Homework 2

15 October 2023 21:12

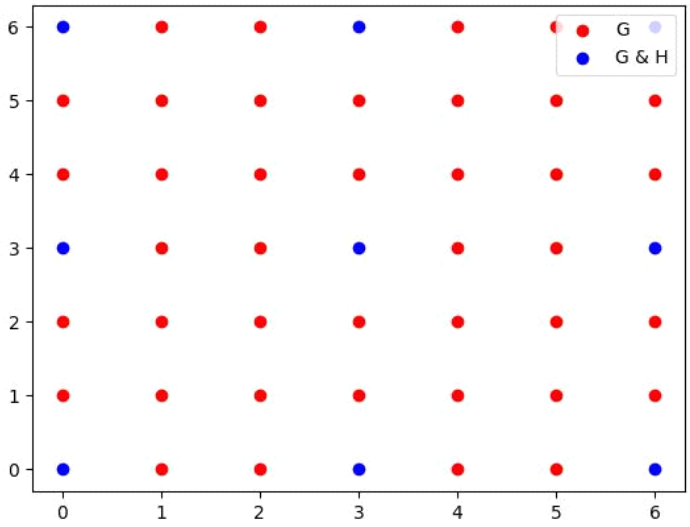
2.1

a) There are 4 element: $G/H = \{0+H, 1+H, 2+H, 3+H\}$

b) 0 and 1

2.2

a)



b) $G = \mathbb{Z}^2$, $H = (3\mathbb{Z})^2$

We have $G/H = \{a+H \mid a \in G\}$

$= \{a+(3\mathbb{Z})^2 \mid a \in \mathbb{Z}^2\}$

G/H has 9 coset leaders. An example of a set of coset leaders is:

$$\left\{ \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}0\\2\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\0\end{bmatrix}, \begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}2\\2\end{bmatrix} \right\}$$

2.3

a)

2.)

a)

- $c_3 + c_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
- $c_2 + c_6 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
- $c_{11} + c_9 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- $c_{14} + c_4 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$
- $c_6 + c_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

b) beach