

1 公式

1.1 平面图形的面积

1. 直角坐标

$$A = \int_a^b f(x)dx$$

$$A = \int_a^b |f(x) - g(x)|dx$$

2. 极坐标

$$A = \int_{\alpha}^{\beta} \frac{1}{2}r^2(\theta)d\theta$$

$$A = \int_{\alpha}^{\beta} \frac{1}{2}[r_2^2(\theta) - r_1^2(\theta)]d\theta$$

1.2 旋转体体积

1. 绕 x 轴旋转

$$V = \int_a^b \pi f^2(x)dx$$

2. 绕 y 轴旋转

$$V = \int_a^b 2\pi x f(x)dx$$

1.3 平面曲线的弧长

1. 直角坐标

$$L = \int_a^b \sqrt{1 + y'^2}dx$$

2. 参数方程

$$L = \int_{\alpha}^{\beta} \sqrt{x_t'^2 + y_t'^2}dt \quad (\alpha \leq t \leq \beta)$$

3. 极坐标

$$A = \int_{\alpha}^{\beta} \sqrt{r^2(\theta) + r'^2(\theta)}d\theta$$

1.4 重要公式

华里氏公式扩展

$$\int_0^{\pi} \sin^n x dx = 2 \int_0^{\frac{\pi}{2}} \sin^n x dx$$

$$\int_0^{\pi} \cos^n x dx = \begin{cases} 0 & n \text{ 为奇数} \\ 2 \int_0^{\frac{\pi}{2}} \cos^n x dx & n \text{ 为偶数} \end{cases}$$

$$\int_0^{2\pi} \sin^n x dx = \begin{cases} 0 & n \text{ 为奇数} \\ 4 \int_0^{\frac{\pi}{2}} \sin^n x dx & n \text{ 为偶数} \end{cases}$$

$$\int_0^{2\pi} \cos^n x dx = \begin{cases} 0 & n \text{ 为奇数} \\ 4 \int_0^{\frac{\pi}{2}} \cos^n x dx & n \text{ 为偶数} \end{cases}$$

积分公式

$$\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \ln(x + \sqrt{x^2 + a^2}) + C$$

$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln(x + \sqrt{x^2 - a^2}) + C$$

$$\int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \arcsin \frac{x}{a} + C$$