1 公式

1.1 平面图形的面积

1、直角坐标

$A = \int_a^b f(x)dx$

$A = \int_a^b |f(x) - g(x)|dx$

2、极坐标

$A = \int_\alpha^\beta \frac{1}{2}r^2(\theta)d\theta$

$A = \int_\alpha^\beta \frac{1}{2}[r_2^2(\theta) - r_1^2(\theta)]d\theta$

1.2 旋转体体积

1、绕 x 轴旋转

$V = \int_a^b \pi f^2(x)dx$

2、绕 y 轴旋转

$V = \int_a^b 2\pi x f(x)dx$

1.3 平面曲线的弧长

1、直角坐标

$L = \int_a^b \sqrt{1 + y^{'2}}dx$

2、参数方程

$L = \int_\alpha^\beta \sqrt{x_t^{'2} + y_t^{'2}}dt\ (\alpha \le t \le \beta)$

3、极坐标

$A = \int_\alpha^\beta \sqrt{r^2(\theta) + r^{'2}(\theta)}d\theta$

1.4 重要公式

华里氏公式扩展

$$\int_0^{\pi} sin^n x dx = 2\int_0^{\frac{\pi}{2}} sin^n x dx$$

$$\int_0^{\pi} cos^n x dx = \begin{cases} 0 & n\text{为奇数} \\ 2\int_0^{\frac{\pi}{2}} cos^n x dx & n\text{为偶数} \end{cases}$$

$$\int_0^{2\pi} sin^n x dx = \begin{cases} 0 & n\text{为奇数} \\ 4\int_0^{\frac{\pi}{2}} sin^n x dx & n\text{为偶数} \end{cases}$$

$$\int_0^{2\pi} cos^n x dx = \begin{cases} 0 & n\text{为奇数} \\ 4\int_0^{\frac{\pi}{2}} sin^n x dx & n\text{为偶数} \end{cases}$$

积分公式

$$\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}ln(x+\sqrt{x^2+a^2}) + C$$

$$\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}ln(x+\sqrt{x^2-a^2}) + C$$

$$\int \sqrt{a^2+x^2}dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}arcsin\frac{x}{a} + C$$