Homework 2

1. Compute each application of a substitution.

   (a) $(p \vee (\neg p))[q/p]$

   (b) $(p \vee (\neg p))[\neg p/p]$

   (c) $(p \to (q \to r))[p/q, p/r]$

   (d) $(p \to (q \to r))[r/q, p/r]$

   Note that atoms are substituted simaltaneously.

2. Show that $\phi \vee (\neg \phi)$ is valid for all formulas ϕ, using the fact that tautologies are closed under substitutions.

3. Compute a conjunctive normal form of each formula, and determine the validity.

   (a) $(p \to q) \wedge p$

   (b) $(p \to q) \vee (\neg p)$

   (c) $(p \to q) \to q$

   (d) $p \to (p \wedge q)$

   (e) $p \to (p \vee q)$

   (f) $(p \wedge q) \vee p$

   (g) $(p \to q) \to p$

   (h) $p \leftrightarrow (q \vee r)$

   (i) $((p \to q) \to p) \to p$

   Recall that conjunctive normal forms are found in three steps;

   (1) eliminate $\to$ and $\leftrightarrow$;

   (2) compute a negation normal form;

   (3) compute a conjunctive normal form.