

2. $\frac{dx}{dt} = x + 5y$

$$\frac{dy}{dt} = -x - 3y$$

$$x(0) = 5, y(0) = 4, \Delta t = \frac{1}{4}$$

$$x(t) = 5e^{-t}(\cos t + 6 \sin t), y(t) = e^{-t}(4 \cos t - 13 \sin t)$$

3. $\frac{dx}{dt} = x + 3y$

$$\frac{dy}{dt} = x - y + 2e^t$$

$$x(0) = 0, y(0) = 2, \Delta t = \frac{1}{4}$$

$$x(t) = -e^{-2t} + 3e^{2t} - 2e^t, y(t) = e^{-2t} + e^{2t}$$

4. $\frac{dx}{dt} = 3x + e^{2t}$

$$\frac{dy}{dt} = -x + 3y + te^{2t}$$

$$x(0) = 2, y(0) = -1, \Delta t = \frac{1}{4}$$

$$x(t) = 3e^{3t} - e^{2t}, y(t) = e^{3t} - 3te^{2t} - 2e^{2t} - te^{2t}$$

5.

$$\vec{x}' = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} \vec{x}, \quad \vec{x}(0) = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

$$\vec{x}(t) = -\frac{8}{5}\mathbf{e}^{-t} \begin{pmatrix} -1 \\ 1 \end{pmatrix} - \frac{4}{5}\mathbf{e}^{4t} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

6.

$$\vec{x}' = \begin{pmatrix} 3 & -13 \\ 5 & 1 \end{pmatrix} \vec{x} \quad \vec{x}(0) = \begin{pmatrix} 3 \\ -10 \end{pmatrix}$$

$$\vec{x}(t) = -2\mathbf{e}^{2t} \begin{pmatrix} \cos(8t) - 8\sin(8t) \\ 5\cos(8t) \end{pmatrix} + \frac{5}{8}\mathbf{e}^{2t} \begin{pmatrix} 8\cos(8t) + \sin(8t) \\ 5\sin(8t) \end{pmatrix}$$