

# Homework 5

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5.2

a)

$x$	$p_X$		$x$	$C(x)$	$l(x)$
1	$\frac{1}{3}$		1	0	1
2	$\frac{1}{3}$		2	10	2
3	$\frac{1}{3}$		3	11	2

$$L(C) = \sum_x p_X(x) l(x) = \frac{1}{3} + \frac{2}{3} + \frac{2}{3} = \frac{5}{3}$$

b)  $p_X(x_1, x_2, x_3) = p_X(x_1) p_X(x_2) p_X(x_3)$

Because  $x_i$  is uniform distributed on  $X = \{1, 2, 3\}$

$$p_X(x_1, x_2, x_3) = \left(\frac{1}{3}\right)^3 = \frac{1}{27} \quad \text{with } x_i \in X$$

c)  $L^* = \sum_{x \in X} p(x) l(x) = 4.8148$

$R^* = \frac{1}{n} L = 1.6049$

d) The single-symbol compression has code rate  $R = L = \frac{5}{3} < R^*$ , so the vector compression is better

As  $n \rightarrow \infty$ , the compression rate reach the lower bound

$$\lim_{n \rightarrow \infty} R = H(X) = H(X) = - \sum_{i=1}^3 p_X(x_i) \log p_X(x_i)$$

$$= - \left( \frac{1}{3} \log \frac{1}{3} + \frac{1}{3} \log \frac{1}{3} + \frac{1}{3} \log \frac{1}{3} \right)$$

$$= \log 3 \text{ bits / symbol}$$