Homework 1

Study Sections 2.1 – 2.3.

1. Consider the following propositional symbols:

   p: $1 + 3 = 4$ q: $1 + 2 = 4$ r: I open the window. s: We will have fresh air.

   Write down a propositional formula corresponding to each statement.

   (a) We have $1 + 3 = 4$ or $1 + 2 = 4$.
   (b) Neither $1 + 3 = 4$ nor $1 + 2 = 4$ holds.
   (c) If I open the window then we will have fresh air.
   (d) If I open the window then $1 + 2 = 4$.
   (e) If $1 + 2 = 4$ then we will have fresh air.

2. Write down a truth table for each formula, and determine validity and satisfiability of the formula.

   (a) $p \wedge (\neg p)$
   (b) $\neg(p \wedge (\neg p))$
   (c) $(p \wedge q) \to (p \vee r)$
   (d) $((p \to q) \to p) \to p$
   (e) $p \to (\neg p)$
   (f) $\neg(p \to (\neg p))$

3. Prove or disprove each logical equivalence.

   (a) $\neg(p \vee q) \approx (\neg p) \wedge (\neg q)$
   (b) $\neg(p \vee q) \approx (\neg p) \vee (\neg q)$
   (c) $(p \to q) \approx (\neg p) \vee q$
   (d) $(\neg p) \approx (p \to \bot)$
   (e) $(\neg p) \approx (\bot \to p)$
   (f) $p \approx (p \to \top)$
   (g) $p \approx (\top \to p)$
   (h) $(p \to q) \wedge (q \to p) \approx (p \leftrightarrow q)$

4. Check if each statement holds. If not, give a counterexample, namely a formula that does not satisfy the statement.

   (a) If a formula ϕ is satisfiable, then $\neg\phi$ is unsatisfiable.
   (b) Every valid formula is satisfiable.
   (c) For every formula ϕ, either ϕ or $\neg\phi$ is satisfiable.
   (d) For every formula ϕ, either ϕ or $\neg\phi$ is valid.

5. Let ϕ be the formula $(p \wedge ((p \to q) \wedge (p \to r))) \to (q \wedge r)$. Verify that ϕ is valid by writing down the truth table, and observe that the propositional logic can be counterintuitive. For instance, read the propositional symbols as follows:

   p: I have 100 yen. q: I can buy an apple. r: I can buy a chocolate.