Homework 2

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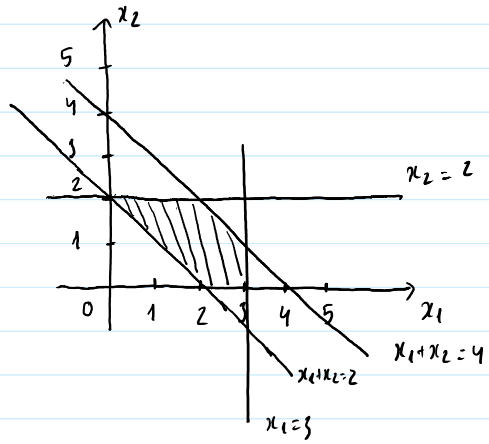
2.1)

$$\begin{aligned}
\text{Maximize} \quad & 2x_1 + x_2 \\
\text{s.t} \quad & x_1 + x_2 \geq 2 \\
& x_1 \leq 3 \\
& x_2 \leq 2 \\
& x_1 + x_2 \leq 4 \\
& x_1, x_2 \geq 0
\end{aligned}$$

a)



b) Standard Form

$$\begin{aligned}
\text{Minimize} \quad & z = -2x_1 - x_2 \\
\text{s.t} \quad & x_1 + x_2 - x_3 = 2 \\
& x_1 + x_4 = 3 \\
& x_2 + x_5 = 2 \\
& x_1 + x_2 + x_6 = 4
\end{aligned}$$

with $A = \begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ 3 \\ 2 \\ 4 \end{bmatrix}$, $c = \begin{bmatrix} -2 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

c) * Choose $B = \{1, 4, 5, 6\}$ as a basis:

$$\begin{array}{c} \left[\begin{array}{cccccc|c} 1 & 1 & -1 & 0 & 0 & 0 & 2 \\ 1 & 0 & 0 & 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 1 & 0 & 2 \\ 1 & 1 & 0 & 0 & 0 & 1 & 4 \end{array}\right] \end{array} \begin{array}{c} (r_1) \\ (r_2) \\ (r_3) \\ (r_4) \end{array}$$

$$\begin{array}{r} \\ r_2 = -r_2 + r_1 \\ \\ r_4 = -r_4 + r_1 \end{array} \left[\begin{array}{cccccc|c} 1 & 1 & -1 & 0 & 0 & 0 & 2 \\ 0 & 1 & -1 & -1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & 0 & 0 & -1 & -2 \end{array}\right]$$

$$\begin{array}{l} r_2 = -r_2 + r_1 \\ \\ r_4 = -r_4 + r_1 \end{array} \begin{bmatrix} 0 & 1 & -1 & -1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & 0 & 0 & -1 & -2 \end{bmatrix}$$

$$\longrightarrow \begin{array}{l} \\ r_2 = -r_2 \\ \\ r_4 = -r_4 \end{array} \begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 2 \\ 0 & -1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 & 2 \end{bmatrix}.$$

- The objective function become:

$$z + 2(2 - x_2 + x_3) + x_2 = 0$$

$$\Leftrightarrow z - x_2 + 2x_3 = -4.$$

- Simplex tabular of the canonical form:

		x_1	x_2	x_3	x_4	x_5	x_6
z	-4	0	-1	2	0	0	0
x_1	2	1	1	-1	0	0	0
x_4	1	0	-1	[1]	1	0	0
x_5	2	0	1	0	0	1	0
x_6	2	0	0	1	0	0	1

$\longrightarrow$ x_3 enter the basis, x_4 leave the basis

		x_1	x_2	x_3	x_4	x_5	x_6
z	-6	0	1	0	-2	0	0
x_1	3	1	0	0	1	0	0
x_3	1	0	-1	1	1	0	0
x_5	2	0	1	0	0	1	0
x_6	1	0	[1]	0	-1	0	1

$\longrightarrow$ x_2 enter the basis, x_6 leave the basis

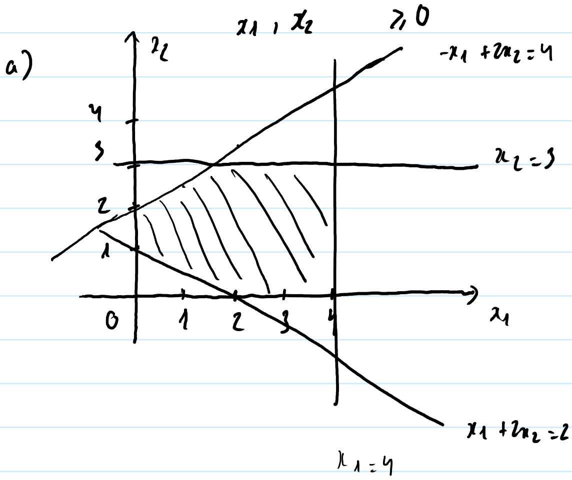
		x_1	x_2	x_3	x_4	x_5	x_6
z	-7	0	0	0	-1	0	-1
x_1	3	1	0	0	1	0	0
x_3	2	0	0	1	0	0	1
x_5	1	0	0	0	1	1	-1
x_2	1	0	1	0	-1	0	1

Optimal solution: $x^* = [3, 1, 2, 0, 1, 0]$, $z^* = -7$

2.2)

$$\begin{array}{ll} \text{Maximize} & x_1 + x_2 \\ \text{S.t} & -x_1 + 2x_2 \le 4 \\ & x_1 + 2x_2 \ge 2 \\ & x_1 \le 4 \\ & x_2 \le 3 \\ & x_1, x_2 \ge 0 \end{array}$$

a)



b) Standard Form:

$$\text{Minimize} \quad -x_1 - x_2$$

$$\begin{aligned}
\text{s.t} \quad -x_1 + 2x_2 + x_3 \quad &= 4 \\
x_1 + 2x_2 \quad - x_4 \quad &= 2 \\
x_1 \quad + x_5 \quad &= 4 \\
x_2 \quad + x_6 &= 3
\end{aligned}$$

with

$$A = \begin{bmatrix} -1 & 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} ; \; b = \begin{bmatrix} 4 \\ 2 \\ 4 \\ 3 \end{bmatrix} ; \; c = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

c) Choose $\mathcal{B} = \{1, 3, 5, 6\}$ as basis:

$$\left[\begin{array}{cccccc|c} -1 & 2 & 1 & 0 & 0 & 0 & 4 \\ 1 & 2 & 0 & -1 & 0 & 0 & 2 \\ 1 & 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 1 & 0 & 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} (r_1) \\ (r_2) \\ (r_3) \\ (r_4) \end{array}$$

$$\begin{array}{l} r_1 = r_2 + r_1 \\ \\ r_3 = -r_2 + r_3 \\ \\ \end{array} \left[\begin{array}{cccccc|c} 0 & 4 & 1 & -1 & 0 & 0 & 6 \\ 1 & 2 & 0 & -1 & 0 & 0 & 2 \\ 0 & -2 & 0 & 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

- The objective function become:

$$z + 2(2 - 2x_2 + x_4) + x_2 = 0$$

$$\Leftrightarrow \quad z - 3x_2 + 2x_4 = -4$$

- Simplex tabular of the canonical form:

		x_1	x_2	x_3	x_4	x_5	x_6
z	-4	0	-3	0	2	0	0
x_3	6	0	4	1	-1	0	0
x_1	2	1	2	0	-1	0	0
x_5	2	0	-2	0	[1]	1	0
x_6	3	0	1	0	0	0	1

$\longrightarrow$

x_4 enter the basis

x_5 leave the basis

		x_1	x_2	x_3	x_4	x_5	x_6
z	-8	0	1	0	0	-2	0
x_3	8	0	2	1	0	1	0
x_1	4	1	0	0	0	1	0
x_4	2	0	-2	0	1	1	0
x_6	3	0	[1]	0	0	0	1

x_2 enter the basis
x_6 leave the basis

		x_1	x_2	x_3	x_4	x_5	x_6
z	-11	0	0	0	0	-2	-1
x_3	2	0	0	1	0	1	-2
x_1	4	1	0	0	0	1	0
x_4	8	0	0	0	1	1	2
x_2	5	0	1	0	0	0	1

Optimal solution $x^* = [4, 5, 2, 8, 0, 0]^T$, $z^* = -7$