Homework 7

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5.7.1.8)

$$\begin{aligned} \text{minimize} \quad & -\log(3+x_1) - \log(9+x_2) && = f(x) \\ \text{s.t} \quad & x_1 + x_2 = 5 && = h(x) \\ & x_1 \geq 0 && = g_1(x) \\ & x_2 \geq 0 && = g_2(x) \end{aligned}$$

a) The first-order KKT conditions follow:

If $x^* = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is a local optimum then exists λ, μ_1, μ_2 which satisfied:

$$\begin{cases} \begin{bmatrix} \frac{-1}{(3+x_1)\ln 10} \\ \frac{-1}{(9+x_2)\ln 10} \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \mu_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \mu_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 & (1) \\ x_1 + x_2 - 5 = 0 & (2) \\ x_1 \geq 0 & (3) \\ x_2 \geq 0 & (4) \\ \mu_i \geq 0 \quad \text{for } i = \{1, 2\} & \\ \mu_1 x_1 = 0 & \\ \mu_2 x_2 = 0 & \end{cases}$$

bcd) * Assume g_1 is active and g_2 is inactive :

(3)(4) $\Leftrightarrow$ $x_1 = 0$ and $x_2 > 0$, $\mu_2 = 0$

(2) $\Leftrightarrow$ $x_2 = 5$

(1) $\Leftrightarrow$ $\begin{bmatrix} \frac{-1}{3\ln 10} \\ \frac{-1}{14\ln 10} \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \mu_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0$

$\Leftrightarrow \begin{cases} \lambda = \frac{1}{14\ln 10} \end{cases}$

$$\Leftrightarrow \begin{cases} \lambda = \dfrac{1}{14\ln 10} \\ \mu_1 = \dfrac{1}{14\ln 10} - \dfrac{1}{3\ln 10} \end{cases}$$

Because $\mu_1 < 0$, reject this solution

* Assume g_1 is inactive and g_2 is active:

(3) (4) $\Leftrightarrow$ $x_1 \geq 0, \mu_1 = 0$ and $x_2 = 0$

(2) $\Leftrightarrow$ $x_1 = 5$

$$(1) \Leftrightarrow \begin{bmatrix} \dfrac{-1}{8\ln 10} \\ \dfrac{-1}{9\ln 10} \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \mu_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0$$

$$\Leftrightarrow \begin{cases} \lambda = \dfrac{1}{8\ln 10} \\ \mu_2 = \dfrac{1}{8\ln 10} \dfrac{-1}{9\ln 10} \end{cases}$$

Because $\mu_2 \geq 0$ and $g_1 > 0$, $x = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$ is a local optimum

* Assume both g_1 and g_2 are active:

(3) (4) $\Leftrightarrow$ $x_1 = 0$ and $x_2 = 0$

Because $h(x)$ is not satisfied, reject this solution

* Assume both g_1 and g_2 are inactive:

(3)(4) $\Leftrightarrow$ $x_1 \geq 0$, $\mu_1 = 0$ and $x_2 \geq 0$, $\mu_2 = 0$

$$(1)(2) \Leftrightarrow \begin{cases} x_1 + x_2 = 5 \\ \dfrac{-1}{(3+x_1)\ln 10} + \lambda = 0 \\ \dfrac{-1}{(9+x_2)\ln 10} + \lambda = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x_1 + x_2 = 5 \\ x_1 = \dfrac{1}{\lambda \ln 10} - 3 \\ x_2 = \dfrac{1}{\lambda \ln 10} - 9 \end{cases}$$

$$\Leftrightarrow \begin{cases} \dfrac{1}{\lambda \ln 10} - 3 + \dfrac{1}{\lambda \ln 10} - 9 = 5 \\ x_1 = \dfrac{1}{\lambda \ln 10} - 3 \\ x_2 = \dfrac{1}{\lambda \ln 10} - 9 \end{cases}$$

$$\Leftrightarrow \begin{cases} \lambda = \dfrac{2}{17 \ln 10} \\ x_1 = \dfrac{11}{2} \\ x_2 = \dfrac{-1}{2} \end{cases}$$

Because $x_2 = \frac{-1}{2}$ not satisfied $g_2(x)$, reject this solution

The final solution is $x^* = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$, $\lambda = \frac{1}{8\ln 10}$, $\mu_1 = 0$, $\mu_2 = \frac{1}{8\ln 10} - \frac{1}{9\ln 10}$