## 平面图形的面积

1. 直角坐标

$$A = \int_a^b f(x)dx$$

$$A = \int_a^b |f(x) - g(x)|dx$$

2. 极坐标

$$A = \int_\alpha^\beta \frac{1}{2}r^2(\theta)d\theta$$

$$A = \int_\alpha^\beta \frac{1}{2}[r_2^2(\theta) - r_1^2(\theta)]d\theta$$

## 旋转体体积

1. 绕 x 轴旋转

$$V = \int_a^b \pi f^2(x)dx$$

2. 绕 y 轴旋转

$$V = \int_a^b 2\pi x f(x)dx$$

## 平面曲线的弧长

1. 直角坐标

$$L = \int_a^b \sqrt{1 + y'^2} dx$$

2. 参数方程

$$L = \int_\alpha^\beta \sqrt{x_t'^2 + y_t'^2} dt, (\alpha \le t \le \beta)$$

3. 极坐标

$$A = \int_\alpha^\beta \sqrt{r^2(\theta) + r'^2(\theta)} d\theta$$

## 重要公式

华里氏公式扩展

$$\int_0^\pi sin^n x dx = 2\int_0^{\frac{\pi}{2}} sin^n x dx$$

$$\int_0^\pi cos^n x dx = \begin{cases} 0, & n为奇数 \\ 2\int_0^{\frac{\pi}{2}} cos^n x dx, & n为偶数 \end{cases}$$

$$\int_0^{2\pi} sin^n x dx = \begin{cases} 0, & n为奇数 \\ 4\int_0^{\frac{\pi}{2}} sin^n x dx, & n为偶数 \end{cases}$$

$$\int_0^{2\pi} cos^n x dx = \begin{cases} 0, & n为奇数 \\ 4\int_0^{\frac{\pi}{2}} sin^n x dx, & n为偶数 \end{cases}$$

积分公式

$$\int \sqrt{x^2 + a^2} dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} ln(x + \sqrt{x^2 + a^2}) + C$$

$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} ln(x + \sqrt{x^2 - a^2}) + C$$

$$\int \sqrt{a^2+x^2}dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}arcsin\frac{x}{a} + C$$