

Homework 12

In these exercises, you may omit derivations of formulas, provided that the formulas are already derived.

1. Show $\mathbf{Q} \vdash \phi$ for each sentence ϕ .
 - (1) $0 + s(0) \doteq s(0)$
 - (2) $s(0) + s(0) \doteq s(s(0))$
 - (3) $\neg(s(0) + s(0) \doteq 0)$
 - (4) $s(0) + s(s(0)) \doteq s(s(0)) + s(0)$
 - (5) $(0 + s(0)) + 0 \doteq 0 + (s(0) + 0)$
 - (6) $0 \cdot s(0) \doteq 0$
 - (7) $s(s(0)) \cdot s(0) \doteq s(s(0))$
 - (8) $\forall x \forall y \forall z (x \doteq y \rightarrow x + z \doteq y + z)$
2. Show $\mathbf{PA} \vdash \phi$ for each sentence ϕ in three steps (a)(b)(c).
 - (1) Let $\phi = \forall x (0 + x \doteq x)$. We perform induction on x . In other words, our induction parameter is x .
 - (a) Show $\mathbf{Q} \vdash 0 + 0 \doteq 0$.
 - (b) Show $\mathbf{Q} \vdash 0 + x \doteq x \rightarrow 0 + s(x) \doteq s(x)$.
 - (c) Show $\mathbf{PA} \vdash \phi$.
 - (1.5) Let $\phi = \forall x \forall y (x + s(y) \doteq s(x) + y)$. We perform induction on y . In other words, our induction parameter is y .
 - (a) Show $\mathbf{Q} \vdash x + s(0) \doteq s(x) + 0$.
 - (b) Show $\mathbf{Q} \vdash x + s(y) \doteq s(x) + y \rightarrow x + s(s(y)) \doteq s(x) + s(y)$.
 - (c) Show $\mathbf{PA} \vdash \phi$.
 - (2) Let $\phi = \forall x \forall y (x + y \doteq y + x)$. We perform induction on y . In other words, our induction parameter is y .
 - (a) Show $\mathbf{PA} \vdash (x + 0 \doteq 0 + x)$.
 - (b) Show $\mathbf{PA} \vdash (x + y \doteq y + x \rightarrow (x + s(y) \doteq s(y) + x))$.
 - (c) Show $\mathbf{PA} \vdash \phi$.
3. Show $\mathbf{PA} \vdash \phi$ for each sentence ϕ . Now you need to find an appropriate induction parameter on your own.
 - (1) $\forall x (0 \cdot x \doteq 0)$
 - (2) $\forall x \forall y \forall z ((x + y) + z \doteq x + (y + z))$