Note

19 January 2023 09:49

* Change of problem (Minimize)
  + Change constraint $b \to b + \Delta b$

$$\underbrace{A_B^{-1} b}_{\bar{b}(CF)} + A_B^{-1} \Delta b \geq 0$$

  + Change objective function $c \to c + \Delta c$

$$\left(c_B^T \underbrace{A_B^{-1} a_j}_{\text{col of } \bar{A}(CF)} - c_j\right) + \left(\Delta c_B^T A_B^{-1} a_j - \Delta c_j\right) \leq 0 \;,\; j \in N$$

* Duality

min $c^T x$	max $b^T y$
$Ax = b$	$A^T y = c$
$x \geq 0$	$A^T y \leq c$
$x \leq 0$	$A^T y \geq c$
x free	$A^T y = c$
$Ax \geq b$	$y \geq 0$
$Ax \leq b$	$y \leq 0$
$Ax = b$	y free

* Complementary Slackness Condition

$$x_i = 0 \quad \text{or} \quad a_i^T y = c_i \;,\; i = 1 \dots n$$

$$y_i = 0 \quad \text{or} \quad A_i x = b_i \;,\; i = 1 \dots m$$

* Canonical form
  + Form: $A_B = I_m$; $c_B = 0^m$

$$x_B = b \;;\; x_N = 0^{n-m}$$

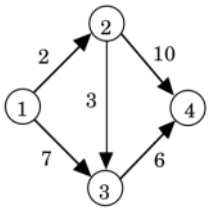
  + Convert standard form to canonical form

$$\min z(x) = d^T b + (c^T - d^T A)x \;;\; d = (A_B^T)^{-1} c_B$$

$$\text{s.t} \quad A_B^{-1} A x = A_B^{-1} b$$

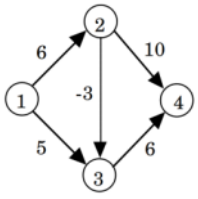
$$x \geq 0$$

* Dijkstra



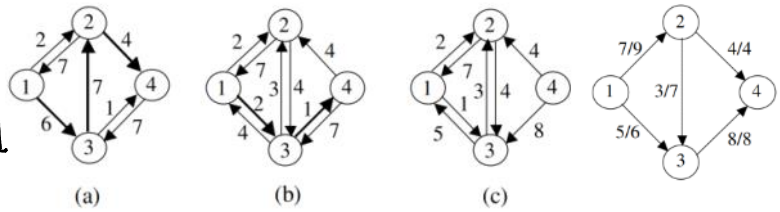
	u				π			
	1	2	3	4	1	2	3	4
0	0	∞	∞	∞	-			
1		2	7	∞	-	1	1	
2			5	12	-	1	2	2
3				11	-	1	2	3

* Bellman-Ford



	u				π				
k	1	2	3	4	1	2	3	4	S^k
0	0	∞	∞	∞	-				1
1	0	6	5	∞	-	1	1		2, 3
2	0	6	3	11	-	1	2	3	3, 4
3	0	6	3	9	-	1	2	3	4
4	0	6	3	9	-	1	2	3	-

* Ford-Fulkerson



(a) (b) (c)