Minor Research Report 4

06/09/2023

Student: TRAN, Thanh Cong
Supervisor: Nao Hirokawa

I. Overview

- Objectives:
  - Translate IMP code into type expressions and generate type constraint equations
  - Perform type inference by using unification algorithm
- Progress: Done
- Source code: https://github.com/thanhtcptit/imp-typescript/blob/main/imp_interpreter/typing.ts

II. Implementation

1. Type terms

Term	**Form**
Type Variable	α
Type Arrow	α → α
Type Application	α α
Type Constructor	Int, Bool, Unit

2. Type environment (**Γ**)

Variable	**Type**
0, 1, 2, …	Int
true, **false**	Bool
+, **-**	Int → Int → Int
>, **>=**, **<**, **<=**, **==**, **!=**	Int → Int → Bool
&&, **||**	Bool → Bool → Bool
!	Bool → Bool
:=	α → α → Unit
if	Bool → Unit → Unit
if-else	Bool → Unit → Unit → Unit
while	Bool → Unit → Unit
;	Unit → α → α

Variable	Type
0, 1, 2, …	Int
true, **false**	Bool
+, **-**	Int → Int → Int
>, **>=**, **<**, **<=**, **==**, **!=**	Int → Int → Bool
&&, **||**	Bool → Bool → Bool
!	Bool → Bool
:=	α → α → Unit
if	Bool → Unit → Unit
if-else	Bool → Unit → Unit → Unit
while	Bool → Unit → Unit
return	α → α

3. Type expressions

Expression (e)	Form	Example
Variable	x	x
Integer	0, 1, 2, …	1
Boolean	**true**, **false**	**true**
λ-abstraction	λx. e	λx. x
Application	e1 e2	λx. λy. (+) x y
Let	let t = e1 in e2	let t = λx. (+) x 1 in t 1

4. Type inference rules

Expression	Deduction form
Variable	$\frac{\mathbf{\Gamma}(x) = \alpha}{\mathbf{\Gamma} \vdash x : \alpha}$
Integer	$\frac{}{\mathbf{\Gamma} \vdash 1 : Int}$
Boolean	$\frac{}{\mathbf{\Gamma} \vdash true : Bool}$

λ-abstraction	$\frac{\mathbf{\Gamma} \cup \{x : \alpha\} \vdash e : \beta}{\mathbf{\Gamma} \vdash \lambda x.\, e : \alpha \to \beta}$
Application	$\frac{\mathbf{\Gamma} \vdash e_1 : \alpha \to \beta \quad \mathbf{\Gamma} \vdash e_2 : \alpha}{\mathbf{\Gamma} \vdash e_1\, e_2 : \beta}$
Let	$\frac{\mathbf{\Gamma} \vdash e_1 : \alpha \quad \mathbf{\Gamma} \cup \{x : \alpha\} \vdash e_2 : \beta}{\mathbf{\Gamma} \vdash let\ x = e_1\ in\ e_2 : \beta}$

5. Type constraints

Expression	**Constraint**
Variable	$\frac{\mathbf{\Gamma}(x) = \bot}{\mathbf{\Gamma} \vdash x : a}\ [a \approx \alpha]$
	$\frac{\mathbf{\Gamma}(x) = \tau}{\mathbf{\Gamma} \vdash x : a}\ [a \approx \tau']$
λ-abstraction	$\frac{\mathbf{\Gamma} \cup \{x : b\} \vdash e : c}{\mathbf{\Gamma} \vdash \lambda x.\, e : a}\ [a \approx b \to c]$
Application	$\frac{\mathbf{\Gamma} \vdash e_1 : b \quad \mathbf{\Gamma} \vdash e_2 : c}{\mathbf{\Gamma} \vdash e_1\, e_2 : a}\ [b \approx c \to a]$
Let	$\frac{\mathbf{\Gamma} \vdash e_1 : b \quad \mathbf{\Gamma} \cup \{x : b\} \vdash e_2 : c}{\mathbf{\Gamma} \vdash let\ x = e_1\ in\ e_2 : a}\ [a \approx c]$