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Homework 2
20 April 2023
a) (pv-p) [q/p] = (qv-q)
b) (\rho \vee \neg \rho) [\neg \rho | \rho] = (\neg \rho \vee \rho)

c) (\rho \rightarrow (q \rightarrow r)) [\rho | q, \rho | r] = (\rho \rightarrow (\rho \rightarrow \rho))
d) (ρ → (q → ())[r/q, p/r] = (p → (r → p))
2. Let 4 = p V -p and 0 = [$1p]
  We have: Ψθ = (ρ ν ¬ ρ)θ = (θ(ρ) ν ¬ θ(ρ)) = φ ν ¬ φ
  Because 4 is valid => $ $ $ 7 $ is valid
a) (p -> q) Ap
 = (7p Vq) Ap
   v= {p-> T, g-> L}, [[(¬ρνq)Λρ]]<sub>v=</sub> L
b) (p → q) V ¬p
= 7pvqv7p
 v= {ρ¬Τ, q¬ ⊥}, [[¬ρνqν¬ρ]]ν= 1
c) (p → q) → q
= 7 (7 p vq) v q
: (p / ¬ q) v q
 = (p vq) ~ (¬q vq)
 v= { p → 1 , q → 1 }, [[ Cp vq) ∧ (¬q vq)]]v= 1
d) \rho \rightarrow (\rho \wedge q)
= 7p v (p 19)
 · (¬pvp) ∧ (¬pvq)
 ν= {p > T, q → 1}, [[(¬pvp) Λ(¬pvq)]]ν= 1
1) p -> (p vq)
 = pvpvq
1) (p/a) Vp
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1) (p/a) Vp  $: (\rho \vee \rho) \wedge (\rho \vee q)$ v= {ρ+1, q-> 1 }, [[(pvp) Λ (pvq)]]v= 1 g) (p -> g) -> p : 7 (7pvg) vp : (p 1 79) Vp = (pvp) \ (7gvp) ν= { q → T, p → ⊥ }, [[ (pvp) Λ (¬q vp)]]v= 1 h) p ( (q V() =[ p -> (qvr)] ~[( qvr) -> q] - [ -p v(qvr)] ~ [ - (qvr) vq] = (¬p~q~r) ~[(¬q~r)~q] = (-p vqv() \ (-q vq) \ (-r vq) v= {p→T, q→ ⊥, r→T}, [[·]]v= ⊥ i) ((p → q) → p) → p : ¬(¬(¬ρνq)νρ)νρ : 7 ( (p / 79) vp) vp : 7 ((pvp) / (7qvp)) vp : (¬(pvp) N¬(¬qvp)) vp  $= (\neg \rho \land \neg \rho) \lor (q \land \neg \rho) \lor \rho$  $= (\neg \rho \land \neg \rho) \lor ((q \lor \rho) \land (\neg \rho \lor \rho))$ = [-p v ((qvp) \ (-pvp))] \ [-pv ((qvp) \ (-pv p))]  $= (\neg \rho \vee q \vee \rho) \wedge (\neg \rho \vee \neg \rho \vee \rho) \wedge (\neg \rho \vee q \vee \rho) \wedge (\neg \rho \vee \neg \rho \vee \rho)$ valid