Chalmers University of Technology

TME290 Autonomous robots

Home problem 3: Sensor fusion for a broken autonomous race car

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1 Introduction

In this assignment, one will working on extended kalman's filter to estimate the road wheel angle (θ) from the rec-file data and the model. The predict model and the observation modrl are 2 main part which one must decide in the extended kalman's filter. The predict model is the model one use to predict next value of each states. As its name, the observation model is used to observe the value predicted in the prediction model comparing with measured data. Then, one use the equation from both model to predict and update which are steps in Extended Kalman's filter (EKF) algorithm. One thing worth mentioned is the model given for this autonomous car as

$$\alpha_f = \theta - \frac{v_y + l_f\dot{\varphi}}{v_x} \tag{1}$$

$$\alpha_r = \frac{l_r\dot{\varphi} - v_y}{v_x} \tag{2}$$

$$\dot{v}_y = \frac{C_f\alpha_f + C_r\alpha_r}{m} - v_x\dot{\varphi} \tag{3}$$

$$\ddot{\varphi} = \frac{l_fC_f\alpha_f - l_rC_r\alpha_r}{I_z} \tag{4}$$

The constant are given as $l_f = 0.8415$, $l_r = 0.6885$, $m = 270$, $I_z = 105$, $C_f = 18748$ and $C_r = 15339$ where α_f and α_r r are the front and rear slip angles, l_f and l_r the distances from COG to front and rear, C_f and C_r the front and rear cornering stiffness, v_x and v_y the longitudinal (forward) and lateral (leftwards) speeds, θ the road wheel angle, φ the yaw angle, m the mass, and I_z the moment of inertia in yaw direction.

2 Modelling

The goal is to estimate the road wheel angle θ. So, one start at θ in equation 1. The θ is the function of v_y, $\dot{\varphi}$, v_x and α_f. From quation 3, α_f is the function of v_x, $\dot{\varphi}$, $\dot{v_y}$, which we consider as v_y, and α_r. In the same way, α_r is the function of $\dot{\varphi}$, v_y and v_x. From substitution, one observe that θ is calculated from v_y, $\dot{\varphi}$ and v_x. Here, v_x is considered as input since it is considered accurate. The states become θ, v_y and $\dot{\varphi}$. The observation values are not working straightforward like the predict model since one get only $\dot{\varphi}$ and $\dot{v_y}$ from the measurement via sensors. For $\dot{\varphi}$, one consider it easy to observe directly from the prediction. From another side, $\dot{v_y}$ can be calculated using those states one decided. So, one set the observation as 2 parameters, which are $\dot{\varphi}$ and $\dot{v_y}$. Now, one model the function for prediction model. Firstly, one look at the θ. One select the most straight forward way as

$$\theta_k = \theta_{k-1} \tag{5}$$

From the kinematic model using ICR, one know that

$$tan(\theta) = \frac{l_r + l_f}{L} \tag{6}$$

and

$$L \approx \frac{v_x}{\dot{\varphi}} \tag{7}$$

From equation 6 and equation 7, one assume that θ is small so $tan(\theta) = \theta$. Then,

$$\theta = \frac{(l_r + l_f)\dot{\varphi}}{v_x} \tag{8}$$

Then, one rearrange equation 5 as

$$\theta_k = \frac{(l_r + l_f)\dot{\varphi}_{k-1}}{v_x} \tag{9}$$

Next, one consider the discrete integral equation for speed

$$v_{yk} = v_{yk-1} + h\dot{v}_{yk-1} \tag{10}$$

Then, one substitute equation 1, equation 2 and equation 3 into this equation. One get

$$v_{yk} = (\frac{hC_f}{m})\theta_{k-1} + (1 - \frac{h(C_f + C_r)}{mv_x})v_{yk-1} + (\frac{h(C_r l_r - C_f l_f)}{mv_x} - hv_x)\dot{\varphi}_{k-1} \tag{11}$$

In the same method as v_y, one know

$$\dot{\varphi}_k = \dot{\varphi}_{k-1} + h\ddot{\varphi}_{k-1} \tag{12}$$

One substitute equation 1, equation 2 and equation 4 as

$$\dot{\varphi}_k = (\frac{hl_f C_f}{I_z})\theta_{k-1} + (\frac{h(l_r C_r - l_f C_f)}{I_z v_x})v_{yk-1} + (1 - \frac{h({l_f}^2 Cf + {l_r}^2 Cr)}{I_z v_x})\dot{\varphi}_{k-1} \tag{13}$$

Now, one can define Matrix F as

$$F = \begin{bmatrix} 0 & 0 & \frac{l_r + l_f}{v_x} \\ \frac{hC_f}{m} & 1 - \frac{h(C_f + C_r)}{mv_x} & \frac{h(C_r l_r - C_f l_f)}{mv_x} - hv_x \\ \frac{hl_f C_f}{I_z} & \frac{h(l_r C_r - l_f C_f)}{I_z v_x} & 1 - \frac{h({l_f}^2 Cf + {l_r}^2 Cr)}{I_z v_x} \end{bmatrix} \tag{14}$$

Similarly, one model the observation matrix H. From equation 3 and the yawrate from state, one get matrix H as

$$H = \begin{bmatrix} \frac{C_f}{m} & -\frac{(C_f + C_r)}{mv_x} & \frac{(C_r l_r - C_f l_f)}{mv_x} - v_x \\ 0 & 0 & 1 \end{bmatrix} \tag{15}$$

Once one finish modelling, one move to the EKF algorithm.

3 Extended kalman's filter algorithm

One issue worth mentioned here is that the frequency of v_x as input and the frequency of $\ddot{\varphi}$ and $\dot{v}_y$ are different with 25Hz and 200Hz. First, one initial matrix P which will be updated every step anyway. Another things are diagonal matrix R and Q which are set as

$$R = \begin{bmatrix} 1000000 & 0 \\ 0 & 10000 \end{bmatrix} \tag{16}$$

$$Q = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 40 \end{bmatrix} \tag{17}$$

Then, one predict the next state using current state and matrix F. Then, one update the P_k. These steps are called prediction step. Then one do the update step. One calculate kalman's gain as G. After that, the predicted states are updated with kalman's gain and the error between measured observation values and the predicted observation values, which are from predicted states and matrix H. In the last step, P_k is updated again with G,H and current P_k.

4 Result and Summary

As a result of the EKF, one plot road wheel angle θ estimated as in figure 1. In addition, one plot θ together with $\ddot{\varphi}$ and $\dot{v}_y$ measured from real sensors as in figure 2. Another plot worth shown here is the comparison between yaw rate predicted and yaw rate measured as in figure 3

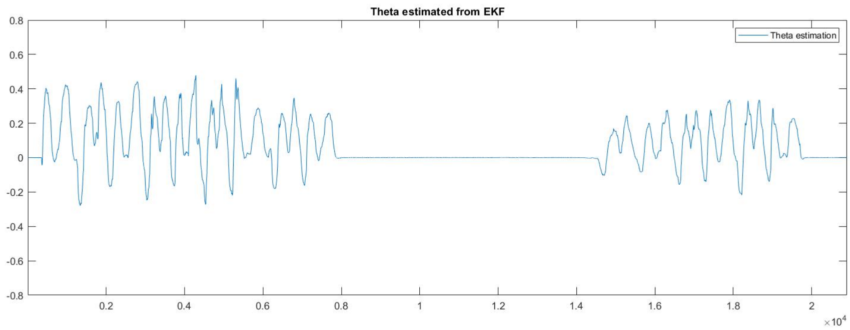

Figure 1: Theta estimated by EKF.

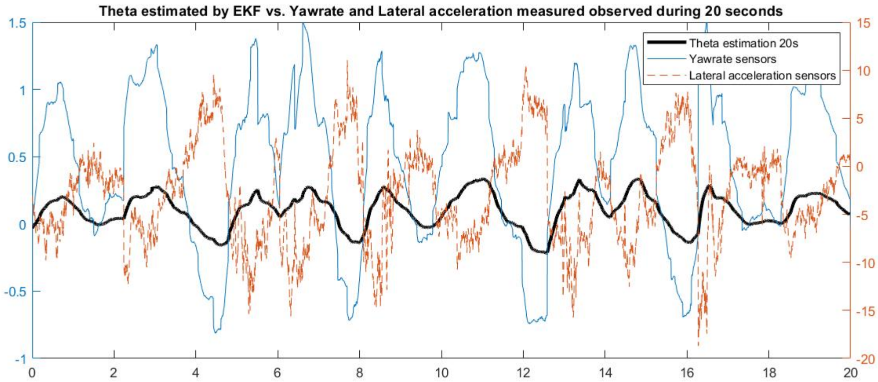

Figure 2: Theta estimated by EKF together with Yaw rate and Lateral acceleration from measured in interesting 20s.

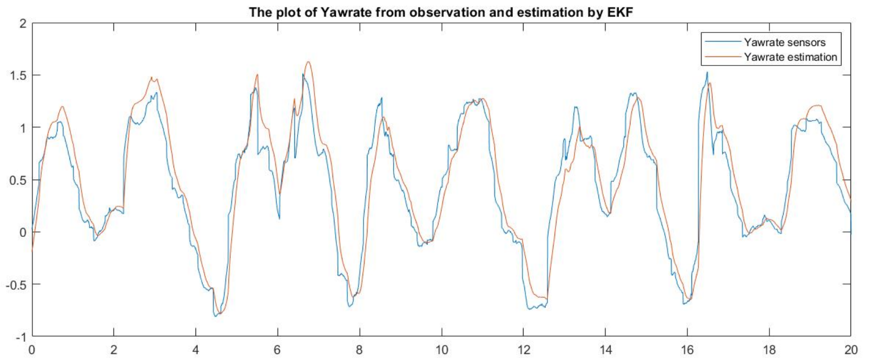

Figure 3: Yaw rate estimated from EKF and Yaw rate measured by sensor.

One can summary here that the road wheel angel θ is estimated from EKF with some trade-off. This trade-off is the trade-off between the smoothness of the signal and the delay. One observe that if one decide to rely on measured data, the estimated states will not be any delay but it will be noisy just like the measurement from sensors. To get rid of the noise, one tune R and Q and the signal estimated will be smoother with some delay comparing to measured signal.