The claim holds

p	q	$(p \to q) \leftrightarrow (\neg p \vee q)$		
F	F	T	T	T

p	q	$(p \to q)$	$\leftrightarrow$	$(\neg p \vee q)$
F	F	T	T	T
F	T	T	T	T
T	F	F	T	F
T	T	T	T	T

d) $\neg p \approx (p \to \bot)$ The claim holds

p	$\neg p$	$\leftrightarrow$	$(p \to \bot)$
F	T	T	T
T	F	T	F

e) $\neg p \approx (\bot \to p)$ Consider $v = \{p \mapsto T\}$

$[[\neg p \leftrightarrow (\bot \to p)]]_v = F$

So the claim doesn't hold

p	$\neg p$	$\leftrightarrow$	$(\bot \to p)$
F	T	T	T
T	F	F	T

f) $p \approx (p \to \top)$ Consider $v = \{p \mapsto F\}$

$[[p \leftrightarrow (p \to \top)]]_v = F$

So the claim doesn't hold

p	p	$\leftrightarrow$	$(p \to \top)$
F	F	F	T
T	T	T	T

g) $p \approx (\top \to p)$ The claim holds

p	p	$\leftrightarrow$	$(\top \to p)$
F	F	T	F
T	T	T	T

h) $(p \to q) \wedge (q \to p) \approx (p \leftrightarrow q)$ The claim holds

p	q	$((p \to q)$	$\wedge$	$(q \to p))$	$\leftrightarrow$	$(p \leftrightarrow q)$
F	F	T	T	T	T	T
F	T	T	F	F	T	F
T	F	F	F	T	T	F
T	T	T	T	T	T	T

4.

a) ϕ: $p \vee q$; $\neg \phi$: $\neg (p \vee q)$ (1)

Consider $v = \{p \mapsto F, q \mapsto F\}$, we have

$[[\neg (p \vee q)]]_v = T$

Consider $v = \{ p \mapsto F,\ q \mapsto F \}$, we have

$[[\neg (p \vee q)]]_v = T$

So $\neg \phi$ is satisfiable

b) True

c) True

d) (1): ϕ and $\neg \phi$ are not valid

5. $(p \wedge ((p \to q) \wedge (p \to r))) \to (q \wedge r)$

p	q	r	p	∧	(p→q)	∧	(p→r)	→	(q∧r)
F	F	F	F	F	T	T	T	T	F
F	F	T	F	F	T	T	T	T	F
F	T	F	F	F	T	T	T	T	F
F	T	T	F	F	T	T	T	T	T
T	F	F	T	F	F	F	F	T	F
T	F	T	T	F	F	F	T	T	F
T	T	F	T	F	T	F	F	T	F
T	T	T	T	T	T	T	T	T	T