SCA1 Homework 1

Thanh Tran Viet-h12430817, Krishna Varun Varma Kalidindi-h12438206, Iryna Ustych-h12427988

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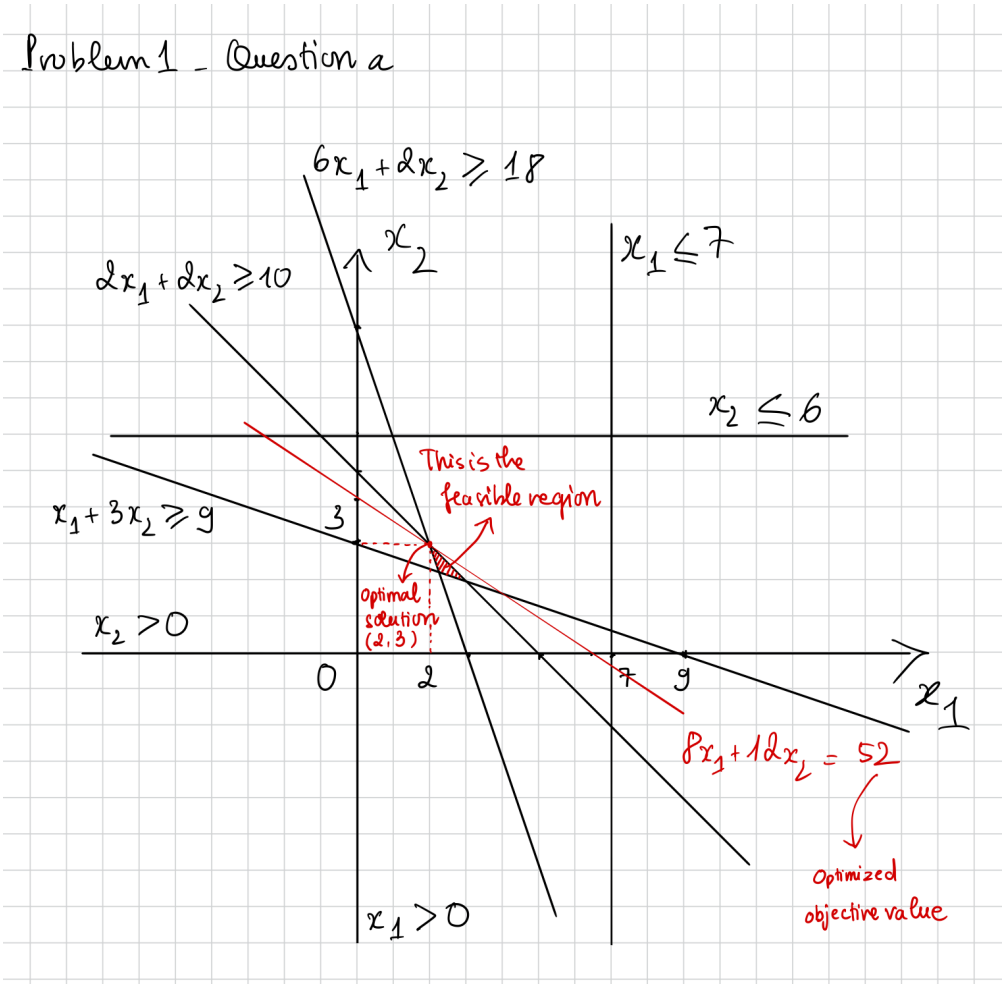
Problem 1

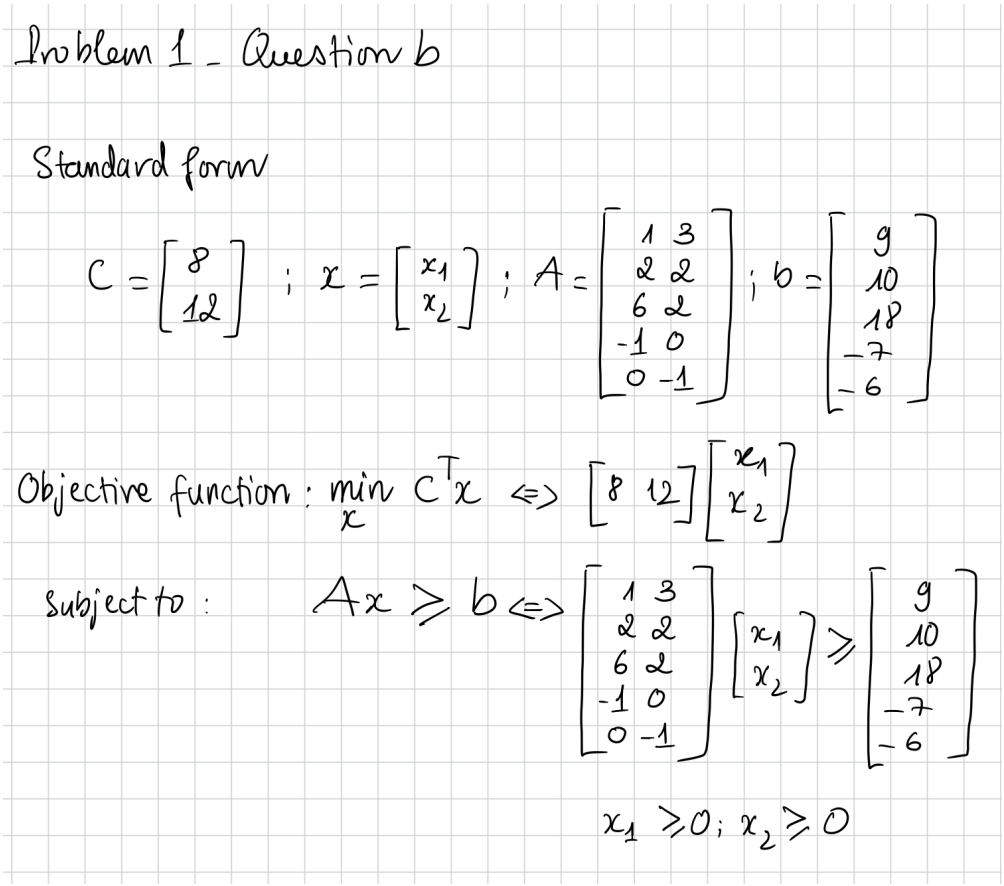
Question a

```
knitr::include_graphics("D:\\WU\\SCA1\\Exercise 1\\Problem_1_Question_a.PNG")
```



Question b

```
knitr::include_graphics("D:\\WU\\SCA1\\Exercise 1\\Problem_1_Question_b.PNG")
```



Question c

Import libraries

```
library(lpSolve)

library(scatterplot3d)
```

Solve the problem

```
optfunc_1 <- c(8,12)

lhs_1 <- matrix(data = c(1,3,2,2,6,2,1,0,0,1), nrow = 5, ncol = 2, byrow = TRUE)

dir_1 <- c(">=",">=",">=","<=","<=")

rhs_1 <- c(9,10,18,7,6)

model_1 <- lp(direction = "min",
              objective.in = optfunc_1,
              const.mat = lhs_1,
              const.dir = dir_1,
              const.rhs = rhs_1,
              compute.sens = TRUE,
              all.int = TRUE)
```

```
print(paste("The solution is", model_1$solution[1], model_1$solution[2]))
```

```
## [1] "The solution is 3 2"
```

Question d

Calculate the lower and upper bounds of the objective function coefficients

```
print(paste("The lower bound of objective function coefficients is",
            model_1$sens.coef.from[1],model_1$sens.coef.from[2]))
```

```
## [1] "The lower bound of objective function coefficients is 4 8"
```

```
print(paste("The upper bound of objective function coefficients is",
            model_1$sens.coef.to[1],model_1$sens.coef.to[2]))
```

```
## [1] "The upper bound of objective function coefficients is 12 24"
```

Because the lower bound of objective function coefficient for x1 is 4, when it changes from 8 to 6, the optimized solution will not change.

```
optfunc_2 <- c(6,12)

model_2 <- lp(direction = "min",
                    objective.in = optfunc_2,
                    const.mat = lhs_1,
                    const.dir = dir_1,
                    const.rhs = rhs_1,
                    compute.sens = TRUE,
                    all.int = TRUE)

print(paste("The solution is", model_2$solution[1], model_2$solution[2]))
```

```
## [1] "The solution is 3 2"
```

However, the lower bound of objective function coefficient for x2 is 8 so when it decreases from 12 to 2, the optimized solution will change.

```
optfunc_3 <- c(8,2)

model_3 <- lp(direction = "min",
                    objective.in = optfunc_3,
                    const.mat = lhs_1,
                    const.dir = dir_1,
                    const.rhs = rhs_1,
                    compute.sens = TRUE,
                    all.int = TRUE)

print(paste("The revised solution is", model_3$solution[1], model_3$solution[2]))
```

```
## [1] "The revised solution is 1 6"
```

Question e

```
sdp_1 <- model_1$duals

for (i in 1:length(sdp_1)) {
```

```
 print(paste("The shadow price for constraint", i, "is", sdp_1[i]))
}
```

```
## [1] "The shadow price for constraint 1 is 2"
## [1] "The shadow price for constraint 2 is 3"
## [1] "The shadow price for constraint 3 is 0"
## [1] "The shadow price for constraint 4 is 0"
## [1] "The shadow price for constraint 5 is 0"
## [1] "The shadow price for constraint 6 is 0"
## [1] "The shadow price for constraint 7 is 0"
```

```
for (i in 1:length(sdp_1[1:5])) {
 print(paste("If the RHS of constraint", i, "increases by 1, the objective value will increase by",
             sdp_1[i]))
}
```

```
## [1] "If the RHS of constraint 1 increases by 1, the objective value will increase by 2"
## [1] "If the RHS of constraint 2 increases by 1, the objective value will increase by 3"
## [1] "If the RHS of constraint 3 increases by 1, the objective value will increase by 0"
## [1] "If the RHS of constraint 4 increases by 1, the objective value will increase by 0"
## [1] "If the RHS of constraint 5 increases by 1, the objective value will increase by 0"
```

```
bd_const_1 <- lhs_1 %*% model_1$solution == rhs_1

for (j in 1:length(bd_const_1)) {
  print(paste("Constraint", j,"is binding?", bd_const_1[j]))
}
```

```
## [1] "Constraint 1 is binding? TRUE"
## [1] "Constraint 2 is binding? TRUE"
## [1] "Constraint 3 is binding? FALSE"
## [1] "Constraint 4 is binding? FALSE"
## [1] "Constraint 5 is binding? FALSE"
```

```
allow_decrease_1 <- model_1$duals.from

allow_increase_1 <- model_1$duals.to

for (k in 1:length(allow_decrease_1[1:5])) {
  print(paste("The lower bound of RHS values for which shadow price", k,
              "holds is", allow_decrease_1[k]))
}
```

```
## [1] "The lower bound of RHS values for which shadow price 1 holds is 5"
## [1] "The lower bound of RHS values for which shadow price 2 holds is 9"
## [1] "The lower bound of RHS values for which shadow price 3 holds is -1e+30"
## [1] "The lower bound of RHS values for which shadow price 4 holds is -1e+30"
## [1] "The lower bound of RHS values for which shadow price 5 holds is -1e+30"
```

```
for (k in 1:length(allow_increase_1[1:5])) {
  print(paste("The upper bound of RHS values for which shadow price", k,
              "holds is", allow_increase_1[k]))
}
```

```
## [1] "The upper bound of RHS values for which shadow price 1 holds is 11"
## [1] "The upper bound of RHS values for which shadow price 2 holds is 15.3333333333333"
## [1] "The upper bound of RHS values for which shadow price 3 holds is 1e+30"
```

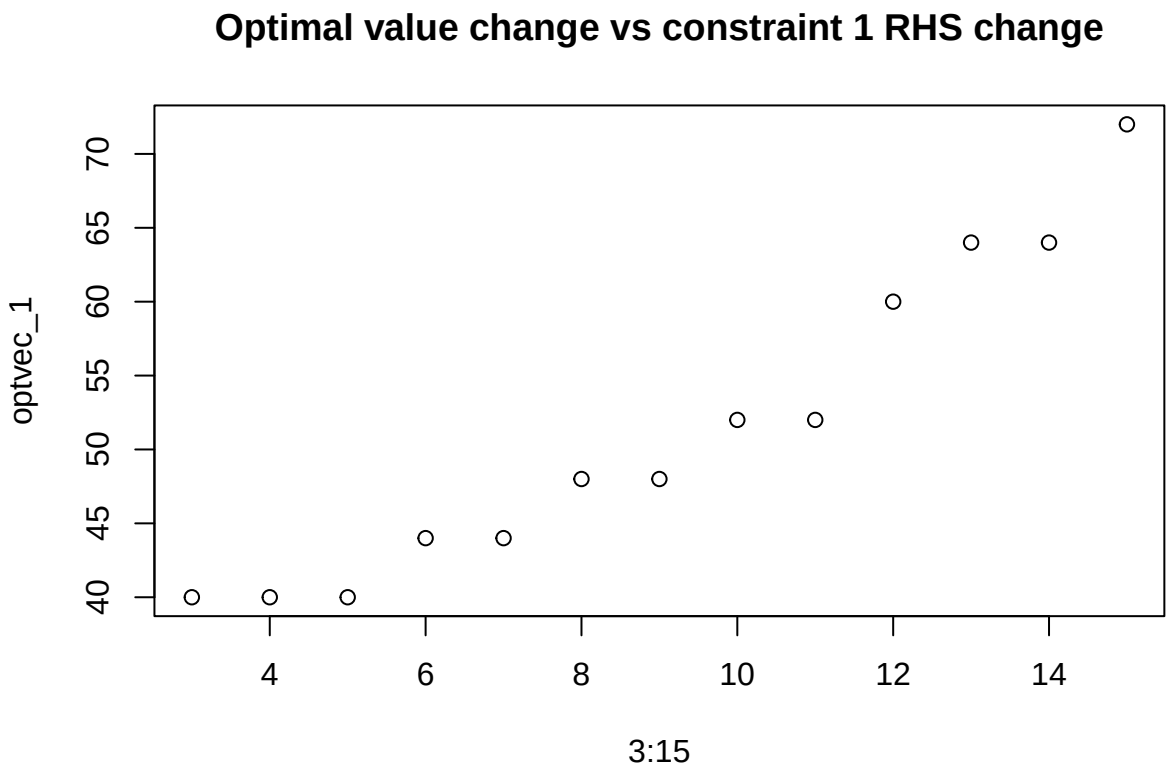
```
## [1] "The upper bound of RHS values for which shadow price 4 holds is 1e+30"
## [1] "The upper bound of RHS values for which shadow price 5 holds is 1e+30"
```

Question f

```
solmat_1 <- matrix(0, nrow = 13, ncol = 2)

optvec_1 <- c()

for (m in 3:15) {
  rhs_2 <- c(m,10,18,7,6)
  model_4 <- lp(direction = "min",
                    objective.in = optfunc_1,
                    const.mat = lhs_1,
                    const.dir = dir_1,
                    const.rhs = rhs_2,
                    compute.sens = TRUE,
                    all.int = TRUE)

  solmat_1[m-2,] <- model_4$solution

  optvec_1[m-2] <- model_4$objval
  }
```

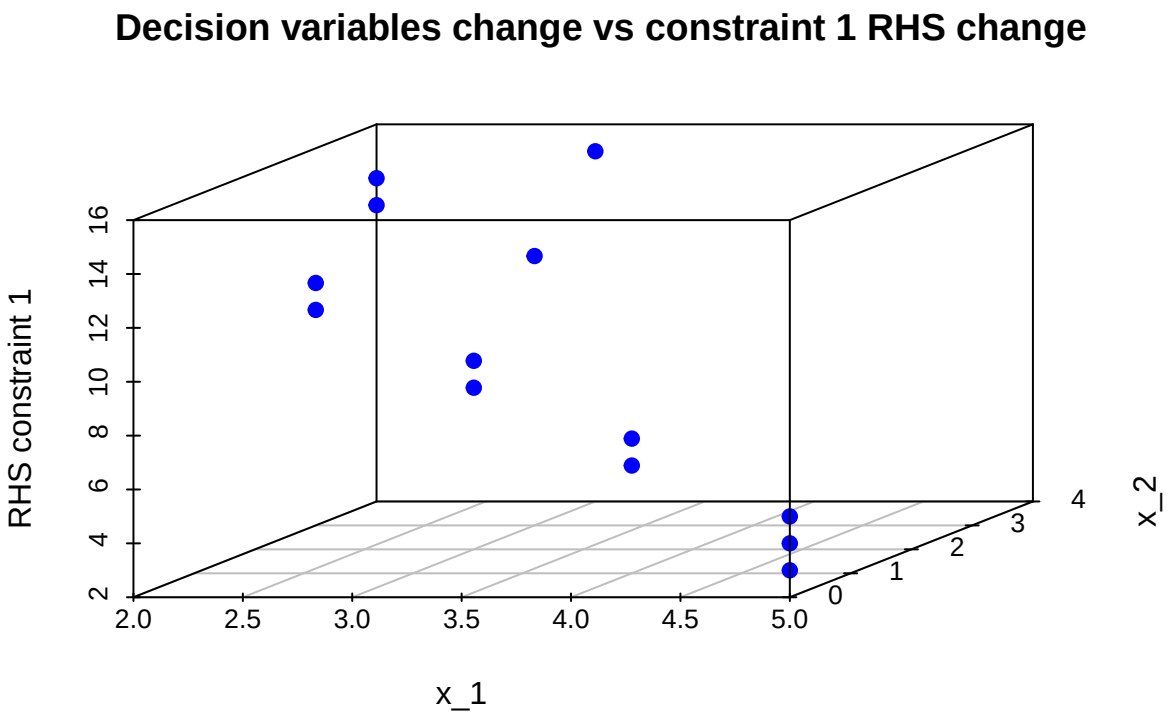
The change of optimal value regarding the change of right-hand side of constraint 1

```
plot(x = 3:15, y = optvec_1,
     main = "Optimal value change vs constraint 1 RHS change")
```



The change of decision variables regarding the change of right-hand side of constraint 1

```
scatterplot3d(solmat_1[,1], solmat_1[,2], 3:15, pch = 19, color = "blue",
              main = "Decision variables change vs constraint 1 RHS change",
              xlab = "x_1", ylab = "x_2", zlab = "RHS constraint 1")
```



Problem 2

Question a

The objective function is minimizing the total supply cost

```
optfunc_4 <- c(285, 290, 295, 250, 270, 250)
```

Decisions variables are the quantity of Indium and Gallium ordered from each supplier.

Constraints are:

- Total demand need to be fulfilled.
- Suppliers' capacity.
- Non-negativity constraint for the quantity of Indium and Gallium ordered from each supplier.

```
lhs_2 <- matrix(c(1, 1, 1, 0, 0, 0,
                  0, 0, 0, 1, 1, 1,
                  1, 0, 0, 0, 0, 0,
                  0, 1, 0, 0, 0, 0,
                  0, 0, 1, 0, 0, 0,
                  0, 0, 0, 1, 0, 0,
                  0, 0, 0, 0, 1, 0,
                  0, 0, 0, 0, 0, 1),
                nrow = 8,
                ncol = 6,
                byrow = TRUE)
```

```r
dir_2 <- c(">=", ">=", "<=", "<=", "<=", "<=", "<=", "<=")

rhs_3 <- c(1000, 800, 350, 500, 450, 750, 250, 350)

model_5 <- lp("min", optfunc_4, lhs_2, dir_2, rhs_3, all.int = TRUE)

print(paste("The quantity of Indium should be bought from each supplier are:",
            model_5$solution[1],
            model_5$solution[2],
            model_5$solution[3]))
```

```
## [1] "The quantity of Indium should be bought from each supplier are: 350 500 150"
```

```r
print(paste("The quantity of Gallium should be bought from each supplier are:",
            model_5$solution[4],
            model_5$solution[5],
            model_5$solution[6]))
```

```
## [1] "The quantity of Gallium should be bought from each supplier are: 750 0 50"
```

```r
print(paste("The total cost is", model_5$objval))
```

```
## [1] "The total cost is 489000"
```

```r
print(paste("The percentage of cost spending on Indium is",
            (model_5$solution[1]*optfunc_4[1]+
               model_5$solution[2]*optfunc_4[2]+
               model_5$solution[3]*optfunc_4[3])/model_5$objval))
```

```
## [1] "The percentage of cost spending on Indium is 0.591002044989775"
```

```r
print(paste("The percentage of cost spending on Gallium is",
            (model_5$solution[4]*optfunc_4[4]+
               model_5$solution[5]*optfunc_4[5]+
               model_5$solution[6]*optfunc_4[6])/model_5$objval))
```

```
## [1] "The percentage of cost spending on Gallium is 0.408997955010225"
```

Question b

In case supplier 1's capacities have shrunk due to production issues, the LP model is adjusted.

```r
rhs_4 <- c(1000, 800, 350*0.4, 500, 450, 750*0.5, 250, 350)

model_6 <- lp("min", optfunc_4, lhs_2, dir_2, rhs_4,
                        compute.sens = TRUE, all.int = TRUE)

print(paste("The quantity of Indium should be bought from each supplier now are:",
            model_6$solution[1],
            model_6$solution[2],
            model_6$solution[3]))
```

```
## [1] "The quantity of Indium should be bought from each supplier now are: 140 500 360"
```

```r
print(paste("The quantity of Gallium should be bought from each supplier now are:",
            model_6$solution[4],
            model_6$solution[5],
            model_6$solution[6]))
```

```
## [1] "The quantity of Gallium should be bought from each supplier now are: 375 75 350"
```

```
print(paste("The total cost now is", model_6$objval))
```

```
## [1] "The total cost now is 492600"
```

```
indium_cost_1 <-
  model_6$solution[1]*optfunc_4[1]+
  model_6$solution[2]*optfunc_4[2]+
  model_6$solution[3]*optfunc_4[3]

print(paste("The total Indium cost spend changed by",
            model_5$solution[1]*optfunc_4[1]+
               model_5$solution[2]*optfunc_4[2]+
               model_5$solution[3]*optfunc_4[3]-indium_cost_1))
```

```
## [1] "The total Indium cost spend changed by -2100"
```

```
gallium_cost_1 <-
  model_6$solution[4]*optfunc_4[4]+
  model_6$solution[5]*optfunc_4[5]+
  model_6$solution[6]*optfunc_4[6]

print(paste("The total Gallium cost spend changed by",
            model_5$solution[4]*optfunc_4[4]+
               model_5$solution[5]*optfunc_4[5]+
               model_5$solution[6]*optfunc_4[6]-gallium_cost_1))
```

```
## [1] "The total Gallium cost spend changed by -1500"
```

```
print(paste("The total cost changed by",model_5$objval - model_6$objval))
```

```
## [1] "The total cost changed by -3600.00000000012"
```

Problem 3

Question a

The objective function is maximizing the total audience contact.

```
optfunc_5 <- c(100000,18000,40000)
```

Variables are total number of advertisements authorized by each medium.

Constraints are:

- Limitation of the total promotional budget.
- Maximum number of advertisements authorized by each alternative.
- Television advertisments should account for at least 10% of the total number of advertisements authorized.
- Radio advertisements must not exceed 50% of the total number of advertisements authorized.
- Non-negativity constraint for number of advertisements authorized by each alternative.

```
lhs_3 <- matrix(c(2000,300,600,
                  1,0,0,
```

```
                  0,1,0,
                  0,0,1,
                  -0.9, 0.1, 0.1,
                  -1,1,-1),
                nrow = 6,
                ncol = 3,
                byrow = TRUE)

dir_3 <- c("<=","<=","<=","<=","<=","<=")

rhs_5 <- c(18200, 10, 20, 10, 0, 0)
```

Question b

```
model_7 <- lp(direction = "max",
              objective.in = optfunc_5,
              const.mat = lhs_3,
              const.dir = dir_3,
              const.rhs = rhs_5,
              compute.sens = TRUE,
              all.int = TRUE)

for (n in 1:length(model_7$solution)) {
  print(paste("The commercial message should be run for medium", n ,"is", model_7$solution[n]))
}
```

```
## [1] "The commercial message should be run for medium 1 is 4"
## [1] "The commercial message should be run for medium 2 is 14"
## [1] "The commercial message should be run for medium 3 is 10"
```

```
for (n in 1: length(model_7$solution)) {
  medium_budget_1 = c()
  medium_budget_1[n] <- model_7$solution[n]*lhs_3[1,n]
  print(paste("Allocated budget for medium", n ,"is", medium_budget_1[n]))
}
```

```
## [1] "Allocated budget for medium 1 is 8000"
## [1] "Allocated budget for medium 2 is 4200"
## [1] "Allocated budget for medium 3 is 6000"
```

```
budget_used_1 <- c(lhs_3[1,1],lhs_3[1,2],lhs_3[1,3]) %*% model_7$solution

print(paste("The whole budget used is", budget_used_1))
```

```
## [1] "The whole budget used is 18200"
```

```
remaining_budget_1  <- rhs_5[1] - budget_used_1

print(paste("The remaining budget is", remaining_budget_1))
```

```
## [1] "The remaining budget is 0"
```

```
audience_reached_1  <- model_7$objval

print(paste("The total audience reached is", audience_reached_1))
```

```
## [1] "The total audience reached is 1052000"
```

Question c

```
bd_const_2 <- lhs_3 %*% model_7$solution == rhs_5

for (o in 1:length(bd_const_2)) {
  print(paste("Constraint", o,"is binding?", bd_const_2[o]))
}
```

```
## [1] "Constraint 1 is binding? TRUE"
## [1] "Constraint 2 is binding? FALSE"
## [1] "Constraint 3 is binding? FALSE"
## [1] "Constraint 4 is binding? TRUE"
## [1] "Constraint 5 is binding? FALSE"
## [1] "Constraint 6 is binding? TRUE"
```

```
sdp_2 <- model_7$duals

for (p in 1:length(sdp_2[1:6])) {
  print(paste("The shadow price for constraint", p, "is", sdp_2[p]))
}
```

```
## [1] "The shadow price for constraint 1 is 51.304347826087"
## [1] "The shadow price for constraint 2 is 0"
## [1] "The shadow price for constraint 3 is 0"
## [1] "The shadow price for constraint 4 is 11826.0869565217"
## [1] "The shadow price for constraint 5 is 0"
## [1] "The shadow price for constraint 6 is 2608.69565217391"
```

```
allow_decrease_2 <- model_7$duals.from

for (p in 1:length(allow_decrease_2[1:6])) {
  print(paste("The lower bound of RHS values for which shadow price", p,
              "holds is", allow_decrease_2[p]))
}
```

```
## [1] "The lower bound of RHS values for which shadow price 1 holds is 14750"
## [1] "The lower bound of RHS values for which shadow price 2 holds is -1e+30"
## [1] "The lower bound of RHS values for which shadow price 3 holds is -1e+30"
## [1] "The lower bound of RHS values for which shadow price 4 holds is 0"
## [1] "The lower bound of RHS values for which shadow price 5 holds is -1e+30"
## [1] "The lower bound of RHS values for which shadow price 6 holds is -16.1"
```

```
allow_increase_2 <- model_7$duals.to

for (p in 1:length(allow_increase_2[1:6])) {
  print(paste("The upper bound of RHS values for which shadow price", p,
              "holds is", allow_increase_2[p]))
}
```

```
## [1] "The upper bound of RHS values for which shadow price 1 holds is 32000"
## [1] "The upper bound of RHS values for which shadow price 2 holds is 1e+30"
## [1] "The upper bound of RHS values for which shadow price 3 holds is 1e+30"
## [1] "The upper bound of RHS values for which shadow price 4 holds is 12.3389830508475"
## [1] "The upper bound of RHS values for which shadow price 5 holds is 1e+30"
```

```
## [1] "The upper bound of RHS values for which shadow price 6 holds is 5.87234042553192"
```

Question d

```
sdp_3 = sdp_2[1]

print(paste("The increased number of audience contact is", sdp_3 * 100))
```

```
## [1] "The increased number of audience contact is 5130.4347826087"
```

```
rhs_6 <- c(18200+100, 10, 20, 10, 0, 0)

model_8 <- lp(direction = "max",
              objective.in = optfunc_5,
              const.mat = lhs_3,
              const.dir = dir_3,
              const.rhs = rhs_6,
              compute.sens = TRUE,
              all.int = TRUE)

for (q in 1: length(model_8$solution)) {
  print(paste("The number of commercial messages should now be run on medium", q,
              "now is", model_8$solution[q]))
}
```

```
## [1] "The number of commercial messages should now be run on medium 1 now is 4"
## [1] "The number of commercial messages should now be run on medium 2 now is 14"
## [1] "The number of commercial messages should now be run on medium 3 now is 10"
```

```
bd_const_3 <- lhs_3 %*% model_8$solution == rhs_6

for (r in 1:length(bd_const_3)) {
  print(paste("Constraint", r,"is binding?", bd_const_3[r]))
}
```

```
## [1] "Constraint 1 is binding? FALSE"
## [1] "Constraint 2 is binding? FALSE"
## [1] "Constraint 3 is binding? FALSE"
## [1] "Constraint 4 is binding? TRUE"
## [1] "Constraint 5 is binding? FALSE"
## [1] "Constraint 6 is binding? TRUE"
```

When adding \$100 to the constraint 1's RHS, while it becomes an unbinding constraint but the change of 100 is much more smaller than the allowable increase of 32000. So the solution {4, 14, 10} is not changed. Nothing "strange" here.

Question e

```
rhs_7 <- c(18200+7500, 10, 20, 10, 0, 0)

model_9 <- lp(direction = "max",
              objective.in = optfunc_5,
              const.mat = lhs_3,
              const.dir = dir_3,
              const.rhs = rhs_7,
              compute.sens = TRUE,
```

```
              all.int = TRUE)


for (s in 1:length(model_9$solution)) {
  print(paste("If $7500 is added, the number of commercial messages should now be run on medium",
              s,"is", model_9$solution[s]))
}
```

```
## [1] "If $7500 is added, the number of commercial messages should now be run on medium 1 is 8"
## [1] "If $7500 is added, the number of commercial messages should now be run on medium 2 is 12"
## [1] "If $7500 is added, the number of commercial messages should now be run on medium 3 is 10"
```

```
audience_reached_2  <- model_9$objval

print(paste("Total audience reached now is", audience_reached_2))
```

```
## [1] "Total audience reached now is 1416000"
```