9/11/22 | EXPERIMENT - 5

Aim : Testing of hypothesis concerning the variance of population.

Experiment :

(i) Suppose that uniformity of a thickness of a part is used in a semiconductor is critical & that measurements of the thickness of a random sample of 18 such parts have the variance $(s^2) = 0.68$. The process is considered to be under control if the variation of thickness is given by $\sigma^2 > 0.36$. Assuming that the measurement constitute a random sample from a normal population. Test the $H_0 : \sigma^2 = 0.36$ against the alternative hypothesis $\sigma^2 > 0.36$ at 5% test of significance.

(ii) Weights in Kg of 10 students are given below

38, 40, 45, 53, 47, 43, 55, 48, 52, 49; can we say that variance of the dist. of weights of all students from which the above sample was taken is equal to 20 square Kg at 5% level of significance.

Theory :

→ Testing the equality of population variance.

To test the H_0 that the variance of a normal pop. equals the given constant given a random sample of size 'n' from a normal pop. we shall have to test the H_0

$$H_0 : \sigma^2 = \sigma_0^2$$

against

$$H_1 : \sigma^2 \neq \sigma_0^2 \;/\; \sigma^2 > \sigma_0^2 \;/\; \sigma^2 < \sigma_0^2$$

Here we use the theorem that if x_1 and x_2 are indep. r.v. x_1 has a chi square dist. with v_1 degree of freedom and $(x_1 + x_2)$ has a chi square dist. with $v > v_1$ df then x_2 has a chi square dist. with $v - v_1$ df thus the critical region for testing the H_0 against the two one-sided alternatives as

$$x^2 \geq x^2_{\alpha,(n-1)}$$

and $x^2 \leq x^2_{(1-\alpha),(n-1)}$ is obtained by using the test statistic $x^2 = \frac{(n-1)s^2}{\sigma_0^2}$

Calculation :

Given

(i) $s^2 = 0.68$
$n = 18$
$\alpha = 0.05$
$\sigma_0^2 = 0.36$

Test the $H_0 : \sigma^2 = 0.36$
against $H_1 : \sigma^2 > 0.36$

$$x^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

$$= \frac{17 \times 0.68}{0.36} = 32.111$$

$$x^2 = 32.111$$

Tabulated value $\chi^2_{0.05, 17} = 27.587$

Since calculated chi-square $\boxed{\chi^2_{cal} > \chi^2_{0.05,17}}$

(ii)

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
38	-9	81
40	-7	49
45	-2	4
53	6	36
47	0	0
43	-4	16
55	8	64
48	1	1
52	5	25
49	2	4
470		280

$$\bar{x} = \frac{470}{10} = 47$$

$$s^2 = \frac{1}{n-1}\left(\Sigma (x_i - \bar{x})^2\right)$$

$$s^2 = \frac{1}{9} \times 280$$

$$s^2 = 31.11$$

$\alpha = 0.05$ $n = 10$ $s^2 = 31.11$ $\sigma_0^2 = 20$

$H_0 : \sigma^2 = 20$
against $H_1 : \sigma^2 > 20$

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

$$= \frac{9 \times 31.11}{20}$$

$$\boxed{\chi^2 = 14}$$

Tabulated value

$$\boxed{\chi^2_{(0.05,9)} = 16.919}$$

$\therefore \chi^2_{cal} = 14 < \chi^2_{(0.05,9)}$

RESULT :

i) We may reject the H_0 & conclude that the variance of a given sample is not equal to 0.36 i.e. the process used in manufacturing of the part must be revised.

ii) We may accept the H_0 and conclude that the variance of the dist. of weights of all student is equal to 20 square kg.