I214 System Optimization
Chapter 5: Nonlinear Programming (Part B)

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Outline

5.3 Unconstrained Optimization

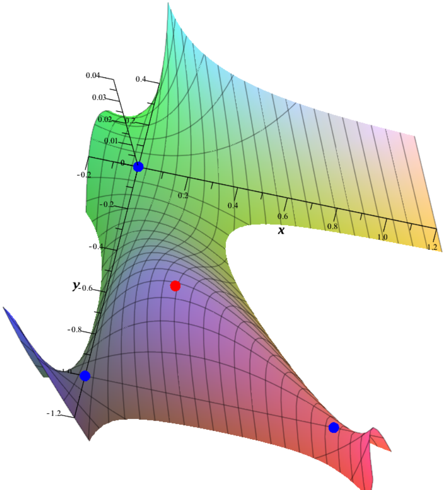
This section considers the case $\mathcal{S} = \mathbb{R}^n$, that is there are no restrictions on the variables, and our goal is to minimize an objective function $f(\mathbf{x})$.

$$\begin{array}{rl} \textbf{Minimize} & f(\mathbf{x}) \\ \textbf{subject to} & \mathbf{x} \in \mathbb{R}^n \end{array} \tag{1}$$

Example

Find and classify the stationary points of the function:

$$f(x_1, x_2) = (x_1 + x_2)(x_1 x_2 + x_1 x_2^2)$$



5.4 Numerical Approaches to Unconstrained Problems*

This section will be skipped.

5.5 Optimization with Equality Constraints

This section considers the constrained problem with only equality constraints:

$$\begin{aligned} \textbf{Minimize} \quad & f(\mathbf{x}) \\ \textbf{subject to} \quad & h_i(\mathbf{x}) = 0 \text{ for } i \in \{1, 2, \ldots, \ell\} \end{aligned} \tag{2}$$

Idea of Lagrange Multipliers

This figure illustrates minimizing $f(x, y)$ subject to one equality constraint $g(x, y) = c$.

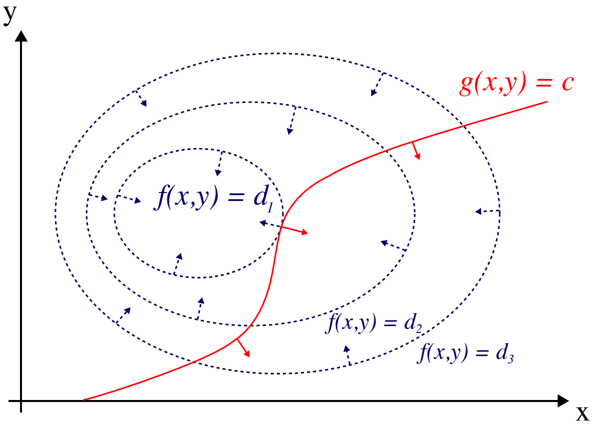
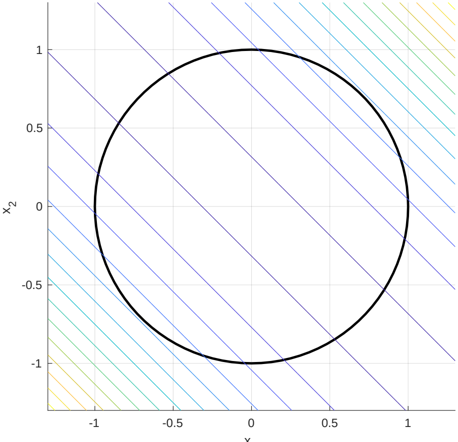


Figure 1: Credit: Wikipedia/Nexcis

Example

Consider the following constrained optimization problem:

$$\begin{aligned} \textbf{Minimize} \quad & (x_1 + x_2)^2 \\ \textbf{subject to} \quad & x_1^2 + x_2^2 = 1. \end{aligned} \tag{3}$$



Class Info

- Midterm exam will be Friday, January 20 13:30–15:10. Paper-based in the classroom.
- Next lecture: Monday, January 23 at 10:50. Continue Nonlinear Programming
- Homework 6. Deadline: Friday, January 27 at 18:00.

Midterm Exam

The midterm is Friday, January 20 13:30–15:10. The exam is closed book. You may use:

- One page of notes, A4-sized paper, double-sided OK.
- Blank scratch paper

You may not use anything else: No printed materials, including books, lecture notes, and slides. No notes (except as above). No internet-connected devices. No calculators — largest matrix inverse will be 2×2.

Exam Content

- Problems similar to Homework 1–5
- Solutions to Homework 1–5 are provided.