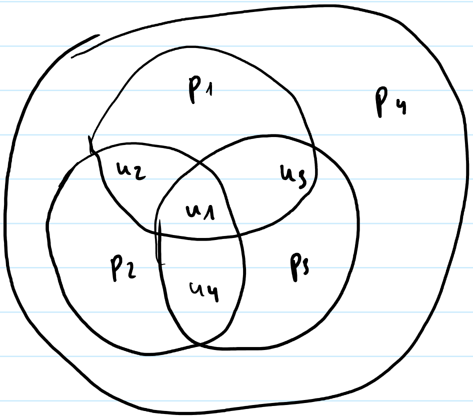
Homework 1

12 October 2023 20:14

1.7

a)



b) $u = [0, 1, 1, 1]$

$\Rightarrow c = [0, 1, 1, 1, 0, 0, 0, 1]$

c) $y_1 = [0, 0, 1, 1, 0, 0, 1, 0]$

$\Rightarrow \hat{c}_1 = [1, 0, 1, 1, 0, 0, 1, 0]$

$y_2 = [0, 1, 1, 0, 0, 1, 1, 1]$

$\Rightarrow \hat{c}_2 = [0, 1, 1, 0, 0, 1, 1, 0]$

d) $y = [?, 1, 0, ?, 0, 1, ?, 0]$

Due to the Hamming code characteristic, we have $u_1 = u_4$ as $u_2 = p_2 = 1$, also because $u_3 = 0$ we have $u_1 = 1$ and $p_3 = 0$. So $\hat{c} = [1, 1, 0, 1, 0, 1, 0, 0]$

1.5

```python
import numpy as np
import scipy

def channels(x, sigma2):
    return np.random.normal(x, np.sqrt(sigma2))

def prob_error(sigma2):
    return 0.5 * scipy.special.erfc(1 / np.sqrt(2 * sigma2))

N = 10000
sigma2_list = [1, 0.5, 0.15]
X = [-1, 1]
p_X = [0.5, 0.5]
for sigma2 in sigma2_list:
    N_err = 0
    for _ in range(N):
        x = np.random.choice(X, size=1, p=p_X)
        y = channels(x, sigma2)
        x_hat = -1 if y < 0 else 1
        if x != x_hat:
            N_err += 1
    print(f"Sigma2: {sigma2} - Estimated P_e: {N_err / N} - "
          f"Analytical P_e: {prob_error(sigma2)}")
```

Sigma2: 1 - Estimated P_e: 0.1584 - Analytical P_e: 0.15865525393145707
Sigma2: 0.5 - Estimated P_e: 0.0834 - Analytical P_e: 0.07864960352514258
Sigma2: 0.15 - Estimated P_e: 0.0042 - Analytical P_e: 0.004911637253759624