

Bad:

a/

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 2 \\ 0 & -4 & -5 \\ 0 & 0 & 1 \end{pmatrix}$$

$\rho(A) = 3.$

b/

$$B = \begin{pmatrix} -2 & 3 \\ 3 & -4 \\ -5 & 7 \\ 1 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} -2 & 3 \\ 0 & 1/2 \\ 0 & 29/2 \\ 0 & 5/2 \end{pmatrix}$$

$\rho(A) = 2.$

c/

$$C = \begin{pmatrix} 1 & 1 & -1 & 4 \\ 2 & 1 & 3 & 0 \\ 0 & 1 & 5 & 8 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & -1 & 4 \\ 0 & -1 & 5 & -8 \\ 0 & 0 & 10 & 0 \end{pmatrix}$$

$\rho(A) = 3.$

d/

$$D = \begin{pmatrix} -1 & 4 & 5 & -2 \\ 1 & 6 & 3 & 4 \end{pmatrix} \longrightarrow \begin{pmatrix} -1 & 4 & 5 & -2 \\ 0 & 10 & 8 & 2 \end{pmatrix}$$

$\rho(A) = 2.$