Homework 9

17 May 2023 21:33

1)

(1) $(x \doteq x)[x/x] = (x \doteq x)$

(2) $(x \doteq x)[y/x] = (y \doteq y)$

(3) $(z \doteq 0)[f(x,y)/y] = (z \doteq 0)$

(4) $(\exists x (y \doteq x))[f(0,y)/y] = (\exists x (f(0,y) \doteq x)$

(5) $(\exists w (f(w,x) \doteq 0))[f(x,y)/z] = (\exists w (f(w,x) \doteq 0))$

(6) $(\forall w (f(x,z) \doteq 0))[f(x,w)/z]$

ϕ': $(\forall y (f(x,z) \doteq 0))[f(x,w)/z] = (\forall y (f(x, f(x,w)) \doteq 0)$

(7) $(\forall w (f(x,z) \doteq 0) \wedge \exists y (z \doteq x))[f(x,y)/z]$

ϕ': $(\forall w (f(x,z) \doteq 0) \wedge \exists k (z \doteq x))[f(x,y)/z] = (\forall w (f(x, f(x,y)) \doteq 0) \wedge \exists k (f(x,y) \doteq x))$

(8) $(\forall u (u \doteq v) \to \forall z (z \doteq y))[f(x,y)/z] = (\forall u (u \doteq v) \to \forall z (z \doteq y))$

2)

(1) $\forall x P(x) \leftrightarrow \exists x Q(x)$

$\approx (\forall x P(x) \to \exists x Q(x)) \wedge (\exists x Q(x) \to \forall x P(x))$

$\approx (\neg \forall x P(x) \vee \exists x Q(x)) \wedge (\neg \exists x Q(x) \vee \forall x P(x))$

$\approx (\exists x \neg P(x) \vee \exists x Q(x)) \wedge (\forall x \neg Q(x) \vee \forall x P(x))$

$\approx \exists y (\neg P(y) \vee Q(y)) \wedge (\forall x \neg Q(x) \vee \forall z P(z))$

$\approx \forall x \forall z \exists y (\neg P(y) \vee Q(y)) \wedge (\neg Q(x) \vee P(z))$

$\forall x \forall z [(\neg P(f(x,z)) \vee Q(f(x,z)) \wedge (\neg Q(x) \vee P(z))]$

(2) $\neg \forall x P(x,y) \vee \forall x R(x,y)$

$\approx \exists x \neg P(x,y) \vee \forall z R(z,y)$

$\approx \forall z \exists x (\neg P(x,y) \vee R(z,y))$

$\forall z (\neg P(f(z),y) \vee R(z,y))$

(3) $\forall x (P(x) \to \neg \exists y R(x,y))$

$\approx \forall x (\neg P(x) \vee \forall y \neg R(x,y))$

$\forall x \forall y (\neg P(x) \vee \neg R(x,y))$

(4) $\exists x \forall y P(x,y) \wedge \forall y \exists x P(y,x)$

(4) $\exists x \forall y\ P(x,y) \wedge \forall y \exists x\ P(y,x)$

$\approx \exists x \forall y\ P(x,y) \wedge \forall u \exists v\ P(u,v)$

$\approx \forall u \exists v \exists x \forall y \left(P(x,y) \wedge P(u,v) \right)$

$\quad \forall u \forall y \left[P(f(u), y) \wedge P(u, g(u)) \right]$

(5) $\neg(\forall x\ P(x) \vee \exists y \neg Q(y)) \vee (\forall z\ P(z) \vee \exists w \neg Q(w))$

$\approx (\neg \forall x\ P(x) \wedge \neg \exists y \neg Q(y)) \vee (\forall z \exists w\ P(z) \vee \neg Q(w))$

$\approx (\exists x \neg P(x) \wedge \forall y\ Q(y)) \vee (\forall z \exists w\ P(z) \vee \neg Q(w))$

$\approx \forall z \exists w \exists x \forall y [(\neg P(x) \wedge Q(y)) \vee (P(z) \vee \neg Q(w))]$

$\quad \forall z \forall y\ [(\neg P(f(z)) \wedge Q(y)) \vee (P(z) \vee \neg Q(g(z)))]$

(6) $\neg \forall x (P(x) \vee \exists y \neg Q(y)) \vee (\forall z\ P(z) \vee \exists w \neg Q(w))$

$\approx \exists x (\neg P(x) \wedge \neg \exists y \neg Q(y)) \vee (\forall z \exists w\ P(z) \vee \neg Q(w))$

$\quad \exists x (\neg P(x) \wedge \forall y\ Q(y)) \vee (\forall z \exists w\ P(z) \vee \neg Q(w))$

(7) $\neg((\neg \forall x\ P(x) \wedge \forall x\ Q(x)) \wedge (\exists x\ R(x) \rightarrow \forall x\ S(x)))$

$\approx \neg(\neg \forall x P(x) \wedge \forall x\ Q(x)) \vee \neg(\neg \exists x\ R(x) \vee \forall x\ S(x))$

$\approx (\forall x\ P(x) \vee \neg \forall x\ Q(x)) \vee (\exists x\ R(x) \wedge \neg \forall x\ S(x))$

$\approx (\forall x\ P(x) \vee \exists y \neg Q(y)) \vee (\exists z\ R(z) \wedge \exists w \neg S(w))$

$\approx \forall x \exists y \exists z \exists w [(P(x) \vee \neg Q(y)) \vee (R(z) \wedge \neg S(w))]$

$\quad \forall x [(P(x) \vee \neg Q(f(x))) \vee (R(g(x)) \wedge \neg (h(x))]$

(8) $\neg(\exists x\ P(x,y) \wedge (\forall y\ Q(y) \rightarrow P(x,x))) \rightarrow \forall x \exists y\ R(x,y)$

$\approx \neg(\neg \exists x\ P(x,y) \vee \neg(\neg \forall y\ Q(y) \vee P(x,x))) \vee \forall z \exists w\ R(z,w)$

$\approx (\exists x\ P(x,y) \wedge (\neg \forall y\ Q(y) \vee P(x,x))) \vee \forall z \exists w\ R(z,w)$

$\approx (\exists u\ P(u,y) \wedge (\exists v \neg Q(v) \vee P(x,x))) \vee \forall z \exists w\ R(z,w)$

$\approx \forall z \exists w \exists u \exists v [(P(u,y) \wedge (\neg Q(v) \vee P(x,x))) \vee R(z,w)]$

$\quad \forall z\ [(P(f(z),y) \wedge (\neg Q(h(z)) \vee P(x,x))) \vee R(z, g(z))]$

(9) $((\forall x\ P(x) \rightarrow \exists y\ Q(x,y)) \rightarrow Q(x,x)) \rightarrow \forall x \exists y\ R(x,y)$

$\approx \neg(\neg(\neg \forall x\ P(x) \vee \exists y\ Q(x,y)) \vee Q(x,x)) \vee \forall z \exists w\ R(z,w)$

$\approx \neg((\forall u\ P(u) \wedge \neg \exists v\ Q(x,v)) \vee Q(x,x)) \vee \forall z \exists w\ R(z,w)$

$\approx (\neg(\forall u\ P(u) \wedge \neg \exists v\ Q(x,v)) \wedge \neg Q(x,x)) \vee \forall z \exists w\ R(z,w)$

$$\approx (\neg (\forall u P(u) \wedge \neg \exists v\ Q(x,v)) \wedge \neg Q(x,x)) \vee \forall z \exists w R(z,w)$$

$$\approx ((\neg \forall u P(u) \vee \exists v\ Q(x,v)) \wedge \neg Q(x,x)) \vee \forall z \exists w R(z,w)$$

$$\approx ((\exists u \neg P(u) \vee \exists v\ Q(x,v)) \wedge \neg Q(x,x)) \vee \forall z \exists w R(z,w)$$

$$\approx \forall z \exists w \exists u \exists v [((\neg P(u) \vee Q(x,v)) \wedge \neg Q(x,x)) \vee R(z,w)]$$

$$\forall z [((\neg P(j(z)) \vee Q(x, g(z))) \wedge \neg Q(x,x)) \vee R(z, h(z))]$$

(10) $\neg \forall x \neg \forall y \neg \forall z\ P(x,y) \vee \neg \exists x \neg \exists y (\neg \exists z\ Q(x,y,z) \to R(x,y))$

$$\approx \exists x \neg (\neg \forall y \neg \forall z\ P(x,y)) \vee \forall x \neg (\neg \exists y (\exists z\ Q(x,y,z) \vee R(x,y)))$$

$$\approx \exists x \forall y \exists z \neg P(x,y) \vee \forall u \exists v \exists t (Q(u,v,t) \vee R(u,v))$$

$$\approx \forall u \exists v \exists t \exists x \forall y \exists z (\neg P(x,y) \vee Q(u,v,t) \vee R(u,v))$$

$$\forall u \forall y (\neg P(j(u), y) \vee Q(u, g(u), h(u)) \vee R(u, g(u)))$$